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Foreign Exchange Interventions in the New-Keynesian Model: Transmission, Policy, and Welfare

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התערבות בשוק מט"ח במודל הניאו-קיינסיאני:
תמסורת, מדיניות ורווחה

יוסי יכין

תמצית

המאמר משתמש במודל ניאו-קיינסיאני סטנדרטי של משק קטן ופתוח למידול התערבות בשוק המט"ח. המאמר בוחן את מנגנון התמסורת של רכישות מט"ח אל המשק, פותר עבור המדיניות האופטימלית, מציע כלל מדיניות בר ביצוע, ומעריך את התרומה הפוטנציאלית לרווחה במשק כתוצאה משימוש נאות בכלי.

המודל התיאורטי שמוצג בנייר מניח שהתערבות בשוק המט"ח משפיעה על המשק דרך ערוץ "מאזן תיק הנכסים". תחת ערוץ זה, רכישות מט"ח דוחקות את החזקתם של נכסים זרים ע"י המגזר הפרטי; כתוצאה מכך פרמיית "שוויון הריביות" (Uncovered Interest rate Parity, UIP) עולה וכך גם התשואה הריאלית האפקטיבית שעומדת בפני השחקנים המקומיים. עליית התשואות מצמצמת את הביקוש המקומי. במקביל, רכישת מט"ח מביאה לפיחות ערך המטבע המקומי, ועל כן לעליית מחירם היחסי של מוצרי יבוא לעומת מוצרים מקומיים. כתוצאה מכך גדל הביקוש ליצוא, והביקוש המקומי ליבוא קטן. ההשפעה על סך הייצור במשק תלויה בגמישות היצע העבודה ביחס להכנסה. מדיניות אופטימלית מבודדת את המשק מהשפעתם של זעזועים פיננסיים כמו תנודות בתנועות ההון וזעזועים לפרמיית הסיכון של המשק. כלל מדיניות ששואף לייצב את פרמיית ה-UIP מביא לשיווי משקל שקרוב לאופטימלי ללא תלות באופי הזעזועים שפוקדים את המשק. המאמר דן בתנאים בהם ייצוב מוחלט של פרמיית ה-UIP הוא אופטימלי. התאמת הפרמטרים של המודל למשק הישראלי מעלה שסך ערכן הנוכחי של התרומות לרווחה מנקיטת מדיניות התערבות אופטימלית בשוק המט"ח, לעומת מדיניות השומרת על רמה קבועה של יתרות, מוערך בכ-2.4% של תצרוכת שנתית.

התוצאות עמידות למגוון שיטות מידול של המגזר הפיננסי, כמוצע בספרות העדכנית.

הדעות המובעות במאמר זה אינן משקפות בהכרח את עמדתו של בנק ישראל.

העבודה מציגה ניתוח תיאורטי, והיא איננה קשורה למדיניות שבה הבנק נקט בפועל.

Foreign Exchange Interventions in the New-Keynesian Model: Transmission, Policy, and Welfare*

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Abstract

The paper introduces foreign exchange interventions (FXIs) to an otherwise standard new-Keynesian small open economy model. The paper studies the transmission mechanism of FXIs, solves for the optimal policy, suggests an implementable policy rule, and evaluates the welfare implications of different policies.

Relying on the portfolio balance channel, a purchase of foreign reserves crowds out private holdings of foreign assets, thereby raising the UIP premium and the effective real return domestic agents face. As a result, a purchase of foreign reserves contracts domestic demand. At the same time, it depreciates the value of the domestic currency, which raises the price of foreign goods relative to domestic goods, thereby expanding foreign demand for home exports and contracting domestic imports. The effect on production depends on the wealth effect on labor supply. Optimal FXIs completely insulate the economy from the effect of financial shocks, such as capital flows and risk premium shocks. A policy rule that aims at stabilizing the UIP premium brings the economy close to its optimal allocation, regardless of the source of the shocks. The paper discusses the conditions under which strict targeting of the UIP premium is optimal. Calibrating the model to the Israeli economy, lifetime welfare gains from following optimal FXI policy, relative to maintaining a fixed level of foreign reserves, amount to 2.4% of annual steady state consumption.

The results are robust to a variety of microstructures of the financial sector suggested in recent literature.

JEL classification: E44, E52, E58, F30, F31, F40, F41, G10, G15.

Keywords: Foreign Exchange Interventions, UIP Premium, Monetary Policy, Open Economy Macroeconomics.

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1 Introduction

The IMF classifies merely one quarter of its inflation targeting members as free floaters; the rest practice some form of foreign exchange intervention (FXI).[1] This paper is for the latter group of countries.

The paper utilizes a standard new-Keynesian small open economy model to analyze sterilized FXIs as an additional policy tool, alongside the monetary interest rate.[2] It examines the transmission mechanism of FXIs, studies their role as a macroeconomic stabilizer, solves for the optimal FXI policy, and proposes an implementable policy rule. An attempt to quantify the potential welfare gains from using FXIs is carried out by calibrating the model to the Israeli economy. The model suggests that FXI policy should seek to stabilize the uncovered interest rate parity (UIP) premium. This policy insulates the economy from the effect of capital flows and risk premium shocks. It is also optimal against real shocks when the economy faces perfectly elastic demand for its exports, as long as monetary policy is able to counteract the effects of nominal rigidities; otherwise, tradeoffs emerge. The potential welfare gains of following optimal FXI policy are not large; nevertheless, they are economically meaningful. All results are robust to a variety of modeling strategies regarding the microstructure of the financial markets.

Before describing the results in more detail, it is important to clarify why sterilized FXIs may affect the exchange rate and other equilibrium outcomes. More generally, this question is related to the conditions under which the size and composition of the central bank balance sheet may matter for equilibrium allocations. Wallace (1981) shows that under complete financial markets open market operations are irrelevant for equilibrium outcomes. Backus and Kehoe (1989) argue for the inefficacy of sterilized FXIs even under incomplete financial markets, provided the central bank faces the same market

[1] IMF (2023). See definitions therein for the classification of exchange rate arrangements. FXIs may be either direct or indirect, can be carried out in the spot market, using financial derivatives, or by means of verbal interventions.

[2] FXIs are sterilized in the sense that the central bank uses the interest rate as an independent policy tool. The model economy is cashless, and hence the central bank cannot change its foreign reserves by altering the supply of domestic money. In the model, any purchase (sale) of foreign reserves is matched by the issuance (redemption) of domestic bonds.

incompleteness as other agents. More recently, Cúrdia and Woodford (2011) demonstrate that the central bank balance sheet has no role in equilibrium determination unless private financial markets are sufficiently impaired. Generally, the conclusion from this strand of literature is that balance sheet policies do not affect equilibrium allocations when the assets traded by the central bank are valued only for their pecuniary return, i.e. when they do not provide special services, such as liquidity, and when all agents can trade these assets freely at the same market price. In the context of sterilized FXIs, this means that one must deviate from the UIP in order have any hope of affecting equilibrium conditions; otherwise agents are indifferent between holding home and foreign assets, and sterilized FXIs are deemed ineffective.

Recent contributions have revived the argument for sterilized FXIs, e.g. Benes et al. (2015), Cavallino (2019), Alla et al. (2020), Fanelli and Straub (2021), Faltermeier et al. (2022) and Itskhoki and Mukhin (2023). To make interventions effective, this literature builds on the portfolio balance channel to generate deviations from the UIP. That is, in these models agents are willing to change the composition of their financial portfolio for a premium, giving rise to deviations from the UIP. While the details of the financial friction differ from one contribution to another, they arrive at similar UIP specifications. In that vein, Yakhin (2022) shows that, to a first order approximation, a simple reduced-form portfolio adjustment cost, as in Schmitt-Grohé and Uribe (2003), is isomorphic to more elaborate modeling strategies that attempt to capture the microstructure of the financial markets. Introducing the financial friction using a simple portfolio adjustment cost is therefore robust to a variety of interpretations regarding the underlying microfoundations of the financial markets.[3] Hence, in this paper I adopt the portfolio adjustment cost of

[3] Yakhin (2022) demonstrates that the simple portfolio adjustment cost is isomorphic, up to a first order approximation, to the financial frictions in Gabaix and Maggiori (2015) and in Fanelli and Straub (2021). In Gabaix and Maggiori (2015) the UIP premium arises due to limited commitment of financial intermediaries to honor their liabilities. In Fanelli and Straub (2021) regulatory exposure limits coupled with participation cost in the international financial markets drive a wedge in the UIP. Uribe and Yue (2006) provide microfoundations for the portfolio adjustment cost as operational costs of the financial sector. In Itskhoki and Mukhin (2021, 2023) risk aversion of financial intermediaries gives rise to a UIP premium. Under standard log-linearization their model is also isomorphic to the simple portfolio adjustment cost (see Appendix A).

Schmitt-Grohé and Uribe (2003) to generate deviations from the UIP condition.[4]

Transmission. In the model, an exogenous rise in foreign reserves is financed partially by a reduction in the private sector holdings of foreign assets. This raises the UIP premium, which, in turn, increases the effective return home agents face on foreign assets. At the same time, the rise in foreign reserves increases demand for foreign currency and depreciates the value of the home currency. The higher return on foreign assets triggers an intertemporal substitution in the consumption of foreign goods from the present to the future; that is, imports fall on impact and rise in subsequent periods. The depreciation of the domestic currency reduces the terms of trade, i.e. home goods become cheaper relative to foreign goods, which also supports the decline in imports demand. Cheaper home goods stimulates demand for exports. Overall net exports rises, which is the other source of financing for the rise in foreign reserves.

Since a purchase of foreign reserves raises effective returns, it has a contractionary effect on demand. On the production side, the effect is ambiguous. A purchase of foreign reserves reduces the terms of trade, which contracts labor demand. Yet at the same time, the fall in consumption may raise labor supply, depending on the specification of preferences. Putting these together, real wages fall but the effect on labor effort is unclear. With additive separable preferences in consumption and labor, as is standard in the new-Keynesian literature, equilibrium labor rises. However, assuming a utility function as in Greenwood et al. (1988), GHH hereinafter, there is no wealth effect on labor supply and equilibrium labor effort falls slightly. Since the model assumes labor is the only factor of production, these results carry into the economy's total output. In sum, while the purchase of foreign reserves unambiguously stimulates exports it does not necessarily expand total production.

Policy. The UIP is an equilibrium condition for efficient risk sharing; hence, deviations from the UIP entail welfare costs that open the door for policy intervention.

[4] It is important to note that aside from allowing a theoretical discussion on sterilized FXIs, these frictions have empirical relevance as well. They help reconcile many of the long-standing exchange rate puzzles: the exchange rate disconnect, the sensitivity of exchange rates to financial flows, the profitability of carry trades and the forward premium puzzle, among others, Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2021).

This paper proposes that central banks should restore efficient risk sharing by adopting a policy rule that stabilizes the UIP premium. That is, FXIs should generally seek to undo the effect of the financial friction. This result resembles the policy recommendation that emerges from standard new-Keynesian models for monetary policy: to eliminate the effect of nominal rigidities and restore the flexible price equilibrium. Optimal policies in Cavallino (2019) and Itskhoki and Mukhin (2023) also support stabilizing the financial wedge in their models.

That said, full stabilization of the UIP premium is not necessarily optimal, just as strict inflation targeting is not. That depends on the structure of the economy and the type of shocks to which the economy is subject. This result is similar to the case of inflation targeting under cost-push shocks, Clarida et al. (1999). In the model, when the economy is hit by capital flow or exogenous "risk premium" shocks, FXIs are able to completely insulate the economy from their effect. That is, not only the UIP premium is fully stabilized in this case, but inflation and production are too. Itskhoki and Mukhin (2023) report a similar result. When the economy is exposed to other shocks, in particular, productivity shocks and exogenous fluctuations in the subjective discount factor, strict targeting of the UIP premium is welfare improving, but it is not necessarily optimal. Its optimality depends on the market imperfections the central bank faces. One imperfection is clearly the financial friction that generates the UIP premium, while another imperfection may result from exports demand. When global demand for the domestic good is downward sloping, the home economy possesses some market power in the global goods market. If domestic exporters are price takers, then they do not internalize the monopolistic power of the economy, and the central bank has an incentive to manipulate the terms of trade in its favor.[5] As a result, the central bank faces a tradeoff between stabilizing the UIP premium and exploiting the economy's market power in the global goods market. When the economy faces a perfectly elastic demand for exports, strict targeting turns optimal provided that monetary policy is able to perfectly counteract the effect of nominal rigidities. Otherwise, the central bank faces yet another tradeoff.

[5] The incentive of the central bank to manipulate the terms of trade in favor of the home economy is emphasized by Corsetti and Pesenti (2001).

The advantage of using the UIP premium as a policy target is that it does not require knowledge of the exact combination of shocks affecting the economy.[6] The paper demonstrates that a policy rule that seeks to stabilize the UIP premium is welfare improving even when traditional monetary policy operates optimally and has exhausted all its potential welfare gains.

UIP deviations provide carry trade opportunities, and therefore are costly for the home economy when exploited by foreigners. e.g. Cavallino (2019), Amador et al. (2020), Fanelli and Straub (2021). Stabilizing the UIP premium reduces carry trade opportunities, and hence reduces, on average, the loss of resources for the home economy. Nevertheless, in this paper I assume that the financial sector is owned entirely by domestic agents, thereby abstracting from welfare gains resulting from this channel. This assumption allows focusing solely on the role of FXIs as a macroeconomic stabilizer, working in tandem with traditional monetary policy.

Welfare. Welfare gains from conducting optimal FXI policy are not large, but are economically meaningful. I compare welfare in an economy with fixed foreign reserves to one where the central bank conducts optimal FXI policy. In both cases, monetary policy sets the interest rate optimally. Hence, this comparison helps in evaluating the role of FXIs over and above that of traditional monetary policy, as it exhausts any potential welfare gains from monetary policy before resorting to FXIs. *Lifetime* welfare gains amount to 2.4% of annual steady state consumption. That is, a representative household living in the fixed-reserves economy would be willing to pay a one-time amount of up to 2.4% of its annual steady state consumption to move to the optimal FXI economy. Comparing to an economy where the central bank follows a policy rule that stabilizes the UIP premium, this value is reduced to 0.8%. Augmenting the policy rule with an argument for smoothing the path of foreign reserves, the welfare gains from optimal FXIs fall to merely 0.1% of annual steady state consumption.[7] These results imply that the

[6] The UIP premium is not directly observed in the data, and therefore using it as a policy target requires estimation. See Kalemli-Özcan and Varela (2023) for recent measurement of UIP deviations and documentation of their properties in both emerging and advanced economies.

[7] In these calculations, the parameters of the model economy are set to match the characteristics of the Israeli economy.

suggested policy rule brings the equilibrium allocation close to the optimal one.

The paper assumes that the entire financial sector is owned by home agents. In the model, welfare clearly declines as the proportion of foreign ownership rises, and the welfare cost amounts to 1.6% of annual steady state consumption when foreigners own the entire financial sector.[8] While this is not a negligible figure, it is smaller than the potential stabilization benefits of following optimal FXI policy when the financial sector is owned by home agents. This result supports the role of FXIs as a macroeconomic stabilizer, rather than a device for stripping carry trade profits from foreigners.

Related literature. This paper joins the growing literature that builds theoretical foundations for sterilized FXIs in general equilibrium macro models. Since the financial markets are central to the transmission and efficacy of sterilized FXIs, some contributions focus solely on policy response to financial shocks, Cavallino (2019), Alla et al. (2020), Chen et al. (2023). The results of this paper justify a special focus on financial shocks, as FXIs are able to insulate the economy from their effect. Nevertheless, the paper finds that FXIs are useful for stabilizing the economy from the effect of other shocks as well.

As mentioned, several authors have highlighted the cost to the economy when foreign financial intermediaries exploit carry trade opportunities, Cavallino (2019), Amador et al. (2020), Fanelli and Straub (2021). The paper shows that while welfare indeed falls as foreigners own a larger portion of the domestic financial sector, FXIs serve as a macroeconomic stabilizer rather than just a means of countering speculative currency trades by foreigners. Moreover, the optimal policy response reduces carry trade opportunities and enhances welfare even when domestic agents own the entire financial sector.

Numerous papers specify FXI policy rules. Faltermeier et al. (2022) let FXIs react directly to inflation and the output gap, highlighting the macroeconomic stabilization role they attribute to FXIs, similar to traditional monetary policy. In a similar vein, Benes et al. (2015) use foreign reserves to target a level of the exchange rate that varies with inflation and the output gap. In addition, their policy rule also smooths exchange rate

[8] The calculation in this exercise assumes that the central bank follows optimal monetary and FXI policies, and compares welfare when the entire financial sector is owned by home agents to the case where it is owned solely by foreigners.

fluctuations, regardless of the state of the economy. In Chen et al. (2023), FXIs counteract movements in the exchange rate, without targeting a specific level. The authors argue that such a policy improves monetary policy tradeoffs especially in emerging economies, because of their limited ability to hedge exchange rate risks and because they experience greater and more persistent exchange rate pass-through to inflation. Lastly, in line with the findings of this paper, Adrian et al. (2021) and Itskhoki and Mukhin (2023) propose that FXIs should focus on stabilizing the UIP premium as it reflects financial inefficiencies. Adrian et al. (2021) emphasize that this policy recommendation improves monetary policy tradeoffs. Similarly, optimal FXIs in this paper aligns with traditional monetary policy objectives. In most instances, the optimal interest rate response is more moderate when FXIs are available, indicating that they improve monetary policy tradeoffs.

The paper attempts to quantify the welfare gains from adopting optimal FXIs. This type of analysis is often missing from this literature. An exception is Faltermeier et al. (2022) who analyze welfare gains from adopting an optimized FXI policy rule but only in the context of commodity price shocks.

Finally, the modeling strategy in this paper is isomorphic to a variety of microfoundations of the financial markets suggested in recent literature, e.g. Gabaix and Maggiori (2015), Fanelli and Straub (2021), Itskhoki and Mukhin (2021, 2023). Therefore, the results are robust to different microstructure interpretations of the financial friction introduced in this paper.

The rest of the paper is organized as follows. The next section presents the model. It introduces foreign reserves through the balance sheet of the central bank, and FXIs as an additional policy tool alongside the monetary interest rate. Section 3 develops the welfare criterion of a utilitarian social planner. Section 4 sets parameter values based on the characteristics of the Israeli economy. Section 5 studies the transmission mechanism of FXIs. Section 6 describes the optimal FXI response to various shocks. Section 7 suggests a policy rule for FXIs. Section 8 explores the conditions under which strict targeting of the UIP premium is optimal. Section 9 conducts welfare analysis, and Section 10 concludes.

2 The Model

The model is a variant of Galí and Monacelli (2005). The world economy is composed of a continuum of symmetric small open economies lying on the unit interval $[0, 1]$. Without loss of generality, the home economy is identified as country 0. All foreign countries are identical, facing exactly the same realization of shocks. This assumption directs attention to the dynamics of the home economy against the rest of the world, without concern of the interaction among foreign economies. It also simplifies the aggregation of foreign quantities without affecting any of the results.

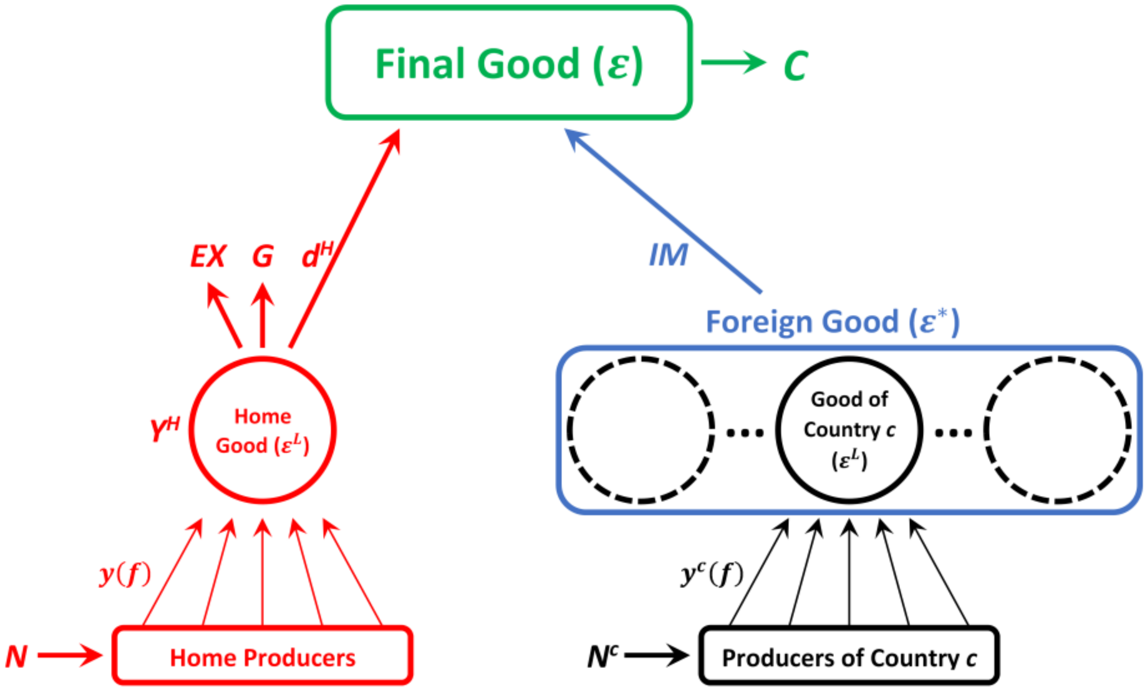
Each economy consists of three types of producers, households, employment agencies, insurance companies and a government.

The structure of production is summarized in Figure 1. Production is organized in three layers. In the first layer, intermediate good producers use labor to produce differentiated goods. Each producer has monopolistic power over supplying its product, while facing nominal price rigidity. In the second layer, assembly lines aggregate the differentiated goods into a homogenous domestic product. The domestic good is used for government consumption, for exports, and as input of production in the third layer. Finally, producers in the third layer use the domestic good together with imported goods to compose a final good. The final good is used for private consumption. The domestic good producers and the producers of the final good are price takers.

The households consume the final good; trade risk-free home and foreign nominal assets, supply labor, and trade wage insurance contracts. Each household is endowed with a differentiated labor skill and has monopolistic power over supplying it to the employment agencies, while facing nominal wage rigidity. Employment agencies aggregate labor skills into homogenous labor services and supply them to the intermediate goods producers. Insurance companies insure households against the risk of not being able to freely adjust their wage. Insurance companies and employment agencies are price takers.

The government consumes the domestic good, subsidizes labor so as to support efficient production in the steady state, intervenes in the foreign exchange markets, and sets the nominal interest rate while supplying any quantity of nominal risk-free bonds to sustain that rate.

Figure 1: The Composition of Goods and Their Uses (elasticities of substitution in parentheses)



Note: N and N^c are labor input in the home economy and in country c, respectively; $y(f)$ and $y^c(f)$ are production of intermediate f in the home economy and in country c, respectively; Y^H is total production of the home good; EX is exports; G is government consumption; d^H is domestic input for the production of the final good; IM is imports; and C is private consumption.

The business cycle is driven by productivity shocks, demand shocks (households' preferences, government expenditure, and world trade shocks), and financial shocks (capital flows and "risk premium" shocks). The law of one price holds, the foreign currency price of foreign goods is normalized to unity, and the world gross interest rate is constant at β^{-1}, where β is the households' discount factor.[9] I assume an internationally symmetric steady state in which trade is balanced and the private sector holds a zero net foreign asset position.

Date t aggregate exogenous events are denoted by s_t, and s^t denotes the history of

[9] For sake of exposition, the derivation below carries foreign prices and foreign interest rates as explicit variables.

events from date zero to date t, that is $s^t = (s_0, s_1, \ldots, s_t)$. Before period 0 the economy starts from steady state. Households are indexed by h, firms by f and countries by c. The exposition below focuses on the home economy.

2.1 Home, Foreign and Final Goods

This section describes the structure of production in the model economy. Focusing on the home country, country 0, let $y_t(f)$ denote domestic production of intermediate good f. Total production of the home good, Y_t^H, is a CES aggregate of $y_t(f)$, $f \in [0,1]$:

$$Y_t^H = \left(\int_0^1 y_t(f)^{\frac{\varepsilon^L - 1}{\varepsilon^L}} df \right)^{\frac{\varepsilon^L}{\varepsilon^L - 1}} \tag{1}$$

where ε^L is the elasticity of substitution between the local intermediate goods.

Y_t^H is used as input in the production of the final good, d_t^H, for government consumption, G_t, and for exports, EX_t:

$$Y_t^H = d_t^H + G_t + EX_t \tag{2}$$

Note that Y_t^H summarizes the total production in the economy. Given the price of each intermediate, $P_t^H(f)$, the demand for intermediate f, $y_t^d(f)$, and the price index of the home good , P_t^H, are given by:

$$y_t^d(f) = \left(\frac{P_t^H(f)}{P_t^H} \right)^{-\varepsilon^L} Y_t^H \qquad f \in [0,1] \tag{3}$$

$$P_t^H = \left[\int_0^1 P_t^H(f)^{1-\varepsilon^L} df \right]^{\frac{1}{1-\varepsilon^L}} \tag{4}$$

where prices are denominated in terms of the home currency.

The final good, is composed of home inputs, d_t^H, and foreign goods (i.e. imports), IM_t. Assuming a CES technology and that the final good is only used for private consumption, C_t, we have:

$$C_t = \left[(1-\lambda)^{\frac{1}{\varepsilon}} \left(d_t^H\right)^{\frac{\varepsilon-1}{\varepsilon}} + \lambda^{\frac{1}{\varepsilon}} \left(IM_t\right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{5}$$

where $\lambda \in [0,1]$ is a measure of openness of the economy; when $\lambda = 0$ the economy is closed. ε is the elasticity of substitution between the home and foreign goods.

Imports are a CES aggregate of imported goods from each foreign country, $IM_t(c)$ for $c \in (0,1]$[10]:

$$IM_t = \left(\int_{0^+}^{1} IM_t(c)^{\frac{\varepsilon^*-1}{\varepsilon^*}} dc \right)^{\frac{\varepsilon^*}{\varepsilon^*-1}}$$

where ε^* is the elasticity of substitution between goods produced in different foreign countries. Letting P_t^F denote the price of total imports and $P_t^F(c)$ the price of imports from country c, both in terms of home currency, demand for imports from each country c, and the price of the foreign composite good are given by:

$$IM_t(c) = \left(\frac{P_t^F(c)}{P_t^F} \right)^{-\varepsilon^*} IM_t \qquad c \in (0,1] \tag{6}$$

$$P_t^F = \left[\int_{0^+}^{1} P_t^F(c)^{1-\varepsilon^*} dc \right]^{\frac{1}{1-\varepsilon^*}} \tag{7}$$

Finally, cost minimization by the final good producers yields the following demand functions for inputs and the price index:

$$d_t^H = (1-\lambda) \left(\frac{P_t^H}{P_t} \right)^{-\varepsilon} C_t \tag{8}$$

$$IM_t = \lambda \left(\frac{P_t^F}{P_t} \right)^{-\varepsilon} C_t \tag{9}$$

$$P_t = \left[(1-\lambda) \left(P_t^H\right)^{1-\varepsilon} + \lambda \left(P_t^F\right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{10}$$

where P_t is the domestic consumer price index (CPI).

2.1.1 The Law of One Price, the Terms of Trade, the Real Exchange Rate, and Export Demand

Let S_t denote the nominal effective exchange rate of the home currency, that is, the price of a basket of the foreign currencies in terms of the home currency.[11] Let P_t^{F*} denote the price of imports, IM_t, in foreign effective terms. Assuming the law of one price holds, suggests:

$$P_t^F = S_t P_t^{F*} \tag{11}$$

[10] Note that each foreign good, $IM_t(c)$, is by itself a CES aggregate of country c's intermediate goods with an elasticity of substitution of ε^L. See Figure 1.

[11] Specifically, define $S_t \equiv \exp\left(\int_{0^+}^{1} \log\left(S_t^c\right) dc \right)$, where S_t^c denotes the exchange rate between the home country and country c. Under the assumption that foreign countries are identical $S_t^c = S_t$ for all $c \in (0,1]$.

P_t^{F*} is exogenous to the home economy and is normalized to unity.

Using the foreign analog of equation (6), the demand for the home good by an arbitrary foreign country c, $EX_t(c)$, is given by (after using the law of one price):

$$EX_t(c) = \left(\frac{P_t^H}{P_t^F}\right)^{-\varepsilon^*} IM_t^c \qquad c \in (0,1]$$

where IM_t^c is total imports of country c[12], and $EX_t(c)$ is their import from the home economy, or equivalently the exports of the home country to country c. Also note that for any two countries i and j, the composition of total imports in country i, IM_t^i, is only infinitesimally different from that of country j, IM_t^j, suggesting that they face the same price for their imports as the home economy, P_t^F. Aggregating over all foreign countries:

$$\int_{0+}^{1} EX_t(c)\,dc = \left(\frac{P_t^H}{P_t^F}\right)^{-\varepsilon^*} \int_{0+}^{1} IM_t^c dc$$

The left-hand side is total exports of the home country, EX_t, and the expression $\int_{0+}^{1} IM_t^c dc$ is total imports in the global economy, or in other words: world trade, WT_t. Define the terms of trade as:

$$TOT_t \equiv \frac{P_t^H}{P_t^F} \tag{12}$$

Suggesting exports demand is given by:

$$EX_t = TOT_t^{-\varepsilon^*} WT_t \tag{13}$$

For future reference, define the price of home and foreign goods relative to consumption price:

$$p_t^H \equiv \frac{P_t^H}{P_t} \tag{14}$$

$$p_t^F \equiv \frac{P_t^F}{P_t} \tag{15}$$

Notice that p_t^F is the CPI-based real exchange rate in the model. This follows by the law of one price, and since home goods take only an infinitesimal portion of foreign consumption, suggesting P_t^{F*} is the foreign price of foreign consumption.

[12] Not to be confused with $IM_t(c)$, which is home imports from country c.

2.2 Intermediate Goods Producers

Each intermediate good producer operates two departments, production and sales. The production department is a price taker. Given factor prices it operates efficiently to satisfy demand at the on-going prices. The sales department sets the price of their good.

2.2.1 The Production Department

Demand for domestic variety f is given by equation (3). Since the production department produces any quantity to satisfy demand we have:

$$y_t(f) = y_t^d(f) = \left(\frac{P_t^H(f)}{P_t^H}\right)^{-\varepsilon^L} Y_t^H$$

where $y_t(f)$ is total production of intermediate f. The production department operates a technology that uses labor as the only production factor. Production technology of firm f is given by:

$$y_t(f) = A_t n_t(f)^\alpha \qquad 0 < \alpha \leq 1 \tag{16}$$

where $n_t(f)$ is the firm's labor input, and A_t is an aggregate, country-specific, productivity shock. Total production cost is given by:

$$TC_t(y_t(f)) = (1 - \tau_w) W_t \left[\frac{y_t(f)}{A_t}\right]^{\frac{1}{\alpha}}$$

where W_t is the wage level employment agencies receive, and τ_w is the rate of labor subsidy. The marginal cost is given by:

$$MC_t(y_t(f)) = \frac{1 - \tau_w}{\alpha} W_t A_t^{-\frac{1}{\alpha}} y_t(f)^{\frac{1-\alpha}{\alpha}} \tag{17}$$

and the real marginal cost in terms of the composite home-good is:

$$RMC_t^H(f) \equiv \frac{MC_t(f)}{P_t^H} \tag{18}$$

2.2.2 The Sales Department and the Phillips Curve

The sales department sets the price of its good, $P_t^H(f)$. However, price setting is staggered across firms as in Calvo (1983), where the probability of price adjustment is $1 - \xi_p$; when a firm cannot freely adjust its price, its price is automatically updated by the steady state gross inflation rate, π_{ss}. Whenever a firm is able to reset its price it maximizes the present discounted value of its expected profits for the duration its new price is expected to remain in effect. Hence, a sales department that can readjust its price in period t solves:

$$\begin{array}{ll} \underset{P_t^H(f)}{Max} & E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \xi_p^s \left\{ \pi_{ss}^s \frac{P_t^H(f)\, y_{t+s}(f)}{P_{t+s}} - \frac{TC_{t+s}\left[y_{t+s}(f)\right]}{P_{t+s}} \right\} \\ s.t. & y_{t+s}(f) = \left(\frac{\pi_{ss}^s P_t^H(f)}{P_{t+s}^H} \right)^{-\varepsilon^L} Y_{t+s}^H \end{array}$$

where $\Lambda_{t,t+s}$ is the stochastic discount factor between time t and $t+s$, and I assume that firms discount future payoffs in accordance with the preferences of their shareholders – the households, that is $\Lambda_{t,t+s} = \beta \frac{U_{c,t+s}}{U_{c,t}}$. Notice that profits are defined in terms of the final good so as to match the units of the discount factor.

The standard solution applies. All firms that can readjust their price at date t set the same price.

Notation 1 *For a firm-specific generic variable X, let $X_{t/t-\tau}$ denote its date t value for firms that last revised their price in period $t-\tau$.*

Optimal price setting results in:

$$p_{t/t}^H = \frac{\varepsilon^L}{\varepsilon^L - 1} \frac{E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \xi_p^s \left(\frac{\pi_{t,t+s}}{\pi_{ss}^s} \right)^{\varepsilon^L} Y_{t+s}^H \left(p_{t+s}^H \right)^{1+\varepsilon^L} RMC_{t+s/t}^H}{E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \xi_p^s \left(\frac{\pi_{t,t+s}}{\pi_{ss}^s} \right)^{\varepsilon^L - 1} Y_{t+s}^H \left(p_{t+s}^H \right)^{\varepsilon^L}} \tag{19}$$

where:

$$\begin{aligned} p_{t/t-\tau}^H &\equiv \frac{P_{t/t-\tau}^H}{P_t} = \frac{\pi_{ss}^\tau}{\pi_t} p_{t-\tau/t-\tau}^H \qquad \tau = 0, 1, 2, 3 \ldots \\ \pi_{t,t+s} &\equiv \frac{P_{t+s}}{P_t} \\ RMC_{t+s/t}^H &= \frac{1}{\alpha} \frac{W_{t+s}}{P_{t+s}^H} A_{t+s}^{-\frac{1}{\alpha}} \left(\frac{\pi_{ss}^s}{\pi_{t,t+s}} \frac{p_{t/t}^H}{p_{t+s}^H} \right)^{-\varepsilon^L \frac{1-\alpha}{\alpha}} \left(Y_{t+s}^H \right)^{\frac{1-\alpha}{\alpha}} \end{aligned}$$

Note that under flexible prices equation (19) boils down to:

$$1 = \frac{\varepsilon^L}{\varepsilon^L - 1} RMC_t^H$$

After taking first order approximation, equation (19) takes the form of a standard new-Keynesian Phillips curve:

$$\widetilde{\pi}_t^H \cong \beta E_t\left(\widetilde{\pi}_{t+1}^H\right) + \frac{\left(1-\xi_p\right)\left(1-\beta\xi_p\right)}{\xi_p}\frac{\alpha}{\alpha+(1-\alpha)\,\varepsilon^L}\widetilde{RMC}_t^H \tag{20}$$

where tiled variables denote log-deviations from deterministic steady state, π_t^H is the gross domestic inflation rate, P_t^H/P_{t-1}^H, and RMC_t^H is the average real marginal cost of intermediate goods in the economy.

2.3 Employment Agencies and the Wage Index

Employment agencies are price takers. They use hours worked of differentiated labor skills supplied by domestic households, $n_t(h)$ $h \in [0,1]$, in a CES aggregator to construct labor input, N_t:

$$N_t = \left[\int_0^1 n_t(h)^{\frac{\varepsilon^N-1}{\varepsilon^N}}\,dh\right]^{\frac{\varepsilon^N}{\varepsilon^N-1}} \tag{21}$$

N_t is then supplied to the domestic intermediate goods producers. Cost minimization results in demand for skill h, $n_t(h)$, and the aggregate wage index:

$$n_t(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\varepsilon^N} N_t \tag{22}$$

$$W_t = \left[\int_0^1 W_t(h)^{1-\varepsilon^N}\,dh\right]^{\frac{1}{1-\varepsilon^N}} \tag{23}$$

where $W_t(h)$ is the wage of labor skill h.

2.4 Insurance Companies

Households receive a binary idiosyncratic shock, $\Upsilon_t(h)$, that signals whether they are able to reset their wage. When $\Upsilon_t(h) = 1$ household h is allowed to adjust its nominal wage, otherwise $\Upsilon_t(h) = 0$ and $W_t(h) = \pi_{ss}W_{t-1}(h)$. The probability of wage adjustment is $1-\xi_w$.

Insurance companies operate in a perfectly competitive market. Every period households and insurance companies meet to sign state-contingent wage insurance contracts against next period's idiosyncratic shocks. Specifically, under each contract household h is obliged to pay the insurance company one unit of the domestic currency in period

$t+1$ if $\Upsilon_{t+1}(s^t, s_{t+1}, h) = 1$, otherwise $\Upsilon_{t+1}(s^t, s_{t+1}, h) = 0$ and the household receives $\psi_t(s^t, s_{t+1})$ units, where the time index highlights that ψ_t is determined at date t when the contract is signed. Let $b_t(s^t, s_{t+1}, h)$ denote the quantity of such contracts associated with household h.

Zero profits condition for the insurance companies pins down ψ_t:

$$\psi_t\left(s^t, s_{t+1}\right) = \frac{1-\xi_w}{\xi_w} \tag{24}$$

which reflects actuarially fair pricing.

2.5 Households

Households consume the final good, trade risk-free home and foreign nominal bonds, supply labor and trade wage insurance contracts.

Domestic bonds, B_t, cost one unit of the domestic currency at date t and pay $1+i_t$ units in $t+1$. Foreign bonds, B_t^*, cost one unit of the effective foreign currency and pay $1+i_t^*$ units in $t+1$.[13] Let $B_t^{HH}(h)$ and $B_t^{*,HH}(h)$ denote home and foreign bonds, respectively, held by household h, and define its foreign asset position in units of foreign goods as:

$$b_t^{*,HH}(h) \equiv \frac{B_t^{*,HH}(h)}{P_t^{F*}}$$

When trading in the international asset market households face a portfolio adjustment cost of $\Theta\left(b_t^{*,HH}(h), \theta_t^*\right)$, measured in units of foreign goods, where θ_t^* is an exogenous aggregate financial shock. Introducing a friction to the international financial markets is necessary because otherwise the uncovered interest rate parity (UIP) holds, and FXIs are deemed ineffective. The choice of a simple portfolio adjustment cost is motivated by Yakhin (2022), who demonstrates that to a first-order approximation this modelling strategy is isomorphic to models with richer microfoundations such as Gabaix and Maggiori (2015) and Fanelli and Straub (2021). Appendix A extends the result to the model

[13] B_t^* is an aggregate of bonds from all foreign countries and i_t^* is their effective return. Specifically: $B_t^* \equiv \frac{1}{S_t}\int_{0+}^{1} S_t^c B_t^c dc$ and $1+i_t^* \equiv \exp\left(\int_{0+}^{1} \log\left(1+i_t^c\right) dc\right)$, where S_t^c is the nominal exchange rate of the home currency against the currency of country c, B_t^c denotes bonds denominated in country c's currency, and i_t^c is the interest rate of those bonds. By symmetry across foreign countries and assuming they face the same shocks, $S_t^c = S_t$, and $i_t^c = i_t^*$.

of Itskhoki and Mukhin (2021, 2023) as well.[14] Hence, the simple and ad hoc portfolio adjustment cost is robust to different interpretations of the source of the financial friction.

I assume the portfolio adjustment cost, $\Theta(\cdot)$, is of the form:

$$\begin{aligned} \Theta\left(b_t^{*,HH}(h), \theta_t^*\right) &= \Theta\left(\widehat{b}_t^{*,HH}(h) - \widehat{\theta}_t^*\right) \\ \text{where} \qquad \widehat{b}_t^{*,HH}(h) &\equiv \frac{b_t^{*,HH}(h)}{TOT_{ss}Y_{ss}^{H,An.}} \\ \widehat{\theta}_t^* &\equiv \frac{\theta_t^*}{TOT_{ss}Y_{ss}^{H,An.}} \end{aligned} \tag{25}$$

where $Y_{ss}^{H,An.}$ is annual GDP, and the function $\Theta(\cdot)$ satisfies:

$$\Theta(\cdot) \geq 0 \qquad , \qquad \Theta''(\cdot) > 0 \qquad , \qquad \Theta(0) = \Theta'(0) = 0$$

That is, $\Theta(\cdot)$ is non-negative and convex, and in steady state its value and the value of its first derivative is zero. A household incurs a cost whenever its foreign asset position, $b_t^{*,HH}(h)$, deviates from some benchmark, θ_t^*. θ_t^* is interpreted as a risk-premium shock, in the sense that a rise in θ_t^* requires households to hold a higher level of $b_t^{*,HH}(h)$ in order eliminate the adjustment cost. Note that "hatted" variables denote their value relative to steady state *annual* GDP. This turns convenient for interpreting impulse response functions, for example, as it provides a better sense of the scale of movement in asset holdings.

The cost $\Theta(\cdot)$ is interpreted as resources captured by the financial sector. Assume that a fraction ϑ of the financial sector is owned by domestic households. Therefore, a fraction ϑ of the *aggregate* portfolio adjustment costs is rebated to domestic households through dividend distribution of financial firms. The households do not internalize this effect when they choose their asset position.

Each household is endowed with a differentiated labor skill, $n_t(h)$, and holds a monopolistic power over supplying it to the employment agencies. Wage setting is staggered as in Calvo (1983), with parameters as described above in Section 2.4.

Consumption of household h is denoted by $c_t(h)$. Households face aggregate preferences shocks, denoted by η_t. Finally, let Π_t denote firms' profits, and T_t denote govern-

[14] Appendix A also shows that the result of Yakhin (2022) is robust to introducing to the model foreign reserves, capital flows and risk-premium shocks, as is the case in this paper.

ment lump-sum transfers.

Household h solves:

$$
\begin{aligned}
& V_{1,t}\left(s^t, \Upsilon_t(h)=1, B_{t-1}^{HH}(h), b_{t-1}^{*,HH}(h), b_{t-1}\left(s^t,h\right)\right) \\
= & \underset{c_t(h),B_t^{HH}(h),b_t^{*,HH}(h),b_t(s^{t+1},h),W_t(h)}{Max}\left\{\begin{array}{c} U\left[c_t(h), n_t(h); \eta_t\right] \\ +(1-\xi_w)\beta E_t\left\{V_{1,t+1}(\cdot)/s^t, \Upsilon_{t+1}(h)=1\right\} \\ +\xi_w\beta E_t\left\{V_{0,t+1}\left(\cdot, \overline{W}_{t+1}(h)\right)/s^t, \Upsilon_{t+1}(h)=0\right\}\end{array}\right\}
\end{aligned}
$$

$$
\begin{aligned}
s.t. \qquad n_t(h) &= \left(\frac{W_t(h)}{W_t}\right)^{-\varepsilon^N} N_t \\
\overline{W}_{t+1}(h) &= \pi_{ss} W_t(h) \\
c_t(h) + \frac{S_t P_t^{F*} b_t^{*,HH}(h)}{P_t} + \frac{B_t^{HH}(h)}{P_t} &= \frac{W_t(h)}{P_t} n_t(h) \\
&+ \frac{S_t\left(1+i_{t-1}^*\right) P_{t-1}^{F*} b_{t-1}^{*,HH}(h)}{P_t} - \frac{S_t P_t^{F*}\Theta\left(b_t^{*,HH}(h), \theta_t^*\right)}{P_t} \\
&+ \frac{\left(1+i_{t-1}\right) B_{t-1}^{HH}(h)}{P_t} + \frac{\Pi_t + T_t - b_{t-1}\left(s^t,h\right)}{P_t}
\end{aligned}
$$

and:

$$
\begin{aligned}
& V_{0,t}\left(s^t, \Upsilon_t(h)=0, B_{t-1}(h), b_{t-1}^{*,HH}(h), b_{t-1}\left(s^t,h\right), \overline{W}_t(h)\right) \\
= & \underset{c_t(h),B_t^{HH}(h),b_t^{*,HH}(h),b_t(s^{t+1},h)}{Max}\left\{\begin{array}{c} U\left[c_t(h), n_t(h); \eta_t\right] \\ +(1-\xi_w)\beta E_t\left\{V_{1,t+1}(\cdot)/s^t, \Upsilon_{t+1}(h)=1\right\} \\ +\xi_w\beta E_t\left\{V_{0,t+1}\left(\cdot, \overline{W}_{t+1}(h)\right)/s^t, \Upsilon_{t+1}(h)=0\right\}\end{array}\right\}
\end{aligned}
$$

$$
\begin{aligned}
s.t. \qquad n_t(h) &= \left(\frac{\overline{W}_t(h)}{W_t}\right)^{-\varepsilon^N} N_t \\
\overline{W}_{t+1}(h) &= \pi_{ss} \overline{W}_t(h) \\
c_t(h) + \frac{S_t P_t^{F*} b_t^{*,HH}(h)}{P_t} + \frac{B_t^{HH}(h)}{P_t} &= \frac{\overline{W}_t(h)}{P_t} n_t(h) \\
&+ \frac{S_t\left(1+i_{t-1}^*\right) P_{t-1}^{F*} b_{t-1}^{*,HH}(h)}{P_t} - \frac{S_t P_t^{F*}\Theta\left(b_t^{*,HH}(h), \theta_t^*\right)}{P_t} \\
&+ \frac{\left(1+i_{t-1}\right) B_{t-1}^{HH}(h)}{P_t} + \frac{\Pi_t + T_t + \psi_{t-1}\left(s^t\right) b_{t-1}\left(s^t,h\right)}{P_t}
\end{aligned}
$$

Where $\overline{W}_t(h)$ is the wage of $n_t(h)$ whenever $\Upsilon_t(h) = 0$. Notice that the difference between V_1 and V_0 is that under the former date t wage level is a choice variable, while under the latter it is taken as given and it is part of the state variables. In addition, the budget constraints differ in the payment to/from the insurance companies.

2.5.1 Households' Euler Equations and the UIP

Full insurance against the idiosyncratic shocks results in equal marginal utilities of consumption across idiosyncratic states for each household:

$$U_{c_t(h)}\left(s^t, \Upsilon_t(h)=0\right) = U_{c_t(h)}\left(s^t, \Upsilon_t(h)=1\right) \qquad \forall h$$

Assuming equal endowment of assets across households in the initial period together with the result above, the optimality condition with respect to domestic bonds suggests equal marginal utilities of consumption across households in every period:

$$U_{c_t(h)}\left(s^t\right) = U_{c_t(g)}\left(s^t\right) \qquad \forall h, g$$

This suggests that we can omit the household index from the marginal utility of consumption and treat it as an aggregate variable. The optimality condition with respect to foreign bonds together with equality of marginal utilities across households implies equality of foreign asset positions across households:

$$b_t^{*,HH}\left(s^t, h\right) = b_t^{*,HH}\left(s^t, g\right) \qquad \forall s^t, h, g$$

Suggesting we can also treat the foreign asset position as an aggregate variable. As a result, the Euler equations for home and foreign bonds read:

$$U_{C_t} = \beta\left(1+i_t\right) E_t\left\{U_{C_{t+1}}\frac{1}{\pi_{t+1}}\right\} \tag{26}$$

$$U_{C_t}\left[1+\frac{\Theta'\left(\widehat{b}_t^{*,HH}-\widehat{\theta}_t^*\right)}{TOT_{ss}Y_{ss}^{H,An.}}\right] = \beta\left(1+i_t^*\right) E_t\left\{U_{C_{t+1}}\frac{\sigma_{t+1}}{\pi_{t+1}}\right\} \tag{27}$$

$$\text{where} \qquad \pi_{t+1} \equiv \frac{P_{t+1}}{P_t} \quad , \quad \sigma_{t+1} \equiv \frac{S_{t+1}}{S_t}$$

Combining these equations yields the modified uncovered interest rate parity condition (UIP). After log-linearization it reads:

$$\widetilde{(1+i_t)} \cong \widetilde{(1+i_t^*)} + E_t\left\{\widetilde{\sigma}_{t+1}\right\} - \frac{\Theta''(0)}{TOT_{ss}Y_{ss}^{H,An.}}\left(\widehat{b}_t^{*,HH}-\widehat{\theta}_t^*\right) \tag{28}$$

and using the law of one price, it can also be written in real terms as:

$$\widetilde{(1+r_t)} \cong \widetilde{(1+r_t^*)} + \left[E_t\left\{\widetilde{p}_{t+1}^F\right\} - \widetilde{p}_t^F\right] - \frac{\Theta''(0)}{TOT_{ss}Y_{ss}^{H,An.}}\left(\widehat{b}_t^{*,HH}-\widehat{\theta}_t^*\right) \tag{29}$$

$$\text{where} \qquad \widetilde{(1+r_t)} = \widetilde{(1+i_t)} - E_t\left(\pi_{t+1}\right)$$

$$\widetilde{(1+r_t^*)} = \widetilde{(1+i_t^*)} - E_t\left(\pi_{t+1}^{F*}\right)$$

From these representations it is clear that the convexity of the portfolio adjustment cost introduces a wedge to the UIP condition. With $\Theta''(0) = 0$ the UIP holds, and exchange rate dynamics are governed by interest rate differentials, suggesting that sterilized FXIs deem ineffective. As demonstrated below, FXIs work through the portfolio balance channel as they affect the private sector holdings of foreign assets, and hence the UIP premium, $-\frac{\Theta''(0)}{TOT_{ss}Y_{ss}^{H,An.}}\left(\widehat{b}_t^{*,HH} - \widehat{\theta}_t^{*}\right)$. In (28), a rise in $\widehat{b}_t^{*,HH}$ raises expectations for depreciation of the domestic currency, though this comes about through an immediate *appreciation* on impact. In (29), the same argument holds for real appreciation[15], implying a rise in the terms of trade on impact.

2.5.2 Optimal Wage Setting

Perfect insurance against the idiosyncratic shocks suggests that all households that can readjust their wage at date t set the same wage, and by the demand function for their labor services, equation (22), they also have the same labor effort.

Notation 2 *For a household-specific generic variable X, let $X_{t/t-\tau}$ denote its date t value for households that last revised their wage in period $t-\tau$.*

Optimal wage setting results in:

$$\frac{W_{t/t}}{P_t} = -\frac{\varepsilon^N}{\varepsilon^N - 1}\frac{\sum_{s=0}^{\infty}\xi_w^s\beta^s E_t\left\{\left(\frac{W_{t+s}}{\pi_{ss}^s}\right)^{\varepsilon^N} N_{t+s}U_{n_{t+s/t}}\right\}}{\sum_{s=0}^{\infty}\xi_w^s\beta^s E_t\left\{\left(\frac{W_{t+s}}{\pi_{ss}^s}\right)^{\varepsilon^N} N_{t+s}\frac{U_{C_{t+s}}}{\frac{\pi_{t,t+s}}{\pi_{ss}^s}}\right\}} \tag{30}$$

Note that under flexible wages $W_{t/t} = W_t$ and:

$$\begin{aligned} w_t &= -\frac{\varepsilon^N}{\varepsilon^N - 1}\frac{U_{N_t}}{U_{C_t}} \\ \text{where} \quad w_t &\equiv \frac{W_t}{P_t} \end{aligned}$$

That is, real wage is set as a constant markup over the marginal rate of substitution between labor and consumption, i.e. labor supply.

---

[15] Recall that p_t^F is the CPI-based real exchange rate in the model.

Let MRS_t denote the average marginal rate of substitution between labor and consumption in the economy:

$$MRS_t = \sum_{s=0}^{\infty} (1-\xi_w)\,\xi_w^s MRS_{t/t-s} = -\sum_{s=0}^{\infty} (1-\xi_w)\,\xi_w^s \frac{U_{n_{t/t-s}}}{U_{C_t}}$$

Using these definitions and taking first order approximation to equation (30) we get:

$$\widetilde{\pi}_t^w \cong \beta E_t \left(\widetilde{\pi}_{t+1}^w\right) - \frac{(1-\xi_w\beta)(1-\xi_w)}{\xi_w} \frac{1}{1+\left(\gamma_{nn} - \frac{\gamma_{nc}\gamma_{cn}}{\gamma_{cc}}\right)\varepsilon^N} \left(\widetilde{w}_t - \widetilde{MRS}_t\right) \tag{31}$$

where tiled variables denote log-deviations from deterministic steady state, and π_t^w is the gross wage inflation rate, W_t/W_{t-1}. $\gamma_{xy} \equiv \frac{U_{xy}}{U_x} Y_{ss}$ is the elasticity of the marginal utility of variable x with respect to variable y evaluated in steady state. The expression $\gamma_{nn} - \frac{\gamma_{nc}\gamma_{cn}}{\gamma_{cc}}$ is the *inverse* of the steady state Frisch elasticity of labor supply (see Appendix B). For an additive separable utility function, i.e. for $\gamma_{nc} = \gamma_{cn} = 0$, equation (31) takes its familiar form, e.g. Galí (2008) chapter 6.

2.6 The Government

The government operates in two arms: a fiscal authority (treasury) and a monetary authority (the central bank).

The fiscal authority consumes the domestic good, its consumption is denoted by G_t. G_t is an exogenous process. The fiscal authority also subsidizes labor at rate τ_w, and receives the central bank profits as transfers, T_t^{CB}. In cases where the central bank incurs losses, the fiscal authority recapitalizes the central bank, i.e. $T_t^{CB} < 0$. The fiscal budget constraint is balanced by lump-sum transfers to the households, T_t, and is given by:

$$T_t^{CB} = P_t^H G_t + \tau_w W_t \int_0^1 n_t(f)\, df + T_t \tag{32}$$

Let $B_t^{*,CB}$ denote foreign bonds held by the central bank[16]; these constitute the stock of foreign reserves. Foreign reserves, FX_t, are measured in units of foreign goods, that is:

$$FX_t \equiv \frac{B_t^{*,CB}}{P_t^{F*}}$$

[16] Here again, $B_t^{*,CB}$ is an aggregate of bonds from all foreign countries.

Similarly to households, the central bank faces a portfolio adjustment cost, $\Theta^{CB}(FX_t)$, also measured in units of foreign goods. The function $\Theta^{CB}(\cdot)$ satisfies:

$$\Theta^{CB}(\cdot) \geq 0 \quad , \quad \Theta^{CB}(FX_{ss}) = \Theta^{CB\prime}(FX_{ss}) = 0 \quad , \quad \Theta^{CB\prime\prime}(\cdot) > 0$$

This adjustment cost is required for imposing stationarity on the system when solving for the optimal FXI policy.[17]

Assuming a cashless economy, the central bank budget constraint is given by:

$$\begin{aligned} & S_t P_t^{F*} FX_t + S_t P_t^{F*} \Theta^{CB}(FX_t) + (1 + i_{t-1}) \left(B_{t-1}^{ROW} + B_{t-1}^{HH} \right) + T_t^{CB} \\ = \; & S_t \left(1 + i_{t-1}^{*} \right) P_{t-1}^{F*} FX_{t-1} + \left(B_t^{ROW} + B_t^{HH} \right) \end{aligned} \tag{33}$$

where B_t^{ROW} is domestic bonds held by the rest of the world and B_t^{HH} is domestic bonds held by domestic households.

Combining the fiscal authority budget constraint, equation (32), with that of the central bank, equation (33), gives the general government consolidated budget constraint:

$$\begin{aligned} & P_t^H G_t + \tau_w \int_0^1 W_t n_t(f)\, df + T_t \\ & + (1 + i_{t-1}) \left(B_{t-1}^{ROW} + B_{t-1}^{HH} \right) + S_t P_t^{F*} \left[FX_t + \Theta^{CB}(FX_t) \right] \\ = \; & S_t P_t^{F*} \frac{1 + i_{t-1}^{*}}{\pi_t^{F*}} FX_{t-1} + \left(B_t^{ROW} + B_t^{HH} \right) \end{aligned} \tag{34}$$

where:

$$\pi_t^{F*} \equiv \frac{P_t^{F*}}{P_{t-1}^{F*}}$$

2.6.1 The Central Banks' Policy Tools

The central bank uses two instruments: the domestic nominal interest rate and sterilized foreign exchange interventions (FXIs). Bonds are supplied at any amount so as to defend the announced interest rate. Note that since the economy is cashless and since the central bank transfers its profits to the treasury in every period, FXIs are sterilized by construction. This follows from the central bank's balance sheet, as under these conditions any adjustment to the level of foreign reserves must be matched with the issuance or redemption of domestic bonds.

[17] See equation (D.15) in Appendix D.

The central bank transfers to the fiscal authority are given by:

$$\begin{aligned} T_t^{CB} &= \left[S_t\left(1+i_{t-1}^*\right)-S_{t-1}\right]B_{t-1}^{*,CB} \\ &\quad -S_tP_t^{F*}\Theta^{CB}\left(FX_t\right)-i_{t-1}\left(B_{t-1}^{ROW}+B_{t-1}^{HH}\right) \end{aligned} \tag{35}$$

Substituting T_t^{CB} into the central bank's budget constraint, equation (33), results in:

$$S_tB_t^{*,CB}-S_{t-1}B_{t-1}^{*,CB}=\Delta\left(B_t^{ROW}+B_t^{HH}\right) \tag{36}$$

This equation dictates that any change in the value of foreign reserves must be matched with the issuance or redemption of domestic bonds, suggesting that FXIs are sterilized.

Let Φ_t denote the central bank purchase of foreign bonds. $B_t^{*,CB}$ evolves according to:

$$B_t^{*,CB}=\left(1+i_{t-1}^*\right)B_{t-1}^{*,CB}+\Phi_t$$

Suggesting:

$$\begin{aligned} FX_t &= \frac{1+i_{t-1}^*}{\pi_t^{F*}}FX_{t-1}+TOT_{ss}Y_{ss}^{H,An.}\widehat{\phi}_t \\ \text{where} \quad \widehat{\phi}_t &\equiv \frac{1}{TOT_{ss}Y_{ss}^{H,An.}}\frac{\Phi_t}{P_t^{F*}} \end{aligned} \tag{37}$$

$\widehat{\phi}_t$ is FXI expressed in percent of annual GDP.

The analysis below assumes that monetary policy sets the interest rate optimally, while exploring different FXI policies.

2.7 Closing the Model: Aggregate Technology and the Balance of Payments

Aggregate labor input is given by:

$$\int_0^1 n_t(f)\,df=N_t=\left[\int_0^1 n_t(h)^{\frac{\varepsilon^N-1}{\varepsilon^N}}dh\right]^{\frac{\varepsilon^N}{\varepsilon^N-1}}$$

Aggregating production technology using (16) and demand for intermediate goods, equation (3), we get:

$$\begin{aligned} Y_t^H &= A_t\left(\frac{N_t}{pd_t}\right)^{\alpha} \\ \text{where} \quad pd_t &\equiv \int_0^1\left(\frac{P_t^H(f)}{P_t^H}\right)^{-\frac{\varepsilon^L}{\alpha}}df \end{aligned} \tag{38}$$

pd_t is a measure of price dispersion in the economy, which is second-order.

To derive the balance of payments note that aggregate firms' profits are given by:

$$\Pi_t = P_t^H Y_t^H - (1-\tau_w) W_t N_t + \vartheta S_t P_t^{F*} \left[\Theta\left(b_t^{*,HH}, \theta_t^*\right) + \Theta^{CB}(FX_t)\right]$$

where the first two terms stand for the profits of the intermediate goods producers, and the last term is the rebate of portfolio adjustment costs from domestically owned financial firms (recall that domestic households own a fraction ϑ of the financial sector).

Aggregating the households' budget constraints and combining the result with the government's consolidated budget constraint and aggregate profits, results in the balance of payments identity:

$$\begin{aligned} FX_t + TOT_{ss} Y_{ss}^{H,An.} \widehat{b}_t^{*,HH} &= \frac{1+i_{t-1}^*}{\pi_t^{F*}}\left(FX_{t-1} + TOT_{ss} Y_{ss}^{H,An.} \widehat{b}_{t-1}^{*,HH}\right) \\ &\quad -(1-\vartheta)\left[\Theta^{CB}(FX_t) + \Theta\left(\widehat{b}_t^{*,HH} - \widehat{\theta}_t^*\right)\right] \\ &\quad + TOT_{ss} Y_{ss}^{H,An.} \widehat{\phi}_t^* + TOT_t EX_t - IM_t \end{aligned} \tag{39}$$

where $\widehat{\phi}_t^*$ is capital inflows to the home economy relative to annual GDP.[18] $\widehat{\phi}_t^*$ is exogenous.

This closes the model. The complete system of equations, approximated to first order, is characterized below.

2.8 Characterizing Equilibrium

This section characterizes the deterministic steady state of the economy, and takes a firs-order approximation to the model's equilibrium conditions around that point.

---

[18] The derivation of (39) uses the law of motion for the accumulation of domestic bonds by foreigners:

$$B_t^{ROW} = (1+i_{t-1}) B_{t-1}^{ROW} + \Phi_t^*$$

where Φ_t^* is the purchase of domestic bonds by foreigners, e.g. through FXIs of foreign central banks. In terms of foreign goods, this equation reads:

$$b_t^{ROW} = \frac{1+i_{t-1}}{\sigma_t \pi_t^{F*}} b_{t-1}^{ROW} + \phi_t^*$$

where $b_t^{ROW} \equiv \frac{B_t^{ROW}}{S_t P_t^{F*}}$ and $\phi_t^* \equiv \frac{\Phi_t^*}{S_t P_t^{F*}}$. Finally:

$$\widehat{\phi}_t^* \equiv \frac{\phi_t^*}{TOT_{ss} Y_{ss}^{H,An.}}$$

2.8.1 Steady State

Consider a symmetric global steady state, in which trade is balanced, prices of home and foreign goods are equal, households hold zero foreign assets position, and inflation rates are equal across countries, that is:

$$
\begin{aligned}
TOT_{ss}EX_{ss} &= IM_{ss} \\
TOT_{ss} &= 1 \\
b_{ss}^{*,HH} &= \theta_{ss}^{*} = 0 \\
\pi_{ss}^{H} &= \pi_{ss}^{F*}
\end{aligned}
$$

where the steady state inflation rates are set at some arbitrary level.

Foreign reserves are held at an exogenous target level, FX^T:

$$FX_{ss} = FX^T$$

and recall that:

$$\frac{1+i_{ss}^{*}}{\pi_{ss}^{*F}} = \beta^{-1}$$

Given these assumptions, the rest of the variables are pinned down using the equilibrium conditions of the model. In particular, unitary terms of trade together with the consumption price index, equation (10), suggest:

$$p_{ss}^{H} = p_{ss}^{F} = 1$$

and symmetry across countries dictates:

$$
\begin{aligned}
\sigma_{ss} &= 1 \\
\widehat{\phi}_{ss}^{*} &= -\frac{1-\beta}{\beta}\frac{FX^T}{Y_{ss}^{H,An.}} \\
IM_{ss} &= WT_{ss}
\end{aligned}
$$

Note that although world trade is exogenous from the point of view of each economy, it is endogenous in the model, and is pinned down through the demand for imports. Also notice that capital inflows are negative in steady state, as they reflect interest payments to foreign central banks for their holdings of domestic bonds, which are part of their foreign reserves.

2.8.2 System of Equations

The following system of equations characterizes equilibrium in the model. Equations are approximated to first-order. Tiled variables denote log deviations from their deterministic steady state.

Optimal price and wage setting:

$$\widetilde{\pi}_t^w \cong \beta E_t\left(\widetilde{\pi}_{t+1}^w\right) - \frac{(1-\xi_w\beta)(1-\xi_w)}{\xi_w}\frac{1}{1+\left(\gamma_{nn}-\frac{\gamma_{nc}\gamma_{cn}}{\gamma_{cc}}\right)\varepsilon^N}\left(\widetilde{w}_t - \widetilde{U}_{N_t} + \widetilde{U}_{C_t}\right) \tag{40}$$

$$\widetilde{\pi}_t^H \cong \beta E_t\left(\widetilde{\pi}_{t+1}^H\right) + \frac{\left(1-\xi_p\right)\left(1-\beta\xi_p\right)}{\xi_p}\frac{\alpha}{\alpha+(1-\alpha)\varepsilon^L}\left[\begin{array}{c}\widetilde{w}_t - \widetilde{p}_t^H \\ -\widetilde{A}_t - (\alpha-1)\widetilde{N}_t\end{array}\right] \tag{41}$$

The Euler equations:

$$\widetilde{U}_{C_t} \cong \widetilde{(1+i_t)} + E_t\left\{\widetilde{U}_{C_{t+1}}\right\} - E_t\left\{\pi_{t+1}\right\} \tag{42}$$

$$\widetilde{U}_{C_t} + \frac{\Theta''(0)}{Y_{ss}^{H,An.}}\left(\widehat{b}_t^{*,HH} - \widehat{\theta}_t^*\right) \cong \widetilde{(1+i_t^*)} + E_t\left\{\widetilde{U}_{C_{t+1}}\right\} + E_t\left\{\widetilde{\sigma}_{t+1}\right\} - E_t\left\{\widetilde{\pi}_{t+1}\right\} \tag{43}$$

Consumption and its composition:

$$\widetilde{C}_t \cong (1-\lambda)\widetilde{d}_t^H + \lambda\widetilde{IM}_t \tag{44}$$

$$\widetilde{d}_t^H \cong \widetilde{C}_t - \varepsilon\widetilde{p}_t^H \tag{45}$$

$$\widetilde{IM}_t \cong \widetilde{C}_t - \varepsilon\widetilde{p}_t^F \tag{46}$$

Exports demand:

$$\widetilde{EX}_t \cong -\varepsilon^*\widetilde{TOT}_t + \widetilde{WT}_t \tag{47}$$

Technology, the resource constraint and the balance of payments:

$$\widetilde{Y}_t^H \cong \widetilde{A}_t + \alpha\widetilde{N}_t \tag{48}$$

$$\widetilde{Y}_t^H \cong (1-\lambda)\frac{C_{ss}}{Y_{ss}^H}\widetilde{d}_t^H + \frac{G_{ss}}{Y_{ss}^H}\widetilde{G}_t + \lambda\frac{C_{ss}}{Y_{ss}^H}\widetilde{EX}_t \tag{49}$$

$$\begin{aligned}\frac{FX_{ss}}{Y_{ss}^{H,An.}}\widetilde{FX}_t + \widehat{b}_t^{*,HH} \cong\ & \beta^{-1}\left(\frac{FX_{ss}}{Y_{ss}^{H,An.}}\widetilde{FX}_{t-1} + \widehat{b}_{t-1}^{*,HH}\right) + \left(\widehat{\phi}_t^* - \widehat{\phi}_{ss}^*\right) \\ & +\beta^{-1}\frac{FX_{ss}}{Y_{ss}^{H,An.}}\left[\widetilde{(1+i_{t-1}^*)} - \widetilde{\pi}_t^{F*}\right] + \frac{\lambda C_{ss}}{Y_{ss}^{H,An.}}\left(\widetilde{TOT}_t + \widetilde{EX}_t - \widetilde{IM}_t\right)\end{aligned} \tag{50}$$

Definitions and identities:

$$\widetilde{U}_{N_t} \cong \gamma_{nc}\widetilde{C}_t + \gamma_{nn}\widetilde{N}_t + \gamma_{n\eta}\widetilde{\eta}_t \tag{51}$$

$$\widetilde{U}_{C_t} \cong \gamma_{cc}\widetilde{C}_t + \gamma_{cn}\widetilde{N}_t + \gamma_{c\eta}\widetilde{\eta}_t \tag{52}$$

$$\widetilde{w}_t - \widetilde{w}_{t-1} \cong \widetilde{\pi}_t^w - \widetilde{\pi}_t \tag{53}$$

$$\widetilde{p}_t^H - \widetilde{p}_{t-1}^H \cong \widetilde{\pi}_t^H - \widetilde{\pi}_t \tag{54}$$

$$\widetilde{p}_t^F - \widetilde{p}_{t-1}^F \cong \widetilde{\sigma}_t + \widetilde{\pi}_t^{F*} - \widetilde{\pi}_t \tag{55}$$

$$\widetilde{TOT}_t \cong \widetilde{p}_t^H - \widetilde{p}_t^F \tag{56}$$

This gives a system of 17 equations in 19 endogenous variables: $\widetilde{U}_{N_t}$, $\widetilde{U}_{C_t}$, $\widetilde{C}_t$, $\widetilde{N}_t$, $\widetilde{w}_t$, $\widetilde{\pi}_t^w$, $\widetilde{\pi}_t$, $\widetilde{Y}_t^H$, $\widetilde{p}_t^H$, $\widetilde{\pi}_t^H$, $\widetilde{(1+i_t)}$, $\widehat{b}_t^{*,HH}$, $\widetilde{\sigma}_t$, $\widetilde{d}_t^H$, $\widetilde{IM}_t$, $\widetilde{p}_t^F$, $\widetilde{TOT}_t$, $\widetilde{EX}_t$, $\widetilde{FX}_t$. The model is closed by specifying policy rules for the interest rate and foreign reserves. The variables $\widetilde{\eta}_t$, $\widetilde{A}_t$, $\widehat{\theta}_t^*$, $\widetilde{(1+i_t^*)}$, $\widetilde{WT}_t$, $\widetilde{G}_t$, $\widetilde{\pi}_t^{F*}$, $\left(\widehat{\phi}_t^* - \widehat{\phi}_{ss}^*\right)$ are exogenous, though by assumption $\widetilde{\pi}_t^{F*} = \widetilde{(1+i_t^*)} = 0$.

3 The Welfare Criterion

This section presents the welfare criterion that a utilitarian policymaker would use for ranking alternative equilibrium outcomes and for the design of optimal policies under commitment. I start by obtaining the rate of labor subsidy that supports efficiency in a decentralized steady state, and then turn to presenting a second order approximation to the welfare function. Centering the economy around an efficient steady state is required for deriving a second-order approximation of the welfare criterion that: (1) can be used as an objective function in a linear-quadratic optimization problem whose solution approximates the solution of the exact problem; and (2) correctly ranks alternative equilibrium allocations that are approximated to first order. See Benigno and Woodford (2012).

3.1 The Optimal Labor Subsidy

To find the rate of labor subsidy, τ_w, that supports the efficient allocation as an equilibrium outcome, I first solve for the constrained-optimal steady state.

A utilitarian social planner seeks to maximize aggregate utility subject to technological constraints and equilibrium conditions. Since the focus here is on the steady state allocation, the optimization is reduced to the following static problem:

$$\begin{aligned}
\underset{\{C_{ss},N_{ss},IM_{ss},d^H_{ss},TOT_{ss}\}}{Max} \qquad & \frac{1}{1-\beta}U\left[C_{ss},N_{ss};\eta_{ss}\right] \\
s.t. \qquad C_{ss} &= \left[(1-\lambda)^{\frac{1}{\varepsilon}}\left(d^H_{ss}\right)^{\frac{\varepsilon-1}{\varepsilon}}+\lambda^{\frac{1}{\varepsilon}}\left(IM_{ss}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
A_{ss}N^{\alpha}_{ss} &= d^H_{ss}+G_{ss}+TOT^{-\varepsilon^*}_{ss}WT_{ss} \\
0 &= \frac{1-\beta}{\beta}\left(FX_{ss}+b^{*,HH}_{ss}\right)+\phi^*_{ss}+TOT^{1-\varepsilon^*}_{ss}WT_{ss}-IM_{ss} \\
\frac{d^H_{ss}}{IM_{ss}} &= \frac{1-\lambda}{\lambda}TOT^{-\varepsilon}_{ss}
\end{aligned}$$

where the first two constraints are dictated by technology and the resource constraint. The third constraint is the balance of payments, and the last is an equilibrium condition for the composition of consumption, which is derived from (8) and (9).

Notice that $b^{*,HH}_{ss}$ and FX_{ss} are not part of the choice variables of the planner. $b^{*,HH}_{ss}$ is determined by the households' Euler equation for foreign bonds, which is a constraint the planner must obey, and therefore can be taken as given in the planner's problem. FX_{ss} is indeterminate in a symmetric global steady state, but it's level does not affect the optimal allocation of the domestic variables.[19] The considerations for the appropriate level of foreign reserves are related to the type of risks the economy faces, which are irrelevant for the *deterministic* steady state allocation.

The solution to the planner's problem is characterized by:

$$-\frac{U_{N_{ss}}}{U_{C_{ss}}C^{\frac{1}{\varepsilon}}_{ss}\left(1-\lambda\right)^{\frac{1}{\varepsilon}}\left(d^H_{ss}\right)^{-\frac{1}{\varepsilon}}}=\frac{\left(1-\lambda\right)\varepsilon+\varepsilon^*-1}{\left(1-\lambda\right)\varepsilon+\varepsilon^*-\left(1-\lambda\right)}\alpha A_{ss}N^{\alpha-1}_{ss} \tag{57}$$

and the four constraints above. Note that (57) imposes symmetry across countries, $TOT_{ss}=1$ and $IM_{ss}=WT_{ss}$, as described in Section 2.8.1.

In the decentralized economy, equilibrium in the labor market suggests:

$$-\frac{\varepsilon^N}{\varepsilon^N-1}\frac{U_{N,ss}}{U_{C,ss}}=\frac{1}{1-\tau_w}\frac{\varepsilon^L-1}{\varepsilon^L}\alpha p^H_{ss}A_{ss}N^{\alpha-1}_{ss}$$

which uses equations (16), (17), (18), (19), and (30). Substituting for p^H_{ss} using (8) and

[19] Higher FX_{ss} implies higher government debt to foreigners, which in turn raises steady state capital outflows due to higher debt service. In the balance of payments, the rise in debt service exactly offsets the effect of higher reserves as $\phi^*_{ss}=-\frac{1-\beta}{\beta}FX_{ss}$.

rearranging results in:

$$-\frac{U_{N,ss}}{U_{C,ss}C_{ss}^{\frac{1}{\varepsilon}}\left(1-\lambda\right)^{\frac{1}{\varepsilon}}\left(d_{ss}^{H}\right)^{-\frac{1}{\varepsilon}}}=\frac{1}{1-\tau_{w}}\frac{\varepsilon^{L}-1}{\varepsilon^{L}}\frac{\varepsilon^{N}-1}{\varepsilon^{N}}\alpha A_{ss}N_{ss}^{\alpha-1} \tag{58}$$

Comparing (57) to (58), it is clear that the social planner can support the efficient steady state as equilibrium by setting:

$$1-\tau_{w}=\frac{\varepsilon^{L}-1}{\varepsilon^{L}}\frac{\varepsilon^{N}-1}{\varepsilon^{N}}\frac{\left(1-\lambda\right)\varepsilon+\varepsilon^{*}-\left(1-\lambda\right)}{\left(1-\lambda\right)\varepsilon+\varepsilon^{*}-1} \tag{59}$$

To understand this result, consider the following special cases. First note that (59) generalizes the formulation in Galí and Monacelli (2005). In their case $\varepsilon^{N}\rightarrow\infty$ and $\varepsilon=\varepsilon^{*}=1$, and the optimal subsidy boils down to:

$$1-\tau_{w}^{GM}=\frac{\varepsilon^{L}-1}{\varepsilon^{L}}\frac{1}{1-\lambda}$$

Second, consider a closed economy, i.e. $\lambda=0$. In that case:

$$1-\tau_{w}^{CE}=\frac{\varepsilon^{L}-1}{\varepsilon^{L}}\frac{\varepsilon^{N}-1}{\varepsilon^{N}}$$

which completely offsets the monopolistic distortions in the model and supports a perfectly competitive equilibrium, as is standard in the new-Keynesian closed economy literature. Finally, consider a perfectly elastic demand for exports, $\varepsilon^{*}\rightarrow\infty$. In that case the optimal subsidy is given by:

$$1-\tau_{w}^{\varepsilon^{*}\rightarrow\infty}=\frac{\varepsilon^{L}-1}{\varepsilon^{L}}\frac{\varepsilon^{N}-1}{\varepsilon^{N}}$$

which coincides with the closed economy case.

Notice that $\frac{(1-\lambda)\varepsilon+\varepsilon^{*}-(1-\lambda)}{(1-\lambda)\varepsilon+\varepsilon^{*}-1}>1$, suggesting that the optimal subsidy in (59) is smaller than the one of a closed economy, and as a result production in steady state is lower than its level in a perfectly competitive equilibrium. The reason is that the planner internalizes the monopolistic power of the economy in supplying its good to the global markets, as it faces a downward sloping demand curve for exports, equation (13). Therefore, the social planner faces a tradeoff between labor market efficiency[20] and exploiting the monopolistic

[20] By labor market efficiency I mean equating the marginal product of labor to the households' marginal rate of substitution between labor and consumption of the home good.

power of the economy in the international markets. This tradeoff is optimally balanced in (59).[21] When $\varepsilon^* \to \infty$, the economy faces a perfectly elastic demand for its good, has no monopolistic power in the global markets, and the planner is left with restoring efficiency in the domestic labor market, just as in the case of a closed economy.

3.2 Second-Order Approximation to the Welfare Criterion

A utilitarian policymaker seeks to maximize welfare in the economy as measured by the aggregate expected discounted utility of domestic households, that is:

$$\mathbb{W} \equiv E_0 \sum_{t=0}^{\infty} \beta^t \int_0^1 U\left(c_t\left(h\right), n_t\left(h\right); \eta_t\right) dh$$

After taking second order approximation, substituting for equilibrium conditions approximated to second-order and using the optimal subsidy (59), the welfare criterion reads:

$$\begin{aligned}
\frac{\mathbb{W}-\mathbb{W}_{ss}}{U_C C_{ss}} &= E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2} y_{1,t}' \Omega_{11} y_{1,t} + \frac{1}{2} y_{2,t}' \Omega_{22} y_{2,t} + x_t' \Omega_{x1} y_{1,t} \right\} \qquad (60) \\
&+ E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \frac{1}{2} \frac{\varepsilon^L}{\alpha} \left[\left(1-\varepsilon^L\right) + \frac{\varepsilon^L}{\alpha} \right] \frac{U_{N_{ss}} N_{ss}}{U_{C_{ss}} C_{ss}} \frac{\xi_p}{1-\xi_p} \frac{1}{1-\beta\xi_p} \left(\widetilde{\pi}_t^H\right)^2 \\ + \frac{1}{2} \varepsilon^N \frac{U_N N_{ss}}{U_C C_{ss}} \left[1 + \left(\gamma_{nn} - \frac{\gamma_{nc}\gamma_{cn}}{\gamma_{cc}} \right) \varepsilon^N \right] \frac{\xi_w}{1-\xi_w} \frac{1}{1-\beta\xi_w} \left(\widetilde{\pi}_t^w\right)^2 \end{array} \right\} \\
&+ t.i.p. + \mathcal{O}\left(\|\cdot\|^3\right)
\end{aligned}$$

where $t.i.p.$ stands for terms independent of policy, and:

$$\begin{aligned}
y_{1,t} &\equiv \left[\begin{array}{ccc} \widetilde{C}_t & \widetilde{N}_t & \widetilde{TOT}_t \end{array} \right]' \\
y_{2,t} &\equiv \left[\begin{array}{cc} \widehat{b}_t^{*,HH} - \widehat{\theta}_t^* & \widetilde{FX}_t \end{array} \right]' \\
x_t &\equiv \left[\begin{array}{ccc} \widetilde{\eta}_t & \widetilde{A}_t & \widetilde{WT}_t \end{array} \right]'
\end{aligned}$$

[21] This formulation corresponds to the standard approach in the literature, e.g. Galí and Monacelli (2005), De Paoli (2009) and Cavallino (2019). However, in principle, it is not clear why the social planner should be constrained by equilibrium conditions, to the extent that they can be altered by taxation. The fact that the optimal subsidy in the text maintains some of the monopolistic power of the economy reflects the social planner's incentive to manipulate the terms of trade in favor of the domestic consumers, as highlighted by Corsetti and Pesenti (2001). Alternatively, one could introduce, in addition to the labor subsidy, a subsidy that directly alters the terms of trade. For example, consider a subsidy, τ_H, to domestic consumption of the home good, d^H. This subsidy discriminates between domestic agents and foreigners, as the latter pay the full price for the same good. In this case the effective terms of trade domestic agents face is $(1-\tau_H) P_t^H / P_t^F$, and the optimal subsidies are given by $1-\tau_H = \frac{\varepsilon^*-1}{\varepsilon^*}$ and $1-\tau_w = \frac{\varepsilon^N-1}{\varepsilon^N} \frac{\varepsilon^L-1}{\varepsilon^L} \frac{\varepsilon^*}{\varepsilon^*-1}$. These subsidies suggest that the planner fully exploits the monopolistic power of the economy while maintaining efficiency in the labor market. In the text I restrict τ_H to zero. Keeping in mind that the model is symmetric across countries, I interpret the subsidy system in the text as an internationally cooperative system that forbids protective tariffs, e.g. a system that is supported by trade agreements.

$$\Omega_{11} = \begin{bmatrix} \gamma_{cc} & \gamma_{cn} & \frac{\lambda\varepsilon(1-\lambda)}{(1-\varepsilon)(1-\lambda)-\varepsilon^*} \\ \gamma_{cn} & \frac{U_{N_{ss}}N_{ss}}{U_{C_{ss}}C_{ss}}\left(\gamma_{nn}+1-\alpha\right) & 0 \\ \frac{\lambda\varepsilon(1-\lambda)}{(1-\varepsilon)(1-\lambda)-\varepsilon^*} & 0 & \frac{\lambda\varepsilon(1-\lambda)}{(1-\varepsilon)(1-\lambda)-\varepsilon^*}\left\{\begin{array}{c}(2-3\lambda)\,\varepsilon-(2-\lambda) \\ +3\varepsilon^*-\frac{\varepsilon^*(1-\varepsilon^*)}{\varepsilon(1-\lambda)}\end{array}\right\} \end{bmatrix}$$

$$\Omega_{22} = (1-\vartheta)\frac{1}{C_{ss}}\frac{\varepsilon^*+\varepsilon\,(1-\lambda)}{(1-\varepsilon)\,(1-\lambda)-\varepsilon^*}\begin{bmatrix} \Theta''\,(0) & 0 \\ 0 & \Theta^{CB\prime\prime}\,(FX_{ss})\,FX_{ss}^2 \end{bmatrix}$$

$$\Omega_{x1} = \begin{bmatrix} \gamma_{c\eta} & \frac{U_{N_{ss}}N_{ss}}{U_{C_{ss}}C_{ss}}\gamma_{n\eta} & 0 \\ 0 & -\frac{U_{N_{ss}}N_{ss}}{U_{C_{ss}}C_{ss}} & 0 \\ 0 & 0 & \frac{\varepsilon\lambda(1-\lambda)}{(\varepsilon-1)(1-\lambda)+\varepsilon^*} \end{bmatrix}$$

The derivation of (60) is detailed in Appendix C. Optimal policies are derived by maximizing (60) while taking linearized equilibrium conditions as constraints. The derivation is detailed in Appendix D.

4 Parameter Values

Parameter values are chosen based on characteristics of the Israeli economy. A period in the model corresponds to one quarter. Values are mostly adopted from the parameterization of the Bank of Israel DSGE model, as reported in Argov et al. (2012), and steady state great ratios of main macro aggregates are calibrated to align with their first moment in the data. Table 1 summarizes the values of calibrated parameters.

Most important is the financial friction parameter, $\Theta''(0)$, as it governs the efficacy of FXIs. Also important are the parameters of the stochastic processes of the exogenous shocks, as they directly affect the second moments of the endogenous variables and hence welfare evaluation. I use Bayesian estimation to evaluate their values.

The sample period for calibration and estimation is mostly based on the decade after the global financial crisis and prior to the COVID-19 crisis, 2010-2019.[22]

[22] I focus on a relatively recent period due to significant structural changes the Israeli economy has gone through over the years. These changes include, among others, transitioning toward a market-based economy, overcoming hyperinflation in the 1980s and disinflation in the 1990s, absorbing a massive wave of immigration after the fall of the Soviet Union, and liberalizing the current account and financial sector.

4.1 Calibration

Symmetric Steady State. The calibration considers a symmetric steady state across countries, suggesting:

$$\begin{aligned}
TOT_{ss} &= 1 \\
EX_{ss} &= IM_{ss} = WT_{ss} \\
b_{ss}^{*,HH} &= 0 \\
\pi_{ss}^{H} &= \pi_{ss}^{F*} \\
\sigma_{ss} &= 1 \\
p_{ss}^{H} &= p_{ss}^{F} = 1 \\
\frac{1+i_{ss}}{\pi_{ss}} &= \frac{1+i_{ss}^{*}}{\pi_{ss}^{*F}} = \beta^{-1}
\end{aligned}$$

Production Function. As in Argov et al. (2012), the elasticity of output with respect to labor is set to two-thirds, i.e. $\alpha = 0.67$. Steady state productivity, A_{ss}, is normalized to unity. N_{ss} is calibrated to match the percent of time households allocate to market activities. Hours worked per employee are trendless at least since 2006, and during the decade prior to the COVID-19 crisis they averaged at 36.1 hours per week. Assuming time allocation of 16 hours per day, employees allocate around 32 percent of their time to work, suggesting $N_{ss} = 0.32$. Using the aggregate production function these values determine the steady state level of domestic production, Y_{ss}^{H}.

Great Ratios. I calibrate the government expenditure share, $\frac{G_{ss}}{Y_{ss}^{H}}$, to 0.3, and trade shares, $\frac{EX_{ss}}{Y_{ss}^{H}}$ and $\frac{IM_{ss}}{TOT_{ss}Y_{ss}^{H}}$, to 0.33. Note that since the model abstracts from capital formation, Y^{H} is interpreted as GDP net of investment. The values above match the sample averages for the period 2010-2019 after the adjustment for investment and imposing balanced trade in steady state. In the data the shares out of GDP are 0.54 for private consumption, 0.22 for fixed capital formation, and 0.22 for government consumption. The export share in the data is 0.32 and the import share is 0.34. Given these values and the steady state level of output, Y_{ss}^{H}, we can pin down G_{ss}, IM_{ss} (using $TOT_{ss} = 1$) and EX_{ss}. d_{ss}^{H} is then pinned down from the domestic resource constraint, equation (2).

Table 1: Calibrated Parameters and Steady State Values, Baseline Parameterization

Panel A: Steady State

Terms of trade	TOT_{ss}	1
Private sector net foreign asset position	$b_{ss}^{*,HH}$	0
Inflation	π_{ss}	$1.02^{1/4}$
Productivity	A_{ss}	1
Labor input	N_{ss}	0.32
Share of government expenditures in domestic output	$\frac{G_{ss}}{Y_{ss}^H}$	0.3
Shares of exports and imports in domestic output	$\frac{EX_{ss}}{Y_{ss}^H}, \frac{IM_{ss}}{TOT_{ss}Y_{ss}^H}$	0.33
Target level of reserves (30 percent of annual GDP)	$\frac{FX^T}{TOT_{ss}Y_{ss}^{H,An.}}$	0.3
Preference shock	η_{ss}	1
Risk premium shock	θ_{ss}^*	0

Panel B: Parameters

Elasticity of domestic output with respect to labor	α	0.67
Subjective discount factor	β	$1.025^{-1/4}$
EoS between home and foreign goods	ε	1.1
EoS between differentiated labor skills	ε^N	13/3
EoS between intermediate goods of the same country	ε^L	13/3
EoS between goods of different countries	ε^*	13/3
Probability of price adjustment	$1-\xi_p$	1/3
Probability of wage adjustment	$1-\xi_w$	0.25
Frisch elasticity of labor supply	$\left(\gamma_{nn} - \frac{\gamma_{nc}\gamma_{cn}}{\gamma_{cc}}\right)^{-1}$	2
Intertemporal EoS	$-\gamma_{cc}^{-1}$	1/3
Share of domestic ownership of the financial sector	ϑ	0.999
2nd derivative of the CB portfolio adjustment cost	$\Theta_{ss}^{CB\prime\prime}$	0.1
Interest rate rule: interest smoothing coefficient	θ_i	0.814
Interest rate rule: inflation coefficient	θ_π	2.538
Interest rate rule: output coefficient	θ_y	0.204

Openness. After solving for domestic uses, d_{ss}^H, and imports, IM_{ss}, the openness parameter, λ, is pinned down from their relative demand, using equations (8) and (9). Specifically, after using $p_{ss}^H = p_{ss}^F = 1$ we get:

$$\lambda = \frac{IM_{ss}}{IM_{ss} + d_{ss}^H}$$

which under the parameterization above yields $\lambda \cong 0.47$. Given λ and IM_{ss}, steady state consumption is pinned down using $C_{ss} = \frac{1}{\lambda} IM_{ss}$.

Discount Factor and Inflation. The subjective discount factor, β, takes the value $1.025^{-1/4}$, which corresponds to an annual steady state real interest rate of 2.5 percent. This is somewhat lower than the value in Argov et al. (2012), who calibrate the steady state real interest rate to 2.9 percent. Using the 10-15 years forward rate of CPI-indexed government bonds as an indicator for the long-run real rate, the data display a downward trend starting the early 2000s. In the period 2010-2019 it averaged 2.5 percent, though at the end of the sample period, during 2019, it averaged only 1.6 percent.

Steady state inflation is set at 2 percent, as it matches the mid range of the inflation target of the Bank of Israel. Home and foreign nominal interest rates are pinned down by the Fisher equation.

Since real prices are constant in steady state, we have:

$$\pi_{ss} = \pi^H_{ss} = \pi^F_{ss} = \pi^w_{ss} = \pi^{F*}_{ss}$$

Elasticities of Substitution (EoS). I adopt elasticities of substitution from Argov et al. (2012). The elasticity of substitution between home and foreign goods, ε, is set to 1.1, and the elasticities of substitution between labor skills, ε^N, intermediate goods, ε^L, and goods of different countries, ε^*, are set to $\frac{13}{3}$. This suggests a markup of 30 percent for home producers and labor suppliers. Following the solution for the efficient steady state allocation, the subsidy rate is set as suggested by equations (59), and real wage is pinned down using labor demand:

$$w_{ss} = \frac{1}{1-\tau_w}\frac{\varepsilon^L - 1}{\varepsilon^L}\alpha p^H_{ss} A_{ss} N^{\alpha-1}_{ss}$$

Price and Wage Stickiness. Ribon and Sayag (2013) conduct a micro-level study of the frequency of price adjustments in Israel during the period 1998-2011. Their study covers all CPI items excluding housing and fruit and vegetables. They report an average price duration of 9.3 months and a median of 7.5 months. I calibrate firms' probability of price adjustment to match an average duration of 9 months, suggesting $1 - \xi_p = \frac{1}{3}$. Using macro data, Argov et al. (2012) report a mode posterior estimate for this parameter of 0.394. Wages typically adjust more slowly to shocks than prices. I assume an average mean wage duration of one year, suggesting $1 - \xi_w = 0.25$. This value deviates substantially from Argov et al. (2012), as they report a mode posterior estimate of 0.544, which suggests that wages are more flexible than prices. My choice of wage adjustment probability is close to that of Smets and Wouters (2007), who estimate it at 0.27 for the American economy, and to Smets and Wouters (2003), who estimate it at 0.263 for the European economy.

Utility Parameters. I consider a standard additive separable utility function:

$$U(C, N; \eta) = \eta \left[\frac{C^{1-\gamma} - 1}{1-\gamma} - \psi \frac{N^{1+\nu}}{1+\nu} \right]$$

Generally, the Frisch elasticity of labor supply is given by $\left(\gamma_{nn} - \frac{\gamma_{nc}\gamma_{cn}}{\gamma_{cc}}\right)^{-1}$, which in this case is reduced to ν^{-1}. Following Argov et al. (2012), the Frisch elasticity is set to 2, suggesting $\nu = 0.5$. The steady state value of the preference shock, η_{ss}, is set to unity. The intertemporal elasticity of substitution (IES) is given by $-\gamma_{cc}^{-1}$, which under the specification of the utility function here equals $\frac{1}{\gamma}$. Havránek (2015) conducts a meta-analysis on reported estimates of the IES in 169 published papers. He concludes that the best calibrated value for the IES is around 0.3-0.4 and considers 0.8 as an upper bound. I use an elasticity of 1/3, that is $\gamma = 3$. Finally, ψ is pinned down using labor supply:

$$\psi = \frac{\varepsilon^N - 1}{\varepsilon^N} w_{ss} N_{ss}^{-\nu} C_{ss}^{\gamma}$$

Foreign Reserves. The target level of foreign reserves, FX^T, is set at 30 percent of annual GDP, which roughly equals the level of reserves in Israel during the decade preceding the COVID-19 crisis. This pins down the scale of foreign exchange interventions in steady state, and by symmetry across countries it also determines the size of capital inflows:

$$\phi_{ss}^* = \phi_{ss} = -\frac{1-\beta}{\beta} FX^T$$

The Central Bank's Adjustment Cost and Domestic Ownership of the Financial Sector. I assume the central bank faces a minor adjustment cost when operating in the foreign exchange markets, and set $\Theta^{CB\prime\prime}(FX_{ss}) = 0.1$. I also assume that the financial sector is owned entirely by domestic agents, suggesting ϑ approaches unity. Specifically I set $\vartheta = 0.999$.[23]

Setting ϑ close to unity allows studying the role of FXIs purely as a macroeconomic stabilization instrument. Importing financial intermediation services is costly for the economy, as financiers are making a profit, on average, due to the UIP premium, e.g. Cavallino (2019) and Fanelli and Straub (2021).[24] This cost is captured by the planner's welfare criterion, equation (60), as it decreases with the variance of $\widehat{b}_t^{*,HH} - \widehat{\theta}_t^*$.[25] Therefore, all else equal, closing the UIP premium improves welfare. Assuming that the financial sector is owned solely by domestic agents eliminates this term from the welfare criterion, and FXIs are motivated solely by their role as a macroeconomic stabilizer.

Interest Rate Rule. The main analysis in this paper assumes that monetary policy follows an optimal interest rate policy; in that case there is no need to specify an interest rate rule. Nevertheless, for the purpose of estimating the remaining parameters, it is useful to rely on an empirically relevant rule, regardless of whether it reflects optimal policy reaction. To that end I adopt the specification of Argov et al. (2012)[26]:

$$\frac{1+i_t}{1+i_{ss}} = \left(\frac{1+i_{t-1}}{1+i_{ss}}\right)^{\theta_i} \left[\left(\frac{\pi_{t-2}+\pi_{t-1}+\pi_t+E_t\left(\pi_{t+1}\right)}{4\pi_{ss}}\right)^{\theta_\pi} \left(\frac{Y_t^H}{Y_{ss}^H}\right)^{\theta_y}\right]^{1-\theta_i} \tag{61}$$

Following Argov et al. (2012), $\theta_i = 0.814$, $\theta_\pi = 2.538$ and $\theta_y = 0.204$.

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[23] In principle, I would like to set $\vartheta = 1$ and $\Theta^{CB\prime\prime}\left(FX_{ss}\right) = 0$. However, as demonstrated in Appendix D, under optimal FXIs this generates unit root dynamics in the social planner's Lagrange multiplier of the balance of payments. This problem is akin to the one of closing small open economy models, as studied in Schmitt-Grohé and Uribe (2003), but instead of having unit root dynamics in the marginal utility of consumption of households it rises in the "marginal utility" of the social planner.

[24] Amador et al. (2020) raise a similar argument regarding the cost of deviations from the covered interest rate parity.

[25] Note that the variance of $\widehat{b}_t^{*,HH} - \widehat{\theta}_t^*$ is proportional to the variance of the UIP premium, and that its coefficient in (60) is negative. This term enters the welfare criterion through the cost of importing financial intermediation services in the balance of payments. See Appendix C.

[26] The interest rate rule in Argov et al. (2012) also includes a reaction to movements in the nominal exchange rate, though with a small coefficient. I omit it from the specification of (61) because, in this paper, policy reaction to developments in the external sector takes a central role through FXIs, which are not included in Argov et al. (2012).

4.2 Bayesian Estimation: Adjustment Costs and the Exogenous Processes

The portfolio adjustment cost parameter, $\Theta''(0)$, is the single most important parameter in the model, as it governs the efficacy of FXIs. Also important are the parameters governing the stochastic process of the exogenous shocks, as they directly affect the relative importance of the shocks in welfare evaluation.

This section employs a Bayesian technique to estimate these parameters. For the estimation I assume that the interest rate follows policy rule (61), adopted from Argov et al. (2012), and that foreign reserves follow an exogenous auto-regressive process. This enables the estimation to rely on empirically relevant processes, without the presumption that policy was conducted optimally during the sample period.

This section describes the choice of prior distributions, and then discusses the dataset for the estimation and the assumptions regarding measurement errors. Technical details regarding the estimation and comparison of prior and posterior distributions are presented in Appendix F. Table 2 presents the estimation results. I use posterior modes as parameter values for the rest of the analysis.

4.2.1 Exogenous Processes

The exogenous variables in the model are productivity, $\widetilde{A}_t$, government expenditure, $\widetilde{G}_t$, the preference shock, $\widetilde{\eta}_t$, world trade, $\widetilde{WT}_t$, capital inflows, $\widehat{\phi}_t^* - \widehat{\phi}_{ss}^*$, and the risk premium shock, $\widehat{\theta}_t^*$. For the purpose of estimation, foreign reserves, $\widetilde{FX}_t$, also follow an exogenous process.

All exogenous variables follow a first-order auto-regressive process of the form:

$$\begin{aligned} X_t &= \rho_X X_{t-1} + \epsilon_t^X \qquad , \qquad \epsilon_t^X \overset{iid}{\sim} N\left(0, \sigma_X^2\right) \\ \text{where} \qquad X_t &\in \left\{\widetilde{A}_t, \widetilde{G}_t, \widetilde{\eta}_t, \widetilde{WT}_t, \widehat{\phi}_t^* - \widehat{\phi}_{ss}^*, \widehat{\theta}_t^*, \widetilde{FX}_t\right\} \end{aligned}$$

I use the beta distribution as prior for the persistence parameters, ρ_X, restricting their value between 0 and 1, and the inverse gamma distribution for the variance of the shocks, σ_X^2. To form priors, I estimate an auto-regressive process for (detrended) productivity[27],

[27] Productivity is measured as $\log(GDP_t) - \alpha \log(N_t)$, where GDP_t is gross domestic product in fixed

$\widetilde{A}_t$, government expenditure, $\widetilde{G}_t$, world trade, $\widetilde{WT}_t$, capital inflows, $\widehat{\phi}_t^* - \widehat{\phi}_{ss}^*$, and foreign reserves, $\widetilde{FX}_t$, using quarterly data for the period $2010-2019$. The estimation is carried out for each series separately, and I use the point estimates and their standard deviation as prior modes and standard deviations of each prior distribution, respectively.[28] For the unobserved processes of the preference shock, $\widetilde{\eta}_t$ and the risk premium shock, $\widehat{\theta}_t^*$, I adopt as prior the estimation results of Argov et al. (2012).[29] Table 2 summarizes the results.

4.2.2 The Portfolio Adjustment Cost

To shape a prior for the distribution of the portfolio adjustment cost parameter, $\Theta''(0)$, I rely on estimates for the effectiveness of the Bank of Israel's FXIs. Ribon (2017), Hertrich and Nathan (2022), and Caspi et al. (2022) find that FXIs have a statistically and economically significant effect on the New Israeli Shekel (NIS) nominal effective exchange rate, at least at the time of the interventions. Ribon (2017) uses data in monthly frequency and estimates the effect of FXIs on the exchange rate using various instrumental variables and specifications. She finds that a purchase of $1 billion by the Bank of Israel is associated with a depreciation of about 0.72 percent of the NIS exchange rate; she also finds little evidence for the erosion of the effect over time. Hertrich and Nathan (2022) use data in daily frequency and estimate the effect using instrumental variables in GMM. They find that a purchase of $1 billion by the Bank of Israel is associated with a depreciation of about 0.82 percent of the Shekel. While the authors report that the point estimate of the effect remains stable over time, it is statistically significant for only 5 trading days. Caspi et al. (2022) estimate the effect of unexpected FXIs on the exchange rate. They identify policy shocks using high frequency, intraday, data and then employ local projection methods to estimate the effect of these shocks on the exchange rate in daily frequency.[30] They find that a typical daily policy surprise

prices and N_t is total hours worked, both seasonally adjusted.

[28] For measurement and data source of each series see Appendix E.

[29] The standard deviation of the risk premium shock is divided by $\Theta''(0)\frac{1}{Y_{ss}^{H,An.}}$ in order to account for the coefficient multiplying it in the UIP equation, because in Argov et al. (2012) that coefficient is 1.

[30] Using minute-by-minute USD-ILS quotes and records from the Bank of Israel dealing-room, Caspi et al. (2022) measure daily policy surprises as the movement of the exchange rate during an "intervention window", just before the intervention starts and immediately after the last intervention transaction on

Table 2: Prior and Posterior Distributions of Estimated Parameters[1]

	Prior Distribution				Posterior Distribution				
	Type	Mode	STD	Implied Mean	Mode	STD	Mean	HPD Interval[2] 5%	HPD Interval[2] 95%
Panel A: Portfolio Adjustment Cost									
2nd derivative of PAC function, $\Theta''(0)$	Inv. Γ	6.350	3.175	8.057	2.569	0.683	2.834	1.7763	3.8435
Panel B: Autocorrelation of exogenous variables									
Productivity, ρ_A	Beta	0.645	0.146	0.616	0.640	0.137	0.618	0.3992	0.8379
Preference shock, ρ_η	Beta	0.782	0.241	0.602	0.657	0.187	0.585	0.2818	0.8737
Government exp., ρ_G	Beta	0.274	0.148	0.324	0.578	0.093	0.571	0.4179	0.7247
World trade, ρ_{WT}	Beta	0.723	0.095	0.703	0.832	0.053	0.824	0.7380	0.9116
Risk premium, $\rho_{\widehat{\theta}^*}$	Beta	0.582	0.105	0.574	0.858	0.055	0.834	0.7475	0.9220
Capital inflows, $\rho_{\widehat{\phi}^*}$	Beta	0.319	0.144	0.355	0.150	0.066	0.169	0.0604	0.2733
Foreign Reserves, ρ_{FX}	Beta	0.913	0.068	0.876	0.868	0.042	0.863	0.7941	0.9329
Panel C: Standard deviation of exogenous shocks[3]									
Productivity, σ_A	Inv. Γ	0.011	0.001	0.011	0.010	0.001	0.010	0.0086	0.0115
Preference shock, σ_η	Inv. Γ	0.012	0.006	0.015	0.020	0.004	0.019	0.0126	0.0257
Government exp., σ_G	Inv. Γ	0.007	0.001	0.007	0.007	0.001	0.007	0.0065	0.0084
World trade, σ_{WT}	Inv. Γ	0.008	0.001	0.009	0.009	0.001	0.009	0.0079	0.0102
Risk premium, $\sigma_{\widehat{\theta}^*}$	Inv. Γ	0.002	0.0004	0.003	0.006	0.001	0.006	0.0053	0.0077
Capital inflows, $\sigma_{\widehat{\phi}^*}$	Inv. Γ	0.005	0.001	0.005	0.006	0.0004	0.006	0.0049	0.0063
Foreign Reserves, σ_{FX}	Inv. Γ	0.018	0.002	0.018	0.018	0.001	0.018	0.0158	0.0204
Panel D: Standard deviation of measurement errors[3]									
GDP	Inv. Γ	0.003	0.002	0.004	0.005	0.001	0.005	0.0032	0.0065
Private Consumption	Inv. Γ	0.005	0.002	0.006	0.004	0.001	0.005	0.0031	0.0065
Exports	Inv. Γ	0.016	0.008	0.021	0.024	0.003	0.025	0.0196	0.0298
Imports	Inv. Γ	0.015	0.008	0.019	0.023	0.003	0.024	0.0192	0.0277
Hours worked	Inv. Γ	0.010	0.005	0.013	0.011	0.003	0.012	0.0071	0.0160
Nominal interest rate	Inv. Γ	0.0003	0.0002	0.0004	0.0004	0.0001	0.0005	0.0003	0.0006
CPI inflation	Inv. Γ	0.003	0.001	0.003	0.005	0.001	0.005	0.0038	0.0058
Nominal depreciation	Inv. Γ	0.015	0.007	0.019	0.022	0.003	0.023	0.0188	0.0269
Terms of trade	Inv. Γ	0.010	0.005	0.012	0.016	0.002	0.016	0.0131	0.0187
Private sector net foreign assets	Inv. Γ	0.013	0.007	0.017	0.022	0.002	0.023	0.0190	0.0270

Notes: (1) Sample period 2010-2019. See Section 4.2 for the choice of priors, observables, and sample period. See Appendix F for technical details regarding the estimation and comparison of prior and posterior distributions. (2) HPD = Highest Posterior Density. (3) Under the prior the variance of each shock follows an inverse gamma distribution, not its standard deviation.

depreciates the NIS by approximately 0.4 percent, and that the effect remains significant for $40 - 60$ days. Cumulating the estimated effect of the interventions starting in 2010 to the end of their sample, suggests that in Caspi et al. (2022) a purchase of \$1 billion is associated, on average, with a depreciation of about 0.94 percent of the NIS exchange rate.[31]

Converting these results to the units of the model, I assess that the effect of a one standard deviation shock to foreign reserves on the exchange rate is about 1.4 percent in Hertrich and Nathan (2022) and in Caspi et al. (2022), and about 1.0 percent in Ribon (2017).[32] That said, given that the effects in Hertrich and Nathan (2022) and Caspi et al. (2022) lose statistical significance during the quarter, their estimates may overstate the effect of FXIs in quarterly frequency. As a benchmark value, I assume that a typical intervention generates a 1.0 percent movement in the exchange rate. In order to get a sense of the order of magnitude of $\Theta''(0)$, I search for its value such that a one standard deviation shock to foreign reserves in the model generates a 1.0 percent depreciation on impact. The resulting value, given the parameterization under the prior modes of the exogenous processes, is approximately 6.35. I use this value as the prior mode for the distribution of $\Theta''(0)$, and a value half that size for its standard deviation. Since $\Theta''(0)$ is restricted to take positive values, I use the inverse gamma as the prior distribution. These choices are summarized in Table 2.

4.2.3 Data and Measurement Errors

As observable variables I use data on GDP (log), private consumption (log), government consumption (log), exports (log), imports (log), total hours worked (log), the return on Bank of Israel 3-month unindexed bill ("Makam", quarterly average), CPI inflation rate (quarter average over quarter average), nominal effective depreciation rate (quarter average over quarter average), the terms of trade (log), net private holdings of foreign

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that day.

[31] I thank the authors for providing the information necessary for converting their results to units of exchange rate movement per \$1 billion of intervention.

[32] In the sample of Hertrich and Nathan (2022) \$1 billion is about 1.0 percent of foreign reserves, in Caspi et al. (2022) it is about 1.2 percent, and in Ribon (2017) it is 1.3 percent. Under the prior, I estimate the standard deviation of the shock to FX_t at about 1.8 percent (Table 2).

assets (relative to trend GDP), foreign reserves (log), world trade (log), and capital inflow to public-sector financial instruments (relative to trend GDP). For the exact definition and data source of each variable see Appendix E.

All series are first-differenced and demeaned. Measurement errors are assigned to all endogenous variables. For the prior distributions of the standard deviations of the measurement errors, I assume that their variance follows the inverse gamma distribution and that they account for one-third of the variation in the data. The exogenous variables ($\widetilde{A}_t$, $\widetilde{G}_t$, $\widetilde{WT}_t$, $\widehat{\phi}_t^*$ and $\widetilde{FX}_t$) are not assigned measurement errors, as these may generate weak identification of the standard deviations of the shocks in their auto-regressive process.

The sample period is the first quarter of 2010 until the fourth quarter of 2019. Toward the end of 2009 the Bank of Israel changed its FXI policy, and moved from purchasing pre-announced daily quantities to discretionary interventions that respond to market conditions.[33] The latter better reflects the role of FXIs in the model, as they are used as a tool for stabilizing the economy against unexpected shocks. I therefore start the sample at the beginning of 2010. The sample ends just before the COVID-19 crisis broke out, which aside from introducing unprecedented economic volatility also triggered, at the beginning of 2021, a large pre-announced program to purchase foreign reserves.[34]

5 The Transmission Mechanism of FXIs

To study the transmission mechanism of FXIs, it is useful to start by focusing on the natural equilibrium, i.e. the equilibrium in an economy with no nominal rigidities. Clearly, nominal rigidities have quantitative effect on equilibrium outcomes, but qualitatively, as shown below, they have little bearing on the impact of FXIs. To further simplify the analysis assume that fluctuations in foreign reserves are white noise:

$$\widetilde{FX}_t = \epsilon_t^{FX} \qquad , \qquad \epsilon_t^{FX} \sim WN$$

Assuming that foreign reserves are exogenous allows analyzing their impact on the economy without concern of feedback effects, from the economy to policy, that complicate the

[33] See Bank of Israel (2010).

[34] See Bank of Israel (2022).

identification of cause and effect. Endogenizing the policy response is the subject of the next section. Assuming a white noise process reveals the degree of persistence the model generates endogenously.

I start by establishing how $\widehat{b}_t^{*,HH}$, the private sector's holdings of foreign assets, comoves with some key variables; and then, building on the results, turn to studying how fluctuations in foreign reserves are transmitted to the economy. The results are summarized in Figure 2. The figure presents the impulse response functions of the system to a temporary rise of 1 percent in foreign reserves.[35]

5.1 The Comovement of $\widehat{b}_t^{*,HH}$ with Key Variables

The considerations regarding the exposure to foreign assets are summarized by the Euler equation for foreign bonds, equation (27). With some manipulation, using the consumption aggregator (5), imports demand (9), and the law of one price (11), it reads:

$$\begin{aligned} U_{IM_t} &= \beta \frac{1+i_t^*}{1+\frac{1}{TOT_{ss}Y_{ss}^{H,An.}}\Theta'\left(\widehat{b}_t^{*,HH}-\widehat{\theta}_t^*\right)} E_t\left\{\frac{U_{IM_{t+1}}}{\pi_{t+1}^{F*}}\right\} \qquad (62)\\ \text{where} \qquad U_{IM_t} &= U_{C_t}\frac{\partial C_t}{\partial IM_t} = U_{C_t}\left(\frac{\lambda C_t}{IM_t}\right)^{\frac{1}{\epsilon}} = U_{C_t}p_t^F \end{aligned}$$

In this form, the Euler equation is written in terms of the marginal utility of imports. Since the second derivative of the adjustment cost is positive, a rise in $\widehat{b}_t^{*,HH}$ reduces the effective return on foreign assets. To maintain the equality, the current marginal utility of imports falls and the expected marginal utility rises. This suggests that changes in $\widehat{b}_t^{*,HH}$ are associated with an intertemporal shift in the consumption of foreign goods. Specifically, a rise in $\widehat{b}_t^{*,HH}$ raises IM_t, or more generally, $\widehat{b}_t^{*,HH}$ and IM_t are positively correlated, at least to the extent that the exogenous variables in (62) remain unchanged.[36]

[35] For the natural equilibrium Figure 2 uses $\xi_w = 0$ and $\xi_p = 0.01$, with the nominal interest rate following the optimal plan. The deviation from pure flexible prices supports a unique equilibrium outcome for the nominal quantities. Nevertheless, the equilibrium allocation of the real variables is practically identical to that of a pure real economy.

[36] This argument slightly abuses the notation U_{IM_t} as if the marginal utility depends solely on imports. However, since the utility function is non-separable in home and foreign goods, this is not the case. Nevertheless, when total consumption and the terms of trade move in the same direction, as is the case here – see Figure 2, income and substitution effects work in the same direction on import demand, and the argument in the text holds.

The terms of trade, TOT_t, and consumption, C_t, also comove positively with imports, IM_t.[37] Intuitively, while a rise in the terms of trade shifts consumption away from home goods and toward imported ones, it also reduces exports demand, thereby clearing resources for domestic uses and moderating any reduction in the consumption of home goods. Overall, the rise in consumption of foreign goods, IM_t, dominates the effect on total consumption, C_t, resulting in a positive comovement between IM_t, TOT_t and C_t. The effect on the consumption of home goods, d_t^H, generally depends on the elasticity of substitution between home and foreign goods, ε. For a large enough ε they move in opposite directions. In our case, ε is close to unity and the consumption of home and foreign goods move in the same direction.

To sum up, we conclude that fluctuations in $\widehat{b}_t^{*,HH}$ induce positive comovement with IM_t, TOT_t and C_t.

5.2 The Transmission Mechanism of FXIs

With the conclusion that $\widehat{b}_t^{*,HH}$, IM_t and TOT_t are positively correlated, we now turn to evaluating the direction in which they move in response to a rise in foreign reserves. To

[37] Using $\widetilde{p}_t^H \cong \lambda \widetilde{TOT}_t$, demand for consumption of home goods, equation (45), demand for exports, equation (47), and technology, equation (48), the resource constraint reads:

$$\widetilde{A}_t + \alpha \widetilde{N}_t \cong (1-\lambda) \frac{C_{ss}}{Y_{ss}^H} \widetilde{C}_t - [(1-\lambda)\varepsilon + \varepsilon^*] \lambda \frac{C_{ss}}{Y_{ss}^H} \widetilde{TOT}_t + \frac{G_{ss}}{Y_{ss}^H} \widetilde{G}_t + \lambda \frac{C_{ss}}{Y_{ss}^H} \widetilde{WT}_t$$

Also, by setting $\xi_w = \xi_p = 0$ in equations (40) and (41), the labor market equilibrium condition reads:

$$\widetilde{U}_{N_t} - \widetilde{U}_{C_t} \cong \lambda \widetilde{TOT}_t + \widetilde{A}_t + (\alpha - 1) \widetilde{N}_t$$

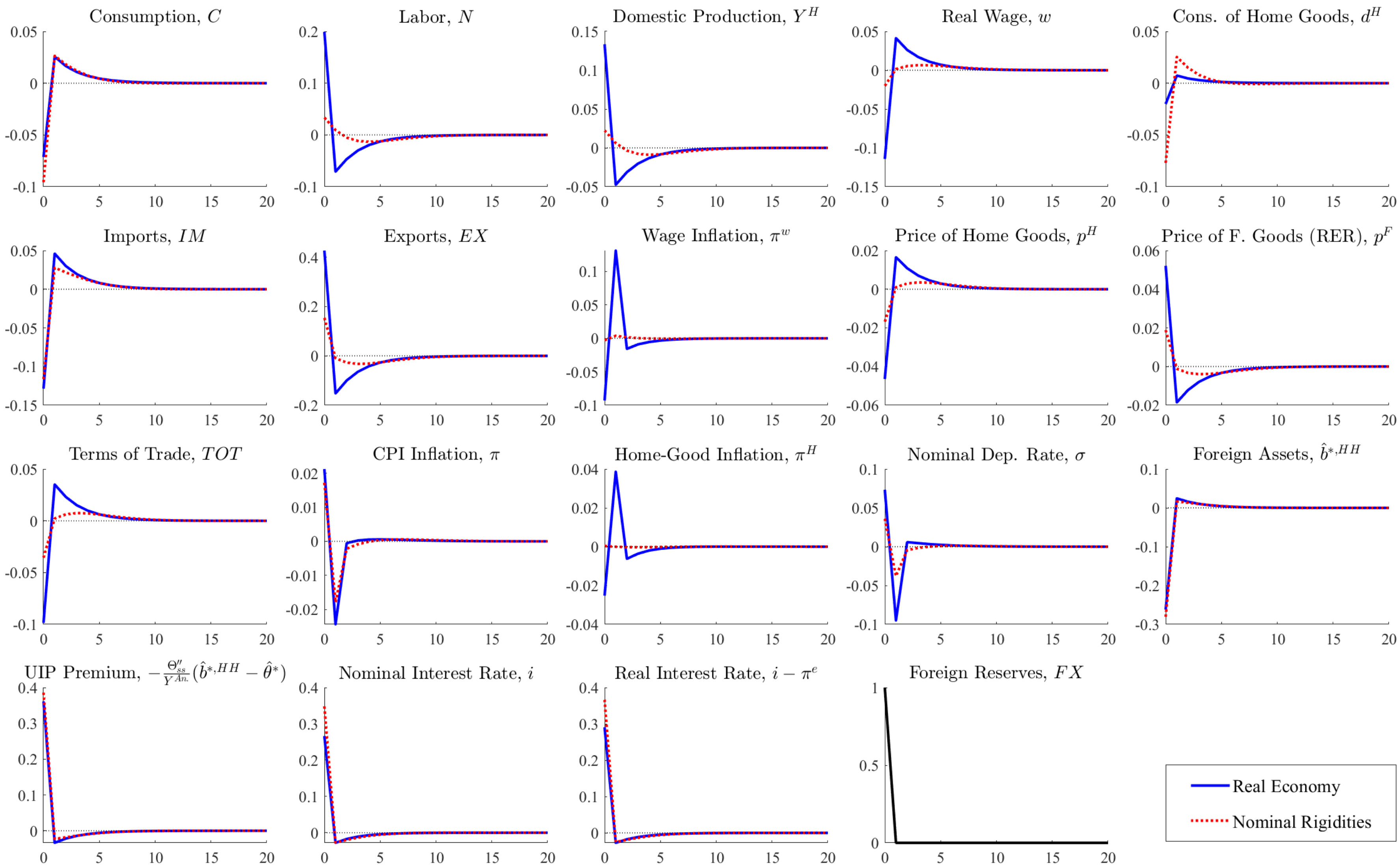
Holding the exogenous variables constant, these equations suggest that the terms of trade, $\widetilde{TOT}_t$ and consumption, $\widetilde{C}_t$, are proportional to each other. A sufficient condition for this result is that $\gamma_{nn} - \gamma_{cn} \geq 0$ and $\gamma_{nc} - \gamma_{cc} \geq 0$. This condition clearly holds for additive-separable preferences in consumption and labor ($\gamma_{cn} = \gamma_{nc} = 0$), but it also holds for other standard utility function such as Cobb-Douglas and GHH.

Finally, using $\widetilde{p}_t^F \cong -(1-\lambda) \widetilde{TOT}_t$, imports demand, equation (46), reads:

$$\widetilde{IM}_t \cong \widetilde{C}_t + \varepsilon (1-\lambda) \widetilde{TOT}_t$$

We therefore conclude that $\widetilde{C}_t$ and $\widetilde{TOT}_t$ comove positively with imports, $\widetilde{IM}_t$. The comovement with labor, $\widetilde{N}_t$, generally depends on the specification of preferences. Under additive-separable preferences, as considered here, the wealth effect on labor supply dominates, and labor comoves negatively with consumption.

Figure 2: The Transmission of FXIs. Response to a 1 Percent Rise in Foreign Reserves, Nominal Rigidities vs the Natural Equilibrium (100 × log points)



Note: In both cases the central bank follows optimal interest rate policy. Under the natural equilibrium $\xi_p = 0.01$ and $\xi_w = 0$. A slight deviation from pure flexible prices is required in order to support a unique equilibrium path for the nominal variables.

that end, observe the balance of payments equation:

$$\frac{FX_{ss}}{Y_{ss}^{H,An.}}\widetilde{FX}_t+\widehat{b}_t^{*,HH}\cong-\frac{\lambda C_{ss}}{Y_{ss}^{H,An.}}\left[(\varepsilon^*-1)\,\widetilde{TOT}_t+\widetilde{IM}_t\right]+EXOG_t \tag{63}$$

where $EXOG_t$ summarizes all variables that do not respond to fluctuations in foreign reserves (exogenous and predetermined variables). The balance of payments equation makes clear that the resources for raising foreign reserves can either come from the financial portfolio of the private sector, $\widehat{b}_t^{*,HH}$, and/or from increasing net exports. In equilibrium, $\widehat{b}_t^{*,HH}$ falls and net exports rises, as we have concluded that $\widehat{b}_t^{*,HH}$, IM_t and TOT_t must move in the same direction. A rise in FX_t crowds out $\widehat{b}_t^{*,HH}$ which, in turn, raises the effective return on foreign assets and reduces imports through the Euler equation for foreign bonds. The fall in imports is associated with a fall in the terms of trade, as described above in Section 5.1. Technically, a rise in FX_t raises the left-hand side of the balance of payments, and equilibrium is restored by reducing $\widehat{b}_t^{*,HH}$, which moderates the rise of the left-hand side, and by a fall in IM_t and TOT_t, which raises the right-hand side. Finally, note that the reduction in the terms of trade raises exports, so that the rise in *net* exports comes about through both a rise in exports and a reduction in imports.[38]

Given the results thus far, we conclude that a rise in foreign reserves crowds out private sector holdings of foreign assets, raises the effective return on foreign assets, reduces the terms of trade, imports and total consumption, and raises exports. The effect on consumption of home goods generally depends on ε. Note also that the fall in total consumption implies, by the Euler equation for domestic bonds, that the real return in terms of the final good rises as well.

In the labor market, the fall in the terms of trade reduces labor demand, since the value of the marginal product, measured in units of the final consumption good, falls. At the same time, labor supply, as measured by the marginal rate of substitution between labor and consumption, rises due to the fall in consumption, although this result depends

[38] Note that in the balance of payments net exports is expressed in units of foreign goods, hence the rise in exports is moderated by the fall in the terms of trade. For $\varepsilon^*>1$, i.e. when foreigners' elasticity of substitution between home and foreign goods is high, as is the case here, export demand is very responsive to the terms of trade and exports rises even when measured in units of foreign goods; for $\varepsilon^*=1$ the rise in demand and the valuation effects cancel out; and for $\varepsilon^*<1$ the valuation effect dominates. Nevertheless, in all cases the fall in imports is sufficiently large to induce a rise in net exports.

on the specification of the utility function. With GHH preferences, labor supply remains unchanged. Putting these together, real wage must fall and the effect on labor is ambiguous. Given the specification of the utility function here, with additive separability in consumption and labor, the rise in labor supply dominates, and labor effort rises. The rise in labor raises domestic production. However, with GHH preferences labor effort falls slightly, as does output (not shown).[39]

In the period immediately after the shock all effects are reversed. This follows directly from the conclusion that the rise in foreign reserves triggers an intertemporal substitution in consumption through its effect on real returns, and from the fact that the shock lasts for only one period. The model then generates modest persistence, and the effects die out after about 6 quarters. The persistence is generated by gradual adjustment of $\widehat{b}_t^{*,HH}$ as agents wish to smooth the portfolio adjustment cost over time.

Finally, recall that under the natural allocation monetary policy is neutral, and the dynamics of nominal prices are of no importance for the real variables. Nevertheless, note that the rise in foreign reserves raises demand for foreign currency and depreciates the domestic currency, i.e. σ rises, as one may anticipate.

To sum up, the transmission mechanism of FXIs works through crowding out private sector holdings of foreign assets, thereby affecting the UIP premium, the return domestic agents face in the financial markets and the exchange rate. The purchase of foreign reserves depreciates the domestic currency and reduces the relative price of home goods, hence stimulating exports demand. On the other hand, it contracts the demand of domestic agents, as it raises the effective real return they face and makes imported goods more expensive. On net, the effect on domestic production is ambiguous, and depends on the wealth effect on labor supply. Under the current parameterization, with additive-separable preferences, the effect on production is expansionary.

[39] Under GHH preferences the utility function is given by:

$$U^{GHH}\left(C_t, N_t; \eta_t\right) = \eta_t \frac{\left[C_t - \psi \frac{N_t^{1+\nu}}{1+\nu}\right]^{1-\gamma} - 1}{1-\gamma}$$

Parameter values are chosen to match the Frisch elasticity of labor supply and intertemporal elasticity of substitution as in Table 1. ψ is then pinned down from the steady state equilibrium condition in the labor market.

5.3 The Transmission with Nominal Rigidities

Now consider the economy with nominal rigidities. In this case monetary policy matters, and I assume the central bank follows the optimal policy. Qualitatively, the model generates dynamics that are similar to those of the natural allocation (the dotted red lines in Figure 2). First notice that π^H and π^w hardly move in this exercise. This is because the shock lasts for only one period, and with little ability to readjust prices and wages in the near future, there is little motive to change them in response to shocks. Second, the rise in foreign reserves depreciates the domestic currency, raises the price of foreign goods and lowers the terms of trade. Note that the optimal monetary policy counteracts this effect by raising the interest rate more aggressively relative to the case of the natural allocation, which also results in a higher *real* interest rate. The fall in the terms of trade is therefore moderated relative to the natural allocation, and the fall in consumption is sharper. These effects amplify the fall in home consumption of domestic goods, and approximately balance each other in their effect on imports. In addition, the moderate fall in the terms of trade results in a smoother path for exports.

In the labor market, real wage in units of consumption, w_t, falls since nominal wages are sticky and the depreciation raises consumption prices. However, the reduction in real wage is muted relative to its movement under the natural allocation, both because labor demand falls by less (following a smaller reduction in TOT_t) and because the rise in labor supply (following the fall in C_t) affects labor market equilibrium only for the fraction of households that can readjust their wage. As a result, real wage and labor effort do not move as much as under the natural allocation. The modest rise in labor suggests only a minor expansion in production.[40] Note that the small increase in supply is matched by a small movement in aggregate demand for home goods, as exports rise only moderately and home demand for domestic goods falls.

Finally, recall that the exogenous rise in foreign reserves crowds out $\widehat{b}_t^{*,HH}$, raises the UIP premium and depreciates the domestic currency. The optimal monetary policy alleviates the depreciation pressures by raising the interest rate rather than by moderating

[40] Here as well, the result is sensitive to the specification of preferences. With GHH utility, labor and production fall slightly.

the rise in the UIP premium. To stabilize the UIP premium, $\widehat{b}_t^{*,HH}$ must rise back towards its steady state level. However, the balance of payments, equation (63), dictates that to that end the rise in foreign reserves must be absorbed by higher net exports, while the Euler equation for foreign bonds, equation (62), suggests that a higher path for $\widehat{b}_t^{*,HH}$ must be associated with higher imports, which reduces net exports. It therefore seems that the monetary interest rate is an improper tool for stabilizing the UIP premium.

6 The Optimal Allocation: Response to Shocks

This section studies the optimal FXI policy and the economy's equilibrium path in reaction to each exogenous shock. In particular, it compares the economy's response under optimal FXI policy to that of an economy where foreign reserves are constrained to remain unchanged. In both cases monetary policy sets the interest rate optimally. This comparison helps in evaluating the role of FXIs over and above that of traditional monetary policy. Appendix D characterizes the equilibrium condition under each allocation. Based on the results, the next section proposes an implementable FXI rule that attempts to bring the equilibrium allocation close to the optimal one.

6.1 Capital Inflow Shock, $\widehat{\phi}_t^*$

Capital inflow shocks are indistinguishable from FXI shocks, except that they work in the opposite direction. To see that, recall the law of motion for foreign reserves, equation (37). After first-order approximation it reads:

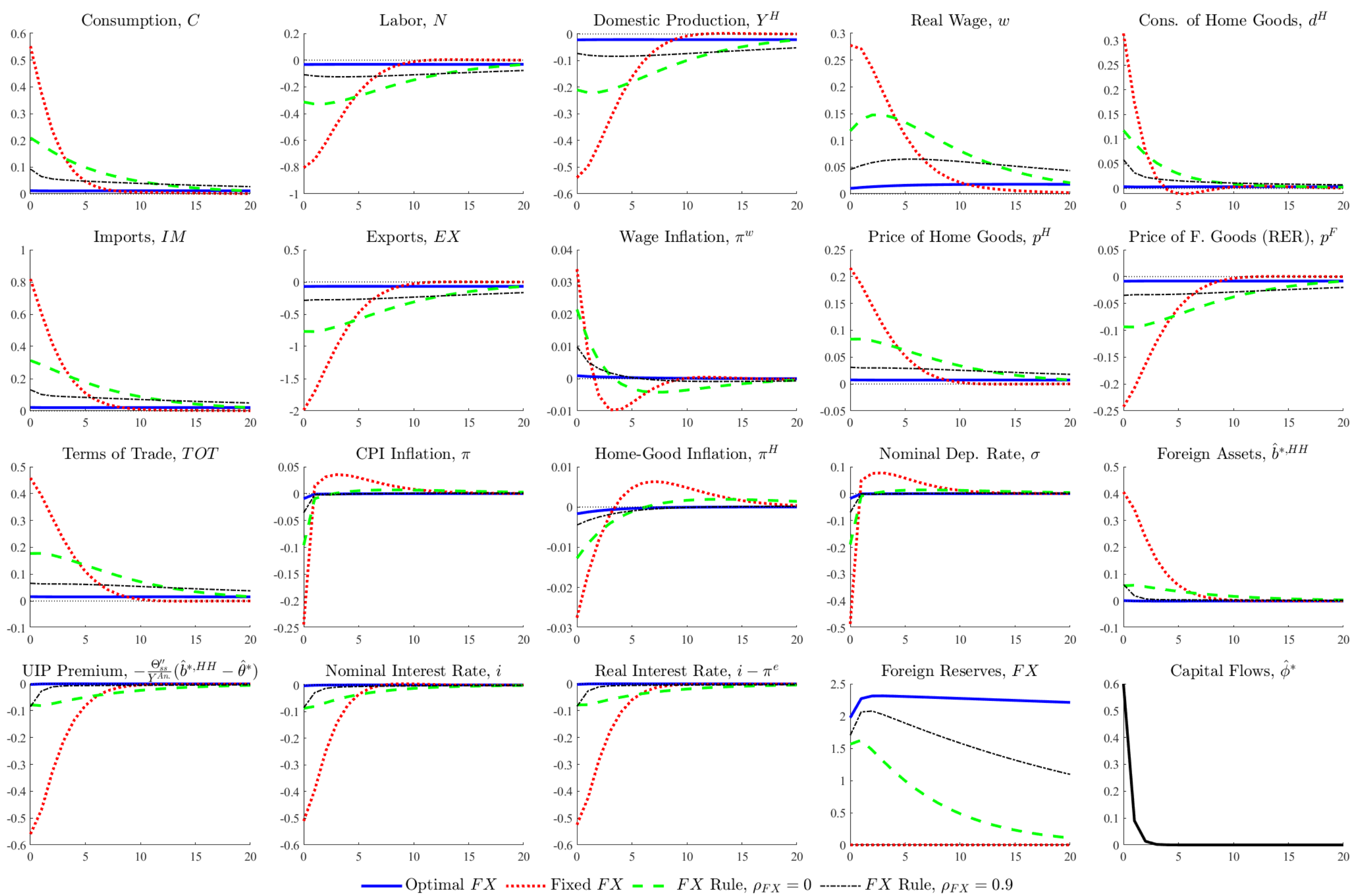
$$FX_{ss}\left(\widetilde{FX}_t - \beta^{-1}\widetilde{FX}_{t-1}\right) \cong Y_{ss}^{H,An.}\left(\widehat{\phi}_t - \widehat{\phi}_{ss}\right) + \beta^{-1}FX_{ss}\left[\widetilde{\left(1+i_{t-1}^*\right)} - \widetilde{\pi}_t^{F*}\right]$$

Substituting into the balance of payments, equation (50), we get:

$$\left(\widehat{\phi}_t - \widehat{\phi}_{ss}\right) + \widehat{b}_t^{*,HH} \cong \beta^{-1}\widehat{b}_{t-1}^{*,HH} + \left(\widehat{\phi}_t^* - \widehat{\phi}_{ss}^*\right) + \frac{\lambda C_{ss}}{Y_{ss}^{H,An.}}\left(\widetilde{TOT}_t + \widetilde{EX}_t - \widetilde{IM}_t\right)$$

From this formulation it is clear that a shock to capital inflows, $\widehat{\phi}_t^*$, is equivalent to a shock of the same magnitude but with the opposite sign to FXIs, $\widehat{\phi}_t$, as both enter the system only through the balance of payments equation. Therefore, when foreign reserves are exogenous, the impulse response functions of a capital inflow shock are similar, but with

Figure 3: Response to a 1 Standard Deviation Shock in Capital Inflows (100 × log points)



the opposite sign, to those of a shock to foreign reserves, which we have just analyzed in Section 5. The only difference is that here we allow for an exogenous inertia. The impulse response functions for the capital inflow shock are depicted in Figure 3.

Start with the case of fixed foreign reserves. In order to comply with the balance of payments, a surge in capital inflow must be absorbed either by the private sector's foreign assets position, $\widehat{b}_t^{*,HH}$, and/or by generating a trade deficit. The impulse response functions suggest that both $\widehat{b}_t^{*,HH}$ rises and net export falls. This is consistent with the analysis of exogenous foreign reserves, which suggested that $\widehat{b}_t^{*,HH}$ and IM_t must move in the same direction. The rise in $\widehat{b}_t^{*,HH}$ lowers the effective return from foreign assets, thereby shifting consumption of foreign goods, IM_t, from the future to the present.

As analyzed in Section 5, the rise in imports is accompanied by an increase in aggregate consumption, C_t, and an improvement in the terms of trade, TOT_t. The rise in the terms of trade is supported by the appreciation of the domestic currency, due to the capital inflow and sticky home prices. The effect on the consumption of home goods, d_t^H, is generally ambiguous, as the rise in total consumption raises the demand for home goods while the improvement in the terms of trade lowers it. When the elasticity of substitution between home and foreign goods, ε, is high enough, d_t^H will tend to move in the opposite direction to IM_t. In our case ε is close to unity and home and foreign consumption comove in the same direction. The rise in the terms of trade, TOT_t, reduces exports, EX_t, and since under sticky prices production is demand-determined, output, Y_t^H, and labor, N_t, fall. Lower demand also reduces home inflation, π_t^H. These effects trigger a monetary expansion by reducing the nominal interest rate, which moderates the effects.

Now turn to optimal FXI policy. Clearly, since the shock to capital inflows, $\widehat{\phi}_t^*$, is equivalent to a shock, with the opposite sign, to FXIs, $\widehat{\phi}_t$, the central bank can neutralize it by absorbing the capital inflows through raising foreign reserves. Therefore, the optimal FXI policy is able to insolate the economy from the effects of capital inflow shocks. The foreign capital is absorbed by foreign reserves, and all other variables in the system are practically unchanged. These results suggest that FXI is a superior policy tool in reacting to exogenous foreign capital flows relative to traditional monetary policy.

6.2 Risk Premium Shock, $\widehat{\theta}_t^*$

Figure 4 presents the impulse response functions to a one standard deviation rise in the risk premium shock, $\widehat{\theta}_t^*$. Again, start by focusing on the impulses of the benchmark model with fixed foreign reserves. A rise in the risk-premium increases the effective return from foreign assets, thereby shifting consumption of foreign goods, IM_t, from the present to the future, and hence IM_t falls. The improvement in the trade balance allows, through the balance of payments, for an increase in the foreign assets position, $\widehat{b}_t^{*,HH}$, thereby endogenously alleviating the pressure from the effective return on foreign assets. The rise in the risk premium and increased demand for foreign currency depreciates the domestic currency and reduces the terms of trade, i.e. σ_t rises and TOT_t falls. With the exception of $\widehat{b}_t^{*,HH}$, these effects are in opposite direction to those generated by a rise in capital inflow, $\widehat{\phi}_t^*$. The rise in $\widehat{b}_t^{*,HH}$ here only moderates the initial rise in the UIP premium but does not reverse it, suggesting the transmission to the rest of the system is similar, but with opposite sign, to that of a rise in capital inflow. Specifically, the fall in the terms of trade raises exports, and elevated demand for home goods raises production, labor and inflation. In reaction, the central bank raises the interest rate.

Consider now the impulse response functions under the optimal FXI policy. Here again, the central bank is able to insulate the economy against the effect of risk premium shocks, similarly to its potency against capital flows. The central bank is able to maintain a stable return on foreign assets by selling foreign reserves to domestic agents. Thereby $\widehat{b}_t^{*,HH}$ rises and offsets the effect of the shock on the UIP premium. Once the premium is stabilized, the transmission of the initial shock to the rest of the economy is disabled, and there is no need for the assistance of monetary policy in stabilizing the economy. In this case as well, FXI is a superior policy tool relative to traditional monetary policy.

6.3 Productivity, A_t

Figure 5 presents the impulse response functions to a one standard deviation shock to productivity, A_t.

All else equal, a rise in productivity raises the supply of home goods, suggesting that total production, Y_t^H, rises in equilibrium even if labor effort, N_t, may fall due to

Figure 4: Response to a 1 Standard Deviation Shock in the Risk Premium (100 × log points)

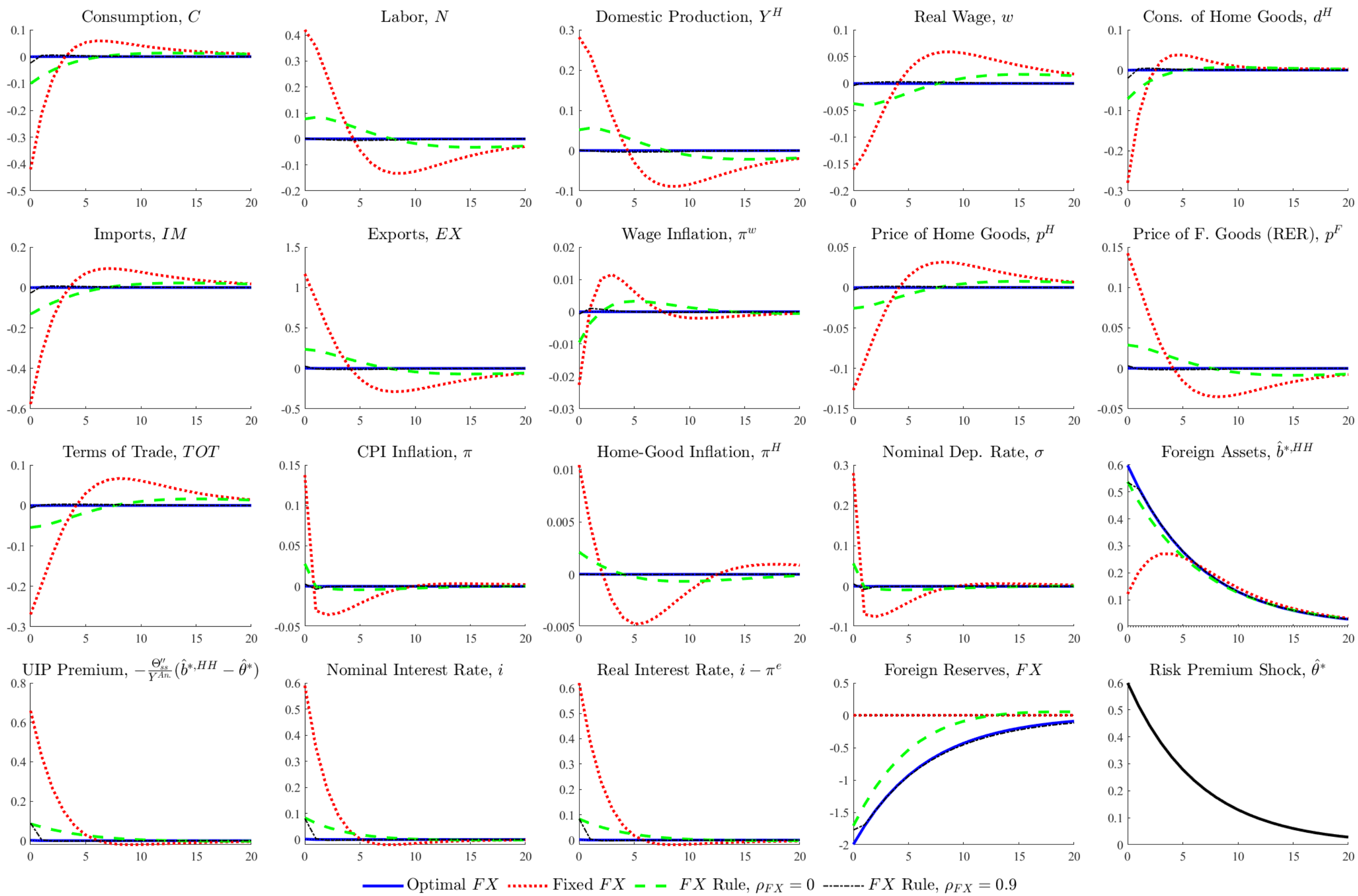
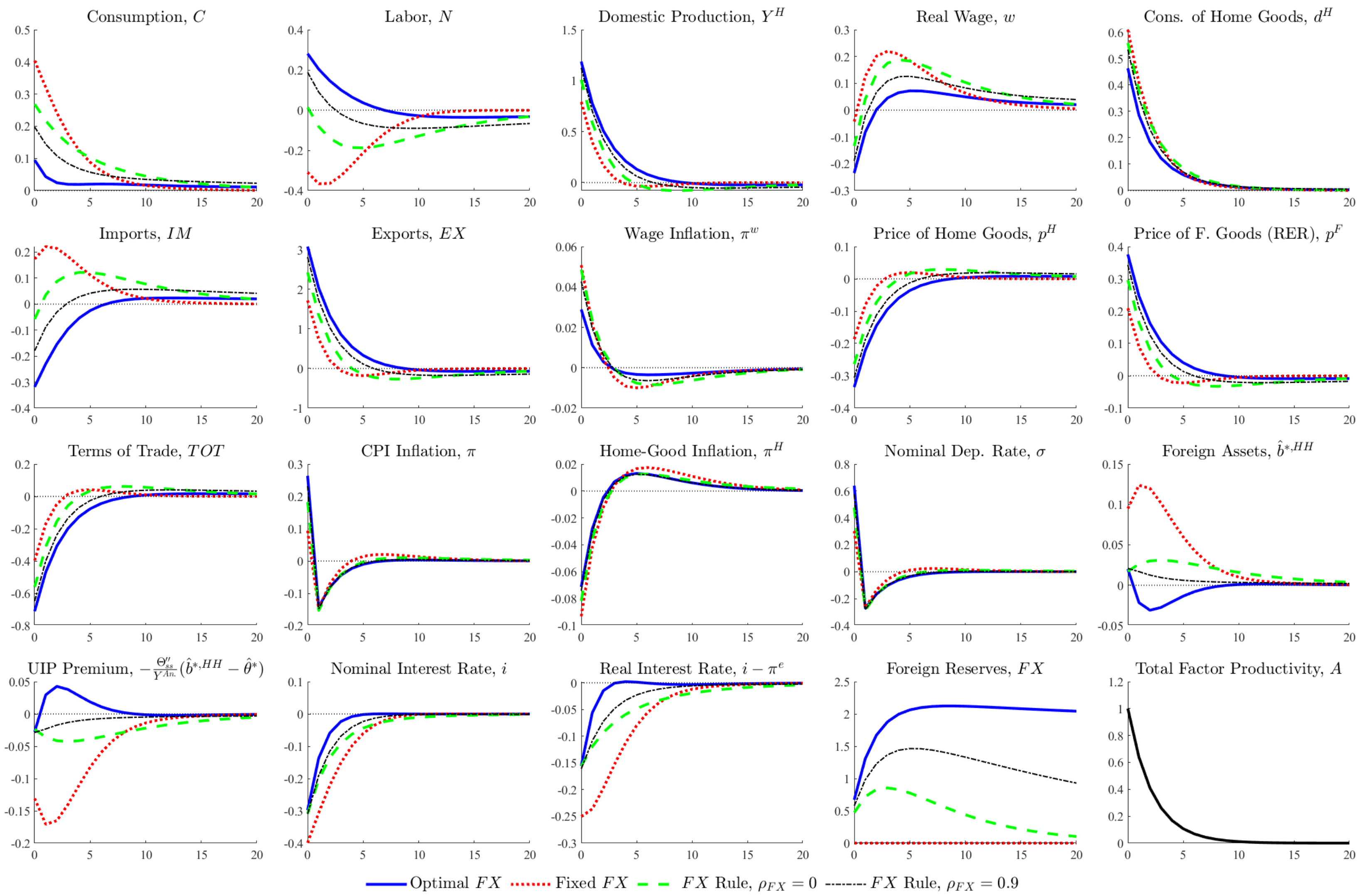


Figure 5: Response to a 1 Standard Deviation Shock in Productivity (100 × log points)



reduction in labor supply. The increase in supply reduces the price of home goods, which is reflected in a fall in the terms of trade, TOT_t, and home-goods inflation, π_t^H, and raises demand-side quantities, consumption of home goods, d_t^H, and exports, EX_t. The optimal monetary policy is expansionary. The reduction in the interest rate suggests that the rise in supply outpaces demand, which justifies a monetary expansion in order to close the "output gap". Qualitatively, these effects are common to both the case of fixed foreign reserves and optimal FXI.

When foreign reserves are fixed, imports and the terms of trade move in opposite directions, contrary to the result in Section 5.1. The reason is that the higher productivity generates additional resources for the economy, and thereby higher demand for consumption, C_t, including consumption of foreign goods, IM_t, even though their relative price is now higher, i.e. TOT_t is lower. Consumers split these additional resources between consumption, C_t, and savings, which raises foreign assets, $\widehat{b}_t^{*,HH}$. The rise in foreign assets reduces their effective return through a lower UIP premium, which also supports imports demand. The rise in total consumption reduces labor supply sufficiently to reduce labor effort, N_t, even though the rise in productivity raises labor demand. This effect is also supported by the expansionary monetary policy.

The rise in productivity raises net exports, which generates appreciation pressures on the domestic currency; in contrast, the rise in foreign assets, $\widehat{b}_t^{*,HH}$, and the monetary expansion push towards a depreciation. On net the currency depreciates in the initial period, i.e. σ_t rises.

Now consider the case of optimal FXIs. With optimal FXIs the central bank accumulates foreign reserves, which depreciates the domestic currency and hence amplifies the fall in the terms of trade. As a result exports rise by more than in the case of fixed foreign reserves. That is, optimal FXIs stimulate foreign demand for home goods. This policy utilizes the rise in productivity for accumulating wealth and achieving a smoother path for consumption. Note that in this case the monetary expansion is less aggressive, compared to the case of fixed foreign reserves, suggesting that optimal monetary and FXI policies work in tandem to stimulate domestic and foreign demand, respectively, so that aggregate demand matches the rise in supply. Nevertheless, optimal FXIs crowd out the rise in $\widehat{b}_t^{*,HH}$ and better stabilizes the UIP premium, as in the case of the financial shocks.

6.4 Government Expenditure, G_t

Figure 6 presents the impulse response functions to a one standard deviation shock to government expenditure, G_t.

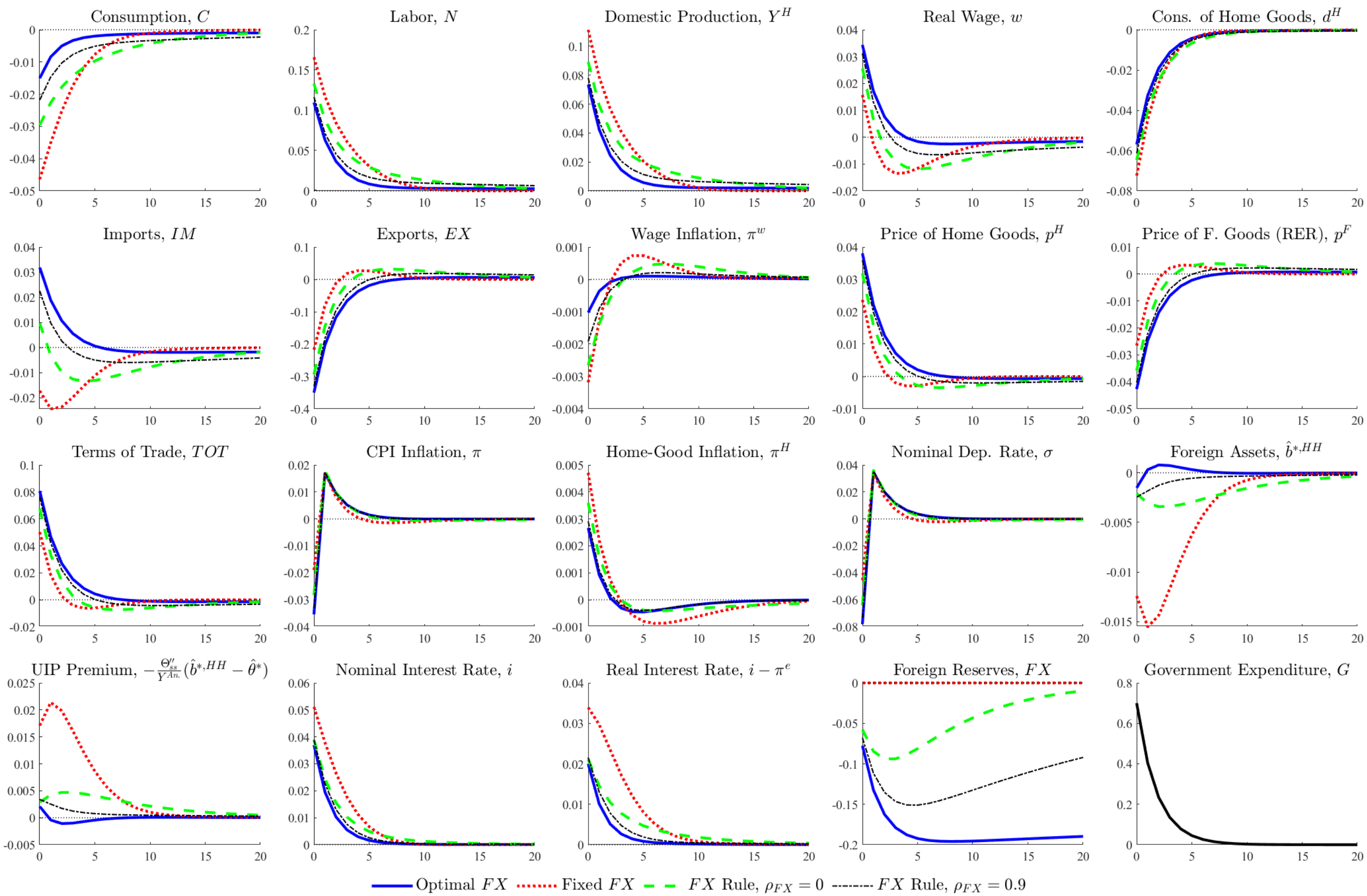
Recall that in the model government consumption is only composed of domestic goods. Therefore, to clear the market for home goods their relative price, i.e. TOT_t, rises, which crowds out exports, EX_t, and domestic consumption of home goods, d_t^H, and raises domestic production, Y_t^H, and labor, N_t. The fall in d_t^H reduces total consumption, C_t.

When foreign reserves are fixed, households smooth consumption by reducing their holdings of foreign assets, $\widehat{b}_t^{*,HH}$, and the fall in $\widehat{b}_t^{*,HH}$ raises the UIP premium. The fall in consumption and the rise in the UIP premium reduces imports demand while the rise in the terms of trade raises it. Under the current parameterization, the expenditure switching effect is not strong enough to lift consumption of imported goods, and imports, IM_t, decline; although this result is sensitive to the value of the elasticity of substitution between home and foreign goods, ε. The fall in net exports generates depreciation pressures, but these are offset by the fall in the private sector's foreign assets position, $\widehat{b}_t^{*,HH}$, and a rise in the interest rate as monetary policy attempts to curb demand. These result in an appreciation of the currency on impact and σ_t falls.

Optimal FXI policy sells foreign reserves, thereby stabilizing $\widehat{b}_t^{*,HH}$ and the UIP premium. Monetary policy is less contractionary, as the sale of foreign reserves helps to appreciate the domestic currency. This, in turn, generates a sharper rise in the terms of trade and further reduces exports. These effects result in higher imports and a smoother path of consumption. With the help of FXIs, the rise in government expenditure is absorbed to a larger extent by a fall in exports rather than a reduction in domestic consumption of home goods, d_t^H.

To sum up, optimal FXIs help monetary policy in stabilizing the economy, as the required monetary contraction is smaller compared to the case where foreign reserves are fixed. Nevertheless, optimal FXIs work to stabilize the UIP premium, as in the case of the financial shocks.

Figure 6: Response to a 1 Standard Deviation Shock in Government Expenditure (100 × log points)



6.5 Preference Shock, η_t

Figure 7 presents the impulse response functions to a one standard deviation rise in the preference shock, η_t.

A rise in η_t shifts demand for consumption from the future to the present, as it raises the current marginal utility of consumption relative to the future. This raises current consumption while increases the ex-ante real interest rate, through the Euler equation for domestic bonds. Higher consumption contracts labor supply, which suppresses production, Y_t^H, and raises real wage (in units of consumption), w_t. At the same time, higher consumption raises demand for both domestic and foreign goods, d_t^H and IM_t.

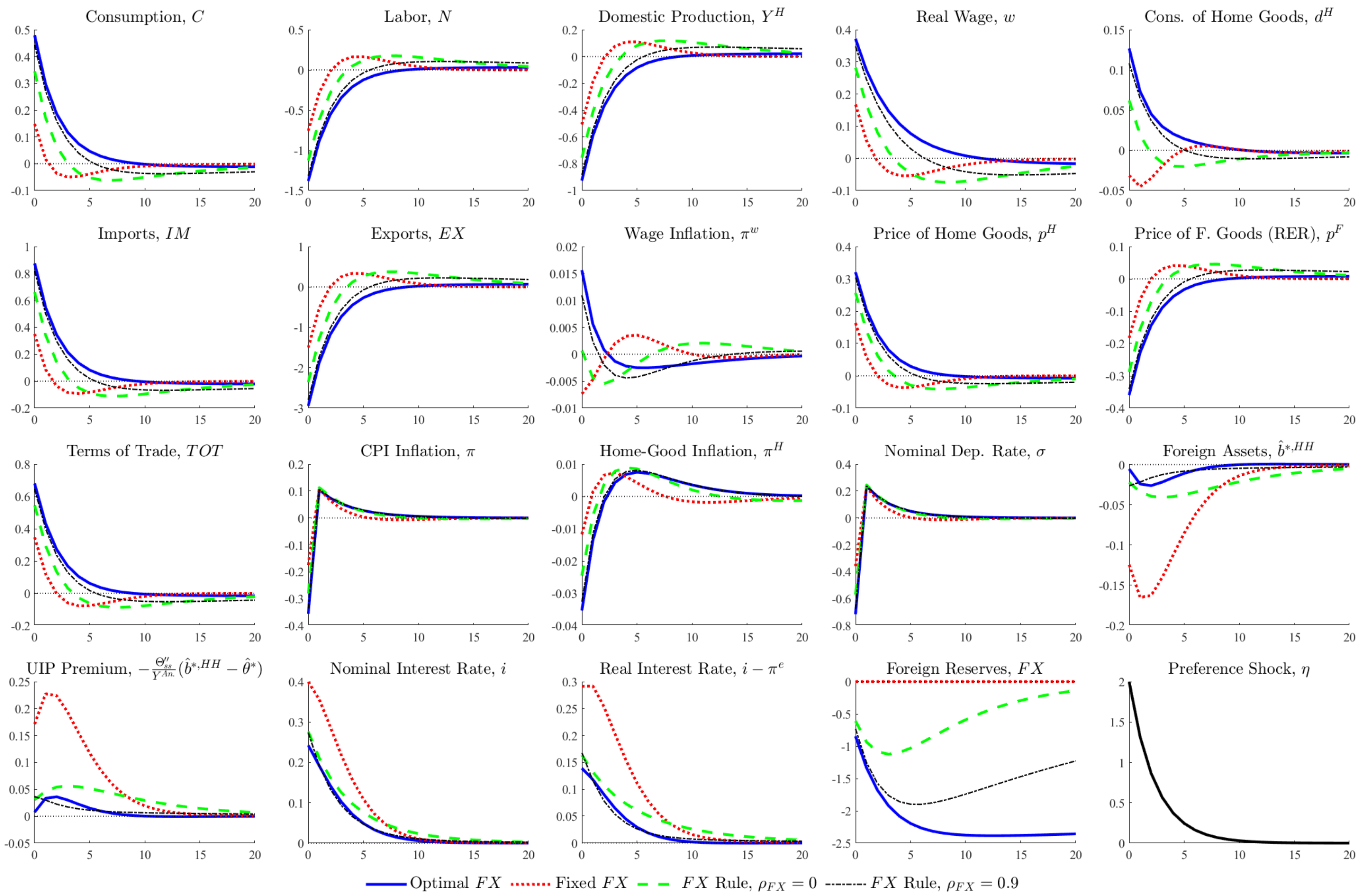
To satisfy higher initial demand for home goods while production is lower, their relative price, TOT_t, must rise. This effect crowds out exports, EX_t, while the total effect on home consumption of domestic goods, d_t^H, is ambiguous as it depends on the elasticity of substitution between home and foreign goods, ε, and on the policy reaction. Higher ε amplifies the expenditure switching effect of the terms of trade, and a sharper rise in the interest rate, to curb demand, moderates the effect of total consumption, C_t, on d_t^H. As for imports, IM_t, both the rise in C_t and the rise in TOT_t stimulate demand for imported goods. When foreign reserves are fixed, households finance the rise in imports by selling foreign assets, $\widehat{b}_t^{*,HH}$, which, in turn, appreciates the domestic currency on impact, i.e. σ_t falls. The rise in demand, following the preference shock, triggers a monetary contraction, which also supports the appreciation of the domestic currency.

Interestingly, home inflation, π_t^H, falls slightly, even though the system is triggered by a positive demand shock. This effect is due to the fall in exports following the appreciation of the domestic currency and the rise in the terms of trade. The stronger the improvement in the terms of trade, the greater is the fall in exports demand and the moderating effect on π_t^H.

Under optimal FXI policy the central bank sells foreign reserves, thereby moderating the fall in the private sector holdings of foreign assets, $\widehat{b}_t^{*,HH}$, and stabilizing the UIP premium. This in turn, lowers the effective return on foreign assets, which raises demand for imported goods, IM_t, relative to the case of fixed foreign reserves.

Consumption and labor are more volatile under the optimal FXI policy relative to

Figure 7: Response to a 1 Standard Deviation Shock in Prefernce Shock (100 × log points)



their path under fixed foreign reserves. This is because the shock is to preferences, and hence in order to smooth the *marginal utility* of consumption, the path of consumption itself must follow a path similar to that of the shock. Indeed, comparing the path of the marginal utility under the two policy regimes (not shown) reveals that it is indeed smoother under the case of optimal FXIs. The sharper movement in consumption is then transmitted to the labor supply. Note also that by equation (60) welfare increases with the covariance of the shock with consumption ($\gamma_{c\eta} = 1 > 0$), and decreases with its covariance with labor ($\frac{U_{N_{ss}} N_{ss}}{U_{C_{ss}} C_{ss}} \gamma_{n\eta} = \frac{U_{N_{ss}} N_{ss}}{U_{C_{ss}} C_{ss}} < 0$). Accordingly, the optimal policy raises the covariance of the shock with consumption and reduces the covariance with labor (makes it more negative).

Finally, notice that under optimal FXIs, the optimal monetary policy is less aggressive than in the case of fixed foreign reserves. This suggests that FXIs and traditional monetary policy work in tandem in this case as well.

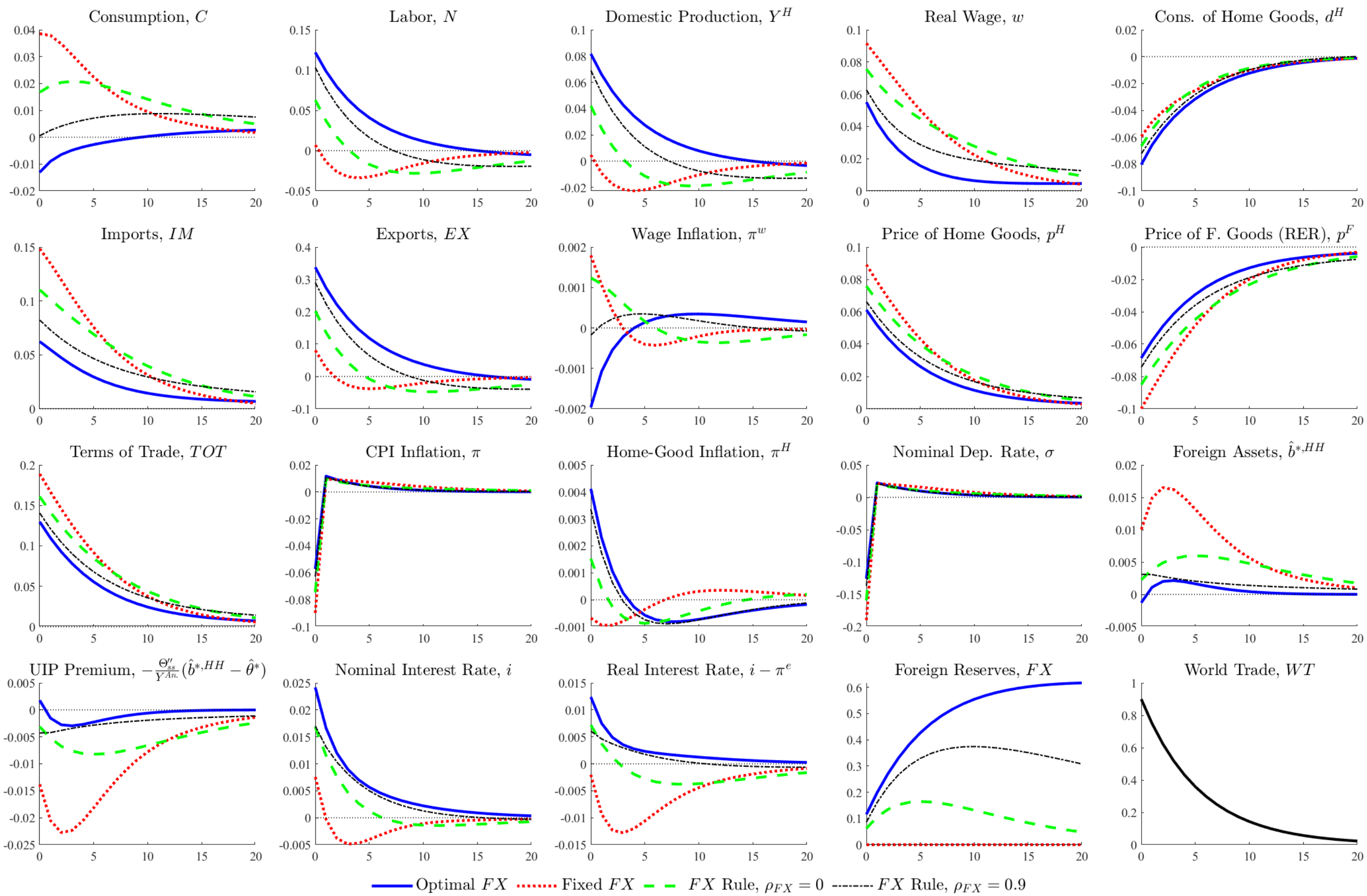
6.6 World Trade, WT_t

Figure 8 presents the impulse response functions to a one standard deviation rise in world trade, WT_t.

A rise in world trade raises demand for exports, EX_t, which, in turn, increases the terms of trade, TOT_t. Higher terms of trade shifts the composition of consumption from home goods to foreign goods, as a result d_t^H falls and IM_t rises.

Under fixed foreign reserves, the inflow of foreign resources, due to the rise in export demand, is split between accumulation of foreign assets, $\widehat{b}_t^{*,HH}$, and a rise in consumption, C_t, though quantitatively these effects are small. The rise in foreign assets lowers their effective return, which reinforces the rise in imports. The inflow of foreign resources also appreciates the domestic currency, and σ_t falls. Labor, N_t, is quite stable as it is affected by two opposing forces; labor demand rises due to the rise in the terms of trade, while labor supply falls following the rise in consumption. These forces raise the real wage, w_t. Domestic production, Y_t^H, follows the same dynamics as labor. Note that although the shock originates in foreign demand for home goods, the actual rise in exports is small, as the reaction of the terms of trade moderates the initial effect. The domestic

Figure 8: Response to a 1 Standard Deviation Shock in World Trade (100 × log points)



economy profits from the improvement in the terms of trade without actually exporting much additional resources abroad. With no much volatility transmitted to the domestic economy, optimal monetary policy practically leaves the interest rate unchanged and lets the free market forces run their course.

Under optimal FXIs, the central bank accumulates foreign reserves, which moderates the initial appreciation and stabilizes $\widehat{b}_t^{*,HH}$. This, in turn, moderates the rise in imports and smoothes the path of total consumption, C_t. These effects moderate the rise in the terms of trade relative to the case of fixed foreign reserves. In the labor market, the rise in labor demand raises labor effort, which increases domestic production.[41] Higher supply of home goods also works to moderate the rise in the terms of trade. Notice that although the optimal reaction of the interest rate is hardly changed relative to the case of fixed foreign reserves, qualitatively monetary policy is now more contractionary. This is the only case where monetary policy becomes more aggressive when FXIs are in place, although quantitatively the effects are nil.

7 FXI Policy Rule

The optimal policy analyzed above tailors the best FXI reaction to each shock; however, in practice, central banks do not have the benefit of observing the composition or the magnitude of the shocks as they hit the economy. An implementable policy recommendation should rely on observables. This section proposes policy principles that aim to support an equilibrium allocation that is close to the optimal one, and that can serve as a practical guide for FXIs.

In response to all shocks, the optimal FXI policy has a common theme: it stabilizes the UIP premium. This is clearly seen in the impulse response functions against capital flows and against the risk premium shock, where policy is able to practically insulate the economy from the effect of the shocks (blue solid lines in figures 3 and 4). Similarly, in the impulses against all other shocks as well, the path of the UIP premium under the optimal FXI policy is always smoother than the one under fixed foreign reserves (blue

[41] In this case labor demand and labor supply work in the same direction, as consumption falls slightly and raises labor supply. This is also reflected by a lower rise in real wage, w_t.

solid line vs dotted red line in figures 5 through 8). This result may come as no surprise, as optimal policies typically strive to neutralize the effect of frictions. Just as monetary policy aims to mitigate the effect of nominal rigidities in standard new-Keynesian models, the optimal FXI policy in this case seeks to mitigate the impact of the financial friction. Nevertheless, under non-financial shocks, optimal policy allows for some variation in the UIP premium and does not stabilize it completely.[42]

Another point that emerges from the figures is that under optimal FXIs foreign reserves, FX_t, are highly persistent. In fact, reserves do not follow a random walk only because they are restricted to be stationary.[43] Intuitively, after a temporary rise in productivity, for example, the social planner would have chosen to raise foreign reserves permanently, and use the return on the additional reserves to raise consumption for perpetuity.

Taking these results together, a natural suggestion for a policy rule is to use FXIs to stabilize the UIP premium while smoothing the path of foreign reserves. Specifically, consider the following policy rule:

$$\frac{FX_t}{FX^T} = \left(1 + \frac{\Theta' \left(\widehat{b}_t^{*,HH} - \widehat{\theta}_t^*\right)}{TOT_{ss} Y_{ss}^{H,An.}}\right)^{\Xi} \left(\frac{FX_{t-1}}{FX^T}\right)^{\rho_{FX}} \tag{64}$$

$$\text{where} \qquad \Xi \gg 0 \quad , \quad 0 \leq \rho_{FX} < 1$$

Recall the households' Euler equation for foreign bonds, equation (27); the term in the first parentheses is the *inverse* of the gross UIP premium. Note that since $\Theta''(\cdot) > 0$ and since the purchase of foreign reserves, FX_t, crowds out private holdings of foreign assets, $\widehat{b}_t^{*,HH}$, the parameter Ξ must be positive in order for the proposed policy rule to stabilize the premium. This rule suggests that the UIP premium should serve as a policy target, much like the role of inflation target in standard Taylor rules. Although the premium is not directly observed in the data, it can be estimated by evaluating deviations from the UIP condition, see discussion in Engel (2014). The second term in (64) affects the persistence of the policy instrument.

[42] The next section addresses the question of why this is the case.

[43] Recall that the central bank faces a minor adjustment cost when trading in foreign bonds.

Setting $\Xi = \rho_{FX} = 0$ brings the model to the case of fixed foreign reserves (red dashed impulses in figures 3 through 8). As $\Xi \to \infty$ the policy completely stabilizes the UIP premium. Technically, however, we cannot eliminate the premium entirely, as it reintroduces unit root dynamics to the model's solution.[44] For the same reason, ρ_{FX} must be strictly smaller than 1, although a sufficiently high value is anticipated to aid in bringing reserves to their optimal path. I simulate the model with the (arbitrary) value:

$$\Xi = 20$$

and experiment with two values for ρ_{FX}:

$$\rho_{FX} = 0 \qquad \text{and} \qquad \rho_{FX} = 0.9$$

The impulse response functions under both parameterizations of (64) are displayed in figures 3 through 8 (green dashed lines for the case of $\rho_{FX} = 0$, and thin black dash-dotted lines for $\rho_{FX} = 0.9$). In both cases the response of foreign reserves, FX_t, always lies between their optimal path (blue solid lines) and zero, suggesting the policy rule indeed pushes foreign reserves toward its optimal response. Moreover, the introduction of reserves smoothing, i.e. $\rho_{FX} > 0$, brings the policy reaction even closer to its optimal path.

This analysis suggests that adopting policy rule (64) seems sensible, at least qualitatively. Section 9 attempts to quantify its welfare gains. However, before turning to welfare analysis, I address the question of why strict targeting of the UIP premium is sub-optimal under non-financial shocks.

8 When Is Full Stabilization of the UIP Premium Optimal?

The financial friction in the model generates the UIP premium, which in turn distorts asset pricing and the equilibrium allocation in the economy. It is therefore reasonable to expect optimal policy to perfectly counteract the effect of the friction, resulting in strict

[44] In Schmitt-Grohé and Uribe (2003) the role of the premium is exactly to eliminate the unit root from the equilibrium dynamics of the model.

targeting of the UIP premium. However, this is true only under financial shocks. When the economy is subject to non-financial shocks, optimal FXI clearly reduces the variation of the UIP premium, relative to the case of fixed reserves, but does not completely eliminate it.

To understand this result, first consider the case of financial shocks. FXIs operate in the financial markets and are able to insulate the economy from the effect of financial shocks regardless of the structure of the rest of the economy. Specifically, the balance of payments implies that a capital inflow shock can be fully offset by increasing foreign reserves by the same amount. This operation does not affect any other equilibrium condition in the model. Similarly, after an exogenous rise in the risk premium, selling reserves can provide just the right amount of funds to domestic households to perfectly stabilize the UIP premium, thereby insulating the economy from the effect of the shock. In this case, only the composition of the financial account in the balance of payments and the composition of the UIP premium are affected, without altering any of the other equilibrium conditions.

When non-financial shocks hit, e.g. changes in productivity or in government consumption, the central bank can still fully stabilize the UIP premium, but it cannot shield the economy from their effect. In these cases, the optimal response depends on the trade-offs the central bank faces, and these are determined by the distortions present in the economy. The model economy incorporates four distortions that cause the equilibrium allocation to deviate from the optimal one. These distortions are: (1) price rigidity, which limits firms from adjusting production optimally in every period; (2) wage rigidity, which similarly constrains labor supply[45]; (3) financial friction, which distorts asset pricing and exchange rate dynamics; and (4) a downward sloping demand for exports, which endows the economy with monopolistic power in the global goods market, while producers of the home composite good are price takers. As emphasized by Corsetti and Pesenti (2001), this creates an incentive for the social planner to manipulate the terms of trade in favor

[45] The optimal labor subsidy, equation (59), accounts for the domestic distortion caused by monopolistic competition in the labor and goods markets. Despite being constant, the subsidy is effective in offsetting the distortion throughout the business cycle due to the assumption of CES aggregators, equations (1) and (21), which result in constant mark-ups.

of the domestic economy.

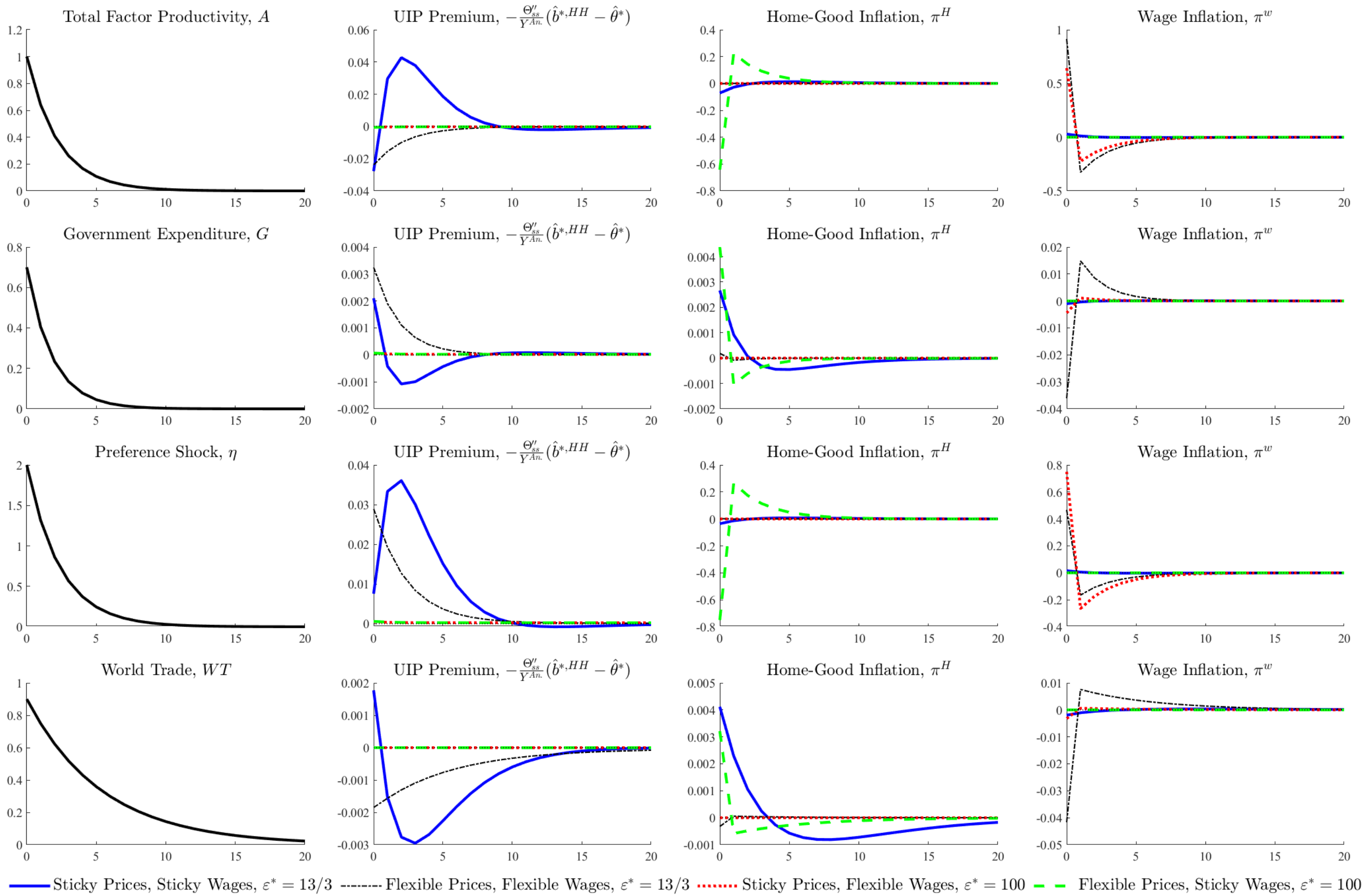
Given that the social planner has only two policy tools – interest rate and FXIs, generally it is not possible to perfectly offset all distortions simultaneously. As a result, tradeoffs emerge, and full stabilization of the UIP premium may not be optimal. Shutting down two of the frictions may support strict targeting of the UIP premium as optimal. Note, however, that in order to maintain the effectiveness of monetary policy, at least one of the nominal rigidities must be preserved, and similarly, in order to maintain efficacy of FXIs the financial friction must be kept in the model. Therefore, with these constraints on policy efficacy, shutting down at least one of the nominal rigidities and removing the monopolistic power of the economy, is expected to eliminate the tradeoffs, and make strict targeting of the UIP premium optimal. This is done by setting $\xi_p = 0$ and/or $\xi_w = 0$ together with $\varepsilon^* \to \infty$. The first two conditions eliminate at least one nominal rigidity, and the third implies that, in the eyes of foreigners, the home good is a perfect substitute for goods from any other country. As $\varepsilon^* \to \infty$, the demand for exports turns perfectly elastic, and the economy loses its monopolistic power in the global markets.

By the same reasoning, with $\varepsilon^* \to \infty$, maintaining price rigidity is expected to give rise to strict targeting of domestic price inflation, π^H, while maintaining wage rigidity is expected to result in strict targeting of wage inflation, π^w.

Figure 9 demonstrates these points. The figure displays the response to non-financial shocks of the UIP premium, domestic price inflation and wage inflation under optimal policies in four cases: (1) the baseline parameterization with price and wage rigidities and a downward sloping demand for exports ($\varepsilon^* = 13/3$); (2) no nominal rigidities while maintaining a downward sloping demand for exports ($\varepsilon^* = 13/3$); (3) sticky prices, flexible wages and (close to) perfectly elastic demand for exports ($\varepsilon^* = 100$); and (4) flexible prices, sticky wages and $\varepsilon^* = 100$. Note that setting ε^* to 100 practically eliminates the monopolistic power of the economy in the global goods market, as exports demand becomes highly elastic. Each row in the figure displays the response to a different shock. The evolution of each shock is displayed in the left column of the figure.

As discussed above, under the baseline parameterization (case 1, blue solid lines) optimal policy does not fully stabilize the UIP premium. This is also the case in the

Figure 9: Optimal Response of Target Variables to Non-Financial Shocks (100 $\times$ log points)



real model with no nominal rigidities (case 2, black dash-dotted lines). The reason is that in this case monetary policy is neutral, which leaves FXIs to both counteract the effect of the financial friction and internalize the monopolistic power of the economy. With one tool and two objectives the planner faces a tradeoff, and the UIP premium is not fully stabilized. With one nominal rigidity and no monopolistic power (case 3 and case 4, dotted red and dashed green lines, respectively), monetary policy addresses the distortion caused by the nominal rigidity and FXIs are free to address the financial friction, resulting in full stabilization of the UIP premium. Moreover, in the case of sticky prices, strict targeting of domestic price inflation, π^H, is optimal, while under sticky wages strict targeting of wage inflation, π^w, turns optimal.

9 Welfare Evaluation

This section evaluates the welfare gains from implementing optimal FXI policy. It compares the welfare under the optimal policy to the welfare under alternative policies: fixed foreign reserves and the FXI rule, equation (64), with $\rho_{FX} = 0$ and with $\rho_{FX} = 0.9$. In all cases, the interest rate is set optimally; hence, this comparison helps in evaluating whether FXIs have an economically significant role over and above that of traditional monetary policy, as it exhausts any potential welfare gains from monetary policy before resorting to FXIs. Table 3 summarizes the results. The table presents the *lifetime* welfare gains, expressed as a percentage of annual steady state consumption. These gains represent the maximum amount that a household residing in an economy with sub-optimal FXI policy would be willing to pay to move to an identical economy where the central bank practices optimal FXIs.[46]

Panel A of Table 3 displays the welfare gains from implementing the optimal policy against the case of fixed foreign reserves. Consider column (1), which corresponds to the

[46] It is important to note that the model's parameters and stochastic processes are chosen to match those of the Israeli economy. As such, some of the results reported below may be specific to Israel and similar countries, while others are more general. For instance, the welfare gains of countering capital flows are heavily influenced by the standard deviation of these shocks, indicating that countries that face greater capital flow volatility may benefit more from implementing optimal FXIs. Other results, such as the superiority of introducing persistence to the FX rule are more general.

benchmark model. Welfare gains are not large, but are economically meaningful; they amount to 2.4% of annual steady state consumption (last row of column (1)).

As expected, FXIs play an important role against financial shocks, capital flow and risk premium shocks, as the central bank is able to insulate the economy from their effects (figures 3 and 4). Nevertheless, productivity and the preference shocks also play an important role. This result may have been anticipated given the magnitude of the effects in the impulse response functions (blue lines vs dotted red lines in figures 5 and 7). Productivity and the preference shock interact with the optimality conditions of households through the Euler equations and through labor supply. The preference shock triggers intertemporal substitution of consumption, and FXIs help with alleviating the cost of shifting resources over time, giving rise to welfare gains.

The role of FXIs against productivity shocks is less straightforward. Productivity affects the households' marginal decisions through the labor market. A positive productivity shock raises the economy's wealth. In the absence of FXIs, saving in the economy rises as households raise their foreign asset position, $\widehat{b}_t^{*,HH}$, to smooth consumption; however, this is costly since increasing households' exposure to foreign assets erodes the return they receive. As a result, households undersave, relative to the optimal allocation, and allocate their additional resources toward consumption, which, in turn, reduces labor supply exactly when labor is most productive. FXIs can increase the economy's savings by accumulating foreign reserves. Higher reserves crowd out $\widehat{b}_t^{*,HH}$, which, in turn, reduces the cost of savings for the economy. The rise in savings allows for a smoother path of consumption, which raises labor supply relative to the case of fixed foreign reserves, thereby better utilizing labor productivity.

Finally, government expenditure and world trade shocks have a negligible impact on welfare. This result corresponds to their small quantitative effects in the impulse response functions (figures 6 and 8). These shocks affect none of the optimality conditions, and hence require little policy intervention.

Columns (2) and (3) in Table 3 present the welfare gains with lower portfolio adjustment cost parameters: 10% of the benchmark value in column (2), and 1% of the benchmark value in column (3). Clearly, welfare gains drop substantially as the finan-

Table 3: Lifetime Welfare Gains from Adopting Optimal FXI Policy
Percent of Annual Steady State Consumption

	(1)	(2)	(3)	(4)
	Nominal Rigidities			Real
$\Theta''(0)$, % of Benchmark Value:	100%	10%	1%	100%
Panel A: Welfare Gains Relative to Fixed Foreign Reserves				
Productivity, A	0.56	0.24	0.05	0.40
Preference shock, η	0.64	0.24	0.04	0.70
Government expenditure, G	0.01	< 0.01	< 0.01	< 0.01
World trade, WT	0.03	0.01	< 0.01	0.03
Risk premium, $\widehat{\theta}^*$	0.34	0.03	< 0.01	0.40
Capital inflows, $\widehat{\phi}^*$	0.87	0.26	0.04	0.91
All shocks	2.44	0.77	0.13	2.44
Panel B: Welfare Gains Relative to FX Rule without Persistence, $\rho_{FX} = 0$				
Productivity, A	0.25	0.17	0.04	0.16
Preference shock, η	0.25	0.16	0.03	0.28
Government expenditure, G	< 0.01	< 0.01	< 0.01	< 0.01
World trade, WT	0.01	0.01	< 0.01	0.01
Risk premium, $\widehat{\theta}^*$	0.03	0.01	< 0.01	0.04
Capital inflows, $\widehat{\phi}^*$	0.27	0.18	0.04	0.27
All shocks	0.81	0.54	0.12	0.77
Panel C: Welfare Gains Relative to FX Rule with Persistence, $\rho_{FX} = 0.9$				
Productivity, A	0.05	0.05	0.02	0.02
Preference shock, η	0.03	0.03	0.01	0.04
Government expenditure, G	< 0.01	< 0.01	< 0.01	< 0.01
World trade, WT	< 0.01	< 0.01	< 0.01	< 0.01
Risk premium, $\widehat{\theta}^*$	< 0.01	< 0.01	< 0.01	< 0.01
Capital inflows, $\widehat{\phi}^*$	0.03	0.03	0.01	0.04
All shocks	0.12	0.12	0.05	0.10

Note: The table presents the lifetime welfare gains resulting from using optimal FXIs compared to fixed foreign reserves (Panel A), and compared to a policy rule that stabilizes the UIP premium with no persistence in reserves, $\rho_{FX} = 0$ (Panel B), and to the same policy rule with persistence, $\rho_{FX} = 0.9$ (Panel C). In all cases, monetary policy sets the interest rate optimally, and the gains are expressed as a percentage of annual steady state consumption. These gains represent the maximum amount that an agent living in an economy with a sub-optimal FXI policy would be willing to pay to move to an identical economy where the central bank practices optimal FXIs. A model with nominal rigidities in columns (1) through (3); real economy in column (4). The portfolio adjustment cost, $\Theta''(0)$, takes its benchmark value, 2.569, in columns (1) and (4), 10% of that value in column (2), and 1% of the benchmark value in column (3).

cial friction becomes smaller. Nevertheless, they are economically meaningful even at 10 percent of the benchmark value, amounting to 0.8% of annual steady state consumption (last row of column (2)). This provides further support for the importance of FXIs.

Column (4) presents the welfare gains in an economy with no nominal rigidities. In this case monetary policy is neutral. Welfare gains under all shocks are of similar magnitude to those of the benchmark economy in column (1). This suggests that monetary policy does little in terms of alleviating the effect of the financial friction. FXIs seem to be a better suited tool on this front.

Panels B and C of Table 3 display the welfare gains from implementing the optimal policy against following policy rule (64) with $\rho_{FX} = 0$ and $\rho_{FX} = 0.9$, respectively. Regardless of the presence of nominal rigidities or the level of financial friction, welfare gains from adopting optimal FXIs are always the smallest under the policy rule with $\rho_{FX} = 0.9$ (Panel C). Nevertheless, even without persistence (Panel B) the rule improves welfare substantially compared to the case of fixed foreign reserves (Panel A). Overall, the policy rule without persistence lowers the welfare cost of deviating from optimal policy from 2.4% of annual steady state consumption to 0.8%, while introducing persistence almost entirely eliminates the cost, reducing it to 0.1% (last row of column (1) in each panel). Importantly, the rule proves useful as a guide for policy. It is welfare-improving regardless of the type of shocks affecting the economy, and its implementation does not require knowledge of the shocks.

Finally, recall that several authors have emphasized that deviations from the UIP are costly for the economy, as they can be exploited by foreigners to take advantage of carry trade opportunities, e.g. Cavallino (2019), Amador et al. (2020), and Fanelli and Straub (2021). Two comments are in order in this regard. First, by stabilizing the UIP premium the optimal FXI policy reduces carry trade opportunities, and therefore, on average, reduces the loss of resources for the economy. Second, in the analysis here, the financial sector is owned entirely by home agents, thereby eliminating any such costs. Therefore, the welfare gains in Table 3 indicate the role for FXIs as a macroeconomic stabilizer alone, rather than a means of stripping intermediation profits from foreigners.

The model permits measuring the welfare benefits from owning the financial sector.

Table 4: Welfare Gains from Owning the Financial Sector
Full vs Partial Ownership under Optimal Monetary and FXI Policies
Percent of Annual Steady State Consumption

	(1)	(2)	(3)
Ownership of the financial sector, ϑ:	90%	50%	0%
Productivity, A	0.13	0.29	0.37
Preference shock, η	0.16	0.35	0.46
Government expenditure, G	< 0.01	< 0.01	< 0.01
World trade, WT	0.01	0.02	0.02
Risk premium, $\widehat{\theta}^*$	0.03	0.12	0.21
Capital inflows, $\widehat{\phi}^*$	0.17	0.42	0.58
All shocks	0.50	1.19	1.65

Note: The table presents the lifetime welfare gains resulting from owning the entire financial sector, $\vartheta = 1$, relative to partial ownership, $\vartheta < 1$. All other parameters take their benchmark values. The central bank follows optimal monetary and FXI policies. Gains are expressed as a percentage of annual steady state consumption. These gains represent the maximum amount an agent living in an economy with partial ownership would be willing to pay to move to an identical economy with full domestic ownership of the financial sector. The ownership share, ϑ, takes the value, 0.9, in columns (1) , 0.5 in column (2), and 0 in column (3).

Table 4 displays the welfare differentials between an economy that owns the entire financial sector, i.e. $\vartheta = 1$, and identical economies that only differ in their ownership share. The exercise in the table assumes that the central bank follows optimal monetary and optimal FXI policies. Welfare clearly falls as foreigners own a larger portion of the financial sector, and it amounts to 1.6% of annual steady state consumption when foreigners own the entire financial sector (last row of column (3)). While this is not a negligible figure, it is smaller than the potential benefits in Table 3 of following optimal FXI policy when the financial sector is owned only by home agents. This result supports the role of FXIs as a macroeconomic stabilization tool, and implies that this role is at least as important as protecting the economy from the cost of carry trades.

10 Conclusion

This paper introduces FXIs as an additional policy tool to an otherwise standard small open economy new-Keynesian model. It studies the transmission mechanism of FXIs through the portfolio balance channel, solves for optimal policy, suggests an implementable FXI rule, and assesses the welfare benefits from following these policies.

Under the portfolio balance channel, the purchase of foreign reserves works by crowding out private holdings of foreign assets, thereby raising the effective return domestic agents face in the financial markets. This, in turn, contracts domestic demand and reduces consumption. The purchase of foreign reserves also depreciates the domestic currency, which raises the price of foreign goods relative to home goods. This effect expands demand for home exports and reduces domestic demand for imported goods. The effect on domestic production is ambiguous and depends on the wealth effect on labor supply.

To make FXIs effective, one must introduce a friction to the operation of the financial markets, which results in deviations from the UIP condition. The UIP is an efficiency condition for the pricing of bonds denominated in different currencies; hence, deviations from the UIP entail welfare costs and open the door for policy intervention. The paper proposes that central banks restore efficiency in the financial markets by adopting a policy rule that stabilizes the UIP premium. Such a rule brings equilibrium outcomes close to the optimal allocation, regardless of the type of shocks to which the economy is subject.

FXIs are most effective against financial shocks, such as capital flows and risk premium shocks, as they are able to perfectly counteract the shocks in the financial markets and insulate the economy from their effect, regardless of the structure of the rest of the economy. Nevertheless, FXIs are also useful against real shocks, such as productivity fluctuations and shocks to the subjective discount factor.

When the economy is subject to real shocks, strict targeting of the UIP premium is not necessarily optimal. If demand for domestic exports is downward sloping while exporters are price takers, then they do not internalize the economy's market power, and the central bank has an incentive to manipulate the terms of trade in favor of the home economy. In this case, the central bank faces a tradeoff between counteracting the distortion in the financial markets and exploiting the economy's market power, and strict UIP premium targeting is not optimal. When the economy faces a perfectly elastic demand for exports, it does not have any market power, and strict targeting turns optimal, provided that monetary policy is able to perfectly counteract the effect of nominal rigidities. Otherwise, the central bank faces yet another tradeoff.

In this paper, as in other contributions that rely on the portfolio balance channel, the financial friction is the only source of UIP deviations, e.g. Benes et al. (2015), Cavallino (2019), Alla et al. (2020), Fanelli and Straub (2021), Faltermeier et al. (2022), Itskhoki and Mukhin (2023). Nevertheless, first-order deviations from the UIP may potentially reflect other factors; for example, the pricing of sovereign default risk. In that case, the risk is driven by fiscal factors and FXIs can probably do little to affect it. Moreover, efficient markets would price that risk properly, and it is not clear that central banks should attempt to restore the UIP condition in that case. In a similar vein, Itskhoki and Mukhin (2023) motivate FXIs by risk aversion with respect to nominal exchange rate fluctuations. However, the existence of exchange rate risk per se does not necessarily indicate financial market inefficiency, e.g. Galí and Monacelli (2005). It is therefore not entirely clear what part of the UIP premium central banks should target.

In this light, further research is needed to refine the policy recommendations of this paper. The research agenda should aim to decompose the UIP premium into components that the central bank should stabilize and those that should be allowed to fluctuate freely. Techniques for estimating these components should be developed as well.

A Appendix: Model Equivalence

This appendix demonstrates that the equivalence result of Yakhin (2022) is robust to introducing foreign reserves, capital flows and risk premium shocks to the model, and extends the result to the model of Itskhoki and Mukhin (2021, 2023) as well. That is, modeling the financial friction using a simple, reduced-form, portfolio adjustment cost, as in the main text, is isomorphic, up to a first-order approximation, to the microfounded modeling strategy of Gabaix and Maggiori (2015), Fanelli and Straub (2021) and Itskhoki and Mukhin (2021, 2023), GM, FS and IM, respectively, hereinafter. Below I only focus on GM and IM, as the extension to FS is immediate[47], and I strip the model from anything that is unrelated to the financial friction. There is no production, differentiated goods, or nominal rigidities. These abstractions do not affect the result.

A.1 The Basic Settings

Consider a small open economy populated by a unit mass of households, a government and a financial sector. The economy is perfectly integrated in the world's goods market. There is one perishable good in the world economy and two currencies, home and foreign. Each period, households in the home economy are endowed with a random allocation of the good, Y_t. Prices are flexible, and law of one price holds. Foreign prices are normalized to 1. Generally, variables are denoted using the same symbols as in the main text. Any deviation is noted explicitly.

The central bank issues domestic risk-free bonds and controls their return, i_t. Let B_t^G denote the government holdings of these bonds. Domestic households hold B_t^{HH} units of the bonds, and foreigners hold B_t^{ROW}. Capital inflows, ϕ_t^*, are exogenous, they are measured in foreign currency, and relate to the foreign holdings of domestic bonds, B_t^{ROW}, by:

$$\phi_t^* = \frac{B_t^{ROW}}{S_t} - \frac{1+i_{t-1}}{\sigma_t}\frac{B_{t-1}^{ROW}}{S_{t-1}} \tag{A.1}$$

The central bank holds foreign reserves, FX_t. Foreign reserves pay the foreign risk-free interest rate, i_t^*.[48] In steady state $1+i_{ss}^* = \frac{1+i_{ss}}{\sigma_{ss}} = \beta^{-1}$.

The consolidated government (monetary and fiscal authorities) budget constraint is given by:

$$(1+i_{t-1})\, B_{t-1}^G + S_t\left(1+i_{t-1}^*\right) FX_{t-1} = B_t^G + S_t FX_t + T_t \tag{A.2}$$

where T_t is lump-sum transfers to the households.

[47] With linear participation cost in FS, their financial friction turns identical to that of GM. Any non-linearity in the cost function is washed away in the first-order approximation. See Yakhin (2022).

[48] Note that here foreign reserves are expressed in units of foreign currency rather than units of foreign goods, as in the text.

A.2 The Portfolio Adjustment Cost Model

Domestic households have access to the international financial markets, but face a convex adjustment cost whenever the level of their foreign asset position, $b_t^{*,HH}$, deviates from some long run target level, $\overline{b}^{*,HH}$, plus a zero-mean noise, θ_t^*. A fraction ϑ of the cost is rebated to the households.

Households The households maximize their expected lifetime utility, $E_0 \sum_{t=0}^{\infty} \beta^t U(C_t)$, subject to the flow budget constraint:

$$\begin{aligned} & S_t C_t + B_t^{HH} + S_t b_t^{*,HH} + S_t \Theta\left(b_t^{*,HH} - \overline{b}^{*,HH} - \theta_t^*\right) \\ \leq \ & S_t Y_t + (1+i_{t-1}) B_{t-1}^{HH} + S_t\left(1+i_{t-1}^*\right) b_{t-1}^{*,HH} + \vartheta S_t \Pi_t + T_t \end{aligned}$$

where $\Theta(\cdot)$ is a convex cost function that satisfies:

$$\Theta(\cdot) \geq 0 \quad , \quad \Theta(0) = 0 \quad , \quad \Theta'(0) = 0 \quad , \quad \Theta''(\cdot) > 0$$

Π_t is the average adjustment cost in the economy and each household is rebated a portion ϑ of that cost. Since the rebate is a function of the economy's average cost, households do not internalize the effect of their choice of $b_t^{*,HH}$ on Π_t.

The first order conditions of households:

$$U_{C,t} = \beta(1+i_t) E_t\left(\frac{U_{C,t+1}}{\sigma_{t+1}}\right) \tag{A.3}$$

$$U_{C,t}\left[1 + \Theta'\left(b_t^{*,HH} - \overline{b}^{*,HH} - \theta_t^*\right)\right] = \beta(1+i_t^*) E_t(U_{C,t+1}) \tag{A.4}$$

Combining the two equations gives the modified UIP:

$$(1+i_t) E_t\left(\frac{U_{C,t+1}}{\sigma_{t+1}}\right)\left[1 + \Theta'\left(b_t^{*,HH} - \overline{b}^{*,HH} - \theta_t^*\right)\right] = (1+i_t^*) E_t(U_{C,t+1}) \tag{A.5}$$

Market Clearing and the BOP In the financial markets:

$$B_t^G + B_t^{HH} + B_t^{ROW} = 0$$

The BOP identity is derived by consolidating the government budget constraint and the households' budget constraint together with the market clearing condition above, while taking into account that a portion ϑ of the portfolio adjustment cost is rebated to the households. This results in:

$$\begin{aligned} FX_t + b_t^{*,HH} &= \left(1 + i_{t-1}^*\right)\left(FX_{t-1} + b_{t-1}^{*,HH}\right) + \phi_t^* \\ &\quad + Y_t - C_t - (1-\vartheta)\,\Theta\left(b_t^{*,HH} - \bar{b}^{*,HH} - \theta_t^*\right) \end{aligned} \tag{A.6}$$

where ϕ_t^* is defined in (A.1).

Closing the Model The households' optimality conditions, equations (A.3) and (A.5), together with the BOP, equation (A.6), result in a system of 3 equations in 5 endogenous variables: C_t, i_t, σ_t, $b_t^{*,HH}$ and FX_t. Y_t, i_t^*, ϕ_t^* and θ_t^* are exogenous. The model is closed by specifying a policy rule for the nominal interest rate, i_t, and for foreign reserves, FX_t.

Log-Linearized Equations Log-linearizing equations (A.3), (A.5) and (A.6), the approximated model is characterized by:

$$\gamma_{cc}\widetilde{C}_t \cong \widetilde{(1+i_t)} + \gamma_{cc}E_t\left(\widetilde{C}_{t+1}\right) - E_t\left(\widetilde{\sigma}_{t+1}\right) \tag{A.7}$$

$$\begin{aligned} E_t\left(\widetilde{\sigma}_{t+1}\right) &\cong \widetilde{(1+i_t)} - \widetilde{(1+i_t^*)} \\ &\quad + \Theta''(0)\left[\left(b_t^{*,HH} - \bar{b}^{*,HH}\right) - \theta_t^*\right] \end{aligned} \tag{A.8}$$

$$\begin{aligned} \frac{FX_{ss}}{Y_{ss}}\widetilde{FX}_t + \frac{b_t^{*,HH} - \bar{b}^{*,HH}}{Y_{ss}} &\cong \widetilde{Y}_t - \frac{C_{ss}}{Y_{ss}}\widetilde{C}_t + \beta^{-1}\frac{FX_{ss} + \bar{b}^{*,HH}}{Y_{ss}}\widetilde{(1+i_t^*)} \\ &\quad + \beta^{-1}\left[\frac{FX_{ss}}{Y_{ss}}\widetilde{FX}_{t-1} + \frac{b_{t-1}^{*,HH} - \bar{b}^{*,HH}}{Y_{ss}}\right] + \frac{\phi_t^* - \phi_{ss}^*}{Y_{ss}} \end{aligned} \tag{A.9}$$

A.3 The GM Model

This section builds on Gabaix and Maggiori (2015). In this model, households only hold domestic risk-free bonds, as they do not have access to the international financial markets. Financial arbitrageurs absorb domestic saving imbalances for a premium. Limited commitment generates deviations from the UIP. Domestic households own a fraction ϑ of the financial firms.

Households The households maximize their expected lifetime utility, $E_0 \sum_{t=0}^{\infty} \beta^t U(C_t)$, subject to the flow budget constraint:

$$S_t C_t + B_t^{HH} \leq S_t Y_t + (1 + i_{t-1}) B_{t-1}^{HH} + \vartheta S_t \Pi_t + T_t$$

where here Π_t represents the dividends from the financiers'. The first order conditions of households is given by:

$$U_{C,t} = \beta (1 + i_t) E_t \left(\frac{U_{C,t+1}}{\sigma_{t+1}} \right) \tag{A.10}$$

which is identical to (A.3).

Financiers Agents are selected at random to operate the financial firms for a single period. The selection process is memoryless. Financiers start each period with no liabilities and a net worth of $\overline{\mathcal{B}}^* + \theta_t^*/\vartheta$, denominated in foreign currency, which is held in foreign bonds. θ_t^* is a zero-mean random shock. They maintain this position through their dividend distribution policy. The quantity $\overline{\mathcal{B}}^* + \theta_t^*/\vartheta$ is interpreted as the financiers' preferred asset position, as they require a premium for deviating from it in order to absorb excess domestic savings.

Let Q_t denote the financiers' holdings of domestic bonds, which can be either positive or negative. The absolute value of Q_t reflects the scale of financial intermediation in the economy. When domestic agents require excess resources, the financiers borrow from abroad in foreign currency and extend a loan of the same value in domestic currency to domestic agents ($Q_t > 0$). When domestic agents wish to save, they lend the financiers in domestic currency ($Q_t < 0$) and the financiers convert these funds into foreign bonds. The asset portfolio of the financial sector is therefore composed of Q_t units of domestic bonds and $\overline{\mathcal{B}}^* + \theta_t^*/\vartheta - \frac{Q_t}{S_t}$ units of foreign bonds.

The financiers' pre-dividend domestic-currency value at the end of their one period term is given by $(1 + i_t) Q_t + S_{t+1} (1 + i_t^*) \left(\overline{\mathcal{B}}^* + \theta_t^*/\vartheta - \frac{Q_t}{S_t} \right)$, and they seek to maximize its expected discounted value, which can be written as:

$$V_t = \left[1 - \frac{1 + i_t^*}{1 + i_t} E_t (\sigma_{t+1}) \right] Q_t + E_t (S_{t+1}) \frac{1 + i_t^*}{1 + i_t} \left(\overline{\mathcal{B}}^* + \frac{\theta_t^*}{\vartheta} \right) \tag{A.11}$$

Financiers are unable to perfectly commit to repay their creditors, and before the end of period t, i.e. before S_{t+1} is realized, they can divert a portion $\Gamma \left| \frac{Q_t}{S_t} \right|$ of their liabilities,

$\Gamma > 0$. Since creditors correctly anticipate the incentives of the financiers, the latter are subject to a credit constraint of the form:

$$V_t \geq E_t(S_{t+1}) \frac{1+i_t^*}{1+i_t} \left(\overline{\mathcal{B}}^* + \frac{\theta_t^*}{\vartheta} \right) + \Gamma \left| \frac{Q_t}{S_t} \right| |Q_t| = E_t(S_{t+1}) \frac{1+i_t^*}{1+i_t} \left(\overline{\mathcal{B}}^* + \frac{\theta_t^*}{\vartheta} \right) + \Gamma \frac{Q_t^2}{S_t} \tag{A.12}$$

The financiers' problem is therefore to choose Q_t so as to maximize V_t, as presented in (A.11), subject to (A.12). Since the objective function is linear in Q_t while the constraint is convex, at the optimum the constraint always binds, and the financiers' demand for foreign assets, in excess of their base position $\overline{\mathcal{B}}^* + \theta_t^*/\vartheta$, is given by:

$$-\frac{Q_t}{S_t} = \frac{1}{\Gamma} \left[\frac{1+i_t^*}{1+i_t} E_t(\sigma_{t+1}) - 1 \right] \tag{A.13}$$

This is the modified UIP equation in the GM model. I will now express it in terms of quantities comparable to those of the portfolio adjustment cost model. Let $b_t^{*,HH}$ denote the value of assets, in units of foreign currency, that domestic households hold through financial intermediaries. These assets are composed of $-Q_t$ home-currency deposits, and a claim to a fraction ϑ of the financiers' net worth, suggesting:

$$\begin{aligned} b_t^{*,HH} &= -\frac{Q_t}{S_t} + \overline{b}^{*,HH} + \theta_t^* \\ \text{where} \quad \overline{b}^{*,HH} &\equiv \vartheta \overline{\mathcal{B}}^* \end{aligned}$$

Substituting for $-\frac{Q_t}{S_t}$ in (A.13) and rearranging, the modified UIP reads:

$$E_t(\sigma_{t+1}) = \frac{1+i_t}{1+i_t^*} \left[1 + \Gamma \left(b_t^{*,HH} - \overline{b}^{*,HH} - \theta_t^* \right) \right] \tag{A.14}$$

Finally, The financiers' distributed dividends are given by:

$$\Pi_t = (1+i_{t-1}^*) \left(\overline{\mathcal{B}}^* + \theta_{t-1}^*/\vartheta - \frac{Q_{t-1}}{S_{t-1}} \right) + \frac{1+i_{t-1}}{\sigma_t} \frac{Q_{t-1}}{S_{t-1}} - \left(\overline{\mathcal{B}}^* + \theta_t^*/\vartheta \right)$$

where the first two terms on the right-hand sides are the gross return on the previous period's holdings of foreign and domestic bonds, and the last term subtracts the financier's net worth that is carried over to the current period.

Market Clearing and the BOP In the financial markets:

$$B_t^G + B_t^{HH} + Q_t + B_t^{ROW} = 0$$

The BOP identity is derived by consolidating the government budget constraint and the households' budget constraint together with the market clearing condition above, while taking into account that a portion ϑ of the financiers' dividends are distributed to domestic households. This results in:

$$\begin{aligned} FX_t + b_t^{*,HH} &= \left(1+i_{t-1}^*\right) FX_{t-1} + \phi_t^* + Y_t - C_t \\ &+ \left[(1-\vartheta)\frac{1+i_{t-1}}{\sigma_t} + \vartheta\left(1+i_{t-1}^*\right)\right] b_{t-1}^{*,HH} \\ &+ (1-\vartheta)\left[\left(1+i_{t-1}^*\right) - \frac{1+i_{t-1}}{\sigma_t}\right]\left(\bar{b}^{*,HH} + \theta_{t-1}^*\right) \end{aligned} \tag{A.15}$$

where ϕ_t^* is defined in (A.1).

Closing the Model The households' optimality condition, equation (A.10), the modified UIP, equation (A.14), together with the BOP, equation (A.15), result in a system of 3 equations in 5 endogenous variables: C_t, i_t, σ_t, $b_t^{*,HH}$ and FX_t. Y_t, i_t^*, ϕ_t^* and θ_t^* are exogenous. The model is closed by specifying a policy rule for the nominal interest rate, i_t, and for foreign reserves, FX_t.

Log-Linearized Equations Log-linearizing equations (A.10), (A.14) and (A.15), the approximated GM model is characterized by:

$$\gamma_{cc}\widetilde{C}_t \cong \widetilde{(1+i_t)} + \gamma_{cc}E_t\left(\widetilde{C}_{t+1}\right) - E_t\left(\widetilde{\sigma}_{t+1}\right) \tag{A.16}$$

$$\begin{aligned} E_t\left(\widetilde{\sigma}_{t+1}\right) &\cong \widetilde{(1+i_t)} - \widetilde{(1+i_t^*)} \\ &+\Gamma\left[\left(b_t^{*,HH} - \bar{b}^{*,HH}\right) - \theta_t^*\right] \end{aligned} \tag{A.17}$$

$$\begin{aligned} \frac{FX_{ss}}{Y_{ss}}\widetilde{FX}_t + \frac{b_t^{*,HH} - \bar{b}^{*,HH}}{Y_{ss}} &\cong \widetilde{Y}_t - \frac{C_{ss}}{Y_{ss}}\widetilde{C}_t + \beta^{-1}\frac{FX_{ss} + \bar{b}^{*,HH}}{Y_{ss}}\widetilde{(1+i_{t-1}^*)} \\ &+\beta^{-1}\left[\frac{FX_{ss}}{Y_{ss}}\widetilde{FX}_{t-1} + \frac{b_{t-1}^{*,HH} - \bar{b}^{*,HH}}{Y_{ss}}\right] + \frac{\phi_t^* - \phi_{ss}^*}{Y_{ss}} \end{aligned} \tag{A.18}$$

A.4 The IM Model

This section adopts the financial structure of Itskhoki and Mukhin (2021). The derivation below builds on Appendix A.4 of their paper. In their model, risk aversion of financial intermediaries generates deviations from the UIP.[49] The households' problem is identical to that of the GM model, so I start with the description of the financial sector.

Financiers As in the GM model, financiers start each period with no liabilities and a net worth of $\overline{\mathcal{B}}^* + \theta_t^*/\vartheta$, denominated in foreign currency, and held in foreign bonds. They maintain this position through their dividend distribution policy. Let Q_t denote the financiers' holdings of domestic bonds. The asset portfolio of the financial sector is composed of Q_t units of domestic bonds and $\overline{\mathcal{B}}^* + \theta_t^*/\vartheta - \frac{Q_t}{S_t}$ units of foreign bonds.

Letting $q_t \equiv -\frac{Q_t}{S_t}$, the present discounted value of the financiers' pre-dividend portfolio, denominated in foreign currency, is given by:

$$V_t = \left[1 - \frac{1+i_t}{1+i_t^*}\frac{1}{\sigma_{t+1}}\right] q_t + \overline{\mathcal{B}}^* + \frac{\theta_t^*}{\vartheta} \tag{A.19}$$

Financial intermediaries optimally choose q_t by maximizing the expected value of a CARA utility, $U(V_t) = -\frac{1}{\omega}\exp(-\omega V_t)$. Note that:

$$E_t U(V_t) = -\frac{1}{\omega} E_t \exp\left\{-\omega\left[1 - \frac{1+i_t}{1+i_t^*}\frac{1}{\sigma_{t+1}}\right] q_t\right\} \exp\left\{-\omega\left(\overline{\mathcal{B}}^* + \frac{\theta_t^*}{\vartheta}\right)\right\}$$

and since $\exp\left\{-\omega\left(\overline{\mathcal{B}}^* + \frac{\theta_t^*}{\vartheta}\right)\right\}$ is positive and known at the time of the portfolio choice, it does not affect the financiers' decision and can be dropped from the objective function. Letting:

$$x_{t+1} \equiv \log(1+i_t) - \log(1+i_t^*) - \log(\sigma_{t+1})$$

The financiers' problem can be written as:

$$\underset{q_t}{Max} \qquad -\frac{1}{\omega} E_t \exp\left[-\omega\left(1 - e^{x_{t+1}}\right) q_t\right] \tag{A.20}$$

At this stage Itskhoki and Mukhin (2021) approximate the problem to its continuous time counterpart. When time periods are short x_{t+1} corresponds to increments of a normal

[49] Itskhoki and Mukhin (2023) adopt a slightly different modelling of the financial sector and resort to a novel approximation technique that leaves their UIP equation non-linear. Nevertheless, under *standard* first order approximation around the deterministic steady state, coupled with the assumption that as the variance of changes in the exchange rate falls the financiers' risk aversion rises proportionally (see Itskhoki and Mukhin (2021, 2023) and below), it is immediate to show that the simple portfolio adjustment cost is isomorphic to the model of Itskhoki and Mukhin (2023) as well.

diffusion process $d\mathcal{X}_t$ with time-varying drift $\mu_t = \log(1+i_t) - \log(1+i_t^*) - E_t[\log(\sigma_{t+1})]$ and time-invariant conditional variance $\sigma_s^2 = var_t[\log(\sigma_{t+1})]$:

$$d\mathcal{X}_t = \mu_t dt + \sigma_s^2 dB_t \tag{A.21}$$

where B_t is a Brownian motion. With short time periods, the solution to (A.20) is equivalent to:

$$\underset{q_t}{Max} \qquad -\frac{1}{\omega} E_t \exp\left[-\omega\left(1 - e^{d\mathcal{X}_t}\right) q_t\right]$$

where $d\mathcal{X}_t$ follows (A.21). Using Ito's lemma the financiers' problem can be written as:

$$\underset{q_t}{Max} \qquad -\frac{1}{\omega} E_t \exp\left[\omega\left(\mu_t + \frac{1}{2}\sigma_s^2\right) q_t + \frac{\omega^2\sigma_s^2}{2} q_t^2\right] \tag{A.22}$$

Taking first order condition and rearranging:

$$q_t = -\frac{\mu_t + \frac{1}{2}\sigma_s^2}{\omega\sigma_s^2}$$

Substituting for μ_t and q_t results in:

$$-\frac{Q_t}{S_t} = -\frac{\log(1+i_t) - \log(1+i_t^*) - E_t[\log(\sigma_{t+1})] + \frac{1}{2}\sigma_s^2}{\omega\sigma_s^2} \tag{A.23}$$

This is the modified UIP equation in the IM model. I will now express it in terms of quantities comparable to those of the portfolio adjustment cost model. Let $b_t^{*,HH}$ denote the value of assets, in units of foreign currency, that domestic households hold through financial intermediaries. These assets are composed of $-Q_t$ home-currency deposits, and a claim to a fraction ϑ of the financiers' net worth, suggesting:

$$\begin{aligned} b_t^{*,HH} &= -\frac{Q_t}{S_t} + \overline{b}^{*,HH} + \theta_t^* \\ \text{where} \qquad \overline{b}^{*,HH} &\equiv \vartheta\overline{\mathcal{B}}^* \end{aligned}$$

Substituting for $-\frac{Q_t}{S_t}$ in (A.23) and rearranging, the modified UIP reads:

$$E_t[\log(\sigma_{t+1})] = \log(1+i_t) - \log(1+i_t^*) + \omega\sigma_s^2\left(b_t^{*,HH} - \overline{b}^{*,HH} - \theta_t^*\right) - \frac{1}{2}\sigma_s^2 \tag{A.24}$$

Itskhoki and Mukhin (2021) assume that as σ_s^2 shrinks, i.e. exchange rate risk falls, the financiers' risk aversion, ω, rises proportionally leaving the product $\omega\sigma_s^2$ constant and nonzero in the limit. This assumption guarantees that the risk premium in (A.24) is first order, and does not wash into the approximation error in the log-linearized system.

Finally, the financiers' distributed dividends are given by:

$$\Pi_t = \left(1+i^*_{t-1}\right)\left(\overline{\mathcal{B}}^* + \theta^*_{t-1}/\vartheta - \frac{Q_{t-1}}{S_{t-1}}\right) + \frac{1+i_{t-1}}{\sigma_t}\frac{Q_{t-1}}{S_{t-1}} - \left(\overline{\mathcal{B}}^* + \theta^*_t/\vartheta\right)$$

which is the same as in the GM model.

Market Clearing and the BOP In the financial markets:

$$B^G_t + B^{HH}_t + Q_t + B^{ROW}_t = 0$$

The BOP identity is derived by consolidating the government budget constraint and the households' budget constraint together with the market clearing condition above, while taking into account that a portion ϑ of the financiers' dividends are distributed to domestic households. This results in:

$$\begin{aligned} FX_t + b^{*,HH}_t &= \left(1+i^*_{t-1}\right) FX_{t-1} + \phi^*_t + Y_t - C_t \\ &+ \left[\vartheta\left(1+i^*_{t-1}\right) + (1-\vartheta)\frac{1+i_{t-1}}{\sigma_t}\right] b^{*,HH}_{t-1} \\ &+ (1-\vartheta)\left[\left(1+i^*_{t-1}\right) - \frac{1+i_{t-1}}{\sigma_t}\right]\left(\overline{b}^{*,HH} + \theta^*_{t-1}\right) \end{aligned} \tag{A.25}$$

which is the same as the BOP in the GM model, equation (A.15).

Closing the Model The households' optimality condition is the same as in the GM model, equation (A.10), together with the modified UIP, equation (A.24), and the BOP, equation (A.25), result in a system of 3 equations in 5 endogenous variables: C_t, i_t, σ_t, $b^{*,HH}_t$ and FX_t. Y_t, i^*_t, ϕ^*_t and θ^*_t are exogenous. The model is closed by specifying a policy rule for the nominal interest rate, i_t, and for foreign reserves, FX_t.

Log-Linearized Equations Log-linearizing equations (A.10), (A.24) and (A.25), the approximated IM model is characterized by:

$$\gamma_{cc}\widetilde{C}_t \cong \widetilde{(1+i_t)} + \gamma_{cc}E_t\left(\widetilde{C}_{t+1}\right) - E_t\left(\widetilde{\sigma}_{t+1}\right) \tag{A.26}$$

$$\begin{aligned} E_t\left(\widetilde{\sigma}_{t+1}\right) &\cong \widetilde{(1+i_t)} - \widetilde{(1+i_t^*)} \\ &\quad + \omega\sigma_s^2\left[\left(b_t^{*,HH} - \overline{b}^{*,HH}\right) - \theta_t^*\right] \end{aligned} \tag{A.27}$$

$$\begin{aligned} \frac{FX_{ss}}{Y_{ss}}\widetilde{FX}_t + \frac{b_t^{*,HH} - \overline{b}^{*,HH}}{Y_{ss}} &\cong \widetilde{Y}_t - \frac{C_{ss}}{Y_{ss}}\widetilde{C}_t + \beta^{-1}\frac{FX_{ss} + \overline{b}^{*,HH}}{Y_{ss}}\widetilde{(1+i_{t-1}^*)} \\ &\quad + \beta^{-1}\left[\frac{FX_{ss}}{Y_{ss}}\widetilde{FX}_{t-1} + \frac{b_{t-1}^{*,HH} - \overline{b}^{*,HH}}{Y_{ss}}\right] + \frac{\phi_t^* - \phi_{ss}^*}{Y_{ss}} \end{aligned} \tag{A.28}$$

A.5 Model Equivalence

Equations (A.16) and (A.26) are identical to (A.7), equations (A.18) and (A.28) are identical to (A.9), and for $\Gamma = \Theta''(0) = \omega\sigma_s^2$ equations (A.17) and (A.27) are identical to (A.8), suggesting the portfolio adjustment cost is isomorphic, up to a first-order approximation, to the GM and IM models.

B Appendix: The Frisch Elasticity of Labor Supply

After deriving the dynamics of wage inflation, equation (31), the text noted that the expression $\gamma_{nn} - \frac{\gamma_{nc}\gamma_{cn}}{\gamma_{cc}}$ is the inverse of the Frisch elasticity of labor supply evaluated in steady state. This appendix shows, more generally, that this expression corresponds to the inverse of the Frisch elasticity of labor supply under flexible wages.

Proposition B.1 *Under flexible wages, i.e. as $\xi_w \longrightarrow 0$, the Frisch elasticity of labor supply is given by:*

$$\left.\frac{\partial N_t}{\partial \omega_t}\right|_{\lambda_t}\frac{\omega_t}{N_t} = \left(\gamma_{nn,t} - \frac{\gamma_{nc,t}\gamma_{cn,t}}{\gamma_{cc,t}}\right)^{-1}$$

where $$\gamma_{cc,t} \equiv \frac{U_{cc,t}}{U_{c,t}}C_t \quad , \quad \gamma_{nn,t} \equiv \frac{U_{nn,t}}{U_{n,t}}N_t$$
$$\gamma_{cn,t} \equiv \frac{U_{cn,t}}{U_{c,t}}N_t \quad , \quad \gamma_{nc,t} \equiv \frac{U_{nc,t}}{U_{n,t}}C_t$$

$\omega_t \equiv \frac{W_t}{P_t}$ is real wage and λ_t is the Lagrange multiplier of the households' intertemporal budget constraint.

Proof. Under flexible wages, the households' optimality conditions are given by:

$$U_{c,t} = \lambda_t \tag{B.29}$$

$$-\frac{\varepsilon^N}{\varepsilon^N - 1} U_{n,t} = \lambda_t \omega_t \tag{B.30}$$

Partially differentiating with respect to the real wage while holding λ_t constant results in:

$$U_{cc,t} \left.\frac{\partial C_t}{\partial \omega_t}\right|_{\lambda_t} + U_{cn,t} \left.\frac{\partial N_t}{\partial \omega_t}\right|_{\lambda_t} = 0 \tag{B.31}$$

$$-\frac{\varepsilon^N}{\varepsilon^N - 1} U_{nc,t} \left.\frac{\partial C_t}{\partial \omega_t}\right|_{\lambda_t} - \frac{\varepsilon^N}{\varepsilon^N - 1} U_{nn,t} \left.\frac{\partial N_t}{\partial \omega_t}\right|_{\lambda_t} = \lambda_t \tag{B.32}$$

By (B.31):

$$\left.\frac{\partial C_t}{\partial \omega_t}\right|_{\lambda_t} = -\frac{U_{cn,t}}{U_{cc,t}} \left.\frac{\partial N_t}{\partial \omega_t}\right|_{\lambda_t}$$

and by the optimality condition for wages, equation (B.30):

$$\lambda_t = -\frac{\varepsilon^N}{\varepsilon^N - 1} \frac{U_{n,t}}{\omega_t}$$

Substituting the results into (B.32) gives:

$$\frac{U_{nc,t} U_{cn,t}}{U_{cc,t}} \left.\frac{\partial N_t}{\partial \omega_t}\right|_{\lambda_t} - U_{nn,t} \left.\frac{\partial N_t}{\partial \omega_t}\right|_{\lambda_t} = -\frac{U_{n,t}}{\omega_t}$$

Rearrange and get the Frisch elasticity of labor supply:

$$\left.\frac{\partial N_t}{\partial \omega_t}\right|_{\lambda_t} \frac{\omega_t}{N_t} = \frac{U_{n,t}}{N_t \left(U_{nn,t} - \frac{U_{nc,t} U_{cn,t}}{U_{cc,t}} \right)}$$

Now rewrite this expression in terms of the elasticities of the marginal utilities, $U_{c,t}$ and $U_{n,t}$, with respect to consumption and labor, C_t and N_t:

$$\left.\frac{\partial N_t}{\partial \omega_t}\right|_{\lambda_t} \frac{\omega_t}{N_t} = \frac{1}{\frac{U_{nn,t}}{U_{n,t}} N_t - \frac{\frac{U_{nc,t}}{U_{n,t}} C_t \frac{U_{cn,t}}{U_{c,t}} N_t}{\frac{U_{cc,t}}{U_{c,t}} C_t}}$$

Suggesting:

$$\left.\frac{\partial N_t}{\partial \omega_t}\right|_{\lambda_t} \frac{\omega_t}{N_t} = \left(\gamma_{nn,t} - \frac{\gamma_{nc,t} \gamma_{cn,t}}{\gamma_{cc,t}} \right)^{-1}$$

■

C Appendix: Second-Order Approximation of the Welfare Function

A utilitarian policymaker seeks to maximize welfare in the economy as measured by the aggregate expected discounted utility of domestic households, that is:

$$\mathbb{W} \equiv E_0 \sum_{t=0}^{\infty} \beta^t \int_0^1 U\left(c_t\left(h\right), n_t\left(h\right); \eta_t\right) dh$$

Taking second order approximation results in:

$$\begin{aligned}
\frac{\mathbb{W}-\mathbb{W}_{ss}}{U_C C_{ss}} = \; & E_0 \sum_{t=0}^{\infty} \beta^t \left(\widetilde{C}_t + \frac{U_{N_{ss}} N_{ss}}{U_{C_{ss}} C_{ss}} \widetilde{N}_t\right) \\
& + \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \begin{bmatrix} \widetilde{C}_t \\ \widetilde{N}_t \end{bmatrix}' \begin{bmatrix} \gamma_{cc}+1 & \gamma_{cn} \\ \gamma_{cn} & \frac{U_N N_{ss}}{U_C C_{ss}}\left(\gamma_{nn}+1\right) \end{bmatrix} \begin{bmatrix} \widetilde{C}_t \\ \widetilde{N}_t \end{bmatrix} \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \left[\gamma_{c\eta} \widetilde{\eta}_t \widetilde{C}_t + \frac{U_{N_{ss}} N_{ss}}{U_{C_{ss}} C_{ss}} \gamma_{n\eta} \widetilde{\eta}_t \widetilde{N}_t\right] \\
& + E_0 \sum_{t=0}^{\infty} \beta^t \begin{bmatrix} \frac{1}{2}\gamma_{cc} Var_h\left[\widetilde{c}_t\left(h\right)\right] + \frac{1}{2}\frac{U_{N_{ss}} N_{ss}}{U_{C_{ss}} C_{ss}}\left(\gamma_{nn}+\frac{1}{\varepsilon^N}\right) Var_h\left[\widetilde{n}_t\left(h\right)\right] \\ +\gamma_{cn} Cov_h\left[\widetilde{c}_t\left(h\right), \widetilde{n}_t\left(h\right)\right] \end{bmatrix} \\
& + t.i.p. + \mathcal{O}\left(\|\cdot\|^3\right)
\end{aligned}$$

where:

$$Var_h\left[\widetilde{c}_t\left(h\right)\right] \equiv \int_0^1 \left(\widetilde{c}_t\left(h\right) - E_h\left[\widetilde{c}_t\left(h\right)\right]\right)^2 dh \quad , \quad E_h\left[\widetilde{c}_t\left(h\right)\right] \equiv \int_0^1 \widetilde{c}_t\left(h\right) dh = \widetilde{C}_t$$

$$Var_h\left[\widetilde{n}_t\left(h\right)\right] \equiv \int_0^1 \left(\widetilde{n}_t\left(h\right) - E_h\left[\widetilde{n}_t\left(h\right)\right]\right)^2 dh \quad , \quad E_h\left[\widetilde{n}_t\left(h\right)\right] \equiv \int_0^1 \widetilde{n}_t\left(h\right) dh = \widetilde{N}_t$$

$$Cov_h\left[\widetilde{c}_t\left(h\right), \widetilde{n}_t\left(h\right)\right] \equiv \int_0^1 \left(\widetilde{c}_t\left(h\right) - E_h\left[\widetilde{c}_t\left(h\right)\right]\right)\left(\widetilde{n}_t\left(h\right) - E_h\left[\widetilde{n}_t\left(h\right)\right]\right) dh$$

Equating marginal utilities of consumption across households, yields:

$$\widetilde{c}_t\left(h\right) - E_h\left[\widetilde{c}_t\left(h\right)\right] = -\frac{\gamma_{cn}}{\gamma_{cc}}\left[\widetilde{n}_t\left(h\right) - E_h\left[\widetilde{n}_t\left(h\right)\right]\right] + \mathcal{O}\left(\|\cdot\|^2\right)$$

Suggesting:

$$\begin{aligned}
Var_h\left[\widetilde{c}_t\left(h\right)\right] &= \left(\frac{\gamma_{cn}}{\gamma_{cc}}\right)^2 Var_h\left[\widetilde{n}_t\left(h\right)\right] + \mathcal{O}\left(\|\cdot\|^3\right) \\
Cov_h\left[\widetilde{c}_t\left(h\right), \widetilde{n}_t\left(h\right)\right] &= -\frac{\gamma_{cn}}{\gamma_{cc}} Var_h\left[\widetilde{n}_t\left(h\right)\right] + \mathcal{O}\left(\|\cdot\|^3\right)
\end{aligned}$$

and using demand for labor skill h, equation (22), we get:

$$Var_h\left[\widetilde{n}_t\left(h\right)\right] = \left(\varepsilon^N\right)^2 Var_h\left[\widetilde{w}_t\left(h\right)\right]$$

Substituting the results into the approximated welfare function, gives:

$$
\begin{aligned}
\frac{\mathbb{W}-\mathbb{W}_{ss}}{U_C C_{ss}} &= E_0 \sum_{t=0}^{\infty} \beta^t \left(\widetilde{C}_t + \frac{U_{N_{ss}} N_{ss}}{U_{C_{ss}} C_{ss}} \widetilde{N}_t\right) \\
&+ \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \begin{bmatrix} \widetilde{C}_t \\ \widetilde{N}_t \end{bmatrix}' \begin{bmatrix} \gamma_{cc}+1 & \gamma_{cn} \\ \gamma_{cn} & \frac{U_N N_{ss}}{U_C C_{ss}} (\gamma_{nn}+1) \end{bmatrix} \begin{bmatrix} \widetilde{C}_t \\ \widetilde{N}_t \end{bmatrix} \\
&+ E_0 \sum_{t=0}^{\infty} \beta^t \left[\gamma_{c\eta} \widetilde{\eta}_t \widetilde{C}_t + \frac{U_{N_{ss}} N_{ss}}{U_{C_{ss}} C_{ss}} \gamma_{n\eta} \widetilde{\eta}_t \widetilde{N}_t\right] \\
&+ \frac{1}{2} \varepsilon^N \frac{U_N N_{ss}}{U_C C_{ss}} \left[1 + \left(\gamma_{nn} - \frac{\gamma_{nc}\gamma_{cn}}{\gamma_{cc}}\right) \varepsilon^N\right] E_0 \sum_{t=0}^{\infty} \beta^t Var_h \left[\widetilde{w}_t(h)\right] \\
&+ t.i.p. + \mathcal{O}\left(\|\cdot\|^3\right)
\end{aligned}
\tag{C.1}
$$

In order to solve for optimal policies, we will seek to maximize the approximated welfare criterion subject to linearized equilibrium conditions. However, Benigno and Woodford (2012) show that for the solution of such a problem to approximate the solution of the exact optimization problem, all endogenous variables in the objective function must be second order. Furthermore, this condition is also required for the approximated welfare criterion to correctly rank alternative equilibrium allocations that are approximated to first order. Hence, in order to derive a valid welfare criterion we must express the linear term in (C.1) as:

$$
E_0 \sum_{t=0}^{\infty} \beta^t \left(\widetilde{C}_t + \frac{U_{N_{ss}} N_{ss}}{U_{C_{ss}} C_{ss}} \widetilde{N}_t\right) = t.i.p. + \mathcal{O}\left(\|\cdot\|^2\right) \tag{C.2}
$$

This can be achieved by choosing the subsidy rate τ_w to support an efficient steady state, and by substituting for the linear term using second order approximation to the balance of payments and the resource constraint of the economy.

Rolling forward the balance of payments, equation (39), setting gross foreign real interest rate to β^{-1}, substituting for EX_t, using (13) and for IM_t, using (10) and (9), we get the intertemporal budget constraint of the economy:

$$
\begin{aligned}
&\beta^{-1}\left(Y_{ss}^A \widehat{b}_{-1}^{*,HH} + FX_{-1}\right) \\
&= \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} (1-\vartheta)\left[\Theta\left(\widehat{b}_t^{*,HH} - \widehat{\theta}_t^*\right) + \Theta^{CB}(FX_t)\right] - Y_{ss}^A \widehat{\phi}_t^* \\ +\lambda\left[(1-\lambda) TOT_t^{1-\varepsilon} + \lambda\right]^{\frac{\varepsilon}{1-\varepsilon}} C_t - TOT_t^{1-\varepsilon^*} WT_t \end{array} \right\}
\end{aligned}
$$

The resource constraint, equation (2), after substituting for Y_t^H using (38), for d_t^H using

(10) and (8), and for EX_t using (13), reads:

$$A_t \left(\frac{N_t}{pd_t}\right)^{\alpha} = (1-\lambda)\left[(1-\lambda)+\lambda TOT_t^{\varepsilon-1}\right]^{\frac{\varepsilon}{1-\varepsilon}} C_t + G_t + TOT_t^{-\varepsilon^*} WT_t$$

Taking second order approximation to both, and combining the results by substituting for $\widetilde{TOT}_t$, we get:

$$\begin{aligned} & \sum_{t=0}^{\infty} \beta^t \left\{ \widetilde{C}_t + \frac{\varepsilon(1-\lambda)+\varepsilon^*-1}{\varepsilon(1-\lambda)+\varepsilon^*-(1-\lambda)} \frac{1}{\Phi} \frac{U_{N_{ss}} N_{ss}}{U_{C_{ss}} C_{ss}} \widetilde{N}_t \right\} \\ = & \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \frac{1}{2} y_{1,t}' \Psi_{11} y_{1,t} + x_t' \Psi_{x1} y_{1,t} + \frac{1}{2} y_{2,t}' \Psi_{22} y_{2,t} \\ -\frac{1}{2}\lambda\phi_1\phi_2 \frac{\varepsilon^L\left[\alpha\left(1-\varepsilon^L\right)+\varepsilon^L\right]}{\alpha} \frac{A_{ss}N_{ss}^{\alpha}}{IM_{ss}} Var_f\left[\widetilde{p}_t^H(f)\right] \end{array} \right\} \\ & + t.i.p. + \mathcal{O}\left(\|\cdot\|^3\right) \end{aligned} \tag{C.3}$$

where:

$$y_{1,t} \equiv \left[\begin{array}{ccc} \widetilde{C}_t & \widetilde{N}_t & \widetilde{TOT}_t \end{array}\right]' \qquad \Phi \equiv \frac{1}{1-\tau_w}\frac{\varepsilon^N-1}{\varepsilon^N}\frac{\varepsilon^L-1}{\varepsilon^L}$$

$$y_{2,t} \equiv \left[\begin{array}{cc} \widehat{b}_t^{*,HH} - \widehat{\theta}_t^* & \widetilde{FX}_t \end{array}\right]' \qquad \phi_1 \equiv \frac{(1-\lambda)\varepsilon+\varepsilon^*}{(1-\lambda)\varepsilon+\varepsilon^*-(1-\lambda)}$$

$$x_t \equiv \left[\begin{array}{ccc} \widetilde{\eta}_t & \widetilde{A}_t & \widetilde{WT}_t \end{array}\right]' \qquad \phi_2 \equiv \frac{(1-\lambda)\varepsilon+\varepsilon^*-1}{(1-\lambda)\varepsilon+\varepsilon^*}$$

and:

$$\begin{aligned} \Psi_{11} & \equiv -\lambda\phi_1 \left[\begin{array}{ccc} 1+\phi_2\frac{1-\lambda}{\lambda} & 0 & \frac{(1-\lambda)\varepsilon}{(1-\lambda)\varepsilon+\varepsilon^*} \\ 0 & -\phi_2\alpha^2\frac{A_{ss}N_{ss}^{\alpha}}{IM_{ss}} & 0 \\ \frac{(1-\lambda)\varepsilon}{(1-\lambda)\varepsilon+\varepsilon^*} & 0 & \begin{array}{c} \varepsilon(1-\lambda)\left[1-\frac{(1-\lambda)(1-\varepsilon)+\lambda\varepsilon}{(1-\lambda)\varepsilon+\varepsilon^*}\right] \\ -(1-\varepsilon^*)^2+\phi_2(\varepsilon^*)^2 \end{array} \end{array}\right] \\ \Psi_{x1} & \equiv \lambda\phi_1 \left[\begin{array}{ccc} 0 & 0 & 0 \\ 0 & \phi_2\alpha\frac{A_{ss}N_{ss}^{\alpha}}{IM_{ss}} & 0 \\ 0 & 0 & (1-\varepsilon^*)+\phi_2\varepsilon^* \end{array}\right] \\ \Psi_{22} & \equiv -\frac{\lambda}{IM_{ss}}\phi_1 \left[\begin{array}{cc} (1-\vartheta)\Theta''(0) & 0 \\ 0 & (1-\vartheta)\Theta^{CB\prime\prime}(FX_{ss})FX_{ss}^2 \end{array}\right] \end{aligned}$$

Comparing (C.3) to (C.2), it follows that the condition for a valid welfare criterion is satisfied if:

$$1-\tau_w = \frac{\varepsilon^N-1}{\varepsilon^N}\frac{\varepsilon^L-1}{\varepsilon^L}\frac{(1-\lambda)\varepsilon+\varepsilon^*-(1-\lambda)}{(1-\lambda)\varepsilon+\varepsilon^*-1}$$

which is exactly the optimal subsidy, in equation (59), that supports the efficient equilibrium in steady state. Under this subsidy we can now use (C.3) to substitute for $\sum_{t=0}^{\infty}\beta^t\left(\widetilde{C}_t+\frac{U_N N_{ss}}{U_C C_{ss}}\widetilde{N}_t\right)$ in (C.1).

The last step is to move from dispersion of wages and prices, $Var_h\left[\widetilde{w}_t\left(h\right)\right]$ and $Var_f\left[\widetilde{p}_t^H\left(f\right)\right]$, to wage inflation and home-good inflation, $\widetilde{\pi}_t^w$ and $\widetilde{\pi}_t^H$. Using proposition 6.3 in Woodford (2003), we get:

$$\begin{aligned}\sum_{t=0}^{\infty}\beta^t Var_f\left[\widetilde{p}_t^H\left(f\right)\right] &= \frac{\xi_p}{\left(1-\xi_p\right)\left(1-\beta\xi_p\right)}\sum_{t=0}^{\infty}\beta^t\left(\widetilde{\pi}_t^H\right)^2 \\ \sum_{t=0}^{\infty}\beta^t Var_h\left[\widetilde{w}_t\left(h\right)\right] &= \frac{\xi_w}{\left(1-\xi_w\right)\left(1-\beta\xi_w\right)}\sum_{t=0}^{\infty}\beta^t\left(\widetilde{\pi}_t^w\right)^2\end{aligned}$$

Following these steps, and using steady state equilibrium relations to simplify coefficients, we get the approximated welfare function as presented in equation (60) in the text.

D Appendix: Characterizing the Optimal Allocation

This appendix characterizes the equilibrium allocations under optimal policies. I consider three cases. First is the fully optimal allocation, where the central bank uses both its tools, FXI and the interest rate, optimally. Second, consider the case where the central bank uses an optimal interest rate policy while holding foreign reserves fixed. In the third case, the interest rate is set optimally while FXIs follow a predetermined policy rule.

D.1 Optimal FXI and Optimal Interest Rate Policy

Before solving for the optimal allocation, I first reduce the system of equilibrium conditions by substituting for $\widetilde{Y}_t^H$, $\widetilde{d}_t^H$, $\widetilde{IM}_t$, $\widetilde{EX}_t$, $\widetilde{p}_t^H$, $\widetilde{p}_t^F$ and $\widetilde{\sigma}_t$, to get the following set of constraints.

Wage inflation dynamics, equation (40), and the change in real wage, equation (53), are:

$$\xi_w\widetilde{\pi}_t^w \cong \xi_w\beta E_t\left(\widetilde{\pi}_{t+1}^w\right) - \frac{\left(1-\xi_w\beta\right)\left(1-\xi_w\right)}{1+\left(\gamma_{nn}-\frac{\gamma_{nc}\gamma_{cn}}{\gamma_{cc}}\right)\varepsilon^N}\left(\widetilde{w}_t-\widetilde{U}_{N_t}+\widetilde{U}_{C_t}\right) \quad \text{(D.1)}$$

$$\widetilde{w}_t-\widetilde{w}_{t-1} \cong \widetilde{\pi}_t^w-\widetilde{\pi}_t \quad \text{(D.2)}$$

Home inflation dynamics, equation (41), and after substituting $\widetilde{p}_t^H \cong \lambda\widetilde{TOT}_t$ into equation (54), we have:

$$\begin{aligned}\xi_p \widetilde{\pi}_t^H &\cong \xi_p \beta E_t \left(\widetilde{\pi}_{t+1}^H\right) \\ &+ \frac{\left(1-\xi_p \beta\right)\left(1-\xi_p\right)\alpha}{\alpha + (1-\alpha)\varepsilon^L}\left[\widetilde{w}_t - \lambda \widetilde{TOT}_t - \widetilde{A}_t - (\alpha - 1)\widetilde{N}_t\right]\end{aligned} \tag{D.3}$$

$$\lambda \widetilde{TOT}_t - \lambda \widetilde{TOT}_{t-1} \cong \widetilde{\pi}_t^H - \widetilde{\pi}_t \tag{D.4}$$

The Euler equation for domestic bonds, equation (42), is:

$$\widetilde{U}_{C_t} \cong \widetilde{(1+i_t)} + E_t\left\{\widetilde{U}_{C_{t+1}}\right\} - E_t\left\{\pi_{t+1}\right\} \tag{D.5}$$

Using $\widetilde{p}_t^F \cong -(1-\lambda)\widetilde{TOT}_t$ and equation (55) to substitute for $\widetilde{\sigma}_{t+1}$ in the Euler equation for foreign bonds, equation (43), it reads:

$$\begin{aligned}&\widetilde{U}_{C_t} + \frac{\Theta''(0)}{Y_{ss}^{H,An.}}\left(\widehat{b}_t^{*,HH} - \widehat{\theta}_t^*\right) - (1-\lambda)\widetilde{TOT}_t \\ \cong\ &\widetilde{(1+i_t^*)} - E_t\left\{\widetilde{\pi}_{t+1}^{F*}\right\} + E_t\left\{\widetilde{U}_{C_{t+1}}\right\} - (1-\lambda)E_t\left\{\widetilde{TOT}_{t+1}\right\}\end{aligned} \tag{D.6}$$

Substituting for technology, exports and demand for home goods, the resources constraint, equation (49), reads:

$$\widetilde{A}_t + \alpha \widetilde{N}_t \cong (1-\lambda)\frac{C_{ss}}{Y_{ss}^H}\widetilde{C}_t + \frac{G_{ss}}{Y_{ss}^H}\widetilde{G}_t + \lambda \frac{C_{ss}}{Y_{ss}^H}\widetilde{WT}_t - \lambda \frac{C_{ss}}{Y_{ss}^H}\left[(1-\lambda)\varepsilon + \varepsilon^*\right]\widetilde{TOT}_t \tag{D.7}$$

Substituting for exports and imports demand, the balance of payments, equation (50), is given by:

$$\begin{aligned}FX_{ss}\widetilde{FX}_t + Y_{ss}^{H,An.}\widehat{b}_t^{*,HH} &\cong \frac{1}{\beta}\left(FX_{ss}\widetilde{FX}_{t-1} + Y_{ss}^{H,An.}\widehat{b}_{t-1}^{*,HH}\right) \\ &+ \frac{1}{\beta}FX_{ss}\left[\widetilde{(1+i_{t-1}^*)} - \widetilde{\pi}_t^{F*}\right] \\ &- \lambda C_{ss}\widetilde{C}_t + \lambda C_{ss}\left[1 - \varepsilon^* - (1-\lambda)\varepsilon\right]\widetilde{TOT}_t \\ &+ Y_{ss}^{H,An.}\left(\widehat{\phi}_t^* - \phi_{ss}^*\right) + \lambda C_{ss}\widetilde{WT}_t\end{aligned} \tag{D.8}$$

This gives a system of 8 periodical equations in 10 endogenous variables: $\widetilde{C}_t$, $\widetilde{N}_t$, $\widetilde{w}_t$, $\widetilde{\pi}_t^w$, $\widetilde{\pi}_t$, $\widetilde{\pi}_t^H$, $\widetilde{TOT}_t$, $\widetilde{(1+i_t)}$, $\widetilde{FX}_t$, and $\widehat{b}_t^{*,HH}$; where we have 2 definitions:

$$\widetilde{U}_{N,t} \cong \gamma_{nc}\widetilde{C}_t + \gamma_{nn}\widetilde{N}_t + \gamma_{n\eta}\widetilde{\eta}_t \tag{D.9}$$

$$\widetilde{U}_{C,t} \cong \gamma_{cc}\widetilde{C}_t + \gamma_{cn}\widetilde{N}_t + \gamma_{c\eta}\widetilde{\eta}_t \tag{D.10}$$

To solve for the optimal allocation, set a Lagrangian using the objective function (60), and the constraints (D.1) to (D.8), and differentiate with respect to each of the endogenous variables. The first order conditions are presented below.

First order condition with respect to consumption, $\widetilde{C}_t$:

$$\begin{aligned}
& \widetilde{U}_{C_t} + \frac{\lambda\varepsilon(1-\lambda)}{(1-\varepsilon)(1-\lambda)-\varepsilon^*}\widetilde{TOT}_t \\
& -\frac{(1-\xi_w\beta)(1-\xi_w)}{1+\left(\gamma_{nn}-\frac{\gamma_{nc}\gamma_{cn}}{\gamma_{cc}}\right)\varepsilon^N}(\gamma_{nc}-\gamma_{cc})\phi_{wInf,t} \\
& -(1-\lambda)\frac{C_{ss}}{Y_{ss}^H}\phi_{RC,t}+\lambda C_{ss}\phi_{BOP,t}+\gamma_{cc}\left(\phi_{hEuler,t}+\phi_{fEuler,t}\right) \\
= & \frac{\gamma_{cc}}{\beta}\left(\phi_{hEuler,t-1}+\phi_{fEuler,t-1}\right)
\end{aligned} \tag{D.11}$$

First order condition with respect to labor, $\widetilde{N}_t$:

$$\begin{aligned}
& \frac{U_{N_{ss}}N_{ss}}{U_{C_{ss}}C_{ss}}\left[\widetilde{U}_{N_t}+(1-\alpha)\widetilde{N}_t-\widetilde{A}_t\right]+\gamma_{cn}\left(\phi_{hEuler,t}+\phi_{fEuler,t}\right)+\alpha\phi_{RC,t} \\
& -\frac{(1-\xi_w\beta)(1-\xi_w)}{1+\left(\gamma_{nn}-\frac{\gamma_{nc}\gamma_{cn}}{\gamma_{cc}}\right)\varepsilon^N}(\gamma_{nn}-\gamma_{cn})\phi_{wInf,t}-\frac{\left(1-\xi_p\beta\right)\left(1-\xi_p\right)\alpha}{\alpha+(1-\alpha)\varepsilon^L}(1-\alpha)\phi_{hInf,t} \\
= & \frac{\gamma_{cn}}{\beta}\left(\phi_{hEuler,t-1}+\phi_{fEuler,t-1}\right)
\end{aligned} \tag{D.12}$$

First order condition with respect to the terms of trade, $\widetilde{TOT}_t$:

$$\begin{aligned}
& \frac{\lambda\varepsilon(1-\lambda)}{(1-\varepsilon)(1-\lambda)-\varepsilon^*}\left\{\left[\begin{array}{c} 3\varepsilon^*-\frac{\varepsilon^*(1-\varepsilon^*)}{\varepsilon(1-\lambda)} \\ +(2-3\lambda)\varepsilon-(2-\lambda)\end{array}\right]\widetilde{TOT}_t+\widetilde{C}_t-\widetilde{WT}_t\right\} \\
& +\lambda\phi_{TOT,t}+\frac{\left(1-\xi_p\beta\right)\left(1-\xi_p\right)\alpha}{\alpha+(1-\alpha)\varepsilon^L}\lambda\phi_{hInf,t}-(1-\lambda)\phi_{fEuler,t}-\beta\lambda E_t\left(\phi_{TOT,t+1}\right) \\
& +\lambda\frac{C_{ss}}{Y_{ss}^H}\left[(1-\lambda)\varepsilon+\varepsilon^*\right]\phi_{RC,t}-\lambda C_{ss}\left[1-\varepsilon^*-(1-\lambda)\varepsilon\right]\phi_{BOP,t} \\
= & -\frac{1-\lambda}{\beta}\phi_{fEuler,t-1}
\end{aligned} \tag{D.13}$$

First order condition with respect to the households' foreign assets position, $\widehat{b}_t^{*,HH}$:

$$\begin{aligned}
& \frac{1-\vartheta}{C_{ss}}\frac{\varepsilon^*+\varepsilon(1-\lambda)}{(1-\varepsilon)(1-\lambda)-\varepsilon^*}\Theta''(0)\left(\widehat{b}_t^{*,HH}-\widehat{\theta}_t^*\right) \\
& +\frac{\Theta''(0)}{Y_{ss}^{H,An.}}\phi_{fEuler,t}+Y_{ss}^{H,An.}\phi_{BOP,t} \\
= & Y_{ss}^{H,An.}E_t\left\{\phi_{BOP,t+1}\right\}
\end{aligned} \tag{D.14}$$

First order condition with respect to foreign reserves, $\widetilde{FX}_t$:

$$\frac{1-\vartheta}{C_{ss}}\frac{\varepsilon^*+\varepsilon(1-\lambda)}{(1-\varepsilon)(1-\lambda)-\varepsilon^*}\Theta^{CB\prime\prime}(FX_{ss})FX_{ss}\widetilde{FX}_t+\phi_{BOP,t}=E_t\left\{\phi_{BOP,t+1}\right\} \tag{D.15}$$

First order condition with respect to home price inflation, $\widetilde{\pi}_t^H$:

$$\frac{\varepsilon^L}{\alpha}\left[\left(1-\varepsilon^L\right)+\frac{\varepsilon^L}{\alpha}\right]\frac{U_{N_{ss}}N_{ss}}{U_{C_{ss}}C_{ss}}\frac{\xi_p}{1-\xi_p}\frac{1}{1-\xi_p\beta}\widetilde{\pi}_t^H-\phi_{TOT,t}+\xi_p\phi_{hInf,t}=\xi_p\phi_{hInf,t-1} \tag{D.16}$$

First order condition with respect to wage inflation, $\widetilde{\pi}_t^w$:

$$\varepsilon^N\frac{U_NN_{ss}}{U_CC_{ss}}\left[1+\left(\gamma_{nn}-\frac{\gamma_{nc}\gamma_{cn}}{\gamma_{cc}}\right)\varepsilon^N\right]\frac{\xi_w}{1-\xi_w}\frac{1}{1-\xi_w\beta}\widetilde{\pi}_t^w-\phi_{wDef,t}+\xi_w\phi_{wInf,t}=\xi_w\phi_{wInf,t-1} \tag{D.17}$$

First order condition with respect to real wage, $\widetilde{w}_t$:

$$\phi_{wDef,t}+\frac{(1-\xi_w\beta)(1-\xi_w)}{1+\left(\gamma_{nn}-\frac{\gamma_{nc}\gamma_{cn}}{\gamma_{cc}}\right)\varepsilon^N}\phi_{wInf,t}-\frac{\left(1-\xi_p\beta\right)\left(1-\xi_p\right)\alpha}{\alpha+(1-\alpha)\varepsilon^L}\phi_{hInf,t}=\beta E_t\left\{\phi_{wDef,t+1}\right\} \tag{D.18}$$

First order condition with respect to CPI inflation, $\widetilde{\pi}_t$:

$$\phi_{wDef,t}+\phi_{TOT,t}+\frac{1}{\beta}\phi_{hEuler,t-1}=0 \tag{D.19}$$

And the first order condition with respect to the interest rate, $\widetilde{(1+i_t)}$:

$$\phi_{hEuler,t}=0 \tag{D.20}$$

where $\phi_{wInf,t}$ is the Lagrange multiplier of wage inflation dynamics, equation (D.1); $\phi_{wDef,t}$ is the Lagrange multiplier of the change in real wage, equation (D.2); $\phi_{hInf,t}$ is the Lagrange multiplier of home inflation dynamics, equation (D.3); $\phi_{TOT,t}$ is the Lagrange multiplier of the change in the terms of trade, equation (D.4); $\phi_{hEuler,t}$ is the Lagrange multiplier of the Euler condition for domestic bonds, equation (D.5); $\phi_{fEuler,t}$ is the Lagrange multiplier of the Euler condition for foreign bonds, equation (D.6); $\phi_{RC,t}$ is the Lagrange multiplier of the resource constraint, equation (D.7); and $\phi_{BOP,t}$ is the Lagrange multiplier of the balance of payments, equation (D.8).

Equations (D.1) through (D.20) characterize the optimal allocation for $\widetilde{C}_t$, $\widetilde{N}_t$, $\widetilde{w}_t$, $\widetilde{\pi}_t^w$, $\widetilde{\pi}_t$, $\widetilde{\pi}_t^H$, $\widetilde{TOT}_t$, $\widetilde{(1+i_t)}$, $\widetilde{FX}_t$, $\widetilde{b}_t^{*,HH}$, $\widetilde{U}_{N,t}$ and $\widetilde{U}_{C,t}$, together with the Lagrange multipliers $\phi_{wInf,t}$, $\phi_{wDef,t}$, $\phi_{hInf,t}$, $\phi_{TOT,t}$, $\phi_{hEuler,t}$, $\phi_{fEuler,t}$, $\phi_{RC,t}$ and $\phi_{BOP,t}$.

Other variables are pinned down using:

$$\begin{aligned}
\widetilde{p}_t^H &\cong \lambda \widetilde{TOT}_t \\
\widetilde{p}_t^F &\cong -(1-\lambda)\,\widetilde{TOT}_t \\
\widetilde{Y}_t^H &\cong \widetilde{A}_t + \alpha \widetilde{N}_t \\
\widetilde{d}_t^H &\cong \widetilde{C}_t - \varepsilon \widetilde{p}_t^H \\
\widetilde{IM}_t &\cong \widetilde{C}_t - \varepsilon \widetilde{p}_t^F \\
\widetilde{EX}_t &\cong -\varepsilon^* \widetilde{TOT}_t + \widetilde{WT}_t \\
\widetilde{\sigma}_t &\cong \widetilde{p}_t^F - \widetilde{p}_{t-1}^F + \widetilde{\pi}_t - \widetilde{\pi}_t^{F*}
\end{aligned}$$

To end this section, a remark on the optimality condition for foreign reserves, equation (D.15), is in order. Notice that if either $\vartheta = 1$ or $\Theta^{CB\prime\prime}(FX_{ss}) = 0$, the Lagrange multiplier of the balance of payments would follow a random walk. Therefore, in order to impose stationarity on the system we have to deviate from these values. This condition is similar to the requirement of a portfolio adjustment cost in order to impose stationarity on the marginal utility of consumption, see Schmitt-Grohé and Uribe (2003). The difference here is that we also have to deviate from full ownership of the financial sector, because, unlike households, the social planner internalizes the fact that the adjustment costs are rebated to the households. Hence, from the standpoint of the social planner, full ownership, i.e. $\vartheta = 1$, is equivalent to no adjustment costs on foreign reserves.

D.2 Optimal Interest Rate Policy and Fixed Foreign Reserves

Now consider the case of optimal interest rate policy with fixed foreign reserves. In that case the equilibrium allocation is characterized by equations (D.1) through (D.20), where the optimality condition with respect to foreign reserves, $\widetilde{FX}_t$, is replaced by:

$$\widetilde{FX}_t = 0 \tag{D.21}$$

Note that formally we should add $\widetilde{FX}_t = 0$ as a constraint, introduce an additional Lagrange multiplier associated with the new constraint, and then solve for the optimal allocation. In this case, all optimality conditions are the same as those in Section D.1, except the one with respect to $\widetilde{FX}_t$, equation (D.15), which is modified slightly as it now contains the new Lagrange multiplier. However, since this is the only equation where the new multiplier shows up and since we are not interested in the multiplier itself, we can drop from the system both the optimality condition with respect to $\widetilde{FX}_t$ and the new multiplier. In other words, to solve for the equilibrium allocation in this case, simply replace the optimality condition (D.15) with the constraint (D.21).

D.3 Optimal Interest Rate Policy and Predetermined FXI Rule

Finally, consider the case where monetary policy is set optimally while FXIs follow a predetermined rule:

$$\frac{FX_t}{FX^T} = \left(1 + \frac{\Theta'\left(\widehat{b}_t^{*,HH} - \widehat{\theta}_t^*\right)}{TOT_{ss}Y_{ss}^{H,An.}}\right)^{\Xi} \left(\frac{FX_{t-1}}{FX^T}\right)^{\rho_{FX}}$$

$$\text{where} \qquad \Xi \gg 0 \quad , \quad 0 \leq \rho_{FX} < 1$$

Taking first order approximation, the policy rule reads:

$$\widetilde{FX}_t \cong \Xi \frac{\Theta''(0)}{TOT_{ss}Y_{ss}^{H,An.}} \left(\widehat{b}_t^{*,HH} - \widehat{\theta}_t^*\right) + \rho_{FX}\widetilde{FX}_{t-1} \tag{D.22}$$

This rule seeks to stabilize the UIP premium, while smoothing the path of foreign reserves. Note that strict targeting of the UIP premium, i.e. $\Xi \to \infty$, introduces unit root dynamics in the approximated system through the households' Euler equation for foreign bonds. However, to substantially stabilize the UIP premium, it is sufficient to set Ξ to a value large enough. I use:

$$\Xi = 20$$

The optimization problem is the same as before, except that (D.22) is added as a constraint. All optimality conditions of section D.1 are the same as before, except those of $\widetilde{FX}_t$ and $\widehat{b}_t^{*,HH}$ – the endogenous variables in (D.22).

The first order condition with respect to foreign reserves, $\widetilde{FX}_t$, now reads:

$$\begin{aligned} & \frac{1-\vartheta}{C_{ss}} \frac{\varepsilon^* + \varepsilon(1-\lambda)}{(1-\varepsilon)(1-\lambda) - \varepsilon^*} \Theta^{CB\prime\prime}(FX_{ss}) FX_{ss}\widetilde{FX}_t + \frac{\phi_{FXRule,t}}{FX_{ss}} + \phi_{BOP,t} \\ = \; & E_t\left\{\phi_{BOP,t+1}\right\} + \frac{\beta\rho_{FX}}{FX_{ss}} E_t\left\{\phi_{FXRule,t+1}\right\} \end{aligned} \tag{D.23}$$

And the first order condition with respect to the households' holdings of foreign assets, $\widehat{b}_t^{*,HH}$, is given by:

$$\begin{aligned} & \frac{1-\vartheta}{C_{ss}} \frac{\varepsilon^* + \varepsilon(1-\lambda)}{(1-\varepsilon)(1-\lambda)-\varepsilon^*} \Theta''(0) \left(\widehat{b}_t^{*,HH} - \widehat{\theta}_t^*\right) \\ & + \frac{\Theta''(0)}{Y_{ss}^{H,An.}} \left(\phi_{fEu,t} - \Xi \phi_{FXRule,t}\right) + Y_{ss}^{H,An.} \phi_{BOP,t} \\ = \; & Y_{ss}^{H,An.} E_t \left\{\phi_{BOP,t+1}\right\} \end{aligned} \tag{D.24}$$

where $\phi_{FXRule,t}$ is the Lagrange multiplier of the FXI policy rule, equation (D.22).

The equilibrium allocation under optimal monetary policy and the FXI rule is characterized by equation (D.22) together with equations (D.1) through (D.20), where equation (D.23) replaces (D.15), and equation (D.24) replaces (D.14).

E Appendix: Data Description

The dataset of the observable variables in the estimation consists of 14 macroeconomic time series in quarterly frequency. The sample period is 2010:Q1 – 2019:Q4. Following is a description of each variable, by categories:

National Accounts Source: Israel Central Bureau of Statistics.

- Gross domestic product. Fixed prices, seasonally adjusted, log first-difference.
- Total private consumption. Fixed prices, seasonally adjusted, log first-difference.
- Government consumption, excluding imported defense. Fixed prices, seasonally adjusted, log first-difference.
- Exports of goods and services, excluding startups and diamonds. Fixed prices, seasonally adjusted, log first-difference.
- Imports of goods and services, excluding imported defense, ships and aircraft, and diamonds. Fixed prices, seasonally adjusted, log first-difference.
- Terms of trade: calculated as the ratio of export prices (excluding startups and diamonds) to import prices (excluding imported defense, ships and aircraft, and diamonds). Log first-difference.

Labor Market Data Source: Israel Central Bureau of Statistics.

- Total labor input (hours) per week. Seasonally adjusted, log first-difference.

Nominal Variables Source: Israel Central Bureau of Statistics (ICBS) and Bank of Israel (BoI).

- CPI inflation rate. Seasonally adjusted, quarter average over quarter average, first difference. (Source: ICBS)
- Nominal 3-month return on Bank of Israel unindexed bill ("Makam"). Average, first difference. (Source: BoI)
- Nominal effective depreciation rate. Quarter average over quarter average, first difference. (Source: BoI)

International Investment Position Source: Bank of Israel.

- Foreign reserves held by the Bank of Israel, expressed in terms of imported goods: calculated by multiplying the quarterly average foreign reserves (in dollars) by the quarterly average ILS/USD exchange rate and dividing by import prices (excluding imported defense, ships and aircraft, and diamonds). Log first-difference.
- Net private-sector (excluding banks) holdings of foreign assets relative to trend GDP, both expressed in terms of imported goods. Quarterly average net assets (in dollars) are multiplied by the quarterly average ILS/USD exchange rate and then divided by import prices (excluding imported defense, ships and aircraft, and diamonds). Trend GDP is calculated as the linear trend of (log) nominal GDP divided by import prices (excluding imported defense, ships and aircraft, and diamonds). First difference.
- Capital inflow to public-sector financial instruments relative to trend GDP, both expressed in terms of imported goods. Capital inflow in dollars is measured using financial investment in public-sector tradable securities. Transformation to units

of imported goods and measurement of trend GDP are the same as in net private-sector holdings of foreign assets above.

World Trade Source: OECD.

- World trade: Total imports of goods and services by OECD countries. Volume index seasonally adjusted (VIXOBSA), log first-difference.

F Appendix: Bayesian Estimation

The estimation was carried out using Dynare version 5.2 and Sims (1999) *csminwel* optimizer. The Markov Chain Monte Carlo (MCMC) Metropolis-Hastings algorithm employed 5 parallel chains with 2 million draws per chain. The first 40 percent of the draws were used as burn-in.

Figures F.1 through F.3 display the prior and posterior distributions for each estimated parameter. Generally, the data seem informative, as posterior distributions differ from priors, though in two cases the effect is questionable (the persistence of productivity shocks and the standard deviation of the measurement error of hours worked). Importantly, the data seems very much informative for the financial friction parameter, $\Theta''(0)$.

Figure F.1: Prior and Posterior Distributions of Model's Parameters

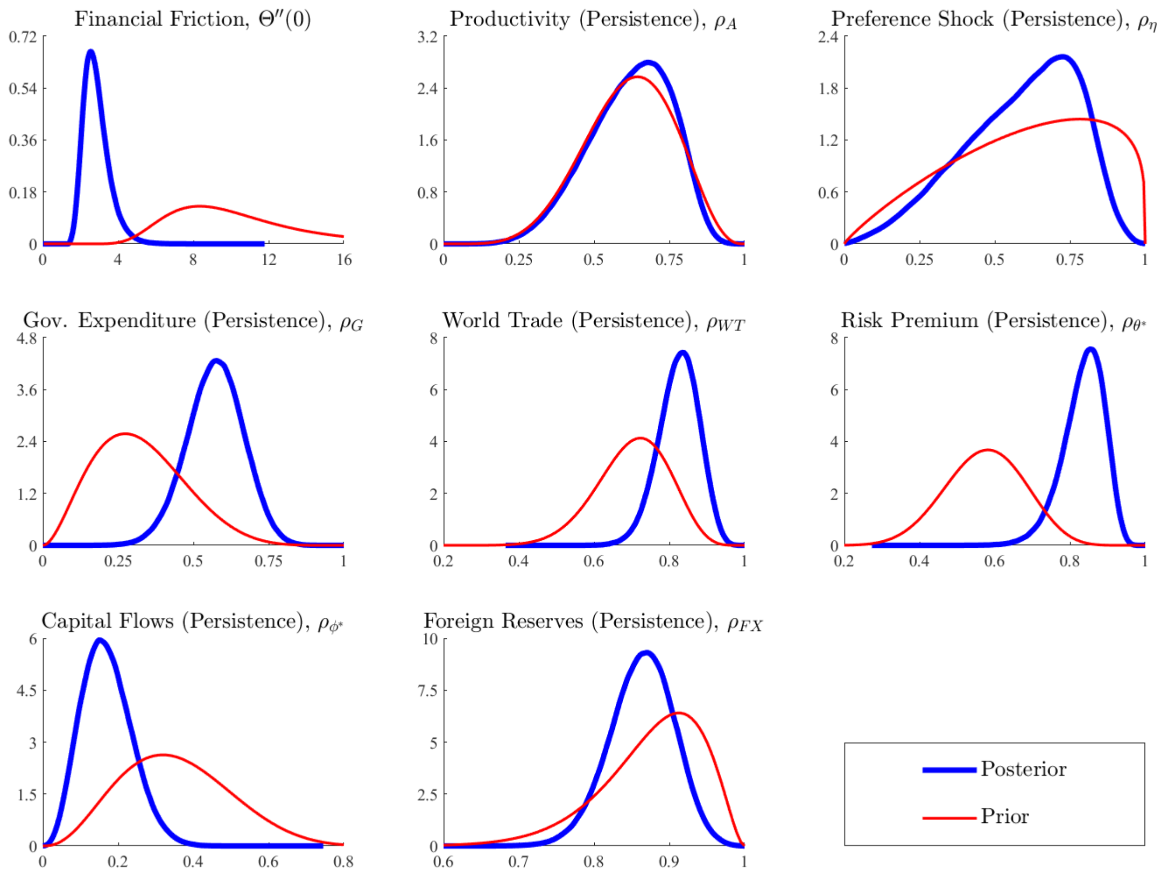


Figure F.2: Prior and Posterior Distributions of the Standard Deviations of the Exogenous Shocks

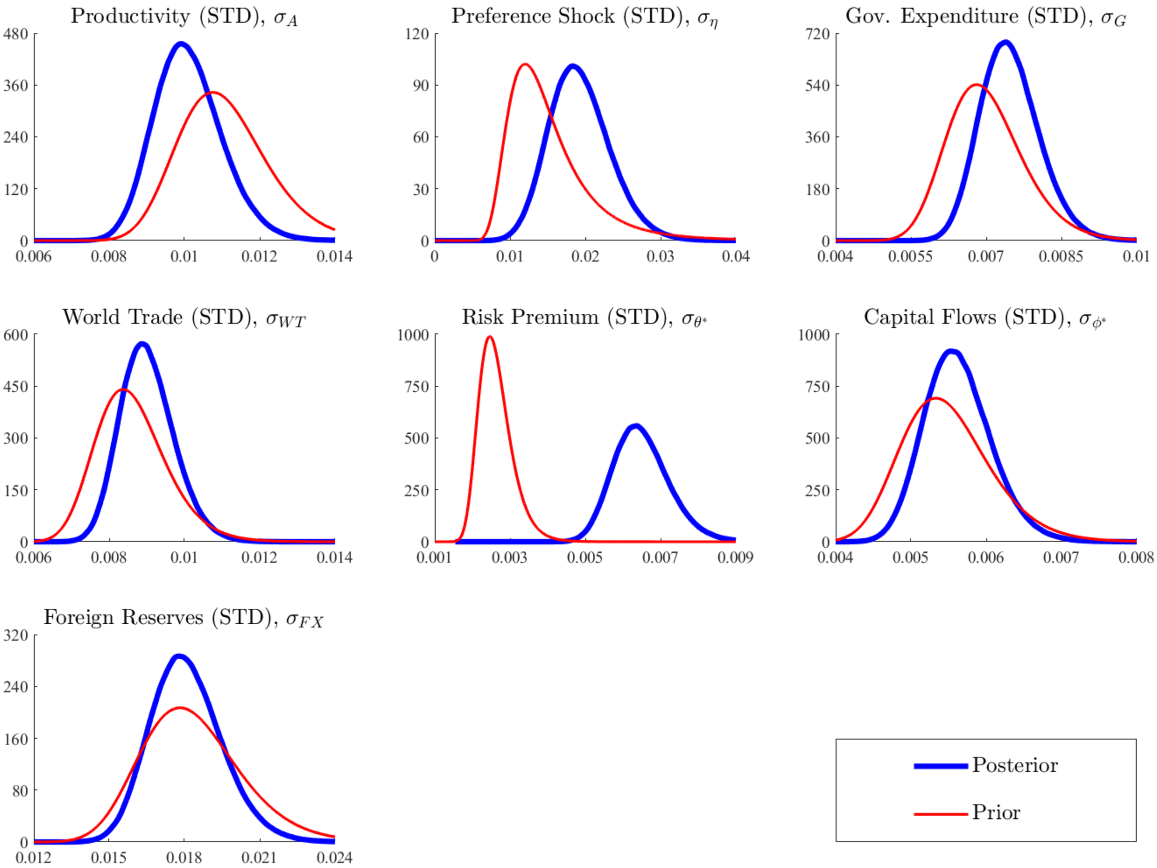
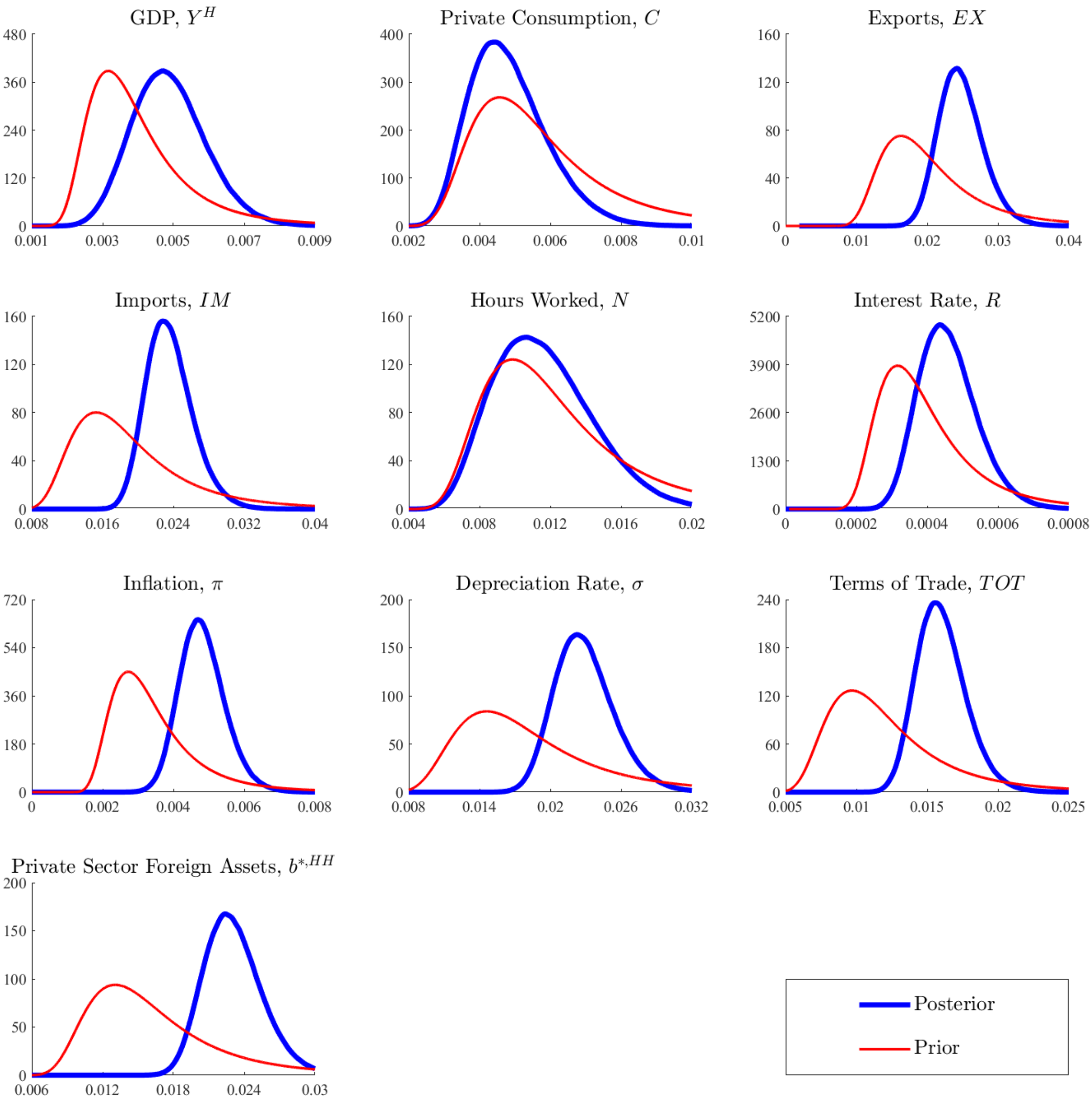


Figure F.3: Prior and Posterior Distributions of the Standard Deviations of the Measurement Errors



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