## **Bank of Israel**



## **Research Department**

# Monetary Policy, Fear, and the Stock Market\* Eliezer Borenstein\*\*

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#### מדיניות מוניטרית, חששות המשקיעים, ושוק המניות

#### אליעזר בורנשטיין

#### תקציר

המחקר מנתח מסגרת שבה ההשפעה של המדיניות המוניטרית תלויה במצב הכלכלה בשל קיומו של ערוץ אינפורמציה אנדוגני. בפרט, מוצג מודל הכולל השקעה בהון מסוכן שבו הבנק המרכזי מחזיק במידע פרטי על מצב הכלכלה וקובע ריבית במטרה לייצב את הביקוש המצרפי. הורדת הריבית ממריצה את ההשקעה דרך הערוץ הסטנדרטי, אך גם מאותתת למשקיעים שהכלכלה פחות חזקה משחשבו, מה שפוגע בביטחון המשקיעים וברצון שלהם להשקיע. ערוץ האינפורמציה זניח כשהכלכלה חזקה, אך נעשה משמעותי כשהכלכלה חלשה. כאשר הכלכלה חלשה מדי, הורדת הריבית דווקא מקטינה את ההשקעה. כך, מדיניות שמטרתה להמריץ את הביקוש המצרפי עלולה, שלא במתכוון, להשיג את התוצאה ההפוכה ולגרום לביקוש להתכווץ עוד יותר. הירידה בביקוש המצרפי אינה יעילה, ומשקפת בעיית תיאום בין המשקיעים. לבסוף, מוצגות עדויות אמפיריות שתומכות בניבוי של המודל לפיו ערוץ האינפורמציה של המדיניות המוניטרית חזק יותר בזמנים שבהם התנאים הכלכליים פחות טובים.

## Monetary Policy, Fear, and the Stock Market

#### Eliezer Borenstein

#### **Abstract**

I analyze a setting in which monetary policy has a state dependent effect due to an endogenously driven information channel. Specifically, I develop a model of investment in risky capital, where a central bank holds private information regarding the state of the economy and sets an interest rate accordingly in order to stabilize aggregate demand. Lowering the interest rate stimulates investment via the standard channel, but also signals weaker economic conditions, which reduces investors' confidence and their desire to invest. The information effect is negligible when the economy is strong, but can become significant when the economy is weaker. In a sufficiently weak economy, reducing the interest rate generates a decline in investment. Thus, a policy aimed at stimulating investment might unintentionally cause the opposite result, weakening aggregate demand even further. The reduction in aggregate demand is inefficient, as it reflects a coordination failure among investors. In line with the model's s prediction, I provide empirical evidence suggesting that the information effect of monetary policy is stronger in times of weaker economic conditions.

Keywords: Central bank information effects, Monetary policy, Financial crisis, Stock market.

JEL Codes: D83, E43, E44, E52, G01.

## 1 Introduction

How should a central bank (CB) respond to a decline in demand? According to the usual prescription, it should ease monetary conditions, e.g. cut interest rates, in order to moderate the decline and minimize the inefficiencies associated with it. Recently, a growing body of literature has documented the presence of an "information effect" (IE) – the idea that actions of the CB convey information which may influence the public's behavior. For example, by lowering the interest rate in order to stimulate demand, the CB might increase pessimism which would have a depressing effect on demand. Thus, to appropriately respond to fluctuations in demand it is important for the CB to take into account the strength of the IE that it is likely to encounter.

The main point of this paper is that the strength of the IE is state dependent. Specifically, I claim that the IE is stronger when the economy is more prone to crises. The state dependent nature of the IE implies that the effectiveness of monetary policy in stabilizing demand is state dependent as well. I show that when the economy is strong, the consequences of not internalizing the presence of the IE are negligible. In contrast, when the economy is sufficiently weak, the consequences of not internalizing the IE are large. The strength of the IE might become so large that reducing the interest rate to offset a negative demand shock might actually cause the opposite result and exacerbate the negative effect of the shock.

I analyze these issues in a model that describes an allocation decision between a risk free asset and risky capital, which could be interpreted as physical capital or as stocks. The stock market is particularly interesting in the context of the IE for a few reasons. First, stock prices seem to be an important driver of aggregate demand, and as such, CB tend to follow them closely (Caballero and Simsek (2024)). Second, the high frequency nature of the stock market enables researchers to gauge its immediate response to CB interest rate decisions, thus allowing a cleaner identification of the effects of monetary policy.

Owing to this high frequency nature of the stock market, it is now common in the literature to look at the stock market in order to identify the effects of monetary policy. In particular, Jarocinski and Karadi (2020) document the reaction of the S&P 500 stock index to monetary surprises. Standard theory suggests that the effect of a monetary surprise on stock prices should be negative – that is, a positive (negative) surprise should decrease (increase) stock prices. However, Jarocinski and Karadi (2020) find that about one third of the observations in their sample were "wrong-sided", where a monetary surprise led to the opposite reaction of stock prices than that predicted by theory. Based on the response of the main macro variables, the authors conclude that a "wrong-sided"

response of stock prices is best explained as being the result of an information shock in which the CB releases information to the market via its monetary announcement. In contrast, a standard response of the stock market suggests that the monetary surprise was the result of a standard monetary shock.

The findings of Jarocinski and Karadi (2020) raise the following question: Why do some monetary surprises behave as standard monetary shocks whereas others behave as information shocks? One possibility, raised by Jarocinski and Karadi (2020), is that the surprises that behave as information shocks are the result of the central banks' statements regarding its view about the economy that accompany the interest rate announcements. In this paper I pursue another, complementary, explanation, where investors extract information from the announced interest decision itself. Under the interpretation I present, the same monetary surprise could result in very different effects on the stock market, depending on the state of the economy. In particular, my model suggests that monetary surprises that occur when the economy is vulnerable to a financial crisis, are more likely to behave as information shocks, since in these states investors are very sensitive to every piece of information.

My model highlights the role of investors' confidence, and the way the CB's actions affect it, in the response of the stock market to interest rate surprises. Lack of investors' confidence is mentioned in the literature as an important driver of declines in the demand for investment and in asset values.<sup>1</sup> As a motivating example, consider the developments in the onset of the Covid-19 crisis: On the background of the evolving Covid-19 crisis, on 3.3.20, the Federal Reserve ("Fed") held an unscheduled emergency meeting in which the federal funds rate was decreased by fifty basis points, to the range 1%–1.25%. After a shortly lived increase, the S&P 500 changed direction and decreased by 2.8% that day. Some examples from the press can give a sense of the effect that the interest rate reduction had on investors' confidence:

"Fears that things may be worse than they look – While an interest-rate cut was good for a knee-jerk positive reaction in equity indexes, gains soon faded as the initial euphoria gave way to questions about just how worried policy makers must be to prompt such a rare move" (MarketWatch 3.3.20).

"The Fed's Rate Cut Brought Fear, Not Reassurance, to the Market – The Federal Reserve probably expected its surprise half-point interest-rate cut to be a "shock and awe" shot in the arm for the stock market. Instead, its impact is turning out to be shockingly awful in terms of inspiring investor optimism" (Barron's 3.3.20).

<sup>&</sup>lt;sup>1</sup>For example, Bollerslev and Todorov (2011) discuss the relation between investors' confidence and asset prices in the context of the 2007-9 financial crisis. Drechsler (2013) shows how time varying uncertainty about the true model of the economy leads to fluctuations in the risk premium that investors in the stock market require.

Fear, or lack of investors' confidence, seems to have played an even bigger role almost two weeks later, on 15.3.20 (Sunday, a day without trading), when the Fed held another emergency meeting in which it announced another interest rate reduction, this time by a hundred basis points, to the range of 0%–0.25% (together with the announcement of additional easing measures). On Monday morning, 16.3.20, the S&P 500 dropped sharply, triggering a trading halt. By the end of the day the S&P 500 fell by 12%, the largest daily decline since 1987. At the same time, as figure 1 shows, the VIX index jumped, demonstrating the role of fear in this stock market crash. Bekaert et al. (2013) decompose the VIX index into two components, investors' uncertainty and investors' risk aversion. They find that lax monetary policy tends to reduce both components. The fact that following a negative monetary surprise the VIX rose, further suggests that at least part of the non-standard response of the stock market to the Fed's announcement can be attributed to the perverse effect that the monetary easing had on investors' confidence.<sup>2</sup>

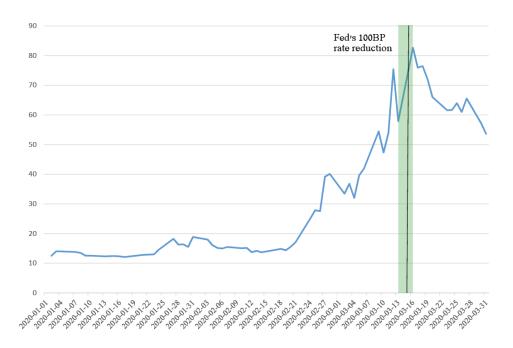


Figure 1: The VIX index 1.1.20 - 31.3.20. Source: Chicago Board Options Exchange, retrieved from FRED, Federal Reserve Bank of St. Louis. The green shaded area indicates the days around the FOMC announcement where there was no trading due to the weekend.

<sup>&</sup>lt;sup>2</sup>Arguably, the strong IE that seemed to take place on these two occasions could be affected by the fact that both decisions took place on unscheduled meetings. Indeed, Paul (2020) finds evidence that the information channel is especially strong around unscheduled Fed announcements, and Lakdawala and Scahffer (2019) find that when FOMC meetings are unscheduled, stock prices rise in response to contractionary shocks that reveal the Fed's forecast.

To analyze the effects of monetary policy on investors' confidence and their desire to invest, I build a model that describes an investment decision of many small investors who base their decision on the return they expect to earn on risky capital, relative to the risk free rate which is set by the CB. The CB's goal is to stabilize aggregate demand around the level it deems as optimal. To this end, the CB varies its interest rate in response to the fundamental – best thought of as a demand shock – such that aggregate demand will attain its optimal level. By changing its interest rate, the CB changes the relative return of the risky alternative, thus affecting the demand for investment. I refer to this channel as the standard channel of monetary policy.

I assume that the CB has an information advantage over the investors, in that it knows the value of the fundamental while investors do not. Realizing their information disadvantage, investors use the interest rate set by the CB to infer the level of the fundamental. The reason that investors care about the fundamental, is that I assume that a large fall of aggregate demand triggers a financial crisis in which investors lose their investment. Importantly, this assumption also generates a strategic complementarity between investors' actions, since the risk that each investor faces will depend on the behavior of other investors. This feature opens the possibility of multiple equilibria, and I apply a global-game technique to analyze which equilibrium will be selected.

In this setup, the interest rate set by the CB plays a dual role: it directly affects the incentive to invest, and also has an indirect effect via the information that investors extract from it. I show that when the state of the economy is sufficiently strong, investors understand that the risk of a financial crisis is negligible, and hence the negative information that an interest rate reduction conveys to them has a negligible impact on their desire to invest. As the state of the economy deteriorates, the risk of a financial crisis increases and becomes more sensitive to the information that is conveyed through the actions of the CB. As investors become less confident, their desire to invest declines, so that a monetary easing is less effective at increasing demand. When the economy weakens even more, the IE strengthens to the degree that its effect dominates that of the standard, stimulative, channel of a monetary easing. In this case interest rate reductions reduce investment. Thus, by reducing the interest rate in response to a negative demand shock – the standard prescription for monetary policy – the CB actually drives aggregate demand further away from its optimal level. This overly low level of aggregate demand is inefficient, and reflects a coordination failure among investors – investors refrain of investing in the risky asset due to their fear that others will refrain from investing.

The CB in my model does not act optimally: when setting the interest rate, it does not take the IE into account. It is likely that policymakers are aware of the possibility that reducing the interest rate will send a negative signal to the market.<sup>3</sup> My analysis demonstrates the importance of taking into account the state dependent nature of the IE, which is likely to be difficult in practice. While the issue of optimal monetary policy in the presence of IE is beyond the scope of this paper, I briefly discuss some implications of my model in this respect. First, the presence of a coordination failure resulting in an overly low level of aggregate demand can potentially justify some sort of information manipulation by the CB. Second, as the only way to manipulate information in my model is by changing the interest rate itself, an optimal policy is likely to entail either a more aggressive response to the fundamental (relative to a policy that does not address the IE), or, alternatively, a policy that does not respond to changes in the fundamental, such that the interest rate ceases to reveal the exact state of the economy.

After presenting the theoretical analysis, I turn to an empirical investigation of the model's main prediction. That is, that the IE is stronger when economic conditions are weaker. The literature on financial crises attributes a central role to the strength of the financial sector in the formation of such crises. Motivated by this literature, I use a proxy for financial distress of financial intermediaries as an indicator for states in which the economy is more vulnerable to crises. I analyze the reaction of stocks (the S&P 500) to interest rate surprises ("news shocks"), and show that positive (negative) news shocks have a smaller negative (positive) impact on stock prices when financial distress is high. Furthermore, when financial distress is extremely high the response of stock prices to news shocks flips sign: positive (negative) news shocks increase (decrease) stock prices. I further show that while negative monetary surprises reduce the VIX indicator, the reduction is significantly smaller on periods of high distress, in accordance with the idea that the presence of the IE might interfere with the calming effect that lax monetary policy has on the stock market, especially at times of financial distress. Thus, the empirical results provide support to the idea that the strength of the IE is state dependent, where times of financial distress are associated with a stronger IE, even to the degree of overturning the standard effect of monetary policy.

Before turning to review the related literature, and in light of the discussion above regarding the implications of the IE, a natural question arises: wouldn't the private information held by the CB eventually become known to the public regardless of the CB's actions? If so, then why is analyzing the implications of the IE important? I note here three reasons.

<sup>&</sup>lt;sup>3</sup>FOMC minutes from the March 15, 2020 meeting reveal that the participants were aware of this issue: "Some participants remarked that the Committee's policy actions [...] could be interpreted as conveying negative news about the economic outlook". "[...] participants noted that a lowering of the target range by 100 basis points, coming so soon after the reduction of 50 basis points less than two weeks earlier, ran the risk of sending an overly negative signal about the economic outlook."

First, it may take quite a while until the public learns this information. The issue is not just about revealing negative information a few months earlier. Suppose that the economy is in a weak state and that the CB privately holds negative information. Revealing it when the economy is susceptible to a crisis might indeed trigger a crisis which would cause long lasting damage. If the revelation of the information would be delayed to a time when the economy was able to partially recover (e.g., a weak financial sector had the time to recapitalize) then a financial crisis could potentially be avoided.

Second, the fact that the information is revealed by the CB at the same time to everyone might matter: when the public learns the information by its own, the process is more
gradual as some agents learn first and some learn later on. The same negative information revealed might cause a stronger impact if transmitted abruptly and simultaneously
to all the agents, especially when this occurs via a process which draws a lot of public
attention.

Finally, there might also be times where the CB indeed precedes the information revelation only by a small amount, say, days. In this case, the IE would likely not be an important policy issue, yet it still has implications in the sense that it could affect estimates of the the role of monetary policy. As I review below, it is common in the literature to look at prices of various financial assets in order to estimate the impact of monetary policy. As the conventional effect of monetary policy is confounded with the IE, estimates that do not control for the IE are likely to yield biased estimates regarding the effect of monetary policy. In this regard, understanding when we would expect a strong IE can potentially help with yielding more accurate estimates.

Related literature My work relates to the literature that documented the presence of an IE in various settings:<sup>4</sup> professional forecasters (Campbell et al. (2012) and Nakamura and Steinsson (2018) (NS18)), the foreign exchange market (Gürkaynak et al. (2021)), the corporate bond market (Smolyanski and Suarez (2021), and in particular the stock market (Jarocinski and Karadi (2020)) and Nunes et al. (2023)).<sup>5</sup> The papers that document the presence of IE in the stock market are a part of a large literature that analyzed the effect of monetary policy on the stock market.<sup>6</sup> Basistha and Kurov (2008) find that the stock market responds stronger to monetary surprises in recessions and in tight credit market conditions. Kurov (2012) finds that news about future Fed funds rates changes tend to move stock prices in the opposite direction when the economy is

<sup>&</sup>lt;sup>4</sup>Romer and Romer (2000) is the seminal paper in this literature.

<sup>&</sup>lt;sup>5</sup>Some papers also questioned the existence or current relevance of the IE: Hoesch et al. (2020) find that the evidence for IE in professional forecasts has weakened. Bauer and Swanson (2023) claim that the results of NS18 reflect that both the Fed and the private forecasters respond to the same news.

<sup>&</sup>lt;sup>6</sup>Seminal papers are Rigobon and Sack (2004) and Bernanke and Kuttner (2005) who find that positive interest rate shocks reduce stock prices, and Gürkaynak et al. (2005) who show that both monetary policy actions as well as statements have an effect on stock prices.

expanding, and in the same direction when the economy is in a recession. Lakdawala and Schaffer (2019) find that stock prices decline in response to positive monetary shocks as well as in response to positive Delphic (news) shocks. They also find that when FOMC meetings are unscheduled, stock prices rise in response to contractionary Delphic shocks. Hubert (2019) shows that the effect of monetary policy on stock prices depends on the macroeconomic information that is published by the CB. Cieslak and Schrimpf (2019) find a significant component of non monetary news. Paul (2020) focuses on monetary surprises which are less likely to include an information effect and documents time variation in the response of stock to the surprises. Miranda-Agrippino and Ricco (2021) show that a monetary tightening reduces stock prices. Pflueger and Rinaldi (2022) build a model featuring time varying risk aversion to explain why Fed announcements have a big effect on stock prices. Elenev et al. (2024) show that the stock market's response to macroeconomic announcements is state dependent and shaped by expectations about the responsiveness of monetary policy to these announcements. My main contribution to this literature is to provide a new explanation for the presence of state dependent effects of monetary policy on stock prices based on an information channel, and to provide evidence suggesting that at least part of this state dependency is related to the endogenous response of investors' confidence to the information conveyed by the Fed's actions.

Technically, the model I present builds on the literature that analyzed global games. This literature, incepted by Carlsson and Van-Damme (1993), describes coordination games in situations of incomplete information, and has applications in various economic contexts such as currency attacks (Morris and Shin (1998)). My model has some similarity with the model of Bebchuk and Goldstein (2011) who analyze self-fulfilling credit freezes. In their model, banks have to decide whether to lend capital to firms, which then use it for investing in projects, or to invest in a risk free asset. Both avenues yield a fixed return, with the return on projects being higher than the risk free rate. Realizing that if other banks decide not to lend firms' projects would fail, banks might be reluctant to lend even if the fundamentals are relatively benign, and a self-fulfilling credit freeze occurs.

My model differs in two main aspects. First, in my model, the return on the risk free asset is endogenous, as it is the result of the policy of the central bank which sets a different rate for each state. Second, the return on the risky asset in my model is also endogenous, as it depends on the amount of investment. The former difference is necessary for analyzing monetary policy which entails an adjustment of the interest rate to changing economic conditions. The latter difference enables the CB to induce different levels of investment according to the level it deems as optimal. Thus, even when fundamentals are sufficiently strong such that the risk of crisis is negligible, investment would change according to the fundamental (rather than staying fixed at its maximal

level where all investors choose to invest).

Applying the global game methodology enables me to explore the potentially highly non-linear nature of the IE, which in my model is negligible in tranquil times but can strengthen to the degree that it dominates the standard channel of monetary policy. This aspect differentiates my paper from previous papers which modelled the IE (Baeriswyl and Cornand (2010), Berkelmans (2011), Tang (2015), Melosi (2017), NS18, Jarocinski and Karadi (2020), Jia (2023)). This literature, which has mostly focused on the implications of the IE on the inflation–output faced by the CB, has embedded the IE in a New-Keynesian framework. These models are typically linearized around a steady state, which does not allow for non-linear effects of the type I model here.

My paper also relates to the literature on CB communication, and in particular to the literature that has investigated the optimal level of CB's transparency (e.g., Morris and Shin (2002), Morris and Shin (2005), Amador and Weill (2010), and Chahrour (2014)). In this literature the CB communicates its views directly, and the trade-off it faces is between providing private agents with a more accurate public signal on the one hand, and the risk that too much public information will result in more uncertainty rather than less. In contrast, in my model the information is conveyed indirectly, via the interest rate it sets.

Thus, the question of how transparent the CB should be about its view on the economy is entangled with the question of what is the appropriate monetary stance, and the trade-off that the CB faces is between reducing the interest rate in order to offset a deterioration in aggregate demand and the risk that reducing the rate will induce a strong IE that will result in an even stronger reduction of aggregate demand.

For example, in my model, when the economy's fundmental deteriorates, the CB would prefer that investors would not know this, since this will increase fear and reduce their demand for investment. If the CB had the ability to signal the information irrespective of the interest rate it sets then it could potentially choose to bias its signal upwards, to increase optimism. But in my model, in order for the CB to hide this information from the public it would have to refrain from reducing its interest rate, which might turn out to be harmful in itself in terms of the demand for investment.

While in this paper I do not analyze the optimal policy of the CB, the model I build provides a setting that can help in examining this question. I believe that an analysis of this kind is important, since it embeds the notion that the interest rate set by a CB reflects its view on the economy, potentially even more than the words it uses to accompany the decision ("Actions speak louder than words").

I now turn to present the analysis in more detail. Section 2 presents the setting of the model and its results. section 3 presents the empirical analysis. Section 4 concludes.

## 2 The model

The model I build is a static one, describing a CB and a continuum of many, small, investors. The model embeds the standard channel under which monetary policy affects investment, where by changing the risk free rate the CB changes the relative attractiveness of investment in the risky asset. To this basic mechanism I add two assumptions which are less standard. First, I assume that investing in risky capital is subject to a risk of a financial crisis which wipes out the investment. Importantly, the crisis I assume that crises are triggered by an overly low level of aggregate demand. Second, I assume that the CB has an information advantage over investors, in that it observes the value of the economy's fundamental which investors do not observe. The combination of these two assumptions gives rise to an information effect of monetary policy on investment. The strength of this effect will be an endogenous outcome of the model. I begin by describing the setting of the model and then turn to analyze the equilibrium obtained.

## 2.1 Setting

The model contains two types of agents: a central bank and investors. I start with describing the central bank and then turn to describe investors.

## 2.1.1 Central bank (CB)

I assume that the goal of the CB is to bring aggregate demand in the economy, D, as close as possible to its optimal level,  $\bar{D}$ , which in the context of this model is a given parameter. To that end, the CB uses its policy tool, the (gross) interest rate R. Formally, I assume that the CB strives to minimize its loss, L, as follows:

$$\min_{R} L = (D - \bar{D})^2 \tag{1}$$

That is, the CB aims to minimize a demand gap, i.e., the difference between actual aggregate demand, D, and the level that the CB deems as optimal,  $\bar{D}$ . Hence, the CB seeks to stabilize aggregate demand around a level it deems as optimal. The loss that the CB entails is specified as a quadratic function, which entails that the CB is equally averse towards both positive gaps and negative ones.

<sup>&</sup>lt;sup>7</sup>The model I develop is inspired by the New-Keynesian literature (See Clarida et. al. (1999) and Galí (2015)) where due to price rigidities, output fluctuates inefficiently in response to shocks, hence the CB finds it optimal to moderate these fluctuations by adjusting its interest rate accordingly. Specifically, it aims to stabilize the level of output around its "nautral level" – the level of output that would prevail if prices were not rigid.

I further assume that aggregate demand, D, is the sum of two components: investment, I, which is endogenous in the model (specifically, it depends on the interest rate), and  $\theta$ , an exogenous component, which I refer to as the fundamental:

$$D = I + \theta \tag{2}$$

The fundamental reflects factors that affect aggregate demand and are exogenous to the model. For example, it could capture changes in the public's preference to consume, in wealth, or in the level of government spending. A decline in these will cause a reduction in demand that the CB would like to offset by reducing the interest rate. The fundamental is randomly drawn from a normal distribution:  $\theta \sim N(\bar{\theta}, \frac{1}{\alpha})$ , where  $\bar{\theta}$  is the mean of  $\theta$ , and  $\alpha$  controls the variance of the fundamental.

Why does the CB care about the level of investment? As I describe shortly, my model does not differentiate between real investments (investment in physical capital) and financial investment (stock purchasing). Rather, it embeds the view that stocks represent claims on firms' assets, and hence the return on stocks is tied to the return on productive capital. Hence, in my model, the CB's motivation to care about investment stems from the fact that real investment is a component of aggregate demand which the CB seeks to stabilize. The CB might also care about financial investment in the stock market since changes in stock prices could affect investors' wealth and hence their demand for consumption – another important component of aggregate demand. Supporting this view, Cieslak and Vissing-Jorgensen (2021) find that a main reason for the Fed paying attention to the stock market is its perception that developments in the stock market drive – not only predict – real developments.<sup>8</sup>

#### 2.1.2 Investors

There is a continuum [0,1] of investors, each indexed by the subscript i. Investors are risk neutral, and hence simply strive to maximize payoffs. Motivated by the literature that ties the stock market return to the return on physical investment (see for example Cochrane (1991)), I assume that, investors choose between saving in a risk free asset and a physical (real) investment. Specifically, each investor has one unit of a good which he can use, besides storing the unit, in one of the following ways:

- 1. Invest the unit in a risk free asset and get a (gross) return R.
- 2. Invest the unit according to the economy's aggregate production function  $Y = I^{\gamma}$

<sup>&</sup>lt;sup>8</sup>See also Caballero and Simsek (2024) for a review of empirical findings regarding the effects of stock prices on the economy.

where I is the amount of aggregate investment,  $I = \int_0^1 I_i \, di$  (that is, the sum of individual investments), and where I've assumed that the initial level of capital is zero, so that the amount of productive capital equals the amount of investment. The production function exhibits diminishing marginal productivity  $(1 > \gamma > 0)$ . Under perfect competition each capital unit holder receives the marginal product of the unit it holds, which is:  $\frac{\partial Y}{\partial I} = \gamma I^{\gamma-1}$ . Hence, assuming zero depreciation, the gross return per unit invested,  $R^k$ , is:

$$R^k = 1 + \gamma I^{\gamma - 1} \tag{3}$$

Note that while the motivation for building the model is explaining the response of the stock market, I have assumed that investors invest directly in productive capital ("real investment"). In the context of this model this is only a technical shortcut – "I" could be thought of as representing the value of stocks purchased by investors ("financial investment"), which is then used by the stock issuing firms to purchase productive capital ("real investment"). Eq. 3 defines the supply curve of the risky asset in the sense that it specifies how much a stock issuing firm would be willing to pay for each additional unit of finance that it acquires. <sup>10</sup>

#### 2.1.3 Risk of a financial crisis

The riskiness of capital in my model stems from the possibility of a financial crisis that results in a severe loss to investors. The causes of financial crises have been a focus of a large literature. Among the factors discussed in this literature, a central role is given to the presence of financial frictions concentrating in the financial sector and their influence on the interaction between the financial sector and the real side of the economy. According to this literature, financial crises typically occur as a result of a negative shock to the net worth of financial intermediaries (that is, a deterioration in the balance sheets of financial intermediaries). As long as the financial sector is well capitalized, the reduction in net worth has no significant impact on the economy. However, if the financial sector is not well capitalized, the reduction in net worth could set in motion an adverse feedback loop that results in large declines in asset prices and hence in big losses to investors:

<sup>&</sup>lt;sup>9</sup>What is crucial for the analysis I present is that increasing investment lowers the return on capital. The exact specification of this relation, however, does not necessarily have to emanate from a production function as I have assumed here. I choose this modelling assumption as a way to bring my model closer to standard macro models (e.g Gertler and Karadi (2011)).

<sup>&</sup>lt;sup>10</sup>A relation of the form presented in eq. 3 is sometimes referred to as describing the demand of firms for capital. Assuming that firms finance the acquisition of capital by issuing stocks (the risky asset), we get that the same relation that defines firms' demand for capital also determines the supply of the risky asset that is accessible to financial investors (purchasing stocks).

<sup>&</sup>lt;sup>11</sup>See Brunnermeier et. al. (2013) for a review on the macroeconomic implications of financial frictions.

the reduction of net worth intensifies financial frictions which lower the financial sector's capacity to obtain funding and hence to supply funding to the real sector. Faced by these funding difficulties, the intermediaries are forced to reduce the amount of funding that they supply to the real sector either by selling assets ("asset fire-sales") or by cutting lending. The reduced level of intermediation results in a decline in real activity that depresses asset prices which further reduces the net worth of the intermediaries, and so on.<sup>12</sup>

Motivated by the literature that highlighted the role of deteriorating balance sheets of financial intermediaries, I assume that a financial crisis is triggered by weak aggregate demand. Specifically, I assume that if aggregate demand is below some critical level,  $D^c$ , a financial crisis occurs, in which investors lose their investment.<sup>13</sup> Formally:

A financial crisis (payoff on risky asset = 0) occurs 
$$if D < D^c$$
 (4)

with  $D^c$  being a parameter of the model.

Eq. 4 captures the idea that a reduction in aggregate demand harms the balance sheets of the financial sector – either due to decreasing the prices of assets held by these institutions or due to an increase in losses on loans supplied to non financial firms who face a decline in demand to their products – and that a sufficiently large deterioration in these balance sheets triggers a financial crisis. The losses that investors suffer in case of a financial crisis could reflect losses on stocks that they own directly. Alternatively, one can view these losses as arriving from a default of financial intermediaries through which the households own these stocks.

Another, slightly different, motivation for the assumption in eq. 4 emerges from viewing the investors in the model as being the financial intermediaries themselves. A large literature suggests that financial intermediaries constitute the marginal investors in various markets.<sup>14</sup> Since intermediaries are often subject to collateral constraints, when their balance sheet deteriorate and they get closer to their constraints, their marginal value of wealth increases which makes them more reluctant to invest in risky assets (see

<sup>&</sup>lt;sup>12</sup>See Bernanke et al. (1999) and Kiyotaki and Moore (1997) for early expositions of these ideas. More recent papers that develop this line of research in the context of a financial sector are Gertler and Kiyotaki (2010), Gertler and Karadi (2011), He and Krishnamurthy (2012), Brunnermeier and Sannikov (2014), and Gertler et al. (2020). Mendoza (2010) develops a model where a financial crash is an endogenous result of an amplification mechanism induced by the presence of collateral constraints. Caballero and Simsek (2020) describe a mechanism that results in a similar adverse feedback loop which does not rely on the presence of financial frictions.

<sup>&</sup>lt;sup>13</sup>The stark assumption that in a case of a financial crisis investors get nothing is made here for simplicity, but is not essential for the qualitative results. The essential feature is that below a certain level of aggregate demand investors that choose to invest in the risky asset will lose a part of their investment.

 $<sup>^{14}</sup>$ See He and Krishnamurthy (2012), Adrian et al. (2014) and He et al. (2017).

for example He and Krishnamurthy (2012) and He et al. (2017)). The assumption in eq. 4 could be interpreted as reflecting this idea, where the damage to the intermediary from violating its collateral constraint – even due to a relatively small loss on investment – is extremely large.

Financial distress and the vulnerability to a financial crisis The parameter  $D^c$  governs how much of a deterioration in aggregate demand the economy can bear before falling into a financial crisis. Following the aforementioned discussion regarding the sources of financial crises and the interpretation of eq. 4, a natural interpretation for  $D^c$  is that it reflects the financial sector's strength prior to the deterioration: if the financial sector is well capitalized,  $D^c$  is low, so that in order for a financial crisis to occur, aggregate demand (D) must deteriorate substantially. In contrast, a high  $D^c$  represents a state where the financial sector is distressed due to having a low amount of capital which brings intermediaries close to their collateral constraints. In this state a crisis could occur even if aggregate demand is relatively high.<sup>15</sup>

The CB and financial crises I assume  $D^c < \bar{D}$  – that is, the critical value of demand that triggers a financial crisis is lower than the level of demand that the CB aims for (eq. 1). This assumption ensures that if the CB achieves its desired demand level, a crisis will not occur. Note, however, that in itself, the occurrence of a financial crisis does not constitute a cost to the CB. The harmful consequences of a financial crisis in my model are only due to the depressing effect that the risk of its occurrence has on the demand for investment and thus on aggregate demand.

Regions of  $\theta$  and the possibility of multiple equilibria The distance of the fundamental from the critical value of demand,  $D^c$ , plays a crucial role in the model: if  $\theta \geq D^c$ , a financial crisis will not occur regardless of the level of investment, since then the level of aggregate demand, D, is always larger than the critical value  $D^c$  (as investment, I, is always non-negative). In contrast, when  $\theta < D^c - 1$  a financial crisis always occurs, as the level of aggregate demand, D, is always smaller than  $D^c$  (since I is bounded at 1). When the fundamental is in an intermediate range, such that  $D^c - 1 \leq \theta < D^c$ , the occurrence of a financial crisis depends on the decisions of investors: if all investors refrain from investing, a crisis would occur for sure, in which case no one finds it optimal to invest, thus constituting an equilibrium. In contrast, if all choose to invest, a financial crisis will never occur. Hence, given that the return on capital is higher than the risk

<sup>&</sup>lt;sup>15</sup>Gertler et al. (2020) describe a similar idea: "Balance sheet conditions affect not only borrower access to credit but also whether the banking system is vulnerable to a run. In this way, the model is able to capture the highly non-linear nature of a collapse: when bank balance sheets are strong, negative shocks do not push the financial system to the verge of collapse. When they are weak, the same size shock leads the economy into a crisis zone in which a bank run equilibrium exists."

free rate, there exists also an equilibrium in which everybody invests.<sup>16</sup> Thus, the region  $D^c - 1 \le \theta < D^c$  exhibits multiple equilibria. The multiplicity of equilibria in this region is the result of the *strategic complementarity* between investors' actions embedded in eq. 4, which generates a dependency of each investor's payoff on the investment decisions of the other investors.<sup>17</sup> The global game literature showed that the existence of multiple equilibria also depends on the presence of common knowledge that agents share regarding the value of the fundamental, and developed the idea that introducing noisy information breaks common knowledge and selects a unique equilibrium. I follow this literature by assuming noisy information as I explain now.

#### 2.1.4 Information

Motivated by the literature documenting the presence of an IE, I assume that the CB has an information advantage over investors. Specifically I assume that investors do not observe  $\theta$  but the CB does. Investors only know the distribution properties of  $\theta$ . This assumption does not mean that investors are oblivious of economic developments. Rather, the information advantage that the CB holds could be thought of as reflecting the *excess* information that the CB holds conditional on all the data that is available to the CB and investors alike.

Recognizing there informational disadvantage, investors will try to deduce  $\theta$  from the interest rate R that the CB sets, and which they observe. The deduction of  $\theta$ , however, is not straightforward as I assume that investors have imperfect knowledge of  $\bar{D}$ , the level of demand that the CB is aiming for. Formally, I assume that investors have a completely uninformative prior regarding  $\bar{D}$  (that is,  $\bar{D}$  is drawn from an improper prior, distributed uniformly over the real line) and that each investor gets a private signal  $x_i \sim N(\bar{D}, \frac{1}{\beta})$ , where  $\beta > 0$  is the precision of the private signal.

Thus, each investor will combine its private signal,  $x_i$ , with the interest rate R to obtain an estimate of  $\theta$ . Due to the heterogeneity in the private signals, each investor will have a different assessment of  $\theta$ . For example, if the CB sets a very low interest rate, some investors will infer that  $\theta$  is very low (that is, the economy is in a bad state), whereas other investors, more optimistic, would attribute the low interest rate to the fact that the CB is aiming for a high level of demand (high  $\bar{D}$ ).

The assumption that investors have imperfect observe D with idiosyncratic noise generates a distribution of beliefs across investors regarding the value of  $\theta$ , and breaks

<sup>&</sup>lt;sup>16</sup>For the ease of exposition I put here aside the fact that the return on the risky investment itself depends on the decisions of investors. In the model I build this is taken into account.

<sup>&</sup>lt;sup>17</sup>To be precise, the complementarity is conditional on the return  $R^k$ . Unconditionally, since  $R^k$  decreases with the level of investment, I, investment decisions actually exhibit strategic substitutability.

common knowledge of the fundamental. The global game literature has shown that the presence of common knowledge when agents' payoffs exhibit strategic complementarities results in multiple equilibria. This limits the ability to analyze the equilibrium obtained, as there would be no way to determine which of the possible equilibria actually prevail. The presence of idiosyncratic noise facilitates the analysis as it selects a specific equilibrium. When I solve for the equilibrium I get a standard condition in the global game literature stating that when the precision of the private signal is sufficiently large the equilibrium is indeed unique.

Finally, I make the simplifying assumption that investors do not make use of the return on capital,  $R^k$ , as an information source. Angeletos and Werning (2006) show that when agents are allowed to learn from prices, models that originally featured a unique equilibrium admit multiple equilibria due to the endogeneity of public information. As I want to retain uniqueness of equilibrium I assume away the possibility of learning from prices.

Technical assumption The analysis focuses on the case where the fundamental is lower than the CB's target,  $\theta < \bar{D}$ , so that the CB needs to induce a positive level investment to achieve its target. However, the response of the CB to realizations where  $\theta \geq \bar{D}$  might complicate the inference problem that investors face even when  $\theta < \bar{D}$ . To see why, suppose that  $\theta_1$  is a realization of the fundamental such that  $\theta_1 < \bar{D}$ , and suppose also that there exists a realization of the fundamental,  $\theta_2 \geq \bar{D}$  for which the CB chooses the same interest rate as in the case  $\theta = \theta_1$  – that is,  $R(\theta_2) = R(\theta_1) \equiv \tilde{R}$ . This implies that observing  $\tilde{R}$  leaves investors ambiguous about the value of  $\theta - \bar{D}$ . Hence, I assume that when  $\theta \geq \bar{D}$  the CB imposes a tax,  $\tau$ , that directly affects aggregate demand, such that  $D = I + \theta - \tau$ . Importantly, the CB announces  $\tau$ . This will allow investors to infer the actual level of  $\theta - \bar{D}$  without ambiguity. In the case I analyze investors do not observe  $\tau > 0$  and hence know for certain that  $\theta - \bar{D} < 0$ .

This completes the description of the model's setting. I now turn to analyze the equilibrium that the model yields.

<sup>&</sup>lt;sup>18</sup>I do not explicitly model the mechanism responsible for investors not learning from the price. Angeletos and Werning (2006) assume the presence of an unobservable shock to the supply of the asset that prevents the price from perfectly revealing the aggregate demand for the asset. I conjecture that the same mechanism, where I assume that the variance of the supply shock goes to infinity (so that agents cannot learn at all from the price) would generate the result that investors do not learn from the return on capital.

<sup>&</sup>lt;sup>19</sup>Introducing this tax also enables the CB to achieve its target when  $\theta \geq \bar{D}$  (by setting  $\tau = \theta + I - \bar{D}$ ) which would not be possible otherwise since investment in my model cannot attain negative values. However, this has no importance for the analysis I conduct.

## 2.2 Equilibrium

I look for a monotone Bayesian Nash Equilibrium. In this equilibrium:

- 1. The CB sets the interest rate R such that it minimizes its loss, given the fundamental  $\theta$  and its belief regarding investors' actions.
- 2. Each investor chooses whether to invest in the risky asset  $(I_i = 1)$  or in the risk free asset  $(I_i = 0)$ , given the interest rate R, the return on the risky asset  $R^K$ , his belief regarding  $\theta$  after observing his private signal  $x_i$ , and given his belief regarding other investors' actions.
- 3.  $R^K$  clears the market for the risky asset.

I start with the behavior of the CB and then turn to investors.

#### 2.2.1 CB

I assume that the CB does not take into account the fact that the strength of the IE is state dependent. That is, it treats the strength of the IE as being constant. For simplicity, I assume that the CB acts as if the strength of the IE is zero.<sup>20</sup> This assumption has the advantage that when the economy is sufficiently strong, the CB will achieve its demand target. Under this assumption, an equilibrium in the market is sustained when investors are indifferent between investing in capital and investing in the risk free rate. Formally, when  $R^K = R$ . Using eq. 3 we have that in this equilibrium,  $I = (\frac{R-1}{\gamma})^{\frac{1}{\gamma-1}}$ . Hence, the CB's optimization problem is:

$$\min_{R} L = (D - \bar{D})^{2} 
\Longrightarrow \min_{R} L = (I + \theta - \bar{D})^{2} 
s.t I = (\frac{R - 1}{\gamma})^{\frac{1}{\gamma - 1}}$$
(5)

which leads to the following policy function:

$$R = 1 + \gamma (\bar{D} - \theta)^{\gamma - 1} \tag{6}$$

Eq. 6 represents a simple policy in which the CB lowers the interest rate when either the fundamental,  $\theta$ , deteriorates or when the level of demand that it deems as optimal,  $\bar{D}$ ,

<sup>&</sup>lt;sup>20</sup>That is, the CB sets its policy without taking into account that its actions will affect investors via their influence on investors' confidence. In other words, it takes the probability that investors ascribe to the possibility of a crisis as exogenous to its actions.

increases. Under the belief of the CB that does not realize the presence of an information channel, setting the interest rate according to eq. 6 would minimize its loss.<sup>21</sup> The policy represented in eq. 6 can be thought of as reflecting a CB which adheres to a standard "prescription" of reducing its interest rate when faced by a decline in demand, without adjusting its policy to internalize the potential impact of the IE.

#### 2.2.2 Investors

To find the level of investment I proceed as following. First, I derive the aggregate demand function for investment as a function of the interest rate R, and then I find the level of investment that clears the market for the risky asset.

I restrict the analysis to monotone strategies.<sup>22</sup> Hence, I assume investors use a threshold strategy: each investor invests iff  $x_i \geq \hat{x}$ . (there will be a different  $\hat{x}$  for every R and  $R^K$ , as I will show soon). In other words, the investors that will choose to invest are the ones that hold a relatively positive view on the economy. Since  $x_i \sim N(\bar{D}, \frac{1}{\beta})$ , aggregate investment is determined as  $I = 1 - \Phi(\sqrt{\beta}(\hat{x} - \bar{D}))$ , where  $\Phi$  is the standard normal cumulative distribution function (CDF). For the investment to be paid back, we must have:  $I + \theta \geq D^c \Longrightarrow \theta \geq D^c - (1 - \Phi(\sqrt{\beta}(\hat{x} - \bar{D})))$ .

From the last inequality we can derive the value of  $\hat{\theta}$ , the critical level of fundamental:

$$\hat{\theta} = D^c - \underbrace{\left(1 - \Phi(\sqrt{\beta}(\hat{x} - \bar{D}))\right)}_{I} \tag{7}$$

so that a default occurs if  $\theta < \hat{\theta}$ .

Now I turn to derive investors' belief about  $\theta$ . Recall the CB sets the interest rate such that  $R = 1 + \gamma(\bar{D} - \theta)^{\gamma - 1}$ . Hence, after observing R, each investor's belief is:

$$E_i(\theta \mid x_i, R) = \frac{\alpha \bar{\theta} + \beta E_i(\bar{D} - \frac{R-1}{\gamma}^{\frac{1}{\gamma-1}})}{\alpha + \beta} = \frac{\alpha \bar{\theta} + \beta (x_i - \frac{R-1}{\gamma}^{\frac{1}{\gamma-1}})}{\alpha + \beta}$$
(8)

Thus, after observing the interest rate R and the private signal  $x_i$ , the posterior distribution of  $\theta$ , as perceived by investor i, is normal with mean  $\frac{\alpha \bar{\theta} + \beta (x_i - \frac{R-1}{\gamma} \frac{1}{\gamma-1})}{\alpha + \beta}$  and precision  $\alpha + \beta$ .

To find the threshold value  $\hat{x}$ , note that optimality of investment requires:

 $<sup>^{21}</sup>$ Since investors have an endowment of 1, the maximal possible demand for investment is truncated at 1. Hence, when  $\theta < \bar{D} - 1$  every policy function that would make the demand for investment to be 1 would be optimal. In this region the loss that the CB entails is positive, as the optimal level of investment is higher than 1.

<sup>&</sup>lt;sup>22</sup>A common result in the global game literature is that a monotone (threshold) equilibrium is the only possible equilibrium. I do not prove this in the context of my paper.

$$\Longrightarrow (1 - \Phi(\sqrt{\alpha + \beta}(\hat{\theta} - \underbrace{\frac{\alpha\bar{\theta} + \beta(x_i - \frac{R-1}{\gamma}^{\frac{1}{\gamma-1}})}{\alpha + \beta}}_{E(\theta|x_i, R)})))R^k \ge R$$
(9)

where  $P_i^{NC}$  denotes the subjective probability that a financial crisis does not occur as perceived by investor i.

From eq. 9 we see that  $P_i^{NC}$  is increasing in  $x_i$  – investors who get a higher signal regarding the goal of the CB are more optimistic with regard to the probability that a financial crisis will not occur. Since the left hand side of eq. 9 is increasing in  $x_i$ , and since investment is optimal when the left hand side is at least as high as R, there must be a threshold of the signal such that investors who receive lower signals choose to invest in the risk free asset whereas investors who get a signal equal to or higher than the threshold choose to invest in the risky asset. Denoting this threshold by  $\hat{x}$  we have:

$$(1 - \Phi(\sqrt{\alpha + \beta}(\hat{\theta} - \underbrace{\frac{\alpha\bar{\theta} + \beta(\hat{x} - \frac{R-1}{\gamma^{-1}})}{\alpha + \beta}}_{PNC})))R^{K} = R$$

$$(10)$$

where  $P^{NC} = (1 - \Phi(\sqrt{\alpha + \beta}(\hat{\theta} - \frac{\alpha\bar{\theta} + \beta(\hat{x} - \frac{R-1}{\gamma} - 1)}{\alpha + \beta})))$  denotes the probability that the marginal investor – the investor that got a signal  $x_i = \hat{x}$  – ascribes to the case that a financial crisis does not occur. This equation describes the optimality condition of the marginal investor, i.e., the investor which is exactly indifferent between investing in risky capital and in the risk free asset. Hence, given that an investor believes that all other agents are investing according to a threshold strategy where  $\hat{x}$  is the threshold, his optimal strategy is also a threshold strategy where  $\hat{x}$  is the threshold.

Eq. 10 enables us to derive the aggregate demand for investment,  $I^D$ , as a function of R. Then, to find the actual level of investment, we need to find the value of  $\hat{x}$  for which  $I^D = I^S$ , where  $I^S$  stands for the supply function of the risky asset, which is defined by the economy's technology (eq. 3). Under certain conditions regarding the precision of the private signal, the demand function is unique. Given the demand function  $I^D(R^K)$ , there is a unique level of  $R^K$  that clears the market and as a result, a unique level of I. This is summarized in the following proposition:

**Proposition 1** For every level of R the level of investment is given by:

$$I = 1 - \Phi(\sqrt{\beta}(\hat{x} - \bar{D})) \tag{11}$$

where  $\hat{x}$  is defined as the solution to the following equation:

$$\underbrace{(1 - \Phi(\sqrt{\alpha + \beta}(D^c - (1 - \Phi(\sqrt{\beta}(\hat{x} - \bar{D}))) - \frac{\alpha\bar{\theta} + \beta(\hat{x} - \frac{R-1}{\gamma}^{\frac{1}{\gamma-1}})}{\alpha + \beta})))}_{R^K})))} \cdot (12)$$

Depending on parameter values, this system could either have a unique equilibrium, or up to three equilibria, out of which one is unstable.

The proof is given in appendix A. The analysis in the appendix shows that eq. 12 has potentially three solutions, meaning that the model potentially exhibits multiple equilibria. The analysis further shows that out of these equilibria, only two are stable. In what follows, I focus on the case where there is a unique equilibrium.

The level of investment as a function of the fundamental Having characterized equilibrium conditions, I now turn to analyze the actual level of investment obtained in equilibrium. The analysis is based on a numerical solution to the model and on a theoretical derivation for the limiting case where the precision of the private signal goes to infinity. Figure 2 presents the level of investment, on the vertical axis, I, as a function of the fundamental,  $\theta$ , on the horizontal axis.<sup>23</sup>

 $<sup>^{23}</sup>$ I solve numerically a system of two equations for the values of  $\hat{x}$  and  $\hat{\theta}$ : eq. 7 and eq. 10 after expressing  $R^K$  as a function of  $\hat{x}$ , and combine it with eq. 6. The numerical solution was conducted using the "Fsolve" function in MATLAB. For  $\beta=0.04$  and  $\beta=4$  it can be shown theoretically (see Appendix A) that there is a unique equilibrium. For  $\beta=40$  I verify numerically that there is a unique equilibrium.

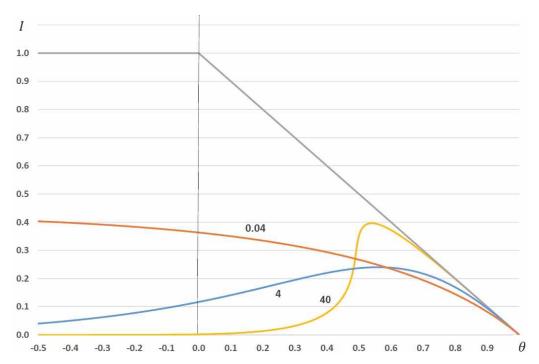


Figure 2: Investment (Y-axis) as a function of the fundamental  $\theta$  (X-axis) – A numerical example:  $D^c = 0.7$ ,  $\bar{D} = 1$ ,  $\beta \in \{0.04, 4, 40\}$ ,  $\alpha = 0.1$ ,  $\gamma = 0.333$ ,  $\bar{\theta} = 3$ .

The figure presents four lines. The grey line describes the level of investment that the CB needs to induce in order to achieve its aggregate demand target. That is, for  $\theta \ge 0$  the grey line depicts the locus  $I + \theta = \bar{D}$ . For  $\theta < 0$  the grey line corresponds to maximal attainable level of investment, which due to the assumption that each investor is endowed with one unit of good, equals one. The grey line also coincides with the level of investment that would be achieved in an economy without risk. The other lines depict the actual level of investment in my model. The red line corresponds to the case where  $\beta = 0.04$ . As can be seen, in this case, lower values of the fundamental  $\theta$  correspond to higher values of I. The reason is that lower values of the fundamental induce the CB to set lower interest rates, hence stimulating investment. Note that the red line is lower than the grey one, that is, actual investment is lower than its optimal value. The reason is that low interest rates signal to agents that the fundamental is low, hence the risk of a default is higher. They are thus less inclined to invest. Since the CB sets its interest rate without taking into account the information that investors learn from it, the interest rate reductions are not strong enough to increase investment to its optimal level. As a result, the demand gap  $(D - \bar{D})$  is negative, even though a zero demand gap was attainable: if investors would invest according to the level the CB aimed for, then for  $\theta \geq -0.3$  a crisis would not occur (since D would be higher than  $D^c$ ). Given that a crisis wouldn't occur,  $I = \min(D - \theta, 1)$  would constitue on equilibrium. In this sense, the level of investment is inefficiently low, reflecting a coordination failure among investors.

The blue line corresponds to the case where  $\beta = 4$ . That is, the precision of the private signal is higher than in the former case. Here, when  $\theta$  is higher than 0.6, lower values of  $\theta$  correspond to higher level of I, as in the former case. However, when  $\theta$  is lower than 0.6, lower values of  $\theta$  correspond to a lower level of I. That is, in this area, lower interest rates reduce investment. What explains this result? We know that in equilibrium, for the marginal investor, the following condition has to hold:  $E_i(P^{NC} \cdot R^k) = R$ . When the CB decreases interest rate, the response of investment depends on the change in  $E_i(P^{NC})$ . When the reduction in the interest rate is moderate, investors believe that  $\theta$ is far enough from the critical value of the fundamental,  $\hat{\theta}$ . Thus,  $E_i(P^{NC})$  stays close to one. The reduction in R in this case translates almost fully to a reduction in return on capital,  $R^k$ , and thus to an increase in investment. In contrast, when the interest rate reduction is strong enough, investors understand that  $\theta$  might be dangerously low (close to the critical value  $\hat{\theta}$ ). In this case  $P^{NC}$  becomes very sensitive to changes in R. Investors now revise their belief of  $P^{NC}$  downwards, so that the reduction in R translates into lower investment. Note also the difference in the level of investment relative to the CB's goal: when the fundamental is sufficiently close to one, the blue line coincides with the grey line, indicating that the CB achieves its target even though it ignores the IE when setting the interest rate. This is because for a sufficiently strong fundamental the IE is negligible. It is only when the economy deteriorates sufficiently that the IE becomes significant.

The yellow line corresponds to the case where  $\beta = 40$ , an even higher precision of the private signal. The level of investment in this case becomes even closer to its desired level when the fundamental is sufficiently strong, but the reduction of investment when the economy weakens, is now stronger.

The economy's resilience and the effects of monetary policy The main idea of this paper is that a given size of an interest rate change could have different effects on investment, depending on the state of the economy and its resilience to a financial crisis. For example, a distressed financial sector, represented by a high value of  $D^c$ , could render the economy more vulnerable to a financial crisis, in which case my model predicts a strong IE and thus a low effectiveness of monetary policy in stimulating investment. Thus, an object of interest is the change in investment following a marginal change of the interest rate by the CB, formally  $-\frac{\partial I}{\partial R}$ . Given the model's solution for the level of investment, an analytical expression for  $\frac{\partial I}{\partial R}$  can be derived, and is provided in appendix B. Here I present a numerical derivation of this object. Figure 3 presents the sensitivity of investments to the risk free rate, for different values of  $D^c$ , based on the numerical simulation that was presented before. The precision of the private signal was set to  $\beta = 4$ .

The figure presents the sensitivity of investment to marginal changes in the interest rate,  $\frac{\partial I}{\partial R}$ , as a function of the state of the world,  $\theta$ . The figure shows four lines. The grey line ("No risk benchmark") corresponds to the case where there is no risk of a financial crisis, and it is derived analytically as follows: In an economy with no risk (which is essentially equivalent to the case where  $D^c$  goes to minus infinity), the return on capital always equals the risk free rate:  $R^K = R$ . Using eq. 3 we have that in this equilibrium,  $I = \left(\frac{R-1}{\gamma}\right)^{\frac{1}{\gamma-1}}$ . From differentiating the level of investment with respect to the risk free rate we have:

$$\frac{\partial I^{\text{No risk}}}{\partial R} = \frac{1}{\gamma} \frac{1}{\gamma - 1} \left( \frac{R - 1}{\gamma} \right)^{\frac{2 - \gamma}{\gamma - 1}} \tag{13}$$

Since by assumption  $\gamma < 1$ , we have that in the riskless economy  $\frac{\partial I}{\partial R}$  is negative (that is, reducing the interest rate increases investment) for each value  $\theta > 0$ .<sup>24</sup>

The other three lines in figure 3 correspond to different values of  $D^c$ , and are derived numerically: I solve the model for different values of  $\theta$  where the size of the jumps between the values is 0.001, and then compute the value of  $\frac{\partial I}{\partial R}$  as  $\frac{I(\theta_1)-I(\theta_0)}{R(\theta_1)-R(\theta_0)}$ .

The green line in figure 3 depicts the case  $D^c = -1$ , that is, the economy is resilient in the sense that it takes a large fall of demand to trigger a crisis. It can be seen that for most values of  $\theta$  this line coincides with the no risk benchmark. For low values of  $\theta$  the line is above the benchmark case. That is,  $\frac{\partial I}{\partial R}$  is smaller in absolute value, but still negative. The difference between the lines reflects the effect of risk. The dark blue line corresponds to the case  $D^c = -0.2$ , that is, an economy which is not as resilient. Here the line lies above the former lines. For low values of  $\theta$  the sign of  $\frac{\partial I}{\partial R}$  flips – interest rate reductions decrease investment rather than increase it. Increasing  $D^c$  to 0.7 (this was the value used in figure 2) produces the same result – under a more vulnerable economy, the value of  $\frac{\partial I}{\partial R}$  increases, and the effect is more noticeable for low values of  $\theta$ . Due to this effect, the range of states where monetary policy has an opposite effect on investment relative to the standard channel increases when the economy is more vulnerable (the value of  $\theta$  for which  $\frac{\partial I}{\partial R}$  switches sign from negative to positive is higher when  $D^c$  is higher).

<sup>&</sup>lt;sup>24</sup>For values of  $\theta$  lower than zero investment is bounded at one (the level of investors' endowment).

 $<sup>^{25}</sup>$ For very low values of  $\theta$  it can be seen that the value of  $\frac{\partial I}{\partial R}$  under  $D^c = 0.7$  is lower than its value under  $D^c = -0.2$ . This is the result of the fact that investment in my model is bounded at zero. Hence, when investment declines sufficiently, it eventually gets to a level where further deteriorations in the fundamental reduce investment by a decreasing rate.

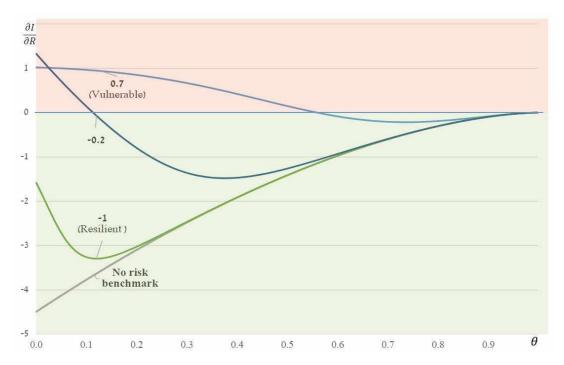


Figure 3: Sensitivity of investment to the risk free rate (Y-axis) as a function of the fundamental  $\theta$  (X-axis), for different values of  $D^c$ . The figure is based on a numerical simulation, with the following parameter values:  $\bar{D} = 1$ ,  $\beta = 4$ ,  $\alpha = 0.1$ ,  $\gamma = 0.333$ ,  $\bar{\theta} = 3$ . The green (red) shaded area marks the region where interest rate reductions increase (decrease) investment.

## 2.3 Policy Considerations

While the goal of this paper is to analyze the state dependent nature of the IE rather than provide a normative analysis of the subject, I briefly discuss in this subsection some potential implications of the results on the way CB should take the IE into account.

NS18 raise the question whether the presence of an IE means that the CB should withhold negative information from the public in order to avoid increasing pessimism. NS18 note two reasons why such a policy may not be optimal. First, an attempt to withhold information is likely to be ineffective, since the public will eventually understand this, and adjust the way they interpret what the CB says accordingly. Second, providing agents with information should enhance welfare by allowing them to prepare gradually to an economic deterioration. The authors further note a possible exception to this intuition, when the CB cannot counteract the negative information it conveys by reducing its interest rate appropriately, for example, due to the zero lower bound.

How should the results in my model be understood in light of this discussion? The first point to note is that in my model demand is inefficiently low due to the strategic complementarity between investment decisions. Hence, there is, at least potentially, scope for an improvement of the outcome. Fear, in my model, is self-fulfilling, and if the CB is

able to convince investors that the a financial crisis will not occur, it will indeed not occur. Thus, in the setting I present here, withholding information might be welfare improving. The second point to note is that in my model, the information that the CB conveys is not an additional policy tool – the only tool is the interest rate. Thus, withholding information would require a manipulation of the interest rate itself. As the investors in my model are rational and understand the incentives of the CB, then, as long as the interest rate is a monotone function of the state of the world (as is the case in my model), they will be able to infer the state of the world even if the CB manipulates its interest rate. Thus, reducing the interest rate by a smaller size (relative to the response that does not internalize the IE) would not have any benefit in terms of avoiding pessimism. Moreover, holding a higher interest rate would drive investment below its optimal level. This means that an optimal policy response is likely to be one of two types: either a more aggressive policy (reduce the rate by *more* than required when ignoring the IE), or alternatively, a policy that commits to regions of "inaction" – ranges of the fundamental for which the CB does not change its policy rate. This would effectively eliminate the public's ability to infer exactly the state of the economy, and could potentially be beneficial.

## 3 Empirical Investigation

In this section I test the theoretical model's prediction that weaker financial conditions result in a stronger IE. Specifically, I test whether the IE is stronger when the financial sector is in distress. I begin with describing the data I use for the analysis, followed by a visual exposition of the main result, and then turn to describe in more detail the econometric methodology I use and the results.

### 3.1 Data

The analysis requires data on stock returns, monetary surprises and a measure for financial distress.

**Stock Returns ("ret")** To measure the response of stock prices to new shocks, I use end of day prices of the S&P 500 stocks,  $p_t$ .<sup>26</sup> I calculate the return (" $ret_t$ "), as

$$ret_t = \frac{p_t - p_{t-1}}{p_{t-1}}$$

Hence, the meaning of  $ret_t = 0.01$ , for example, is that the S&P 500 increased by one

<sup>&</sup>lt;sup>26</sup>Source: S&P Dow Jones Indices LLC, retrieved from Bloomberg.

percent during day t.<sup>27</sup>

News Shocks ("NS") For the news shocks (monetary surprises) I use a series of news shocks constructed by Acosta and Saia (2020) who extend the series that were originally calculated by NS18, following their methodology. The news shock is constructed as the first principal component of the unanticipated change over the 30-minute windows discussed above in five different interest rates.<sup>28</sup> The data spans from 2000 to 2019, omitting some observations during the crisis of 2008-9, and containing only scheduled FOMC meetings, for a total of 151 observations. As the news shocks don't have a natural scale, NS18 scale them such that their impact on the one year nominal treasury yield is one. For example, a news shock of magnitude 0.01 means that the surprise was such that the nominal 1 year yield moved by 0.01.

Figure 4 illustrates the construction of stock returns and of the news shock. The initials "FFRF" in the figure refer to the federal funds rate futures which are an important, although not the sole, determinant of the news shock.

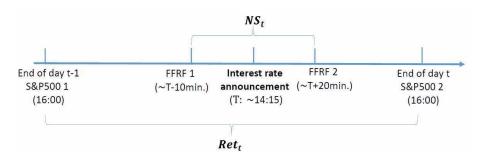


Figure 4: Construction of data on returns and on news shocks, timeline.

Financial Distress ("Noise") The discussion highlighted the role of the financial sector's strength of the financial sector as an important determinant of the vulnerability of the economy to financial crises, and hence on the strength of the IE. As a measure for the strength of the financial sector I use an index calculated in Hu et al. (2013). The indicator exploits mispricings in the US bond market, referred to as "noise", to identify periods in which investors' capital (particularly institutional investors) became

<sup>&</sup>lt;sup>27</sup>Due to data limitations, I do not use intradaily data as is common in the literature (i.e. looking at stock prices reaction in a tight window around the interest rate announcement). However ,this should not bias the coefficient on the explanatory variable (the news shock). The implication of this could be a higher standard error due to noise that is added to the dependent variable (a period of almost a day in which other factors, not related to the monetary news shock, could affect the return of the S&P 500). As I show below, the results of regressing the daily S&P 500 return on monetary news shocks are similar in magnitude to those obtained in NS18 (where a narrow 30 minute window was used).

<sup>&</sup>lt;sup>28</sup>The five interest rates are: the Fed funds rate immediately following the FOMC meeting, the expected Fed funds rate immediately following the next FOMC meeting, and the expected three-month eurodollar interest rates at horizons of two, three, and four quarters. NS18 use data on Fed funds futures and eurodollar futures to measure changes in market expectations about future interest rates at the time of FOMC announcements.

scarce. As the authors note, when capital is scarce the lack of sufficient arbitrage capital limits arbitrage forces and assets can be traded at prices significantly away from their fundamental values, producing the observed noise in prices.<sup>29</sup> To ease the interpretation of the results, I standardize the index of Hu et al. (2013) by subtracting the mean of the index and dividing it by its standard deviation. Figure 5 presents the series of the noise indicator, after standardization, for the 151 observations in my sample.

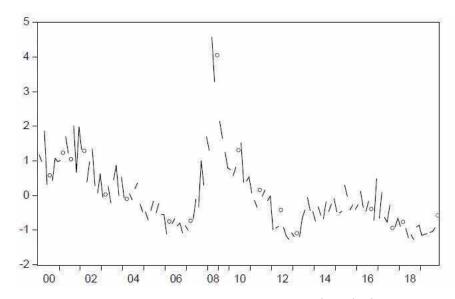


Figure 5: Noise Indicator, 2000-2019, Based on Hu et al. (2013), (retrieved from authors' website) after standardization.

Visual exposition of main result Figure 6 presents a visualization of the main result of the analysis. The figure shows a binned scatter plot where the Y-axis shows the daily S&P 500 return and the X-axis shows the news shock, based on the methodology of NS18. Every dot corresponds to a certain interest rate announcement of the Fed. The observations are divided into two groups: the blue dots represent observations where the level of noise was low, whereas the red dots correspond to observations where the level of noise was high. I have defined noise levels that are higher than one standard deviation of the noise distribution as "high noise", and all other observations as "low noise". The figure contains two regression lines, corresponding to each of the defined groups. As can be seen, when the noise is low, the slope of the line is negative, in accordance with the standard result that stock prices react negatively to monetary surprises. However, the novel finding in this paper, is that the level of noise has a positive effect on the slope, and for high enough noise levels the slope even becomes positive. That is, when the level of

<sup>&</sup>lt;sup>29</sup>He et al. (2017) construct a measure that proxies for intermediaries' financial distress. However, using this measure in the empirical analysis yielded no significant results. One possible explanation is that the data are not suited for the analysis – the data up to 2020 is only available in a monthly frequency instead of daily.

financial distress in the market is high, the information channel dominates the standard channels of monetary shocks. I now turn to present the formal methodology and the empirical results in more detail.

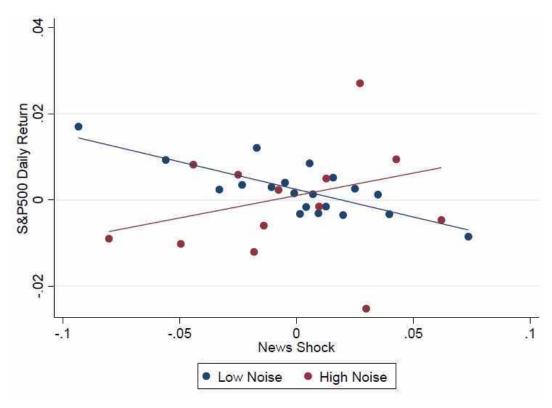


Figure 6: Binned Scatter Plot of S&P500 Daily Return and the News Shock, High Noise vs. Low Noise.

## 3.2 Methodology

I start my empirical inquiry by replicating a standard regression in the literature in order to measure the average response of stock prices to news shocks. Formally, I start with the following basic regression ("regression 1"):

$$ret_t = \alpha + \beta \cdot ns_t + \varepsilon_t \tag{14}$$

To test my hypothesis I check whether the response of stocks to news shocks is more positive when financial distress is high. Formally, I run regressions of the form ("regression 2"):

$$ret_t = \alpha + \beta \cdot ns_t + \gamma \cdot noise_{t-1} \cdot ns_t + \varepsilon_t \tag{15}$$

where  $noise_{t-1}$  refers to the level of the noise indicator a day before the news shock, in order to avoid a possible endogeneity issue. The hypothesis that the IE is stronger in

times of financial distress would be corroborated by finding that  $\gamma > 0$ . To understand the logic behind the test, suppose that the average response of stock prices to interest rate surprises ( $\beta$  in equation 14) is combined of two opposite channels: First, the standard channel in which positive monetary surprises decrease stock prices. Second, the IE, in which positive monetary surprises increase stock prices due to the positive information they reveal to investors. Now, suppose that when financial distress is high investors are more sensitive to information. This means that the second channel, which generates a positive relation between stock returns and news shocks, becomes stronger. Thus, we would expect to see that following a positive surprise, the return would be higher than in the case where financial distress is low (the analysis is symmetric for the case of a negative monetary surprise).

#### 3.3 Results

Table 1 summarizes the results. Column 1 reports the results of the regression 1. According to the results, a news shock in the magnitude of 100 basis points, reduces the value of the S&P 500 by 6.3%. For comparison, NS18 find that a positive news shock reduces the value of the S&P 500 by 6.5%. Thus, to the degree that interest rate surprises affect stock prices via the two opposite effects, the standard one and the IE, the standard effect is dominant on average, as found in the literature. Column 2 presents the results of regression 2, which is the main regression. In this regression I add the interaction of the news shock (NS) with the level of noise (Noise). The regression also includes  $noise_{t-1}$  itself as a control variable. As can be seen, the coefficient of the news shock becomes more negative, -0.099. The coefficient of the interaction is positive, 0.091, and significant at a 1% level. Quantitatively, the interpretation of these results is as follows: a positive news shock in the magnitude of 100 basis points, holding the level of noise at its average level (which is zero, by construction), leads to a decline of 9.9 percent in the S&P 500. However, if the level of noise is higher from its average value by one standard deviation, then the reduction in the S&P 500 is expected to be much smaller: -9.9% + 9.1% = -0.8%. In words, the information channel is stronger in situations of high noise, hence the suppressing effect of interest rate hikes on stock prices is largely mitigated by the positive information that such a hike conveys to investors.

#### 3.3.1 Robustness and further investigation

The rest of the columns in table 1 are dedicated for a few robustness checks, which will be described now.

Table 1: Regression Results – S&P 500 Index

|               | (1)      | (2)       | (3)       | (4)       | (5)       | (6)      | (7)       |
|---------------|----------|-----------|-----------|-----------|-----------|----------|-----------|
| Const.        | 0.002**  | 0.002***  | 0.002***  | 0.002***  | 0.002**   | 0.003*** | 0.002*    |
|               | (0.001)  | (0.002)   | (0.001)   | (0.001)   | (0.001)   | (0.001)  | (0.001)   |
| NS            | -0.063** | -0.099*** | -0.100*** | -0.076*** | -0.094*** | 0.213*** | -0.187*** |
|               | (0.031)  | (0.027)   | (0.026)   | (0.027)   | (0.028)   | (0.036)  | (0.0266)  |
| Noise         |          | 0.001     | -0.001    |           | 0.001     | 0.002    | 0.002     |
|               |          | (0.001)   | (0.001)   |           | (0.001)   | (0.001)  | (0.001)   |
| Noise*NS      |          | 0.091***  | 0.063*    |           | 0.096**   | 0.053**  | 0.02      |
|               |          | (0.028)   | (0.032)   |           | (0.03)    | (0.026)  | (0.037)   |
| Dum_crisis    |          |           | 0.022***  |           |           |          |           |
|               |          |           | (0.006)   |           |           |          |           |
| Dum_crisis*NS |          |           | 0.46***   |           |           |          |           |
|               |          |           | (0.149)   |           |           |          |           |
| VIX           |          |           |           | 0.001     | 0.001     |          |           |
|               |          |           |           | (0.001)   | (0.001)   |          |           |
| VIX*NS        |          |           |           | 0.042*    | -0.009    |          |           |
|               |          |           |           | (0.025)   | (0.03)    |          |           |
| Observations  | 151      | 151       | 151       | 151       | 151       | 60       | 91        |
| R-squared     | 0.042    | 0.12      | 0.185     | 0.069     | 0.12      | 0.47     | 0.43      |
| Adj.R-squared | 0.035    | 0.1       | 0.157     | 0.05      | 0.09      | 0.44     | 0.41      |

Note: The table presents regression results for the S&P 500 index (percentage change). Standard errors (in parentheses) are calculated using the HAC (Newey-West) method. \* p<0.1; \*\*\* p<0.05; \*\*\*\* p<0.01.

Controlling for the financial crisis of 2008-9 As figure 5 shows, the level of the noise index is particularly high during the financial crisis of 2008-9. In parts of the crisis,

the index rose to even higher values, but these are not included in my sample since NS18 intentionally did not include them while computing their series of news shocks. Even after omitting this period, there are a few observations with high values during 2008-9. To test whether the results of regression 2 are driven entirely by the financial crisis, I define a dummy for four months during the crisis that include the highest four values of the index in the sample. Column 3 presents the results of a regression which adds this control. A few results should be noted. First, the dummy itself has a value of 0.022 which is significant at a 1% level, reflecting a higher return on FOMC decision days during the financial crisis.<sup>30</sup> Second, the interaction between news shocks and the crisis is positive and very large – 0.46, as well as significant at a 1% level. Thus, during the financial crisis the information effect was particularly large – large enough to overcome the standard effect of interest rate surprises. Third, the interaction of news shocks and noise is lower in this regression -0.063 as opposed to 0.091 in regression 2, and its significance is lower. However, it is still marginally significant at a 5% level (p-value of 0.0546), indicating that the results in regression 2 are significantly, yet not entirely, driven by the episode of the financial crisis.

Financial distress or uncertainty? In column 4 I re-estimate regression 2, but instead of "noise" being the explanatory variable, I use the VIX index, which is a common measure of uncertainty.<sup>31</sup> When the VIX is used, the coefficient of the interaction between the news shock and VIX is smaller than the coefficient obtained when "news" was used (0.042), and is now also significant only at a 10% level (p-value of 0.094). The explanatory power of the regression is also lower. In column 5 I include both the noise indicator as well as the VIX. The results show that the coefficient on Noise\*NS is still significant at a level of 5%, and almost the same in magnitude as in regression 2. In contrast, the coefficient on the variables related to VIX are not significant, suggesting that the effect found in regression 2 is more associated with "noise" than with the VIX.

To understand these results I first check whether these two variables are correlated, and find that they are – a regression of "noise" on a constant and on the VIX index delivers a highly significant coefficient on VIX and an R-squared value of 0.32. To further investigate why information sensitivity seems to be more affected by changes in "noise" rather than in the VIX, I investigate the relation between these two indicators to a measure constructed by He et al. (2017) which is intended to capture the degree of financial intermediaries' distress. In contrast with the noise measure of Hu et al. (2013),

<sup>&</sup>lt;sup>30</sup>Recent literature documents high stock returns that accrue during FOMC decision days. See for example Lucca and Moench (2015).

<sup>&</sup>lt;sup>31</sup>Source: Chicago Board Options Exchange, retrieved from FRED, Federal Reserve Bank of St. Louis. I standardize the VIX variable by subtracting its sample mean and dividing it by its standard deviation.

the measure developed by He et al. (2017) looks directly at the financial state of the intermediaries: based on He and Krishnamurthy (2012), He et al. (2017) show that the price of risk faced by financial intermediaries is related to their leverage ratio. Specifically, they show that the price of risk faced by financial intermediary is linear in the squared reciprocal of the capital ratio of the intermediary sector (hereafter, intermediary leverage). Low intermediary capital, reflected in high leverage, reduces the risk bearing capacity of the intermediary sector and thus reduces their willingness to purchase risky assets. Hence, I check whether my results might be driven by the relation between the measures I use and the risk bearing capacity of financial intermediaries'. As regression 2 uses daily data, the following analysis would ideally be conducted using daily data as well, but the data on intermediary capital exists (for the relevant sample) only at a monthly frequency, so I conduct the analysis at this frequency. Specifically, I use monthly data from 2000m01 to 2020m12 (252 observations) to estimate the following regression:

$$Intermediary\_leverage_t = \alpha + \underbrace{\beta}_{0.23} \cdot VIX_t + \underbrace{\gamma}_{0.5} \cdot Noise_t + \varepsilon_t \qquad (16)$$

$$\underbrace{0.23}_{0.066} \qquad \qquad (0.067)$$

where all variables were standardized by subtracting their sample mean and dividing them by their standard deviation. As the results show, both coefficients are highly significant, but the coefficient on "Noise" is about twice the size of the coefficient on VIX. Hence, "noise" seems to be more correlated with intermediary leverage than the VIX index, which might suggest that it is a better proxy for the strength of the financial sector, and hence for the vulnerability of the economy to a crisis – which is the relevant state variable according to my model.

Information effect or illiquidity\financial frictions? The explanation that I provided for the state-dependency of stock prices was that the IE is state-dependent: weak when the financial sector is robust, and strong when the financial sector is distressed. Here I briefly discuss an alternative explanation: perhaps the state dependency has nothing to do with an information effect, but stems from the fact that the financial sector's distress generates market frictions that hamper the normal functioning of the stock market (e.g. due to a reduction in liquidity). In other words, my findings could potentially be the result of a state-dependency in the standard channel of monetary policy rather than in the IE.

I believe that this alternative explanation does not fit my findings well: to the degree that financial frictions of various types inhibit the financial sector from adjusting its position in response to monetary policy shocks, we would expect the stock prices response to monetary shocks to be weaker – not to flip sign, as my results indicate when financial distress is very high. To see why, suppose that there is no information effect, and that the CB has reduced its interest rate. Following the reduction, investors' demand for stocks would go up and hence result in an increase in stock prices. If investors are constrained due to collateral or liquidity constraints, they will be reluctant (or unable) to obtain resources for acquiring stocks, and hence their demand for stocks would increase by less than it would in normal times. In an extreme situation when collateral\liquidity constraints are fully binding, their demand for stocks might remain unchanged. Even in this extreme case, their demand for stocks is not expected to decrease, as is implied by the reduction in stock prices that I document, following interest rate reductions that occur when distress is sufficiently high.<sup>32</sup>

Dividing the sample into "right/wrong sign" sub-samples In 40% of the observations, the stocks respond to monetary shocks in an opposite way to expected. Jarocinski and Karadi (2020) show that "right-sign" and "wrong-sign" observations are systematically different: Interest rate hikes (reductions) that are accompanied by a stock price decrease (increase) are followed by developments that are consistent with a standard monetary shock. Interest rate hikes (reductions) that are accompanied by a stock price decrease (increase) are followed by developments that are consistent with an information shock. Thus, if the results in regression 2 actually capture an attribute of the information channel, it should be more pronounced for the "wrong-sign" observations. Hence, I re-estimate regression 2 on two sub-samples: "wrong-sign" observations (column 6 of Table 1) and "right-sign" observations (column 7). The results indicate that the coefficient of the interaction Noise\*NS are indeed insignificant for the "right-sign" sample and significant for the "wrong-sign" sample.

The effect of monetary policy on VIX The theoretical model's prediction that the effect of an interest rate reduction on investment would be weaker at times of high

<sup>&</sup>lt;sup>32</sup>Some notable models of financial frictions actually imply an *amplification* of the effects of monetary policy rather than an attenuation. For example, in Gertler and Karadi (2011) who model a banking sector subject to a financial friction, an interest rate reduction increases asset prices, strengthens the balance sheets of the financial sector, and enables them to increase their investment even further, bidding asset prices up by more than they would in a frictionless case. An additional amplifying effect could occur due to the behavior of the non financial sector: the improvement in the balance sheets of stock issuing firms is likely to result in a reduction in the risk premium that investors demand, which would contribute to a further increase in their stock prices (Bernanke et al. (1999)). Thus, in times of financial distress one would expect to see an amplified effect of the standard channel, rather than a dampened one. The results presented in table 2, in which interest rate reductions during periods of financial distress increase stock prices by less are inconsistent with such an explanation.

distress, was based on the idea that at these occasions, reducing the interest rate would substantially increase investors' fear. To check whether the data supports this causal chain, I run the same regression as regression 2, but with the VIX indicator (percentage change) as the dependent variable. Table 2 shows that while negative monetary surprises reduce the VIX indicator (consistent with the findings in Bekeart et al. (2013)), the reduction is significantly weaker on periods of high distress, as can be seen by the negative coefficient on the interaction between noise and NS. This is in accordance with the idea that the presence of the IE might interfere with the calming effect that lax monetary policy has on the stock market, especially at times of financial distress.

Table 2: Regression Results – VIX

| Const.        | -0.022*** |  |
|---------------|-----------|--|
|               | (0.004)   |  |
|               |           |  |
| NS            | 0.598***  |  |
|               | (0.161)   |  |
|               |           |  |
| Noise         | -0.007    |  |
|               | (0.005)   |  |
| Noise*NS      | -0.368**  |  |
| Noise No      |           |  |
|               | (0.166)   |  |
| Observations  | 151       |  |
| R-squared     | 0.09      |  |
| Adj.R-squared | 0.07      |  |

**Note:** The table presents regression results for the VIX index (percentage change). Standard errors (in parentheses) are calculated using the HAC (Newey-West) method. \* p<0.1; \*\* p<0.05; \*\*\* p<0.01.

## 4 Conclusion

This paper presented a model that describes the effects of monetary policy on investment, with an emphasis on the informational effect of monetary policy on investors' confidence. I showed that if the central bank holds private information regarding the state of the economy, reducing its interest rate is less effective in stimulating investment, and hence less effective in stabilizing aggregate demand. I further showed that the moderating effect on investment is stronger when the economy is weaker. When the economy is sufficiently weak, the negative information effect is strong enough to overturn the standard expansionary effect of interest rate reductions, resulting in a reduction of investment. As

these declines in investment are the result of a coordination failure among investors, they are inefficient. In line with the model's main prediction, I provided evidence, based on the reaction of the US stock market to interest rate surprises, suggesting that the information effect of monetary policy on stock prices is higher when the economy – specifically, the financial sector – is in a weaker state.

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# 6 Appendix A: Proof of proposition 1 and equilibrium analysis

**Proposition 1** For every level of R the level of investment is given by:

$$I = 1 - \Phi(\sqrt{\beta}(\hat{x} - \bar{D}))$$

where  $\hat{x}$  is defined as the solution to the following equation:

$$(1 - \Phi(\sqrt{\alpha + \beta}(D^{c} - (1 - \Phi(\sqrt{\beta}(\hat{x} - \bar{D}))) - \frac{\alpha\bar{\theta} + \beta(\hat{x} - \frac{R-1}{\gamma}^{\frac{1}{\gamma-1}})}{\alpha + \beta}))) \cdot (1 + \gamma(1 - \Phi(\sqrt{\beta}(\hat{x} - \bar{D})))^{\gamma-1}) = R$$

Depending on parameter values, this system could either have a unique equilibrium, or up to three equilibria, out of which one is unstable.

As I focus on monotone strategies, the level of investment simply states the fraction of investors that get a signal  $x_i > \hat{x}$ , under the assumption that  $x_i \sim N(\bar{D}, \frac{1}{\beta})$ , this is reflected in the first equation of the proposition. The second equation is derived as by inserting eq. 7 into eq. 10 and by inserting the functional form of the supply function.

To analyze existence and uniqueness, define  $G(\hat{x}) = P_{NC}(\hat{x})R^K(\hat{x}) - R$ . As  $\hat{x} \to \infty$ ,  $P_{NC} \to 1$  and  $R^K \to \infty$ . Hence,  $G(\hat{x}) \to \infty$ . As  $\hat{x} \to -\infty$ ,  $P_{NC} \to 0$  and  $R^K \to 1 + \gamma$ . Hence,  $G(\hat{x}) \to -R$ . Thus, an equilibrium always exists. To analyze the number of solutions, differentiate  $G(\hat{x})$  with respect to  $\hat{x}$ :

$$\frac{\partial P^{NC}}{\partial \hat{x}} R^K + \frac{\partial R^K}{\partial \hat{x}} P_{NC}$$

with  $\phi(\cdot)$  denoting the probability density function of the standard normal distribution, evaluated at the relevant point:

$$\phi(\cdot) = \phi(\sqrt{\alpha + \beta}(D^c - (1 - \Phi(\sqrt{\beta}(\hat{x} - \bar{D})))) - \frac{\alpha\bar{\theta} + \beta(\hat{x} - \frac{R - 1}{\gamma}^{\frac{1}{\gamma - 1}})}{\alpha + \beta}))$$

 $\phi(\circ)$  denoting the probability density function of the standard normal distribution, evaluated at the relevant point:

$$\phi(\circ) = \phi(\sqrt{\beta}(\hat{x} - \bar{D})) > 0$$

$$\frac{\partial P^{NC}}{\partial \hat{x}} = -\phi(\cdot)\sqrt{\alpha + \beta}(\phi(\circ)\sqrt{\beta} - \frac{\beta}{\alpha + \beta}) \leq 0$$
and
$$\frac{\partial R^K}{\partial \hat{x}} = -\phi(\circ)\sqrt{\beta}\gamma(\gamma - 1)(1 - \Phi(\sqrt{\beta}(\hat{x} - \bar{D}))^{\gamma - 2} > 0$$

Given the sign of these expressions, a sufficient condition for  $\frac{\partial G(\hat{x})}{\partial \hat{x}}$  to be positive is for  $\frac{\partial P^{NC}}{\partial \hat{x}}$  to be non negative. This will be the case if  $\frac{\sqrt{\beta}}{\alpha+\beta} \geq \phi(\circ)$ . The maximum value attainable by the standard normal PDF is  $\frac{1}{\sqrt{2\pi}}$ , and so  $\frac{\sqrt{\beta}}{\alpha+\beta} \geq \frac{1}{\sqrt{2\pi}}$  ensures that  $\frac{\partial P^{NC}}{\partial \hat{x}}$  is non negative. The last condition can be developed as follows:  $\beta^2 + 2(\alpha - \pi)\beta + \alpha^2 \leq 0$ . The roots of this cubic equation are given by  $\pi - \alpha \pm \sqrt{\pi(\pi - 2\alpha)}$ . Thus for values of  $\beta$  such that  $\pi - \alpha + \sqrt{\pi(\pi - 2\alpha)} \geq \beta \geq \pi - \alpha - \sqrt{\pi(\pi - 2\alpha)}$ , the value of  $\hat{x}$  is unique. This case is represented figure 1a, which shows the value of G (as a function of the level of investment, I, which is negatively related to  $\hat{x}$ ). Given the parameters used in the numerical exercise, the roots of the cubic function are 0.002 and 6.081.

Case 1: Monotone  $G(\hat{x})$ , Unique Equilibrium

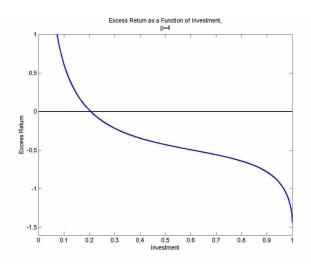


Figure 1a: Excess return (Y-axis) as a function of investment I (X-axis).  $\theta = 0.342$ .  $D^c = 0.7, \bar{D} = 1, \beta = 4, \alpha = 0.1, \gamma = 0.333, \bar{\theta} = 3$ .

For values of  $\beta$  outside this range, the sign of  $\frac{\partial G(\hat{x})}{\partial \hat{x}}$  could become negative, and hence there might be more then one  $\hat{x}$  for which  $G(\hat{x}) = 0$ .

I now show that there are, at most, three equilibria, of which two that are stable. As shown above, as  $\hat{x} \to -\infty$ ,  $G(\hat{x}) \to -R$ . In addition, as  $\hat{x} \to -\infty$ ,  $\phi(\circ) \to 0$  and  $\phi(\cdot) \to 0$ . Hence,  $\frac{\partial P^{NC}}{\partial \hat{x}}$  which converges to zero from above (i.e., it is positive), and  $\frac{\partial G(\hat{x})}{\partial \hat{x}} \to \frac{\partial R^K}{\partial \hat{x}} P_{NC}$  which converges to zero from above as well. Note that  $\frac{\partial G(\hat{x})}{\partial \hat{x}}$  can become negative only if  $\frac{\partial P^{NC}}{\partial \hat{x}}$  is negative, which happens iff  $\frac{\sqrt{\beta}}{\alpha+\beta} < \phi(\circ)$ . By the properties of the normal PDF we know that this holds only for values of  $\hat{x}$  that are sufficiently close to the center of the

distribution  $(\bar{D})$ . Hence, the sign of  $\frac{\partial P^{NC}}{\partial \hat{x}}$  changes twice (when  $\hat{x}$  increases): once from positive to negative, and then from negative to positive. Hence, if  $\frac{\partial G(\hat{x})}{\partial \hat{x}}$  changes sign, it will happen twice: from positive to negative and then to positive again. In this case,  $G(\hat{x})$  may attain a value of zero up to three times: once in the first increasing part of  $G(\hat{x})$ , one in the decreasing part, and another one in the second increasing part of  $G(\hat{x})$ . Figure 2a and 3a present two different cases of a non-monotone  $G(\hat{x})$ , in figure 2a the function crosses the zero line only once (unique equilibrium), and in figure 3a it crosses it three times (three equilibria).

Case 2: Non-Monotone  $G(\hat{x})$ , Unique Equilibrium

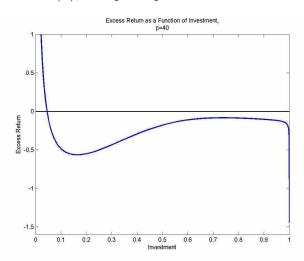


Figure 2a: Excess return (Y-axis) as a function of investment I (X-axis).  $\theta = 0.342$ .  $D^c = 0.7, \bar{D} = 1, \beta = 40, \alpha = 0.1, \gamma = 0.333, \bar{\theta} = 3$ .

Case 3: Non-Monotone  $G(\hat{x})$ , Multiple Equilibria

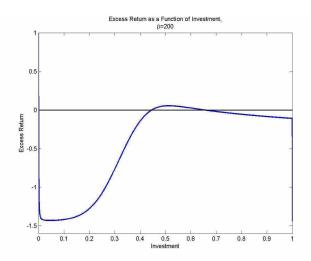


Figure 3a: Excess return (Y-axis) as a function of investment I (X-axis).  $\theta=0.342$ .  $D^c=0.7, \bar{D}=1, \beta=200, \alpha=0.1, \gamma=0.333, \bar{\theta}=3$ .

Finally, note that the middle solution represented in figure 3a is unstable in the sense that even the slightest change in the level of investment (equivalently, in  $\hat{x}$ ) will move the system to a different equilibrium: suppose that the system in the intermediate equilibrium and that for some reason there's an arbitrarily small increase in investment. Excess return increases and so more investors will find it profitable to invest in the risky asset, which will move us to the equilibrium with the highest amount of investment (which is roughly the level aimed for by the CB in this example –  $\bar{D}$  –  $\theta \approx 0.66$ ). Similarly, suppose that the system in the intermediate equilibrium and that there's an arbitrarily small decrease in investment. Excess return decreases and so less investors will find it profitable to invest in the risky asset, which will move us to the equilibrium with the lowest amount of investment (essentially zero in this example).

# 7 Appendix B: An analytical expression for the effectiveness of monetary policy

We are interested in finding how much investment changes as a result of a marginal change in the interest rate set by the CB. From eq. 11 we have:

$$\frac{\partial I}{\partial R} = -\phi(\circ)\sqrt{\beta}\frac{\partial \hat{x}}{\partial R} \tag{17}$$

with  $\phi(\circ)$  denoting the probability density function of the standard normal distribution, evaluated at the relevant point:

$$\phi(\circ) = \phi(\sqrt{\beta}(\hat{x} - \bar{D})) > 0$$

In order to find how a change in the interest rate affects investment, we thus need to look at the effect it has on  $\hat{x}$ . I use eq. 12 to define:

$$F(R, \hat{x}) \equiv P^{NC}(\hat{x}, R)R^{K}(\hat{x}) - R = 0$$

Using the implicit function theorem we have:

$$\frac{\partial \hat{x}}{\partial R} = -\frac{\frac{\partial F(R,\hat{x})}{\partial R}}{\frac{\partial F(R,\hat{x})}{\partial \hat{x}}} = \frac{1 - \frac{\partial P^{NC}}{\partial R} R^K}{\frac{\partial P^{NC}}{\partial \hat{x}} R^K + \frac{\partial R^K}{\partial \hat{x}} P^{NC}}$$
(18)

Hence, combining eq. 18 and eq. 17, we have:

$$\frac{\partial I}{\partial R} = \phi(\circ)\sqrt{\beta} \frac{\frac{\partial P^{NC}}{\partial R}R^K - 1}{\frac{\partial P^{NC}}{\partial \hat{x}}R^K + \frac{\partial R^K}{\partial \hat{x}}P^{NC}}$$
(19)

where:

$$P^{NC} = 1 - \Phi(\sqrt{\alpha + \beta}(D^c - (1 - \Phi(\sqrt{\beta}(\hat{x} - \bar{D}))) - \frac{\alpha\bar{\theta} + \beta(\hat{x} - \frac{R-1}{\gamma} \frac{1}{\gamma-1})}{\alpha + \beta})) > 0$$

$$R^K = 1 + \gamma(1 - \Phi(\sqrt{\beta}(\hat{x} - \bar{D})))^{\gamma - 1} > 0$$

$$\frac{\partial P^{NC}}{\partial R} = -\phi(\cdot) \frac{\beta}{\sqrt{\alpha + \beta}} \frac{1}{\gamma - 1} \frac{1}{\gamma} (\frac{R - 1}{\gamma})^{\frac{2 - \gamma}{\gamma - 1}} > 0$$

$$\frac{\partial R^K}{\partial \hat{x}} = -\phi(\circ) \sqrt{\beta} \gamma (\gamma - 1) (1 - \Phi(\sqrt{\beta}(\hat{x} - \bar{D}))^{\gamma - 2}) > 0$$

$$\frac{\partial P^{NC}}{\partial \hat{x}} = -\phi(\cdot) \sqrt{\alpha + \beta} (\phi(\circ) \sqrt{\beta} - \frac{\beta}{\alpha + \beta}) \leq 0$$

For values of  $\beta$  such that  $\pi - \alpha + \sqrt{\pi(\pi - 2\alpha)} > \beta > \pi - \alpha - \sqrt{\pi(\pi - 2\alpha)}$ , the last expression is positive since  $\sqrt{2\pi} > \frac{\alpha + \beta}{\sqrt{\beta}}$ , and hence  $\phi(\circ)\sqrt{\beta} < \frac{\beta}{\alpha + \beta}$ , with  $\phi(\cdot)$  denoting the probability density function of the standard normal distribution, evaluated at the relevant point:

$$\phi(\cdot) = \phi(\sqrt{\alpha + \beta}(D^c - (1 - \Phi(\sqrt{\beta}(\hat{x} - \bar{D}))) - \frac{\alpha\bar{\theta} + \beta(\hat{x} - \frac{R-1}{\gamma}^{\frac{1}{\gamma-1}})}{\alpha + \beta})) > 0$$