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Abstract

This research examines a model of an economic-financial crisis caused by a sudden debt deleveraging in the economy. In this type of a crisis, demand is contracted, and the monetary interest rate may drop to its effective lower bound – a phenomenon called the "**liquidity trap**" – in such a way, that the monetary policy is restricted in its response (Eggertsson and Woodford, 2003). At the same time, there are other mechanisms that may intensify the crisis, such as the mechanism of the "**financial accelerator**" (Bernanke et al., 1999), and the mechanism of the "**debt deflation**" (Eggertsson and Krugman, 2012). Therefore, we are induced to question, what is the "contribution" of those various mechanisms to this crisis, and in particular - what is the interaction between those mechanisms.

For this purpose, a general equilibrium model has been built in a Neo-Keynesian framework with two types of representative agents – a borrower and a saver – where the financial spread of the borrower depends on his or her level of leverage (the ratio of debt to the value of assets). The model is solved without linearization, emphasizing the monetary policy rule, which includes an effective lower bound on the interest rate. This is to enable an analysis of the interactions between the various mechanisms.

From the analysis of the reaction of the economy to the debt deleveraging in the various situations, it was found that the interaction of the various mechanisms is extremely significant. For example, when the economy enters the "liquidity trap", the effect of the "financial accelerator" is intensifying the crisis to a great extent, much more than it occurs in a situation where the interest rate is not subject to the effective lower bound. In fact, the analysis illustrates the importance of an effective monetary policy in the course of a financial crisis, because monetary expansion is critical in this situation and prevents a crisis which is much more acute.

Keywords: liquidity trap; the effective lower bound (*ELB*); financial friction; monetary policy; financial crisis; financial crisis; debt deleveraging; credit market; financial accelerator, debt deflation

ניתוח מנגנוני ההגבר בתהליך הורדת מינוף

נמרוד כהן

תקציר

מחקר זה בוחן מודל של משבר כלכלי־פיננסי הנגרם בעקבות הורדת מינוף פתאומית במשק. במשבר מסוג זה הביקושים נפגעים והריבית המוניטרית עלולה לרדת למגבלתה התחתונה – תופעה שנקראת **"מלכודת הנזילות"** – כך שהמדיניות המוניטרית מוגבלת בתגובתה (Eggertsson and Woodford, 2003). במקביל, ישנם מנגנונים נוספים שעלולים להעצים את המשבר, כמנגנון **"המאיץ הפיננסי"** (Bernanke et al., 1999), ומנגנון **"דפלציה של החוב"** (Eggertsson and Krugman, 2012). נשאל אם כן, מהי "תרומתם" של המנגנונים השונים למשבר, ובפרט – מהי האינטראקציה ביניהם.

לצורך כך נִבְנָה מודל של שיווי משקל כללי במסגרת ניאו־קֵיינְסִיאָנִית עם שני סוגי פרָטִים מייצגים – לווה וחוסך – כאשר המרווח הפיננסי של הלווה תלוי ברמת המינוף שלו (יחס החוב לשווי הנכסים). המודל נפתר ללא לינאריזציה, בדגש על כלל המדיניות המוניטרית, אשר כולל מגבלה תחתונה לריבית. זאת כדי לאפשר ניתוח של האינטראקציות בין המנגנונים השונים.

מניתוח תגובת הכלכלה להורדת מינוף במקרים השונים, נמצא כי האינטראקציה של המנגנונים השונים משמעותית ביותר. לדוגמה, כאשר הכלכלה נכנסת ל"מלכודת הנזילות", האפקט של "המאיץ הפיננסי" מעצים משבר עד מאוד, הרבה מעבר למקרה שבו הריבית אינה מוגבלת. למעשה, הניתוח ממחיש את החשיבות של מדיניות מוניטרית אפקטיבית בשעת משבר פיננסי, כיוון שהרחבה מוניטרית במצב זה היא קריטית ומונעת משבר חריף בהרבה.

מילות מפתח: מלכודת נזילות; מגבלת הריבית התחתונה; חיכוך פיננסי; מדיניות מוניטרית; משבר פיננסי; משבר כלכלי; הורדת מינוף; שוק אשראי; מאיץ פיננסי, דפלציית חוב

1. INTRODUCTION

Many economic crises had been their intensified or were caused by the Overleveraging of borrowers, who during a crisis had to suddenly reduce debt level (Mendoza, 2010; Uribe, 2006a). In fact, the posing of a limit on the leverage of borrowers is intended to discipline them, so that they do not evade the repayment of their debt. However, at the macroeconomic level, such a limitation during a crisis may actually trigger a mechanism that is intensifying it. This is because it can force borrowers to drastically reduce their expenditures, so that aggregate demand will be negatively impacted. Moreover, a direct shock to the leverage limit may by itself trigger a sudden debt deleveraging and create a financial crisis, as we will be able to discover in this research.

A shock to the leverage limit represents a situation where lenders fear that the financial situation is worse than they thought to be and that assets were overpriced, and there is a prevalent concern about debt repayment, so that the leverage limit is too loose, and therefore the lenders decide to tighten it. Such a shock is reflected in the reduction of the credit supply in the economy and in the increase of the spread, which lead to a sudden debt deleveraging on the part of the borrowers.

Borrowers who are obligated to produce quick repayments of their debts, are forced to reduce their uses. In the event that the shock is broad and powerful, it may lead to a decrease in aggregate demand, which is reflected in a decrease in the GDP and in the inflation, while the central bank may lower the interest rates according to the situation, in order to stimulate households (especially savers), to increase their consumption, so as to make up for the slack in the demand on the part of the borrowers.

For a strong enough shock, the monetary interest rate may get stuck at its effective lower bound (hereforth, ELB), a situation called the "liquidity trap" (Eggertsson and Woodford, 2003). In this case the central bank is restricted, and it cannot lower the monetary interest rate anymore, and the real interest rate does not decrease to a sufficiently lower rate. Therefore, the households which are among the savers, do not compensate for the laxity on the part of the borrowers, triggering the further fall of the aggregate demand, as well as of the GDP and the inflation.

In this situation, the incomes of the borrowers are being reduced, so that they have even a greater difficulty than before in repaying their repayment of their debts. Moreover, the decrease in inflation increases the real value of the (nominal) debt and it further burdens the borrowers, a mechanism called "debt deflation" (Mendoza, 2010). In fact, when the nominal interest rate stops at the ELB, then the drop in inflation leads to an increase in the real interest rate, which in turn intensifies the crisis.

Another mechanism that may intensify a crisis is the "financial accelerator" (Bernanke et al., 1999), which results from the fact that the debt limit depends on the value of the collaterals. When the value of the assets of the borrowers which are tied as a collateral is diminishing, then the leverage increases, and the risk premia also increases accordingly, which further burdens the borrowers and intensifies the crisis. Admittedly, this is a very well-known mechanism, but it has not been yet studied in depth in debt deleveraging models, in which the economy enters the "liquidity trap".

Also, the "financial accelerator" is mainly studied in models which consist of a representative agent and are under the assumption that there is perfect consumption insurance. On the other hand, this research will discuss the model of two agents – one borrower and one saver – where there is no consumption insurance (and also incomplete markets). This point is significant in general when considering a reaction to an asymmetric shock, and specifically when considering a shock that causes debt deleveraging, which impairs directly only the expenditures of the borrowers. Since the borrowers strive to "smooth out consumption" (as much as they can, households avoid the drastic reduction of their consumption), but as said, they do not have a consumption insurance, and they therefore are drastically reducing their spending on investments. The sharp drop in the demand for investment reduces the price of capital, and reduces in turn the value of assets (physical capital), so that the "financial accelerator" culminates in an intensified reaction, in compared to the situation of a representative agent model.

In essence, this research will examine, given a debt deleveraging shock, what is the contribution of the various mechanisms – the liquidity trap, the debt deflation and the financial accelerator – to the formation of the crisis and its intensification, and what is the interaction between them. The main innovation in this research is the integration of these three amplification mechanisms, without linearization, and in particular within a model with an heterogeneity of borrower-savers (without consumption insurance), subject to the influence of a debt deleveraging shock.

It was found that after a debt deleveraging shock, when the ELB on the interest rate is ignored, i.e., outside the "liquidity trap", the "financial accelerator" has a relatively moderate amplifying effect. That is, by comparing the reaction of a model in which the debt limit is exogenous (without a "financial accelerator"), against a model in which the debt limit is endogenous (in the presence of a "financial accelerator"), it is found that when the monetary interest rate is free to reduced without any bound, it succeeds in the mitigation of the crisis in both situations. The explanation for this is that although in the presence of a "financial accelerator", when the risk premium increases greatly, it may push borrowers to quickly reduce their debt. But a sharp reaction of the monetary interest rate manages to sharply lower the risk-free real interest rate and thus lower the interest rates on credit. That is, the policy manages to moderate the reaction of borrowers; and encourages savers in addition to increase consumption, so that there is an almost full compensation for the decrease in the consumption of the borrowers.

However, when the economy enters the "liquidity trap" and the monetary policy is restricted, the effect of the "financial accelerator" obtained is greatly increased, i.e., the reaction to a debt deleveraging shock is greatly increased and the duration of time in the "liquidity trap" is prolonged (to a moderate extent). In this situation, the real interest rate rises, thus greatly exacerbating the crisis, because it pushes the borrowers to reduce their debt faster, and therefore to sharply reduce their uses, and on the other hand – it does not push the savers to compensate for the slack in demand on the part of the borrowers. The decrease in demand for investments, which is reflected in the decrease in the value of the capital, reduces the value of the collaterals, and therefore greatly increases the effects which are resulting from the shock of debt deleveraging. Also, the decrease in demand leads to a decrease in the GDP and to weakening the incomes of the borrowers, who are stressed even more in the process of the debt deleveraging.

The research layout is as follows: after the introduction and the literature review, the model will be presented in detail, focusing on the less conventional parts of it. Next, the reaction to a debt deleveraging shock will be analyzed. At the end, main conclusions and possible broadenings will be presented.

1.1. Literature Review

The literature on economic and financial crises is extremely broad, so we will focus on the literature that discusses models in which the crisis includes the entering of the "liquidity trap" and there is a "financial accelerator" mechanism. The issue of the interaction between the "financial accelerator" and the monetary policy has already been discussed before (Bernanke et al., 1999), and it is argued that the more the policy can stabilize GDP, the smaller the role of the "financial accelerator" in increasing the business cycle (this is true for any of the amplification mechanism). However, in the presence of the "financial accelerator" only a slight countercyclical reaction is needed in order to reduce the volatility in the GDP.

Other researchers strengthen this claim in their research (Gertler et al., 2007) in their argument that countercyclical monetary policy can potentially prevent a financial crisis. Lowering interest rates during a recession, for example, is a measure that helps stabilize changes in asset prices, and therefore stabilizes the balance sheets of the borrowers. External restrictions on monetary policy, on the other hand, is constraining this option for the stabilization of the cycle.

The first formal models that have discussed the "liquidity trap" are standard Neo-Keynesian models of a representative agent (for example Eggertsson and Woodford, 2003). Within them, a shock to the time preferences (intertemporal consumption preferences) of the individuals, increases their desire to save, and it is accordingly damaging the aggregate demand, triggering the creation of an economic crisis. Typically, the central bank responds to this situation by lowering the interest rates, which pushes real interest rates down and reduces the desire to save. But when the shock is big enough, the monetary interest rate reaches its ELB, thus restricting the ability of the central bank to act in response. Thus, the decrease in demand leads to a decrease in inflation, which in turn is leading to a higher real interest rate, which encourages the savings of the household even more. And the result, as mentioned, is a "liquidity trap" in which the GDP drops considerably.

However, as discussed in (Eggertsson and Krugman, 2012), the shock to the preferences can be considered as an abstraction of a more realistic shock in the debt market. This shock reduces the debt limit of the borrowers and forces them to a quick action of debt deleveraging, by reducing their consumption; A process that could be harmful to the aggregate demand and put the economy into recession, with a drop in the GDP and the inflation. Meanwhile, the nominal interest rate may reach its ELB. The decrease in inflation increases the real value of the (nominal) debt and further burdens the borrowers, a mechanism called "debt deflation". The researchers emphasize that the distribution of debt between different individuals is critical (a model with heterogeneity, one that separates the different types of agents, in contrast to a representative agent model), when analysing the implication of leverage and financial friction on the economy.

In this context, against the backdrop of the failure of the standard framework of the Neo-Keynesian model to explain the "Great Recession" – the 2008 financial crisis – and to predict the crash in asset prices, the need to incorporate financial frictions into the model had risen.

In another research that examines the consequences of the "liquidity trap" along similar lines, (Guerrieri and Lorenzoni, 2017) get the separability between borrower and saver based on idiosyncratic income shocks. Therefore, they get dynamic distribution of incomes, and accordingly the distribution of debt and saving positions (saving has the role of insurance – precautionary saving) and their dynamics.

In another research, (Korinek and Simsek, 2016) analyzes a macro-stability policy for debt market with similar stracture.

The research of (Benigno et al., 2020) expands the model of (Eggertsson and Krugman, 2012), so within it a borrower may exceed the debt limit but is forced to pay a risk premium which is increasing with the level of the leverage. In such a situation, the reaction to the shock is a dynamic debt deleveraging since it is carried out over a period of time. The researchers show that in the United States, starting in 2009, there is a development of debt deleveraging, and that during that period it is possible to notice a sharp decrease in the economic activity and the inflation, which is accompanied by a decrease in the risk-free interest rate, along with a widening of the risk margin. Their model does manage to replicate the crisis, however, the duration of the "liquidity trap" is much shorter compared to the duration of time in the real world. Therefore, it is likely that the basic model lacks internal stabilizing mechanisms, such as: consumption habits, investments (including adjustment costs), a "financial accelerator" and friction in the labor market.

The current research takes a similar approach, but makes a further expansion and includes physical capital that is used both as a factor of production and as collateral for taking up credit. Accordingly, in the model that we will present in this research, the leverage of the borrowers is relative to the value of their assets, which is determined endogenously in the model (and not a leverage relative to a constant value of the steady state), that is, the debt capacity depends on the value of the physical capital held by the borrower, and is not exogenous as in (Benigno et al., 2020). In this way, a very important mechanism was added to the model – a "financial accelerator", which causes the persistance and amplification of the crisis, and as mentioned, is at the core of the research question. Accordingly, the credit in the model includes household credit (consumption and housing), as in (Benigno et al., 2020), but also business credit for capital financing (investments).

An important point which is hardly discussed in the literature on the "financial accelerator" is with regard to the assumption of a perfect consumption insurance and the use of the representative agent model. According to the assertion of the current research, there is a significant effect for the non-existence of consumption insurance, especially under an asymmetric shock[1]; which is a shock that impacts in a different way on the consumption path of different agents.

An example of this is a shock that shifts sources from borrowers to savers, which is a financial shock, and without the presence of financial friction it has no effect, because it is a shift of sources "within the family". The Net Worth shock and the "debt deleveraging" shock are such in the case of an individual. In the latter, the shock causes individual borrowers to be in distress and pressures them to reduce their expenditures but since they do not have any consumption insurance and want to "smooth out" their consumption, they mainly reduce investments. On the other hand, when there is a consumption

[1] (Debortoli and Galí, 2018) show that in a simple framework (a closed economy without investments), ceteris paribus, the aggregate effect of a shock depends on the degree to which the shock results in the shift of resources (redistribution) between the households that are subject to a constraint (borrowers) and those that are not subject to a constraint (saving).

insurance, the individual borrowers receive help (sources), which allows them to "smooth out" their consumption, and thus the damage to investments is weaker. In conclusion, in an economy with no consumption insurance the reaction to asymmetric shocks is increased compared to the situation with consumption insurance (the representative agent model).

The article by (Mertens and Ravn, 2011) also investigates a "liquidity trap" crisis in a model with two representative agents – one saver and one borrower – and a "financial accelerator" mechanism. However, in their model the economy moves between a state of a "liquidity trap" ($i = 0$) and a normal state (when $i > 0$ according to the Taylor rule) according to an exogenous state variable, so that the effective *duration of time in the liquidity trap is determined as we wish* (calibrated to fit other researches). On the other hand, in the model that we will present here, *the duration of time in the liquidity trap is endogenous* and is determined in a general equilibrium according to the Taylor rule which includes an ELB on the interest rate. Also, within their model it is not possible to examine how much the ELB on the interest rate contributes to the crisis, because according to them the crisis has no meaning without the "liquidity trap", since the crisis formed by the fear of the agents that the economy will enter the "liquidity trap" (and it indeed does enter it). It should also be noted that the shock according to them is symmetrical, so that both types of agents become pessimistic and reduce their demand function accordingly. Therefore, it is not of great importance that there is no consumption insurance, and the result is similar to a"representative agent model.

In the models which are described above, the focus is on the leverage of households and the impact of their debt deleveraging process on their consumption. So, these are mostely models of two representative agents – one saver and one borrower – without consumption insurance. They do not include investments and the leverage limit is exogenous. In addition, there is a complementary and extensive literature that focuses on the leverage of firms (or financial institutions), and on the effect of the deleveraging process on investments (Del Negro et al., 2017a). In this literature, the mechanism of the "financial accelerator" is at the core of the discussion, but on the other hand, for simplicity, the existence of a perfect consumption insurance between borrowers and savers is often assumed, and the representative agent model is often used, which produces a relatively weak reaction to an asymmetric shock, as discussed above.

Furthermore, when the financial shock is purely on the firms' side, and the dominant detriation is to the supply (in the products market), then there is no decrease in inflation and no entry into the "liquidity trap". For example, in (Gertler and Karadi, 2011), a crisis caused by a shock to the capital quality of the firms which reinforced through the balance sheets of the banks (financial accelerator)[2]. But the effect of

[2] It should be noted that this study also examines a "news" shock, according to which capital quality is expected to decrease in the future, but in the end this does not materialize. In such a situation the "news" detriments the value of the assets, but because the shock is not actually detrimental, thus there is no direct impact on the level of effective capital. This is how the researchers succeed in separating the

the "financial accelerator" is only slightly impacted by the existence of the ELB on the interest rate.

Another research within the framework of a representative agent model with a "financial accelerator" (Carrillo and Poilly, 2014), shows that the impact of a Net Worth shock is very weak, so the chance that this shock will deliver an economy into the "liquidity trap" is almost zero. According to them, the consumption of the households (the representative agent) compensates for the decrease in the investments of the firms (the entrepreneurs, who do not consume). Conversely, in a model with borrower-saver heterogeneity without consumption insurance, the entrepreneurs also consume, and, in this situation, they reduce consumption and therefore the aggregate consumption decreases. Additionally, (Merola, 2012) examines a representative agent model with a "financial accelerator" subject to a financial shock. His findings consist of a very weak influence of the "liquidity trap" – the gap between a situation with an ELB on the interest rate and a situation without an ELB.

Another common shock in the literature dealing with the "financial accelerator" is the risk premium shock which increases the spread between the monetary interest rate and the household interest rate and is also expressed in the increase in the required return on capital (Smets and Wouters, 2007). In reaction to this shock, a simultaneous decrease in investment and consumption is obtained. This is in contrast to the reaction to a Net Worth shock where investment drops,

"asset value" impact (financial impact) from the normal impact of a shock to the quality of capital (the real impact of erasing sources).

but consumption rises. (Carrillo and Poilly, 2014) show that after risk premium shock the interest rate drops and the economy enters the liquidity trap. (Merola, 2012) also shows that the reaction in this situation is indeed more significant than the reaction to the Net Worth, and in particular the implication of the "liquidity trap" on the reaction. We should note that this is a symmetric shock because it impacts households and firms in a similar way. Therefore, borrower-saver heterogeneity is not very important for analysis risk premium shock.

There is extensive literature that studies financial crises from an empirical point of view, and in particular this includes studies dealing with debt deleveraging processes and financial crises (see Mian et al., 2013, 2014, 2017; Mian and Sufi, 2010a, 2010b).

2. The Model

The model that will be analyzed in this research is a macroeconomic model of general equilibrium in a Neo-Keynesian framework with two agents – one borrower and one saver. The premise of this research is that the rationale for the heterogeneity of the households is that only the borrower has access to investment options (therefore he is also called an entrepreneur), and he therefore needs credit financing[3]. Also, compared to a representative agent model, the assumption in

[3] Firms cannot finance themselves (self-finance), in the long term, because they have a finite time horizon. This is equivalent to the assumption that an entrepreneur is less patient, and that there is a difference in the rate of time preference between a saver and a borrower, see (Eggertsson and Krugman, 2012). Borrower and saver distribution can also be explained by idiosyncratic shocks that are not insured (incomplete markets) (Guerrieri and Lorenzoni, 2017); Difference in risk aversion and more.

this research is that there is no consumption insurance. In addition to this, the research assumes that there is a limit on the amount of debt that a borrower can take relatively to the collateral which is in his possession; A limitation that is designed to prevent him from taking off with the credit. It should be noted that the collaterals here consist of physical capital, so the limit depends on the value of the capital owned by the borrower.

In accordance with the research question, the effects of the various mechanisms will be tested, and in particular the effect of the ELB on the interest rate, therefore a non-linear model will be solved[4]. We will start with a real model with a single (homogeneous) product, and later we will expand to a Neo-Keynesian framework with price rigidity. The model contains the follows frictions and disturbances: financial friction that is expressed in a limit on debt; nominal friction that manifests itself in price rigidity, and in addition interference with the nominal interest rate which is blocked from below by the ELB; and the last friction is a real one - and is expressed in adjustment costs to changes in investments.

2.1 The Households

As mentioned, let us assume that there are two types of households, the first one is a "**saver**" type and the second one is a "**borrower**" type – a leveraged household which is an entrepreneur and has access to

[4] Will be solved under full certainty (deterministic simulation). For this purpose, an initial staedy state is defined that simulates the state of the economy before the shock, and a final staedy state is defined to which the economy converges. On top of that, other shocks are defined that are detected at time $t = 0$ (exogenous process). So, the reaction is actually the path of convergence to the final stable state, given that the exogenous process known in advance.

investment in physical capital. Acorrdenly, these are two representative agents, that is, all savers in the economy are the same and are represented by a representative agent marked by the index S, and the borrowers are also identical and are represented by a representative agent marked by the index B. The weight of borrowers in the economy is χ^B, and the weight of savers in the economy is χ^S, and all variables in the model will be per a single household. We will describe the behavior of the different households, $j \in \{S, B\}$, as the maximization of the following multi-period utility:

$$V_{t=0}^{j} = \max_{\{C_t^j, L_t^j, K_t^j\}} E_0 \sum_{t=0}^{\infty} \left(\beta^j\right)^t U\left(C_t^j, C_{t-1}^j, L_t^j\right)$$

For both types of agents, we will assume a separable periodic utility between labor and consumption and a CRRA (constant relative risk aversion) functional form, including consumption habits.

$$U\left(C_t^j, C_{t-1}^j, L_t^j\right) = \frac{\left(C_t^j - h^j C_{t-1}^j\right)^{1-\sigma^j}}{1-\sigma^j} - \vartheta^j \frac{{L_t^j}^{1+\omega^j}}{1+\omega^j}$$

for $\sigma^j \in [0,1)$ .and in the special case $\sigma^j = 1$, $\ln\left(C_t^j - h^j C_{t-1}^j\right)$.

The periodic budget constrain of the representative agents is:

$$w_t L_t^j + q_t K_t^j + \frac{b_t^j}{R_t^j} + T^j = C_t^j + Q_t I_t^j + b_{t-1}^j$$

Let us denote by b_t^j the debt to be paid at the next period, with the real interest rate denoted by R_t^j (we will use the following ***convention*** for *inter-period variabels*: $x_t = x_{t \to t+1}$, such as interest rate, return, discount factor, but except for inflation). We denote by T_t^j a payment (lump-sum) that a household receives from firms or banks which it owns and which are not under its control. In particular, let us assume that only the saver owns the bank, so the profits are being recorded (as a dividend) in his budget constrain $T^S = \Pi_t^{Bank}$. The entrepreneur is the owner of all the firms.

The capital accumulation process is defined as follows:

(1) $$K_{t+1}^j = I_t^j + (1-\delta)K_t^j$$

We assume as mentioned that only the borrower (the entrepreneur) has access to investments in physical capital, so that he accumulates capital and leases the capital stock each period to a producing firm, at the price q_t. He can sell the capital from the previous period after depreciation, $(1-\delta)K_t^j$, and buy new capital, K_{t+1}^j, both at the current market price Q_t. That is, the net demand for capital is $Q_t I_t = Q_t\left(K_{t+1}^j - (1-\delta)K_t^j\right)$. As we will see later on, the supply of investments comes from capital producers.

The development of the **first order conditions of the households** is detailed in the appendix. Here we present the main equations. First order conditions **according to the consumption**:

(2) $$\lambda_t^j = \left(C_t^j - h^j C_{t-1}^j\right)^{-\sigma^j} - \beta^j h^j E_t\left[\left(C_{t+1}^j - h^j C_t^j\right)^{-\sigma^j}\right]$$

And we will define the stochastic discount factor (SDF):

(3) $$m_t^j = \beta^j \frac{\lambda_{t+1}^j}{\lambda_t^j}$$

First order conditions **according to the labor**:

(4) $$\vartheta^j {L_t^j}^{\omega^j} = \lambda_t^j w_t$$

Expresses the optimal labor supply, and it embodies within it the fact that the marginal rate of substitution between labor and consumption equals the real wage (for each type).

First order conditions **according to the capital** (only borrowers have access to the capital market, i.e., $j = B$):

(5) $$1 = E_t[m_t^B \cdot \tilde{R}_t^K]$$

And the return on capital investment (ex ante) is

(6) $$\tilde{R}_t^K = \frac{q_{t+1} + (1-\delta)Q_{t+1}}{Q_t}$$

We should note that the price of capital, Q_t is determined according to expectations. For example, in the situation where the return on capital increases (an increase in financing costs for the entrepreneur), given that everything else is constant, then the demand for investments decreases and the price of capital decreases.

2.2 The Credit Market with Financial Friction (endogenous credit limit)

First, the *saver* can make a deposit at a risk-free interest rate, R_t^S. A first order condition by debt (savings) gives the Euler condition for consumption:

(7) $$1 = E_t[m_t^S R_t^S]$$

In contrast, the *borrower* is faced with the *credit supply* of the banks in the form:

(8) $$R_t^B = R_t^S \cdot \Phi\left(b_t^B / \bar{b}_t^B\right)$$

Where $\Phi(\cdot)$ is the spread function that the bank offers, which represents the risk management of the bank, and in particular a premium for bankruptcy risk (please see the appendix). The premium is a function of the nominal value (principal and interest), since this is the amount that the borrower has to repay (this is accepted in financial models such as the Merton model). $\bar{b}_t^B$ represents the **debt capacity** of the borrower, which is the value of the assets he owns that can be considered to be a collateral. Other things being constant, the spread increases as the **level of leverage**, $b_t^B / \bar{b}_t^B$, increases

It should be noted that in (Benigno et al., 2020; Eggertsson and Krugman, 2012) the leverage is in relation to an absolutely exogenous value, while in the current model the leverage is in relation to the value of the assets that are determined endogenously in the model, therefore a "financial accelerator" can be created, as will be detailed later on.

The assumption of the research is that the borrower has an inventory of physical capital, therefore the expected inventory in the next period is used as collateral for the debt, similar to what is indicated in the research of (Iacoviello and Minetti, 2006). Thus, we will define the debt capacity as follows:

(9) $$\bar{b}_t^B = \Theta_t E_t Q_{t+1} K_{t+1}^B$$

The price of capital, Q_{t+1} is determined in a perfecte market, so it is exogenous from the perspective of the lender, but endogenous in the economy[5]. The capital to collateral conversion factor Θ_t is exogenous and expresses the part of the capital that the lender estimates that he will be able to take over and sell in the situation of insolvency. It also embodies the internalization of high liquidation costs during "pressure". That is, part Θ_t of the assets is confiscated in situation of non-payment of debt.

A debt deleveraging shock will be expressed as a negative shock to Θ_t ,which embodies a story in which borrowers change estimates for the expected value of their collateral liquidation. The background story – a continuous period of stable economic growth and an increase in asset prices which encourage an easing of the leverage ratio (a period of optimism and complacency). During this period, borrowers take loans and increase expenditures through a process of leverage. But at a certain point in time there may be a **sudden** tightening in relation to leverage – an event called the "Minksy Moment". Many lenders realize that assets that were overpriced and the collateral limit of

[5] Therefore, an external effect can arise as a result of changes in the price of capital, as follows: during high growth period (booms), the price of capital increases as a result of a collective decision of the individuals. That is why the credit facilities increase and this incentivizes individuals to take loans and increase overborrowing. Although this makes it possible to increase consumption, which in itself increases the expansion phase, however, when growth is moderating and the price of capital drops, it forces borrowers to start a debt deleveraging process and reduce their consumption, which contributes to the deepening of the recession. Thus, we are getting an inefficient equilibrium dynamic of a very sharp growth and contraction. Although every individual understands that the increase in the price of capital is inefficient and leads to over-leveraging, the individual cannot prevent this, because he is negligible in relation to the economy. Please see also (Uribe and Schmitt-Grohé, 2017).

the borrowers is too "loose", therefore the collateral requirement is tightened. Therefore, there is a transition from a process of increasing leverage to a process of a debt deleveraging – and the credit available to the lenders is decreasing, which may lead to a crisis[6;7].

As mentioned before, the innovation in the current research compared to (Benigno et al., 2020) is in the assumption that the leverage is relative to the value of the capital stock of the borrower, which is endogenous in the model. In this situation – the more capital the borrower holds, the more it decreases the level of leverage, and makes it possible to increase debt, and therefore it is increasing investments even more. But in the other direction, when the demand for investments is decreasing, and accordingly - the value of the capital is decreasing, this process is increasing the leverage ratio and makes financing costs more expensive. In this way the demand for investments is decreasing even more, and so on, the negative amplification is created – which is called a "**financial accelerator**". To examine a situation **without a financial accelerator**, we will look at a model where the debt

[6] A similar story can be a banking crisis in which there is an increase in the funding costs of banks or a shock to the capital of the banks, see (Gertler and Karadi, 2011). So, the banks are forced to reduce their credit offerings and this leads to a recession.

[7] The call for debt deleveraging is coordinated because lenders see others making this process and this encourages them to do as well. In addition, when the value of assets (collateral) drops then there is a spilover effect. We should note that when many lenders are debt deleveraging together, it will have a cumulative effect. They do not internalize external influences that the debt deleveraging may produce, such as an economic crisis, which in the end may hurt them more severely. As (Yellen, 2009) described it:
"...A process of balance sheet debt deleveraging has spread to nearly every corner of the economy. ... Minsky understood this dynamic. He spoke of the paradox of debt deleveraging, in which precautions that may be smart for individuals and firms—And indeed essential to return the economy to a normal state—nevertheless magnify the distress of the economy as a whole".

capacity is exogenous $\bar{b}_t = \Theta_t \bar{Q}\bar{K}^B$.

Let us assume that the spread function is convex, and its form is:

$$R_t^B = R_t^S \, exp\left(\phi \cdot \hat{b}_t^B\right) \tag{10}$$

for $\phi > 0$. The extreme situation where $\phi \to 0$ describes the limit where there is no financial friction, and the interest rate on loans is equal to the interest rate on deposits. Where, $\hat{b}_t^B$ is the difference between the debt and the debt capacity (in percentages): $\hat{b}_t^B = b_t^B / \bar{b}_t^B - 1$.

First order conditions of the borrower according to debt (Euler conditions):

$$1 = E_t\left[m_t^B \tilde{R}_t^B\left(\hat{b}_t^B\right)\right] \tag{11}$$

Where the effective interest rate as *perceived* by the borrower is:

$$\tilde{R}_t^B = R_t^S \, exp\left((\phi + \nu)\hat{b}_t^B\right) = R_t^B \, exp\left(\nu \hat{b}_t^B\right) \tag{12}$$

An increase in the level of debt results in an increase in the spread, as reflected in formula (10), and this in turn incentivizes the borrower to reduce his debt, as it is reflected in formula (11) which represents the demand for credit. Since the borrower recognizes the effect of his debt level on the spread that he is paying[8], then a term is added that embodies the elasticity of the spread in relation to the debt, ν, which depends on structural parameters as detailed in the appendix. When, $\nu \to 0$, approaches the situation where the borrower does not take into

[8] Called "internal" debt-elastic interest rate, compared to a model without internalization called "external" debt-elastic interest rate. see (Schmitt-Grohé and Uribe, 2003).

account the effect of the debt level on the interest that he is paying. A borrower who realized that his debt level impacts the spread, will be inclined to take up less debt compared to a borrower who doesn't re-alize.

Let us note that the no-arbitrage condition with respect to the bor-rower results with $\tilde{R}_t^B\left(\hat{b}_t^B\right) = \tilde{R}_t^K$.

2.3 Firms, the Capital Market and the Labor Market

A firm produces a final product using labor and capital formelized as Cobb–Douglas production function:

(13) $$Y_t = A_t {K_t}^{\alpha} L_t^{1-\alpha}$$

Where A_t is the total factor productivity. Let us assume the existence of a perfect competition in the labor market and the capital market, such that the firm accepts as a given the wage, w_t, and the price for hiring capital, q_t.

In each period, given the wage and the price for hiring capital, the firm maximizes its profit (determines optimal capital and labor):

(14) $$\Pi_t = Y_t - w_t L_t - q_t K_t$$

First order conditions according to capital (the demand for capital):

(15) $$q_t = (Y_t)'_K = \alpha A_t (K_t/L_t)^{\alpha-1}$$

First order conditions according to labor (demand for labor):

(16) $$w_t = (Y_t)'_L = (1-\alpha) A_t (K_t/L_t)^{\alpha}$$

The firm compares the marginal product by production factor (MPL and MPK), to the price of the production factor (w_t and q_t respectively). Now, divide the equations, and we will get the optimal ratio of capital to labor in the terms of the firm:

(17)
$$\frac{K_t}{L_t} = \frac{\alpha}{1-\alpha}\frac{w_t}{q_t}$$

2.4 The Capital Producer

The capital producer produces new capital, I_t, through the input of a final product (at a fixed price 1), and has to cover adjustment costs. That is, it provides the entrepreneurs with the demand for investments I_t, at the price Q_t. From optimization we are getting (see appendix):

$$Q_t = 1 + f\left(\frac{I_t}{I_{t-1}}\right) + f'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}} - E_t m_t^B f'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2$$

While we are assuming adjustment costs of the form: $f\left(\frac{I_t}{I_{t-1}}\right) = \frac{\gamma_{II}}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$. When the coefficient ,$\gamma_{II}$, is very large, we get the case where the capital is fixed.

2.5 Aggregate Values and Markets Clearing

As mentioned, the part of borrowers (and savers) of the total population is χ^B(and χ^S). Accordingly, the aggregate consumption (per capita) is $C_t = \chi^B C_t^B + \chi^S C_t^S$. And similarly, for total labor input, total capital, total investment and total adjustment costs. Note that the sum of all credit and savings is zero: $\chi^B b_t^B + \chi^S b_t^S = 0$. Hence a useful expression for the relationship between debt and household savings is: $b_t^B = -b_t^S \chi^S/\chi^B$.

If we add the budget constraint of the borrower and saver by weighting their share in the population, and if we set definitions of aggregate values, we will get the aggregate budget constraint:

$$w_t L_t + q_t K_t + \chi^B T_t^B + \chi^S T_t^S = C_t + Q_t I_t - \chi^S \frac{b_t^S}{R_t^S} - \chi^B \frac{b_t^B}{R_t^B}$$

As mentioned, the saver is the owner of the banks, while the borrower is the owner of the firms. Thus, the saver receives the (aggregate) profit resulting from the bank brokerage differences:

$$\chi^S T_t^S = \Pi_t^{Bank} = -\chi^S \frac{b_t^S}{R_t^S} - \chi^B \frac{b_t^B}{R_t^B}$$

We will use the expression that we have found above for the relationship between debt and savings (per capita) and we will get an expression for the profit of the banks:

$$T_t^S = \Pi_t^{Bank}/\chi^S = -b_t^S \left(\frac{1}{R_t^S} - \frac{1}{R_t^B}\right) \tag{18}$$

Now, the aggregate budget constraint:

$$w_t L_t + q_t K_t + \chi^B T_t^B = C_t + Q_t I_t$$

The profits of the firms which are producing the products and producing the capital, go only to the borrower, and are defined as follows:

$$\chi^B T_t^B = \Pi_t^{Firm} = Y_t - (w_t L_t + q_t K_t) + Q_t I_t - I_t\big(1 + f(I_t/I_{t-1})\big)$$

That is,

$$T_t^B = \Pi_t^{Firm}/\chi^B$$

Therefore, when the profits of the firms are placed in the aggregate budget constraint, there is an offset with the incomes of the households from labor and the hiring of capital, and we are getting on the sources side:

$$Y_t = C_t + I_t\big(1 + f(I_t/I_{t-1})\big) \tag{19}$$

2.6 Extension to the Neo-Keynesian framework

The above model is a purely real model with a homogeneous product and with absolutely flexible prices, and as we know, in such a model the monetary policy is not relevant. But we are interested in the effect of the policy when the nominal interest rate reaches the ELB. Therefore, we will move to a model where "prices are sticky" (there is a price rigidity), so that the price level will change slowly, and thus some of the real variables will depend on inflation, which is necessary to "destroy" the classical dichotomy[9]. In other words, this is how inflation will have a real effect, and therefore monetary policy will have a real effect.

In order to have a model with "sticky prices", first, the model must be changed to allow the existence of "price setters". This is by assuming a state of a monopolistic competition, and secondly, friction must be added to the decision process of the setting of the prices, so that not all prices are set every period.

But let us start with a preliminary step – and define a nominal debt,

[9] This is a concept developed by Prof. Dan Patenkin. When the classical dichotomy exists, the real economy will determine real variables and the monetary system will determine only nominal variables. This occurs when new variables of nominal interest and inflation are introduced into a real model in such a way that they do not enter into the equations of the existing model (we are just addint the "Fisher's rule" and the "Taylor's rule").

integrate it into the budget constraint and get the "Fisher rule" (detailed development in the appendix).

2.6.1 Nominal Debt and Fisher's Rule

Assuming that the debt is nominal, $b_t^{nom,j}$, then the budget constraint will be updated accordingly:

$$w_t L_t^j + q_t K_t^j + T_t^j + \frac{b_t^j}{R_t^{nom,j}} = C_t^j + Q_t I_t^j + b_{t-1}^j \frac{1}{\Pi_t} \tag{20}$$

While we are defining the **nominal debt in real terms** as follows: $b_t^j = b_t^{nom,j}/P_t$, and noting the difference in respect to real debt, where the former is expose to inflation shocks.

We should note that now the profit of the banks that goes to savers is:

$$\Pi_t^{Bank} = -b_t^S \left(\frac{1}{R_t^{S,nom}} - \frac{1}{R_t^{B,nom}} \right) \tag{21}$$

Now, assuming that the risk premium is a function of the real debt (debt limits are relative to the real debt), we get the "Fisher rule" from the first order conditions of the borrower, which links the nominal interest rate to the real interest rate and inflation expectations:

$$R_t^{nom,j} = R_t^j E_t[\Pi_{t+1}] \tag{22}$$

This is actually an ex-ante condition of an impossibility for arbitrage – investing in the nominal channel and the real channel are *expected* to yield the same return.

2.6.2 Monopolistic Competition (with flexible prices)

According to the assumption of a monopolistic competition as formulated by Dixit-Stiglitz, each firm produces a differentiated product for which it is the one that determines the price. The solution is a standard, so only the important points for updating the model will be noted (a detailed development appears in the appendix).

We shall assume a sequence of identical firms, each producing a different intermediate product, which give the firm a monopolistic power and allows the firm to set the price. The various intermediate products are "packaged" into a single final product used for consumption and investment, similar to the real model. The packaging is carried out by a competitive firm, and from the optimization of the packaging the relative demand function for each intermediate product is derived.

We shall return to the intermediate goods firms. All firms hire capital and labor from a perfect market and face the same hiring prices for capital and wages. Also, they all have the same Cobb Douglas production function. But each firm produces a different intermediate product, and therefore can set the price by itself, when it knows what is the scope of the demand that it is facing. For each price p_i it sets, it knows in advance what the demand for the product $y_i(p_i)$ will be. Therefore, the firms optimize in two parts.

In the first step, the firm minimizes real cost. And we are getting that all the firms will hire capital and labor in the same optimal ratio, since the price of the factors of production are the same. Also, the ratio does not depend on the choice of the quantity that the firm will produce:

$$\frac{K_t}{L_t} = \frac{\alpha}{1-\alpha}\frac{w_t}{q_t} \tag{23}$$

Also, the real marginal cost of a firm is:

$$MC_t = \frac{1}{A_t}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{q_t}{\alpha}\right)^{\alpha} \tag{24}$$

The marginal cost is also the same for all the firms and depends only on the prices of the factors of production, and not on the quantity that the firm chooses to produce.

We should note that the labor demand equation is:

$$w_t = MC_t \cdot (1-\alpha)A_t\left(\frac{K_t}{L_t}\right)^{\alpha}$$

And the capital demand equation is:

$$q_t = MC_t \cdot \alpha A_t\left(\frac{K_t}{L_t}\right)^{\alpha-1}$$

Similar to the model of perfect competition, the price of a factor of production is related to the marginal product according to the factor of production, but here there is also a factor of the real marginal cost.

In the second stage, the firms are performing maximization according to the price, and we get that all the firms set the same price, which is a fixed margin in percentages (markup) above the nominal marginal cost. Therefore, the price that the firm sets is the same as the general price level, and therefore, the real marginal cost is fixed and equal to the inverse of the markup: $MC_t = \mu^{-1}$. This is the appropriate condition for the situation of absolutely flexible prices. The adding of a monopolistic competition by itself has no important effect if the prices remains absolutely flexible. Since the markup is fixed, then the effect will be on the value of the steady state, and not on the dynamics.

Therefore, in the next section, an element of price rigidity will be introduced.

2.6.2 Price Rigidity

Now a price rigidity component will be added to the model, in accordance with Calvo's assumptions (Calvo, 1983). In each period, a firm in a monopolistic competition gets an opportunity to change the price with the probability of $1-\ \theta$ (where θ is a measure of the price stickiness, and in the limit $\theta \to 0$ we return to a world of absolutely flexible prices). Therefore, when a firm maximizes its discounted profit flow, it takes into account the probability of a price update. And the optimization problem is:

$$\max_{p_t^j}\left\{E_t \sum_{\tau=0}^{\infty} \theta^\tau\, Q_{t,t+\tau}\ \cdot \left(p_t^j - p_{t+\tau} MC_{t+\tau}\right)\left(p_t^j / p_{t+\tau}\right)^{-\nu} Y_{t+\tau}\right\}$$

Where $Q_{t,t+\tau}$ is the "stochastic discount factor" (SDF) of the owners of the firms. We accept that the firm sets a price according to the optimal pricing rule (after log-linearization, while we are defining $\widehat{mc}_t = MC_t/MC - 1$,$\hat{\pi}_t = \pi_t - \bar{\pi}$):

(25) $$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \widehat{mc}_t$$

This is the "Phillips curve", where the elasticity of inflation relative to the real marginal cost is $\kappa = (1-\theta)(1-\beta\theta)/\theta$.The parameter β is the subjective (effective) "discount factor" of the owners of the firms (the borrower). This equation replaces the condition that real marginal cost is constant for an economy with flexible prices and monopolistic pricing (the previous section). That is, price rigidity was introduced into the model by replacing the equation of constant marginal cost (23) with the equation of a Neo-Keynesian "Phillips curve" (25).

The current equation embodies a situation where price determination is a forward-looking process, and the fact that inflation responds more strongly to changes in marginal cost when prices are highly flexible. In the limit of flexible prices $\theta \to 0$, actually $\kappa \to \infty$, but at the same time $\widehat{mc}_t \to 0$ i.e., the Phillips curve (AS) is vertical, and the marginal cost (~output gap) is absolutely rigid. And there is no longer a transmission between inflation expectations and actual inflation, because the firms can update prices without any limit.

It is possible to write the "Phillips curve" as a sum:

$$\hat{\pi}_t = \kappa \sum_{\tau=0}^{\infty} \beta^{\tau} E_t \widehat{mc}_{t+\tau} \tag{26}$$

That is, inflation is a discounted sum of the deviation of the marginal cost. It should be noted that on the left side there is a nominal variable, while on the right side there is a real variable. In the situation of absolutely flexible prices, the marginal cost is constant, so there are no deviations in it, and we are getting zero inflation gap.

It is possible to understand how the nominal side has an effect on real variables, if we assume that for some reason there is a change in the nominal interest rate. In a model of flexible prices this changes only inflation ("Fisher rule"), without any other effects. On the other hand, in a Neo-Keynesian model, such a change in inflation can only occur in equilibrium if the real marginal cost has changed (see equation 25 above). That is, when we added the "Phillips curve", we added to the model a relationship between a nominal variable (inflation) and a real variable (the real marginal cost). That is, we have broken the classic dichotomy.

2.6.3 Monetary Policy

Let us assume a policy guided by an inflation target, which takes into account the ELB on the interest rate (say 0%) as follows:

$$r_t^{s,nom} = max\left[0, (\rho^S + \pi) + \phi_\pi(\pi_t - \pi) + \phi_y \hat{y}_t\right] \tag{27}$$

Where we define: $\widehat{y_t} = Y_t/\bar{Y} - 1$, $1 + \rho^S = (\beta^S)^{-1}$ and assume that $\phi_\pi > 1$ to satisfy the "Taylor condition". When inflation and output gap are at the target: $r_t^{S,nom} = \rho^S + \pi$.

Let us assume that the central bank manages to maintain the inflation target precisely ($\pi_t = \pi$) and the ELB on the interest rate does not exist. Such a world can be interpreted as *equivalent to an equilibrium allocation in a world where all prices are flexible*. Mechanically, if a policy manages to maintain the inflation target precisely, then the "Phillips equation" becomes degenerative, in the sense that it is obtained that the real marginal cost is constant $0 = \widehat{MC}_t$, thus $MC_t = \mu^{-1}$. That is, it is equivalent to the replacement of the "Phillips equation" back into an equation of condition for perfectly flexible prices (see Eggertsson and Krugman, 2012). The reason why this interpretation works is that a debt deleveraging shock – as can be seen in the equations (2) and (25) – does not produce a trade-off between the inflation and the output in the event that there is no ELB on the nominal interest rate. In conclusion, a central bank that manages to maintain the inflation target produces an allocation like in a world with flexible prices.

2.7 Steady State and Prameterization

The main contribution of the present research is in theoretical analysis; therefore, parameterization of the model allows to obtain results that make economic sense and are related to real world.[10] The main source on which we will base the choice of parameters is the article by (Benigno et al., 2020), which were also based on parameters accepted in the literature, with the exception of the parameters unique to the credit model, for which they performed a calibration. The unique parameters are the size of the shock, the debt deleveraging process and parameters of the elasticity of the spread in relation to the debt. Parameters from the research of (Gertler and Karadi, 2011) were also examined, and the results remained very similar. Below is the list of parameters and their values in most of the simulations that we have performed:

Parameter	Value	Reference
ω	0.5	were also tested {0.276 , 0.8}
$\bar{L}$	0.25	
γ	0.5	were also tested {0.3 , 1.7}
χ^B	0.61	(Benigno et al., 2020)
h	0	was also tested 0.8
Θ_t^B	0.64 → 0.56	`
β^S, β^B	0.9963	(Benigno et al., 2020)
σ	0.66	(Benigno et al., 2020)
ϕ	0.055	(Benigno et al., 2020)
ν	0.159	(Benigno et al., 2020)

[10] "*The point is that if you have a conceptual model of some aspect of the world, which you know is at best an approximation, it's ok to see what that model would say if you tried to make it numerically realistic in some dimensions.*"(Eggertsson and Krugman, 2012) .

Parameter	Value	Reference
α	0.3	
δ	0.025	
κ	0.02	
ϕ_π	1.5	
ϕ_Y	1	
$\bar{\pi}$	0.005	in quarterly terms

The steady state was calculated for the high level of leverage, which is a starting condition for the simulation, and also calculated for the low level of leverage to which the simulation should converge after the shock of the debt deleveraging process (see details in the appendix).

Next, the reaction of the model will be examined under a specific set of parameters. However, at the same time, sensitivity analyzes of the reactions to different parameters were performed (mainly in the vicinity of the selected parameters), and it was checked whether there is a significant change in the results if the parameters are changed. The debt deleveraging shock was calibrated in terms of intensity and persistence, so that it would match the decline in consumer and business (non-financial) credit relative to the GDP for the United States.

3. Analysis - Financial Crisis

The purpose of this research, as mentioned, is to examine a financial crisis that begins as a reaction to a debt deleveraging shock, and during which the economy enters the "liquidity trap", in particular when the crisis intensifies due to mechanisms such as the "financial accelerator" and “debt deflation”. Since the research intends to examine the contribution of various mechanisms, the analysis will be split into several stages.

First, to simplify the issue, we will try to understand the effect of the "financial accelerator" in a situation where the ELB on the interest rate is ignored. In addition, we assume that debt is real (indexed), so there is no "debt deflation", and likewise, the assumption is that the capital stock is fixed. The reference case here will be a situation without a "financial accelerator" (when the debt capacity is exogenous, as we discussed in section 2.2). This examination will make it possible to understand the basic mechanism of the accelerator in a cleansed way.

In the second stage, an ELB on the interest rate will be taken into account, and this will make it possible to test the mechanism of the "financial accelerator" when the debt deleveraging process pushes the economy into the "liquidity trap". In the third stage, we will add dynamics of the capital stock. And in the last stage, a situation of a nominal debt will be examined, so that an amplifying mechanism of "debt deflation" is added, and it will be possible to compare between the contribution of the "financial accelerator" and that of the "debt deflation" mechanism. Only then will it be possible to get a complete picture of the various contributions to the financial crisis (in our calibrated model).

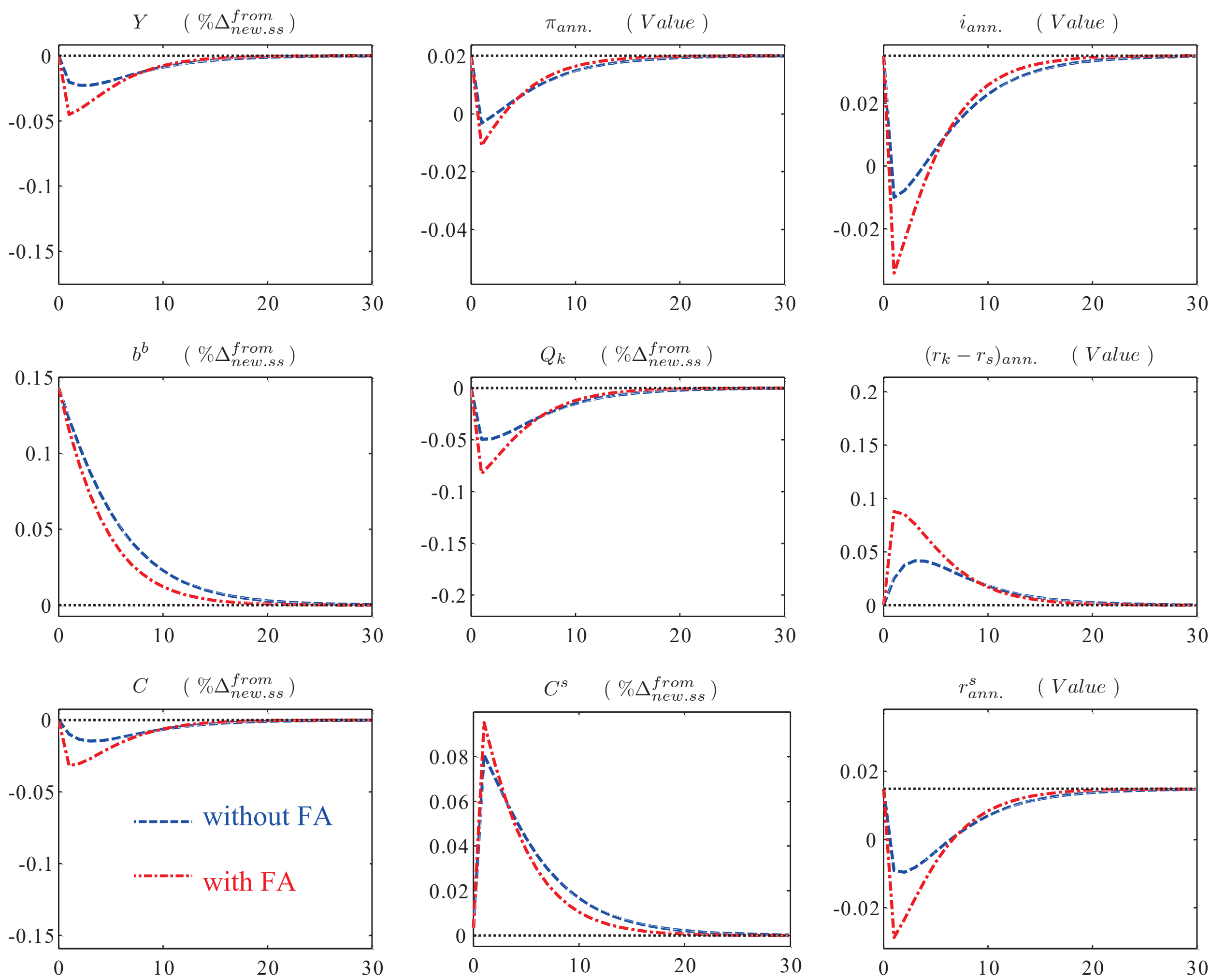
3.1 Ignoring the ELB

As mentioned, let us start with a situation where we ignore the ELB on the interest rate, and that the debt is real (indexed) so there is no "debt deflation". Let us also assume that the capital stock is constant, this is under the assumption of an absolutely rigid investment supply. For this purpose, the adjustment costs to the change in the level of investments were determined to be very high, so that the level of investments always remains constant, and accordingly the capital stock as well.

Figure 1 shows a reaction to a debt deleveraging shock in two situations – with a "financial accelerator" (the red line), and without a "financial accelerator" (the blue line). In both situations the shock causes a decrease in the credit supply and an increase in the credit spread. These two phenomena are "pushing" borrowers to reduce their debt by reducing their expenditures - consumption and investments. The slackness in the demand of the borrowers impacts the aggregate demand in a negative way, therefore, there is a decrease in the output and in the inflation. The central bank lowers the interest rates to a negative territory (currently we are ignoring the ELB on the interest rate), thus managing to lower the real interest rates sufficiently enough to "push" savers to increase consumption, thus partially offsetting the laxity on the part of borrowers. That is, the monetary policy succeeds in carrying out an expansion and greatly moderates the crisis.

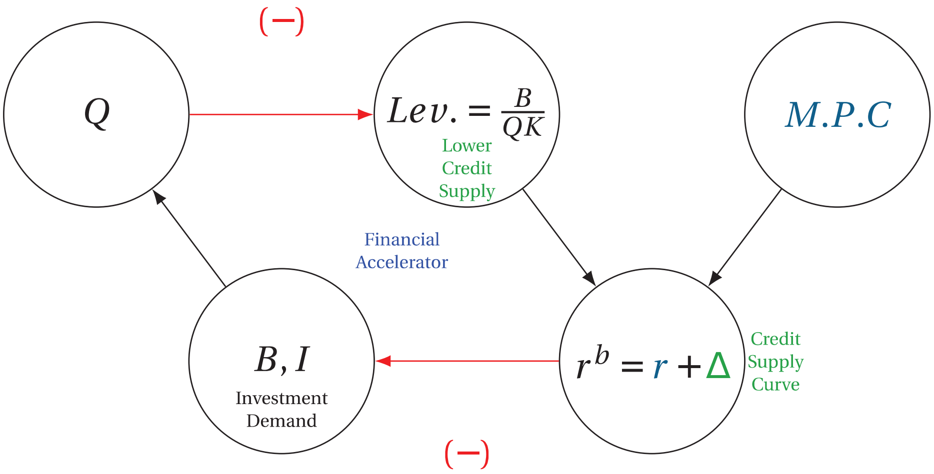
The decrease in demand for investments causes a decrease in the price of capital and a decrease in the value of the assets owned by the borrower (physical capital stock), which are used as a collateral. In a situation where the debt capacity of the borrower is endogenous (the blue line), there is a decrease in his debt capacity, meaning he is perceived as riskier. This manifests itself in an increase in the leverage of the borrower, and therefore his credit spread increases, which in itself further decreases the demand for investments, thus creating a spiral effect. That is, when the debt capacity is endogenous and depends on the value of the assets, then we are getting an empowering mechanism of a financial accelerator (see Figure number 2), in relation to the situation where the debt capacity is fixed and there is no financial accelerator. Quantitatively, it is possible to distinguish in Figure 1, that in the situation of a "financial accelerator", the output is two times smaller compared to the situation without an accelerator (the blue line versus the red line).

Figure 1: Reaction to a debt deleveraging shock, when the capital stock is fixed, and when the ELB on the interest rate is ignored. In blue: a situation without a financial accelerator, and in red: the presence of a financial accelerator mechanism (FA abbr. for Financial Accelerator)



We emphasize that the monetary policy succeeds in "softening" the "financial accelerator", because the monetary interest rate drops sharply into a negative territory and "pushes" the real interest rate down a lot (a decrease of about 4 percentage points), which offsets part of the increase in the spread (a decrease of about 9 percentage points).

Figure 2: Schematic of the "financial accelerator"



3.2 Takeing Into Account the ELB on the Interest Rate

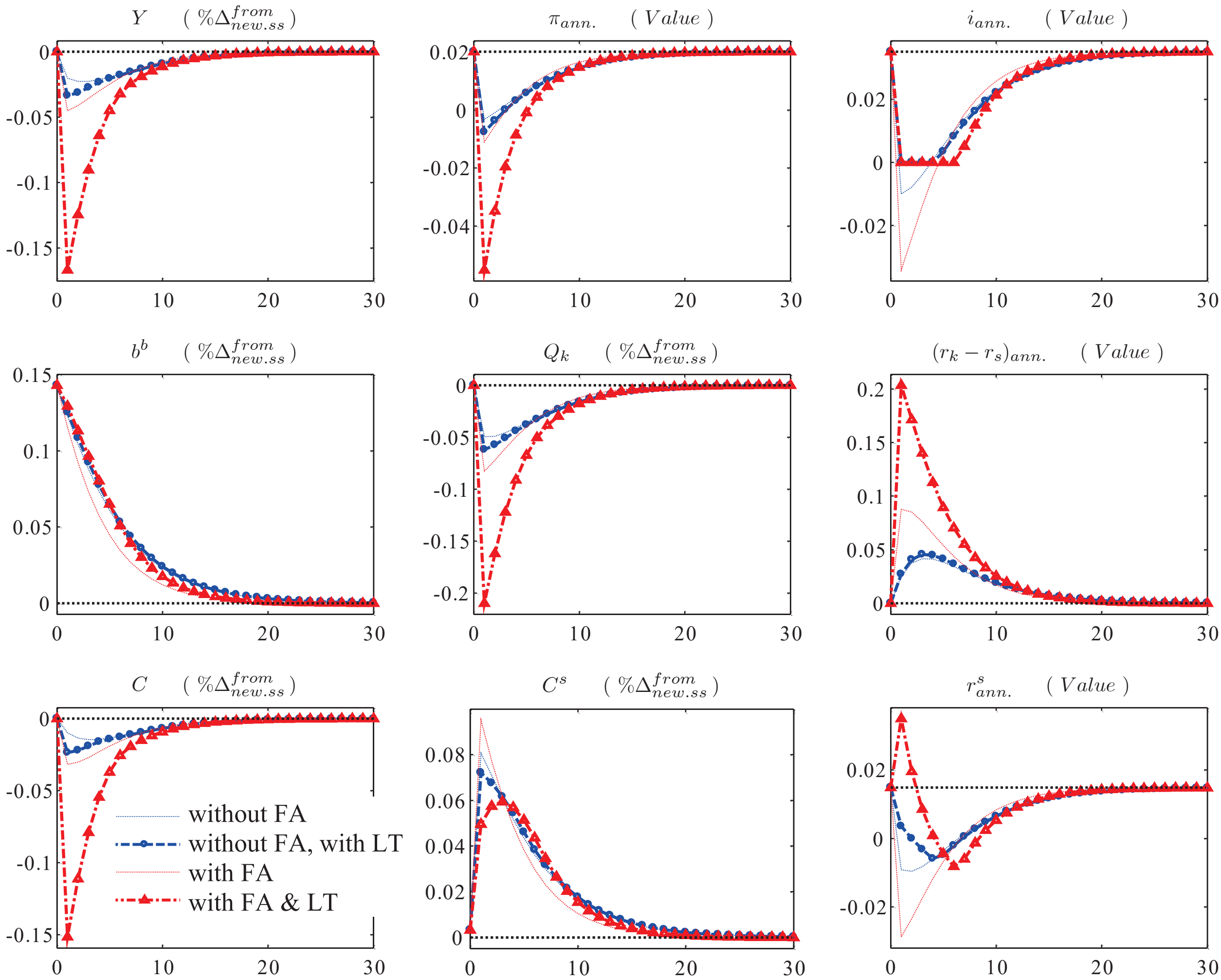
Now we will examine a crisis of debt deleveraging when there is an ELB on the interest rate. As in the previous examination, there is still the assumption that the debt is in real terms, so that there is no "debt deflation", and the assumption is that the capital stock is fixed. The response is shown in Figure 3.

The general story has not changed compared to the previous situation. In the model **without a financial accelerator** (the blue line with circles) we saw in the previous situation that the monetary interest rate fell only slightly below zero. Now, the monetary interest rate is "stuck" at the ELB, and as a result the real interest rate falls less than in the situation without an ELB. That is, the policy is less expansionary, and accordingly, the recession is slightly larger.

Now, we will discuss the situation **with the "financial accelerator"** (the red line with triangles) when we first saw the mechanism of the "financial accelerator" that causes an increase in the credit spread, and discussed the monetary policy that manages to offset part of this increase through a sharp drop in interest rates. However, when there is an ELB on the monetary interest rate, it does not manage to "push" the real interest rate down, and in fact, the real interest rate even rises in an initial reaction against the backdrop of the sharp drop in inflation. The reaction of the borrowers is impacted by the sum of the real (risk-free) interest rate and the credit spread, so that their effective interest rate skyrockets. At the same time, their income is greatly impacted, and although they have a very large incentive to de-leverage, they are unable to do so faster, and are therefore forced to reduce consumption expenditures in a very extreme way.

The above situation, within the framework of a model of heterogeneous agents, well illustrates the significance of the lack of consumption insurance (or the assumption of incomplete markets), which leads to a very extreme behavior of the borrowers, and as a result, an acute economic crisis is obtained. Such a story is not established in standard models of a representative agent, which actually assume consumption insurance of the various households, and produce much more moderate reactions.

Figure 3: Reaction to a debt deleveraging shock in an economy with a fixed capital stock, when taking into account the ELB on the interest rate (LT abbr. for Liquidity Trap)



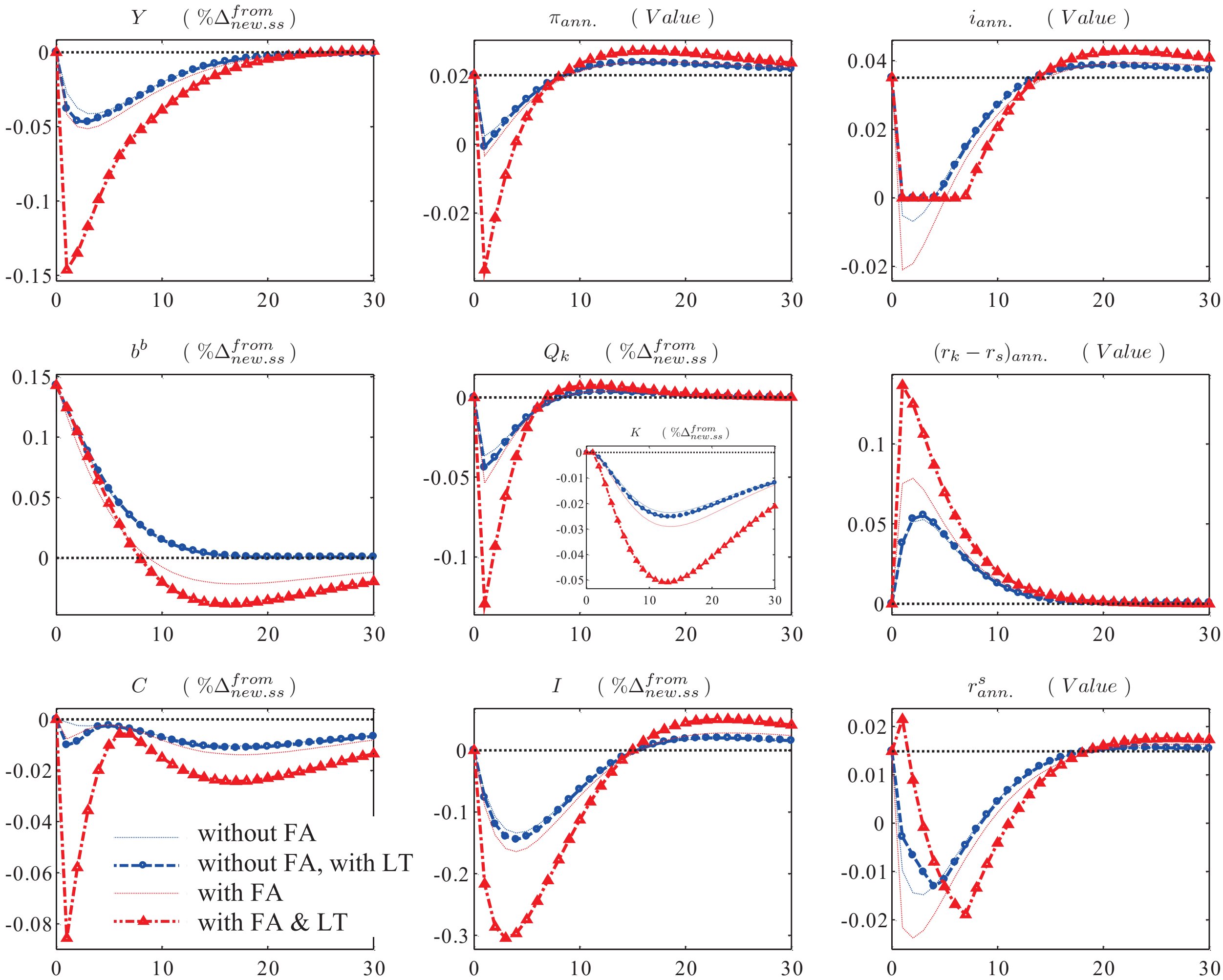
In conclusion - when the economy enters the "liquidity trap" and the monetary policy is very much restricted, the mechanism of the "financial accelerator" – the feedback between the decrease in the price of capital and the increase in the credit spread – increases the crisis significantly. In fact, it can be seen that in reaction to the deleveraging, (1.) the damage resulting from having the "liquidity trap" only (without a "financial accelerator") is about **4 basis points**, and (2.) the damage from having the "financial accelerator" only (without the "liquidity trap") is about **5 basis points**, and (3.) the damage in the situation of entering the "liquidity trap" in the presence of the "financial accelerator" is about **16 basis points**, which is significantly larger than the sum of (1.) and (2.). This indicates that there is an *important interaction between the mechanisms*. On the other hand, the "liquidity trap" without a "financial accelerator" lasts for 4 (four) quarters, compared to 6 (six) quarters when there is a "financial accelerator", meaning, a moderate increase of the period of "liquidity trap".

3.3 Flexible Capital Stock

Now we will examine the real-world situation where the capital stock is changing during the crisis, as can be seen in figure number 4. The picture is similar to what we saw earlier, and in particular it is possible to see that when the economy enters the "liquidity trap" and the "financial accelerator" operates (the red line with triangles), then we are getting a much more acute crisis compared to the other situations (without an accelerator or without an ELB on the interest rate).

In relation to the situation of fixed capital, when capital stock is flexibe, then in a situation where the borrowers need to deleverage, they also reduce expenditures by reducing investments, and accordingly we witness a sharp drop in investments, which is indeed a typical phenomenon in past crises. They also reduce consumption expenditures, but the reduction in investments allows them to better "smooth out" the consumption path, and thus the aggregate consumption decreases by about 9% compared to 15% in the previous situation. But on the other hand, the decrease in investments obviously harms the capital stock and the development of the output with the continuation of the crisis. That is, a slightly moderate crisis is obtained in terms of intensity, but a much more persistent one.

Figure 4: Reaction to a debt deleveraging shock in an economy with a flexible capital stock



The dynamics of the credit capacity of the borrowers depends directly on the value of the capital stock that is used as a collateral, and it can be split into two components: one part is the price of capital, which reacts quickly and drops drastically at the beginning of the crisis, but quickly corrects itself and returns to its level of about a year and a half after the beginning of the crisis. Whereas the other part, is the capital stock, which reacts very slowly and reaches a minimum after about two years, and it takes a long time to return to its pre-crisis level. This is also the reason that in this situation, during the crisis, the debt level falls below the new debt level (overshooting), since the persist impairment to the stock of physical capital impairs the debt capacity over time, while the impairment to the price of capital is very temporary.

3.4. Nominal Debt and “Debt Deflation"

Until now, the assumption is that the debt is in real terms (indexed), so it is not exposed to the risk of a decrease in inflation. Now, we will analyse a case of nominal debt – which is more common in the market – and exposed to the risk of a decrease in inflation. That is, during a crisis in which there is a surprising drop in inflation, since the debt is nominal then the drop in inflation increases the value of debt in real terms – the "Fisher debt deflation" effect. Since the debt burden in real terms increases, it makes it even more difficult for borrowers to reduce debt, a deflationary spiral is created, and thus the crisis intensifies, as it is analyzed by (Eggertsson and Krugman, 2012). In this section, a comparison will be made between the contribution of the "financial accelerator" and that of the "debt deflation" mechanism.

Figure 5 below shows the main macro variables in a debt deleveraging crisis, when the debt is in real terms, so there is no "debt deflation". This is in contrast to Figure 6 that appears immediately after, which depicts nominal debt and therefore there is a "debt deflation" effect. In both illustrations, the economy enters the "liquidity trap", while the "financial accelerator" is in action. It can be seen that when there is also "debt deflation", the crisis is deeper by about 2 percentage points (a contraction of 8% instead of 6%).

Figure 5: Reaction to a debt deleveraging shock in an economy where the debt is in real terms indexed (In the presence of a financial accelerator and an ELB on the interest rate)

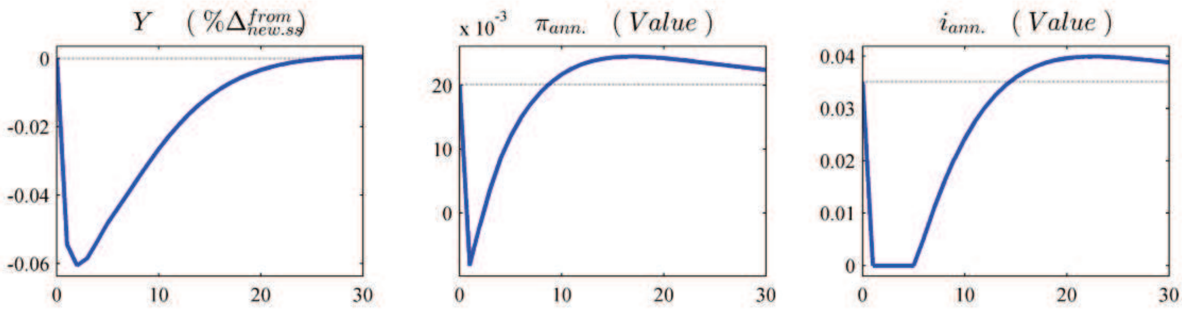
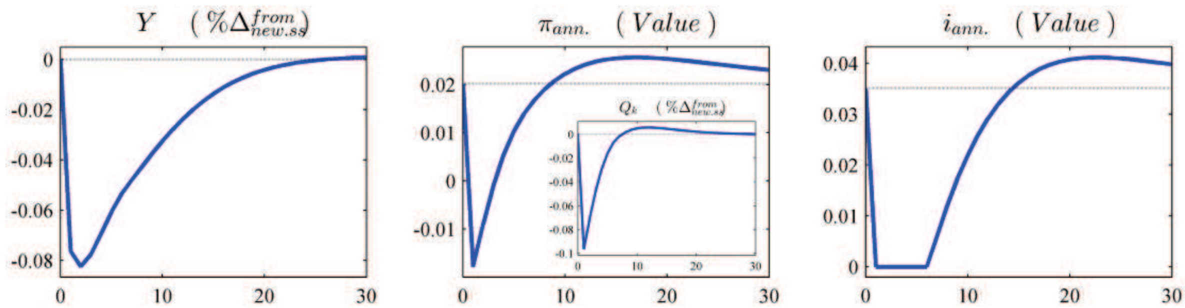


Figure 6: Reaction to a debt deleveraging shock in an economy where the debt is nominal, therefore there is "debt deflation" (In the presence of a financial accelerator and an ELB on the interest rate)



From a quantitative analysis for the tested calibration and its environment, it appears that the "financial accelerator" mechanism is more significant than the "debt deflation" mechanism. We will note that in Figure number 6, inflation decreases by approximately 4 percentage points, therefore the "debt deflation" effect increases the leverage of the borrowers by approximately 4 percentage points. On the other hand, the price of capital decreases by about 10 percentage points, so the leverage increases accordingly by about 10 percentage points. Hence, as mentioned, the effect of the "financial accelerator" is dominant.

3.5 Further Analyzes and Extensions

Another examination carried out is the "**Forward Guidance**" analysis, in which we left the interest rate at the ELB for a longer period of time than was derived from the "Taylor rule"[11]. As mentioned, in the solution of the model, there is an assumption of absolute certainty (deterministic simulation), that is, beyond the opening point which embodies a surprise in the credit capacity, there are no surprises in the future, and the individuals know the interest rate path in advance. Therefore, when it is defined that the interest rate will remain within the ELB for a specified number of quarters, this embodies the intention of the central bank (guidance) with regard to the future interest rate.

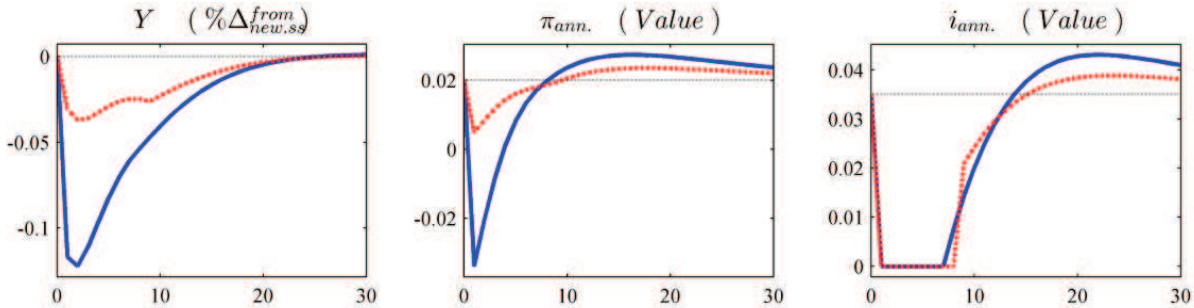
Figure number 7 shows a policy that declares a zero-interest rate for two years. It can be seen that within the model framework **the "Forward Guidance" has a very large effect**, and if we are extending the length of time that the interest rate is at the ELB it results in reduction of the crisis. This result, according to which there is a very strong "forward guidance" effect in the Neo-Keynesian model, while on the other hand, in the real world, the effect is much more moderate is known as “The Forward Guidance Puzzle” (Del Negro et al., 2017b; Kutai, 2020). And this criticism is, of course, relevant to further researches with regard to the subject (Benigno et al., 2020).

In fact, in the simulation above we embody an assumption of a "full credibility", meaning that all individuals in the economy absolutely believe the statement of the bank regarding the future interest rate

[11] Such guidance is called Odyssean F.G. since the bank announces in advance a period and is not conditioning it subject to developments; unlike the Delphic F.G., which depends on developments and is suitable for discretionary policy.

path. However, in the real world we are often in a situation of "partial credibility", that is, there are individuals in the economy who do not believe in the guidance, and as a result behave in a way that is less expansive. In the limit case where there is no confidence in the statement of the bank, the interest rate path will indeed remain at the ELB, but we will expect a much weaker effect of the policy[12], so that the crisis will be much more severe.

Figure 7: reaction to a debt deleveraging shock under a policy according to the Taylor rule and an ELB on the interest rate (the blue line) and under a "Forward Guidance" policy for 8 (eight) quarters (the dotted red line)



Further with regard to the "Forward Guidance", it should be noted that a commitment to keep a low interest rate for a long period of time (or a large purchase of long-term bonds) might perhaps curb a crisis in the short term, since it supports the value of assets, and this stops the "financial accelerator" mechanism. However, this kind of policy may produce a large future expansion (excessive leverage), and a future risk to stability, that is, sowing the seeds of the next crisis.

Another question that was not directly examined here, since the

[12] This case corresponds to a simulation in which the monetary shocks are surprising every period (called MIT shocks). Compared to the simulations shown in the figure, which can be interpreted as equivalent to a simulation in which the shocks are known in advance.

model is not suitable for such an examination[13], but one can get an intuition about it from the model: what is the risk that may be caused by the increase in leverage in the economy? On the face of it, as the leverage in the economy increases beyond a certain level (when there is no structural improvement in the economy), one can assume that the chance of a debt deleveraging shock increases. Also, given a shock, the intensity of the shock can also be larger, and the chance of falling into the "liquidity trap" increases. In fact, when within the framework of the model we increased the intensity of the shock, the damage increased very significantly (to the point of not being resolved). Therefore, an increase in leverage increases the risk of a debt deleveraging crisis.

Finally, we mention two extensions that could be interesting for further research. First, the work does not discuss an optimal monetary policy, and it may be worthwhile to examine this point in depth. In this context, it is important to note that the natural interest rate in the model is endogenous and depends inversely on the credit spread, similar to the model of (Benigno et al., 2020). Another direction that is interesting to examine is the case of a small and open economy ((Benigno and Romei, 2014; Borenstein and Ilek, 2021; Schmitt-Grohé and Uribe, 2003)), where a situation is possible in which the greater the financial openness, the more restricted the monetary policy is in its response, and therefore the crisis may worsen. But it depends of course

[13] See (Adrian et al., 2020) which presents a model in which the strength of the shock depends on the leverage.

a lot on where the openiness is, whether on the side of the lenders (savers) or on the side of the borrowers.

4. Conclusions

In this work we have examined a model of a financial crisis which was caused by a sudden debt deleveraging process. The model includes physical capital, such that the debt capacity and the leverage are endogenous (depending on the value of the capital stock), and impact the financing spread of the borrowers. Accordingly, the model contains a "**financial accelerator**" amplifying channel, since, as the value of the capital decreases, the spread increases, which results in a further decrease in demand and therefore - in the value of the capital.

In the situation where the debt is nominal, then in a crisis, when inflation decreases, the real debt burden increases – "**debt deflation**" – and makes it even more difficult for borrowers, such that the crisis is intensified. The premise of this research is that there is **no consumption insurance**, compared with models of a representative agent, such that in times of crisis, the borrowers cannot be supported by the households which are among the savers, therefore they react extremely and impact the aggregate demand in a negative way. And when **the monetary interest rate reaches the ELB** it is restricted in its support, and the crisis is intensified even more.

The model is solved precisely (without a "Taylor approximation"), within a deterministic simulation, in order to examine the various amplification channels and their interaction during a crisis, when the economy deviates significantly from the steady state. That is, because the model equations are not linear, such as the monetary policy rule that includes an ELB on the interest rate, and likewise, the equations include interaction terms between the various variables. Therefore, an accurate solution of the model allows us to capture, **non-linear effects and interaction between mechanisms**. The model was tested under different specifications, in which some of the mechanisms are disabled, and thus we were able to find **contributions** of the mechanisms and the interactions between them.

It was found that there is a strong interaction between the presence of an ELB on the interest rate and the "financial accelerator" mechanism, so that a crisis in which both mechanisms operate, is much greater than the sum of the impairment from each mechanism separately. It was found that in a standard calibration, the contribution of the "financial accelerator" mechanism is dominant in relation to the contribution of the "debt deflation" mechanism.

We therefore conclude that the more we maintain an effective monetary policy during a financial crisis, the more this may make a huge difference with regard to the depth of the crisis. In other words, the monetary policy is significant during a crisis in which spiral mechanisms such as the "financial accelerator" and “debt deflation” may operate with great force. Maintaining the effectiveness of the monetary policy has a particular importance in an environment of low interest rates as we have known in recent years, both against the backdrop of an environment of low inflation and also after a recession (or when growth expectations are moderate).

4.1. Additional Material for Reference

For further reading please see (Justiniano et al., 2015) (Amano and Shukayev, 2012) (Uribe, 2006b) (Razin, 2014) (Martin and Philippon, 2017)

Please see appendices on GitHub or download here. Reference materials and the model are here.

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