A simple theory-based estimate of the real natural rate of interest in open economies[1]

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Abstract

We propose a simple methodology to estimate the short-term natural rate of interest (NRI) in small open economies. Following Clarida et al. (2002), the NRI depends on the expected growth of (1) domestic potential output and (2) output abroad. We use observable expectations within an estimated central bank's policy rules in Israel, Sweden, and Canada to derive NRI estimates. Our estimates possess strong common dynamics: they fall during crises and rise during booms. Our estimates also imply that monetary policy has been accommodative since the global financial crisis.

שיטה פשוטה לאמידה של הריבית הריאלית הטבעית של הטווח הקצר למשקים פתוחים

אלכס אילק וגיא סגל

תקציר

אנו מציעים שיטה פשוטה לאמידה של הריבית הטבעית של הטווח הקצר למשקים פתוחים. לפי Clarida et al. (2002), ריבית זו תלויה בצמיחה הצפויה של התוצר הפוטנציאלי המקומי ובצמיחה הצפויה (בפועל) של שותפות הסחר. אנו משתמשים בנתונים ובהערכות לגבי הציפיות של שני הרכיבים, וגוזרים את הריבית הטבעית עבור שלושה משקים קטנים ופתוחים, ישראל, קנדה ושוודיה, מתוך כלל ריבית של בנק מרכזי של כל מדינה. האומדנים שלנו לריבית הטבעית עבור שלוש המדינות מאופיינים בדינאמיקה דומה: הריבית הטבעית יורדת בתקופות משבר או מיתון ועולה בתקופות של גאות כלכלית. מהמחקר עולה כי לאחר המשבר הפיננסי הגלובלי בשנים 2007-08 המדיניות המונטרית בשלושת המשקים הייתה מרחיבה ובכך תמכה בפעילות הריאלית וביציבות המחירים.

1 Introduction

Although the natural rate of interest (NRI, hereinafter) is a theoretical concept, it has real implications on the stance and conduct of monetary policy. Setting the monetary real interest rate in accordance with the NRI leads, all else equal, to output at its potential level together with price stability. Hence, the NRI is a benchmark indicator for setting the monetary policy rate. In the New Keynesian (NK, hereinafter) model, the NRI is derived as the interest rate that would have been obtained in a counterfactual economy, where prices and wages are flexible (see, e.g., Galí (2015); Woodford (2003)). As the NRI is unobserved, there is a need to estimate it.

According to the canonical NK model, in a closed economy the NRI is affected by the time preferences and by the expected growth of the potential output, called the flexible-price economy output (see, e.g., Woodford (2003); Laubach and Williams (2003); Galí (2015)). In an open economy, the NRI is also determined by the expected growth of actual (rather than potential) output abroad (see, e.g., Clarida et al. (2002); Gali and Monacelli (2005)). This augmentation results from the terms of trade that together with the real wage affect the real marginal cost.

In this paper we estimate the NRI in three open economies, Israel, Sweden, and Canada, by relying on theoretical specification. Our main contribution is that we derive the natural rate from an estimated policy rule of the central bank (CB, hereinafter). Specifically, at the first stage, we specify the NRI based on the theoretical specification of Clarida et al. (2002), which enables us to specify the NRI using observable data on expected growth abroad and on a proxy for the expected potential output in the domestic economy, obtained from surveys of professional forecasters or internal forecasts of CBs. This is in contrast to Zhang et al. (2021), who treat expectations for a foreign economy's growth as unobservable, and derive them within their model. At the second stage, we estimate an augmented Taylor-type rule for the CB, where the specified components of the NRI from the first stage serve as some of the explanatory variables in the policy rule. Thus, we derive the estimator of the natural rate using observed data as a proxy for the relevant variables suggested by the literature. Hence, the estimated policy rule, which also includes the theory-based NRI, also leads to a good description of the monetary policy. At the final stage, after estimating the NRI from the policy rule, we test its plausibility by relying on the equilibrium conditions of the IS equation within the NK framework for small open economies.

The dependence of the NRI in open economies on foreign expected output is consistent with the empirical finding of Holston et al. (2017). They point out the existence of interdependence in natural rates across the UK, Canada, U.S., and the euro area. In the former two economies, which are more open than the other two, the dependence of the domestic natural rates on their counterparts in the U.S. and in the euro area is found to be much greater than in the reverse case.

To the best of our knowledge, Elkayam and Segal (2018) are the first to estimate an open-economy short-term NRI using observed (proxy) data on expected growth abroad. The estimation we present here significantly extends Elkayam and Segal (2018) by also relating to the Effective Lower Bound (ELB, hereinafter). Elkayam and Segal (2018) estimated the NRI for Israel between 1995 and 2015, that is, until the Bank of Israel (BoI, hereinafter) interest rate was cut to its ELB of 0.1 percent. Our sample for Israel ends in 2019:Q4, including the binding ELB period between 2015 and 2018, and hence we estimate it using IV-Tobit.

Aronovich and Meldrum (2020) also use data (from surveys of professional forecasters) to estimate the NRI. They estimate nonlinear regressions of 5-to-10-years-ahead forecasts of the short-term nominal interest rate and CPI inflation on U.S. treasury yields, and interpret the difference between the forecasted values as a measure of the NRI. Aronovich and Meldrum's (2020) approach is essentially similar to Clarida (2009) and Beenstock and Ilek (2010) who derive the NRI from the real forward rates in the capital market. However, all their estimates relate to the long-term NRI, and thus their approach is incompatible with our goal of estimating the short-term NRI.

Our methodology of deriving the NRI estimator from the CB's policy rule leads to a CB's perceived estimator of the short-term NRI. The assumption behind the use of this methodology is that monetary policy is conducted "properly," in the sense that the monetary committee reacts to relevant variables to achieve price stability and stabilize real activity.[1] That is, there is no need to assume that the monetary committee assesses the NRI directly. Hence, our NRI estimator is an interpretation of a linear combination of variables that are in the information set of the policy committee, and that according to theory represent the NRI.

Our use of the CB policy rule has another advantage. The high uncertainty of NRI estimates is well documented (e.g., Laubach and Williams (2003); Barsky et al. (2014); Neri and Gerali (2019)). The high uncertainty of NRI estimators stems from the high uncertainty of the specifications of the models used, as the NRI is model-dependent. To illustrate this point, Neri and Gerali (2019) present various NRI estimates in the U.S. and the euro area, and find that these estimates have a strong heterogeneity (see Figures 3 and 4 in Neri and Gerali (2019)). Beyer and Wieland (2019) find a very high uncertainty of NRI estimates derived from a multi-equational model à la Laubach and Williams (2003). The high uncertainty also stems from their attempt to estimate several unobservable variables at the same time in that model.[2] By contrast, although our NRI estimate is also policy-rule dependent, there is a wide consensus in the literature about the specification of the CB policy rule, which accounts for the inflation gap, the output gap, and the NRI (and may also include the exchange rate gap in an open economy), as well as about the key parameters

[1] This is a highly reasonable assumption in the three tested economies. It is reflected, for example, in the inflation expectations for two years and longer, which are well anchored within the bounds of the inflation target through most of the tested samples.

[2] Beyer and Wieland (2019) attribute the high uncertainty of NRI estimators primarily to their high sensitivity to small changes in the econometric specification or in the data series.

in the policy rule. An additional notable advantage of estimating the NRI from the CB policy rule according to our approach is that we use observable data of two key variables, namely the expected growth of trade partners and a simple proxy for potential output growth, instead of extracting these variables from a model by using the Kalman filter, as in all the studies cited above. Thus, we consider our proposed approach of estimating the NRI as a complement, rather than a substitute, to alternative approaches existing in the literature.

Our paper differs substantially from Elkayam and Segal (2018) and Zhang et al. (2021) as it uses the original specification of the NRI as derived by Clarida et al. (2002), according to which the NRI responds both to the expected growth of domestic potential output (rather than to the expected TFP, as in Elkayam and Segal (2018) and Zhang et al. (2021)), and to the expected growth abroad. While in Clarida et al. (2002) these two specifications are identical, because in their model the NRI is affected only by productivity shocks, we contend that the original specification is more appropriate, because in practice the economy is affected by various shocks, which are better reflected by the domestic potential output than by the TFP alone.

Under complete information, which is widely assumed in standard NK models (see, e.g., Galí (2015)), the NRI included in the policy rule is the same as the NRI that affects the output gap in the IS equation. Under incomplete information, which is a more realistic assumption, the CB does not necessarily fully observe the true NRI of the economy, and therefore an estimator that is derived from the policy rule may deviate from the (unobserved) true NRI. Even if all of the structural equations are correctly specified, but the CB's policy rule erroneously reacts to the same NRI that appears in the IS equation (e.g., Zhang et al. (2021)), the estimate of the NRI could be biased. As we estimate the CB's perceived NRI from one equation only, namely, the CB's policy rule, we test the plausibility of our NRI estimate by relying on the equilibrium conditions of the IS equation within the NK framework for small open economies, as shown in Laxton et al. (2006). In this framework, the output gap is positively affected by the real exchange rate gap, and negatively affected by the monetary stance, defined as the gap between the real interest rate and the NRI. However, in contrast to the complete-information assumption mentioned above, we test the IS equation using the most up-to-date data on the output gap and the real exchange rate gap. We find robust evidence that in all three countries considered that the monetary stance, which is based on our estimate for the NRI, negatively affects the output gap. The monetary stance, together with the real exchange rate gap, explain quite well the output gap in the sample period.

We find that the estimated rules in the countries considered fit the data remarkably well, similar to the findings in Zhang et al. (2021), where their time-variant short-term NRI estimator significantly improves the fit of the model, compared to a specification with a constant NRI over time. Similarly, Elkayam and Segal (2018) show that a use of the time-variant NRI estimator fits better the BoI interest rate than other measures of the NRI (e.g., long-term real forward rates).

Because the NRI, according to our specification, is affected by the expected growth abroad directly, it is procyclical w.r.t. expected global activity: it declines when a global recession is expected and rises when a global boom is expected. For example, in Canada and in Israel, the estimated NRI declined in 2003, following the recession in the U.S. at that time, rose afterwards, dropped sharply at the outbreak of the Global Financial Crisis (GFC, hereinafter) in 2008–2009, and rose dramatically after 2009 in tandem with the recovery of the economy. The NRI estimate of Sweden shows the most dramatic drop following the GFC, as Sweden is more of an open economy than Israel and Canada, as reflected by the share of imports and exports in the GDP. The NRI in Israel and in Sweden dropped again in 2012 due to the euro sovereign debt crisis, and rose afterwards, although to a lower level than before the financial crisis. This procyclicality of the estimated NRI is similar to that in Curdia et al. (2015) and Barsky et al. (2014) for the U.S.[3]

The rest of the paper is organized as follows. Section 2 summarizes the theory of the NRI in an open economy, Section 3 describes the background of monetary policy in Israel, Sweden, and Canada, Section 4 describes the data, Section 5 presents the estimation results of the policy rules in each country, Section 6 reports the estimated NRI, Section 7 tests the plausibility of the NRI estimates using the NK framework, and Section 8 concludes.

2 The natural rate of interest in an open economy

According to Clarida et al. (2002), the "supplements" to the NRI in an open economy are the expected growth abroad and the expected growth of the (domestic) potential output. Clarida et al. (2002) and Gali and Monacelli (2005) show, using the canonical NK open economy model, that the NRI, r_t^n, is given by[4]

$$r_t^n = \gamma_y E_t \left\{ \Delta \overline{y}_{t+1} \right\} + \gamma_* E_t \left\{ \Delta y_{t+1}^* \right\}. \quad (1)$$

$E_t \left\{ \Delta \overline{y}_{t+1} \right\}$ is the expected growth of potential output ($\overline{y}$ is the domestic potential output, that is, the output that would have been obtained in a perfectly flexible-price economy). $E_t \left\{ \Delta y_{t+1}^* \right\}$ is the expected growth of (actual) foreign output. γ_y and γ_* are structural parameters, each being a function of, among other things, the degree of openness, measured by the weight of imported goods in the CPI. Equation (1) lays the groundwork for our analysis. It implies that the unobserved NRI can be estimated by using the observed expected growth abroad along with an (observed) proxy for the expected growth of domestic potential output.

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[3]Barsky et al. (2014) estimate the NRI in the U.S., using a NK model that also includes the time-variant (conditional) variance of the natural output.

[4]In Clarida et al. (2002) the dependence of potential output and natural rate on foreign output stems from an assumption of complete markets that directly links consumption in an open economy to consumption (output) abroad. However, in Galí (2015) and in practice, the dependence on foreign output stems also from the demand for exports.

However, the NRI cannot be derived from Eq. (1), because its parameters are unknown. The trick that we use is to rely on the policy rule of the CB, one of whose components is the NRI. In this way, we can estimate the RHS of Eq. (1) jointly with the parameters of the other explanatory variables in the policy rule.

The augmented Taylor rule and the natural rate of interest

The schematic policy rule is given by

$$i_t = \rho_1 i_{t-1}+\rho_2 i_{t-2}+(1-\rho_1-\rho_2)[\overline{\pi}+r_t^n+\tau_\pi \left\{[\omega\widetilde{\pi}_{t-1} + (1-\omega)E_t \left\{\widetilde{\pi}_{t+4}\right\}]\right\}+\tau_x E_{t-1}\left\{\widetilde{x}_t\right\}+\tau_q \tilde{q}_{t-1}], \tag{2}$$

where ρ_i is the interest-rate smoothing parameter,[5] and $\overline{\pi}$ is the inflation target. $\widehat{r}_t^n$ is the theory-based estimated NRI defined in Eq. (1), i.e.,

$$r_t^n = \gamma + \gamma_y E_t \left\{\Delta\overline{y}_{t+1}\right\} + \gamma_* E_t \left\{\Delta y_{t+1}^*\right\}. \tag{3}$$

τ_π is the coefficient of the inflation (environment) gap; We define the inflation (environment) gap as a deviation of the weighted average of: (1) the annual inflation rate (with weight ω), and (2) the expected inflation for the next year (with weight $1-\omega$) from the inflation target. A variable $\tilde{z}$ denotes gap. Finally, the schematic augmented Taylor rule also includes the expected output gap from the previous period,$E_{t-1}\left\{\widetilde{x}_t\right\}$, and the real exchange rate gap from the previous period, q_{t-1}.

Based on the estimated parameters in Eq. (3), we can derive the estimated natural interest rate r_t^n. Note, that Eq. (3) differs from Eq. (1) by the constant γ, because Eq. (3) is the empirical version of the theoretical Eq. (1) , which is denoted in terms of deviations from steady-state.

3 Background of monetary policy

We apply our methodology to three inflation-targeting (IT) small open economies: Canada, Israel, and Sweden.

Israel: During 1992–2000 Israel experienced a successful disinflation process, where the annual inflation rate declined from a two-digit rate to a one-digit rate, and with a target range that was set to 1–3 percent from 2003 onward. The first annual inflation target of 14–15 percent was announced at the end of 1991, along with a change of the exchange rate regime from a horizontal to an upward-sloping exchange rate band. In the next few years the inflation target was updated downward every few years. While the inflation target was first announced in 1991, the monetary interest rate became the main monetary policy tool only a few years later in 1997, when the BoI stopped intervening in the exchange rate market (for over a decade, until the GFC). In 2001–2002 the BoI sharply and unexpectedly changed the interest rate several times, leading

[5] For Israel we find that a policy rule with one lag of the interest rate fits the data better than a rule with two lags.

to severe deterioration of its credibility (for a discussion on the causes and the consequences of those surprises see BOI (2002)). Therefore, our Israeli sample starts from 2003:Q2, when credibility was regained along with price stability. The sample ends in 2019:Q4, just before the outbreak of Covid-19 worldwide.

A correct assessment of the ELB for Israel, as well as for Sweden and Canada (see below), is critical for the estimation procedure in Section 5. From 2015:Q2 to 2018:Q3 the BoI's interest rate was set at its historically lowest level of 0.1%. The long period of about four years at this level strongly supports the conjecture that the level of 0.1% was indeed the ELB of the BoI interest rate. BOI (2016) also provides support for this argument.

Sweden: Sweden's annual inflation rate had been around 2 percent for a few years when the Riksbank introduced an inflation target of 2 percent in January 1993, and began to apply it in January 1995. The target was introduced as part of a move to a floating exchange rate. The Riksbank has since become more independent by the Riksbank Act (1988:1385) as of January 1999. Our Swedish sample starts in 2006:Q1 and ends in 2019:Q4. The late starting date is due to data limitations: an internal Riksbank's time series of Sweden's international trade partners' expected growth, a significant component of the NRI according to theory, starts only in 2006:Q1. Following the GFC, the Rikskbank gradually cut the repo interest rate from 4.5% in 2008:Q3 to 0.25% in 2010:Q2. Then the repo rate was raised to 2% in 2011, but then it was reduced to its lowest negative rate (since the 1970s) of -0.5% from 2016:Q2 to 2018:Q4. The Riksbank's publications support that the level of -0.5% was not a binding ELB. For example, the press release of the Riksbank on February 2016, when the repo rate was cut to -0.5%, noted that "*There is still scope to cut the repo rate further.*" In the press release on December 2016 it was written: "*The repo rate is retained at -0.50 per cent and there is still a greater probability that the rate will be cut than that it will be raised in the near term.*" Nevertheless, Appendix A shows an NRI estimate for Sweden when one assumes that the ELB was binding, an estimate that is less likely.

Canada: Canada introduced an inflation target in February 1991; the CPI inflation target was set at 3 percent by the end of 1992, at 2.5 percent by mid-1994, and at 2 percent from the end of 1995 onward. This inflation target has been renewed by the federal government and the BoC once every few years.

Our Canadian sample starts in 1999:Q1 and ends in 2014:Q1 due to data limitations. When estimating the BoC's interest rate rule, we use real-time staff forecast data (Champagne and Sekkel (2018); BoC, 2020), which is publicly available. Some data is available only from 1999, and some data is published after a considerable lag.[6] The Bank of Canada cut the interest rate in the midst of the GFC to its lowest level of 0.25% for three quarters in 2009:Q1–2009:Q3. In its monetary policy report published on April 2009, it was noted that "*the Bank lowered its target for the overnight rate by one-quarter of a percentage point to 0.25 per cent, which the Bank judges to be the effective lower bound for that rate.*"

[6] The reference to data for Israel, Sweden, and Canada is shown in Appendix B.

4 Data

All estimated policy rules include the inflation environment gap, the components of the NRI (Equation 1), and one or two lags of the monetary interest rate.

The inflation environment is constructed using a weighted average of the deviations of two variables from the (mid-range in Israel) target $\overline{\pi}$: (1) the annual inflation rate, $\widetilde{\pi}_{t-1}$ (with weight ω), and (2) the expected inflation for the next year, $E_t\left\{\widetilde{\pi}_{t+4}\right\}$ (with weight $1-\omega$). For Canada and Israel the deviation of the expected inflation is from the previous quarter, while for Sweden it is from the current quarter.

We follow Ince and Papell (2013) and use the one-quarter-ahead one-sided Hodrick–Prescott (HP, hereinafter) filtered output to proxy the real-time expected domestic potential output in the theory-based NRI. Ince and Papell (2013) show that the one-sided HP filter on the revised GDP time series in the U.S., UK, and Germany is a very good proxy of real-time output gap (which is based on real-time domestic potential output, and therefore can be used to measure the real-time growth of potential output). As for the expected foreign output growth, for Canada we proxy it using expected growth in the U.S., which is Canada's largest trade partner: in 2019, the U.S.'s share in Canada's exports was about 75%, and 50% in imports. In particular, we use the expected U.S. real GDP growth one quarter ahead, based on the survey of professional forecasters conducted by the federal reserve of Philadelphia.[7] For Israel, we add the expected real GDP growth in the euro area one quarter ahead, based on the survey of professional forecasters conducted by the ECB.[8] The euro area and the U.S. together account for about half of Israel's exports and imports.[9] For Sweden, as mentioned above, we use the Riksbank's internal time series of expected growth of Sweden's trade partners. This series is comprised of more than 20 countries, based on Sweden's exports share to each country.

The estimated policy rule of the Bank of Israel includes the interest rate of the previous quarter. The estimated policy rules of the Riksbank and the Bank of Canada[10] include the interest rate of the previous two quarters. The inclusion of two lags of the interest rate for Sweden is in line with Lindé et al. (2009), Chappell and McGregor (2017), and Chappell and McGregor (2018). Note that the estimated sum of the two interest rate smoothing parameters in both economies is positive and below one.[11] The Bank of Canada's policy rule also includes the expected output gap from the previous period for the current period, $E_{t-1}\widetilde{x}_t$. For Sweden, Cúrdia and Finocchiaro (2005) and Jansson and

[7] Appendix B.

[8] Appendix B.

[9] UK's expected growth, third in its trade-weight between 2003 and 2019, should have also be included in Israel's proxy for the foreign growth. However, UK is omitted from Israel's proxy due to a short available data of the expected growth.

[10] We tried to include (nominal and real) housing prices as well as (nominal and real) exchange rate depreciation into Eq. (2) for all countries considered, but none of them were significant.

[11] This is in contrast to super-inertial rules where the sum is above one; see, e.g., Giannoni and Woodford (2003) and Giannoni (2014).

Vredin (2003) obtain a very small and barely significant effect of real activity on the interest rate. They emphasize that after implementing the inflation targeting regime, the Riksbank is much more concerned with inflation stability than with the real economy. In our model, once we condition on the NRI, the Swedish output gap becomes insignificant (the same is true for Israel). As we explain later, it might be that there is a problem in identifying the time-variant NRI jointly with the output gap, due both to a strong positive correlation between the (real-time) potential output growth and the (real-time) output gap and to a relatively short sample.

The Bank of Israel's policy rule also includes one lag of the real exchange rate gap $\widetilde{q}_{t-1}$(the gap between the actual and the fundamental real exchange rate, based on Frish (2016)), and the interest rate in the previous quarter (one lag).

5 Estimation results

In this section we describe the CB policy rules and NRI estimates for Canada, Israel, and Sweden.

Two econometric issues require special attention: ELB and endogeneity.

From 2015:Q2 to 2018:Q3 the BoI interest rate was set at its historically lowest level of 0.1%, which was indeed the ELB (Section 3). Given that the BoI interest rate as a dependent variable in the policy rule is bounded from below, the estimation requires a corner solution method (Tobit). In Canada, the ELB of 0.25% was reached in the midst of the GFC, but for three quarters only (2009:Q1–2009:Q3). Since this period is very short, and more importantly, it is unclear whether the ELB was indeed binding during that period, we apply a standard approach of GMM without considering the corner solution. As we explained in Section 3, there is no evidence at all that the ELB was binding in Sweden.

Endogeneity stems from the fact that we proxy for the expected growth of the potential output $E_t \left\{\Delta\overline{y}_{t+1}\right\}$ using the lead of the realized potential output growth $(\Delta\overline{y}_{t+1})$. To overcome both of these econometric issues, we estimate the BoI policy rule by IV-Tobit, which requires two stages. In the first stage, we derive the "exogenous" component of the endogenous variable $\Delta\overline{y}_{t+1}$ by regressing it on the predetermined/exogenous variables and several additional instrumental variables. In the second stage, we estimate the policy rule by a standard Tobit, where the explanatory variable $\Delta\overline{y}_{t+1}$ is now replaced by the fitted value from the first stage.

We estimate the Riksbank's policy rule by GMM, because the (corresponding) one-year inflation expectations are likely to be affected simultaneously by the monetary interest rate in the same quarter, and because of the endogeneity mentioned above. Although there is no evidence that the ELB was binding in Sweden, one can claim that the ELB was indeed binding, when the Riksbank's rate was at it's negative level of -0.5% from the second quarter of 2016 until the third quarter of 2018. Appendix A shows a sensitivity analysis of Sweden's

NRI estimate using an IV-Tobit. The policy rule for Canada is estimated by GMM since the Bank of Canada interest rate reached its lowest level of 0.25% for three quarters only and it is unclear whether this was binding level even for this very short period.

Table 1 shows the estimation results; an estimated parameter is denoted by $\widehat{\tau}$.

The estimated parameter of the inflation environment gap is greater than one in all estimated policy rules, satisfying the Taylor principle. In Israel, the estimated parameter ($\widehat{\tau}_\pi = 2.2$) is close to that obtained in Argov et al. (2012). However, in Canada and Sweden the estimated parameter is much higher (above 4) than what is usually obtained in the literature.[12,13] To test for the robustness of the parameter $\widehat{\tau}_\pi$ in Sweden we estimate a policy rule with a constant NRI and one lagged (real-time) output gap. The sample is longer, from 2000:Q3 to 2019:Q4. We obtain a significant (p-value = 0.02) $\widehat{\tau}_\pi$ of 2.9. The parameter of the output gap is close to one and is also significant (p-value = 0.1).[14] When we omit the output gap we obtain $\widehat{\tau}_\pi = 1.8$, but the fit of the rule falls considerably. Hence, we conclude that the estimated $\widehat{\tau}_\pi$ is highly dependent on the policy rule specification (in the range of 2 to 4), and that $\widehat{\tau}_\pi$ in our original specification could be plausible, conditional on the time-varying NRI.

The relative weight of past inflation in all estimated economies is low ($\widehat{\omega} \approx 0.25$), implying that monetary policy is more forward-looking than backward-looking w.r.t. inflation.

Our main interest is to estimate the parameters related to the NRI. The responses of the interest rate to the expected growth of domestic potential output and to the expected foreign output(s) are positive, as expected, and significant in all estimated economies. Note that the parameter of the expected growth of Sweden's trade partners ($\widehat{\gamma}_{y*}$) is much higher than its counterparts in Israel and Canada. When instead of using Riksbank's internal series of Sweden's trade partners, we apply the expected growth in the U.S. and the euro area, based on the survey of professional forecasters, we obtain positive parameters, around 0.8, similar to the estimated parameters in Israel and in Canada, but the former parameters are highly insignificant. One explanation could be that these two measures are poor proxies for the true and relevant expected growth of Sweden's trade partners.

The response of the Canadian interest rate to the (expected) output gap is strong ($\widehat{\tau}_x = 0.85$) and consistent with other estimated responses to real activity for Canada (Dong (2013); Nikolsko-Rzhevskyy (2011); Lam and Tkacz (2004)).

[12] For example, the estimates of Dong (2013), Nikolsko-Rzhevskyy (2011), Lam and Tkacz (2004), and Lubik and Schorfheide (2007) are in the range of 1.3 to 2.1. Champagne and Sekkel (2018) find strong responses of the Canadian interest rate to inflation and real activity after the implementation of the IT regime, but the estimated response to inflation in their model is much lower than in our model.

[13] A similar parameter for Sweden was found also in Owusu (2020) Table 5 for the sample 2003–2018.

[14] The list of the instrumental variables was slightly changed due to the change in the specification of the policy rule, and also to verify that the rank condition was satisfied.

This response is absent in Israel and Sweden, unless a constant NRI is assumed, implying that with the estimated time-varying NRI, there is no omission of the output gap in the estimated policy rules for Israel and Sweden, and hence there is no endogeneity problem and estimation bias from that respect.

It should be noted that there is a strong positive correlation between the potential output growth and the output gap (both derived with the one-sided HP filter) in the sample period in Israel and Sweden. Given the relatively short sample period for these countries, it is hard to identify the effect of the output gap along with the time-variant NRI. Nevertheless, when we compare the fit of the model in both countries, we find that the model with a time-varying NRI (excluding output gap) fits the data much better than the model with the output gap (excluding the time-varying NRI).

The Bank of Israel's interest rate responds positively and significantly to the real exchange rate gap. The significant response was expected, as the Bank of Israel's monetary committee consistently emphasizes the key role of the exchange rate on inflation and real activity. Moreover, since 2008 the Bank has intervened in the foreign exchange market in order to stabilize large deviations of the exchange rate from its fundamental level. Cúrdia and Finocchiaro (2005) included the exchange rate as a component of the Riksbank policy rule before 1992 when it had a target zone regime and the exchange rate played a significant role in policy decisions. After 1992, the exchange rate was excluded from the policy rule, as the Riksbank and the government announced a regime switch to an explicit inflation targeting (IT) and floating exchange rate. Incorporating the real exchange rate gap into the Riksbank's policy rule leads to a negative estimated coefficient, which is highly unreasonable. A negative sign for the exchange rate is also obtained in Chappell and McGregor (2017) and in Chappell and McGregor (2018). For Canada, we find that either the exchange rate effect on the monetary interest rate is insignificant (in terms of changes in the nominal exchange rate) or it has a negative sign (in terms of the real exchange rate gap). This finding is consistent with Champagne and Sekkel (2018), who show that after 1992, the estimated response to the exchange rate is close to null. A similar result is obtained by Dong (2013). Lubik and Schorfheide (2007) do find a significant and a notable effect of the exchange rate on the Canadian interest rate, but their sample includes the pre-IT period.[15]

One interesting finding obtained in several studies is the significant response of the Canadian interest rate to the Fed interest rate (Champagne and Sekkel (2018); Hofmann and Takáts (2015); Vanderhart (2003)). In our model we exclude the Fed interest rate. Although the Fed interest rate improves the fit of the Canadian policy rule, it harms the identification of the NRI. The reason is that the expected activity in the U.S. most likely affects the Fed interest rate. Therefore, once the Fed interest rate is included together with the NRI, the former absorbs, to some extent, the NRI in Canada, resulting in an NRI undervaluation.[16]

[15] A significant effect of the exchange rate in the pre-IT period was also found by Champagne and Sekkel (2018).

[16] To see this, assume that the Fed interest rate is explained by the expected growth in the

Table 1 - The estimated Augmented Taylor Rule for Israel (IV-Tobit), Sweden and Canada (GMM)[17]

Parameter/Country	IS	SW	CA
$\widehat{\tau}_\pi$	2.18^{**} (6.14)	4.05^{**} (5.24)	4.56^{**} (3.00)
$\widehat{\omega}$	0.25^{*} (2.33)	0.21^{**} (10.91)	0.24^{**} (6.53)
$\widehat{\rho}_1$	0.83^{**} (55.96)	0.81^{**} (11.86)	1.09^{**} (10.56)
$\widehat{\rho}_2$		$0.093^{\dagger}$ (1.80)	-0.21^{**} (-2.53)
$\hat{\gamma}$	-2.81^{*} (-1.92)	-0.98 (-1.36)	$-2.42^{\dagger}$ ($-1..83$)
$\widehat{\gamma}_y$	0.40^{*} (2.06)	$0.46^{\dagger}$ (1.72)	$0.77^{\dagger}$ (1.63)
$\widehat{\gamma}_{*,us}$	$0.65^{\dagger}$ (1.87)		$0.75^{\dagger}$ (1.73)
$\widehat{\gamma}_{*,eu}$	0.50^{*} (2.07)		
$\widehat{\tau}_{y*}$		1.81^{*} (2.19)	
$\widehat{\tau}_x$			0.85^{*} (1.97)
$\widehat{\tau}_q$	0.17^{**} (2.79)		
R^2	0.99	0.99	0.98
$D.W.$	1.80	1.27	1.03
$J.stat.$		5.00 (p=0.42)	5.60 (p=0.34)

* and ** indicate statistical significance at the 5 percent and 1 percent levels, respectively. † indicates statistical significance level at the 10 percent level. The value in parentheses represents the *t-value* of the estimated parameter (corrected by the Newey-West methodology). DW presents the Durbin-Watson statistic for serial correlation in residuals. $J.stat.$ is a J-statistic testing for validity of instrumental variables (below its *p-value*).

Instrumental variables for Israel: $E_t\left\{\Delta y^{*US}_{t+1}\right\}, E_t\left\{\Delta y^{*EU}_{t+1}\right\}, \widetilde{\pi}_{t-1}, E_{t-1}\widetilde{\pi}_{t+4}, \widetilde{q}_{t-1}, i_{t-1}, ygap_{t-1}, \Delta y^W_{t-1}, \Delta y^W_{t-2}, \Delta y^W_{t-3}, \Delta y^{imp}_t$, and a constant.

Instrumental variables for Sweden: $E_t\left\{\Delta y^{*}_{t+1}\right\}, \widetilde{\pi}_{t-1}, E_{t-1}\widetilde{\pi}_{t+4}, E_{t-2}\widetilde{\pi}_{t+4}, i_{t-1}, i_{t-2}, \Delta y^W_t, \Delta y^W_{t-1}, \Delta y^W_{t-2}, \Delta y^W_{t-3}, \Delta^H rp_{t-1}$, and a constant.

Instrumental variables for Canada: $E_t\left\{\Delta y^{*US}_{t+1}\right\}, \widetilde{\pi}_{t-1}, E_{t-1}\widetilde{\pi}_{t+4}, E_{t-1}\widetilde{x}_t, i_{t-1}, i_{t-2}$,

U.S., $E_t\Delta y^{US}_{t+1}$, and other variables, Z_t, $i^{FED}_t = \theta_1 E_t \Delta y^{US}_{t+1} + \theta_2 Z_t$. According to Clarida et al. (2002) (Eq. (1)), the NRI for Canada is given by $r^n_t = \alpha E_t\left\{\Delta\overline{y}_{t+1}\right\} + \beta E_t\left\{\Delta y^{US}_{t+1}\right\}$ (for simplicity assume that the U.S. is the only trade partner of Canada). If we insert i^{FED}_t into the NRI for Canada we obtain $r^n_t = \tilde{\alpha} E_t\left\{\Delta\overline{y}_{t+1}\right\} + \tilde{\beta} E_t\left\{\Delta y^{US}_{t+1}\right\} + \tilde{\gamma}\underbrace{\{\theta_1 E_t \Delta y^{US}_{t+1} + \theta_2 Z_t\}}_{i^{FED}_t}$.

It can be seen that the estimated parameter $\tilde{\beta}$ is equal to $\beta - \tilde{\gamma}\theta_1$, which biases the NRI estimator downward.

[17] For Israel the sample is 2003:Q2-2019:Q4, for Sweden the sample is 2006:Q1-2019:Q4., and for Canada the sample is 1999:Q3-2014:Q1.

$\Delta y_t^W, \Delta y_{t-1}^W, \Delta y_{t-2}^W, \Delta y_{t-3}^W, \Delta rer_{t-1}, \Delta^H rp_{t-1}$, and a constant.[18]

6 Evolution of the estimated NRI in Israel, Sweden, and Canada

This section examines the estimates of the NRI and the implied monetary stance in Israel, Canada, and Sweden. The monetary stance (MS) is an indicator of the degree of policy accommodation or tightness with respect to real activity. It is defined as the gap between the CB's real interest rate and the NRI, $MS_t = r_t - NRI_t$, where $r_t = i_t^{CB} - E_t\pi_{t+1}$ (the CB's nominal interest rate, i_t^{CB}, minus the expected inflation, $E_t\pi_{t+1}$). According to the NK dynamic IS equation, a negative MS means that the monetary policy is accommodative, i.e., it leads to a higher output gap, *ceteris paribus*, whereas a positive MS means that the monetary policy is contractionary. Note that after the GFC, all three countries took unconventional policy measures that went beyond the conventional policy on the CB's interest rate: all three CBs applied forward guidance, the BoI intervened in the foreign exchange market, and the Riksbank also applied quantitative easing programs. To the extent that these policies were effective, their effect is reflected in the monetary stance (via inflation expectations and expected real activity). Thus, even when the CB conducts an unconventional monetary policy, the measured monetary stance is a good indicator of the degree of policy accommodation or tightness. Indeed, Section 7 shows that the measured MS, which also covers the post-GFC period, behaves in accordance with the dynamic NK output gap equation.

Figure 1 shows the dynamics of the estimated NRI in Israel, Canada, and Sweden. It is evident that there is high comovement among the NRI estimates: the correlation among the NRI in Israel, Sweden, and Canada is very high – about 0.8. This high comovement in the three countries' NRI estimates, which is emphasized by Holston et al. (2017), suggests an important role for "*global factors affecting real interest rates.*" In our specification, following the model of Clarida et al. (2002), the global factor is reflected in the economy's trade partners' expected growth, which affects the NRI in a small open economy directly and indirectly, through real marginal costs, and affects the domestic potential output (see Eq. (1)). Hence, our NRI estimates, by construction, are procyclical: in most of our sample, except for the GFC, the estimated NRI in the three examined economies is positive, reflecting positive expected growth worldwide. During the GFC, the NRIs declined sharply into negative

[18] The additional instrumental variables were verified as having strong and significant effect on the endogenous variables. Therefore, the rank condition is satisfied for each country. The variables are: $ygap$ –the output gap derived from a one-sided HP filter; Δy^W– output growth in the G4 countries (which, based on calculations of the BOI reseach department, is a proxy for output growth in developed countries); Δy^{imp} – growth of imports in OECD countries; $\Delta^H rp$ – the change in home prices (in real terms); Δrer – the real depreciation of the Canadian dollar.

territory for a relatively short period, after which they rose. The Swedish NRI shows the sharpest decline during the GFC. This sharp decline stems from two factors: (1) during the GFC the decline in the expected growth of Sweden's trade partners was much larger than the decline in the expected growth in the U.S. and the euro area; (2) the estimated response to the expected growth of the trade partners is much higher in Sweden than in Israel and Canada. This result is plausible, as Sweden is a more open economy than Canada and Israel, as reflected in the ratio of trade volume to GDP. If we compare our NRI estimate for Sweden with Armelius et al.'s (2018) estimate, we find a sizable difference between them during the GFC and afterwards. During the GFC their NRI estimate declined to -0.5%, in contrast to our estimate of a record low -18%, . After the GFC, the NRI in Armelius et al. (2018) continued the negative trend and reached -2% at the end of 2016 (end of sample). By contrast, according to our estimate, after the GFC the NRI rose to positive values of around 1% on average. The difference between the two estimates comes mainly from the different evolution of the global factors in each model. In our model the global factors are observable variables and represent the expected growth of the trade partners, which was very negative during the GFC and positive afterwards. In Armelius et al. (2018) the global factors are unobservable variables and are interpreted economically outside the model. According to Armelius et al. (2018), these estimated global factors were moderately negative during the GFC, and even more strongly negative afterwards.

The estimated NRI in Israel declined in 2003, following the recession in the U.S. at that time, rose afterwards, dropped sharply at the outbreak of the GFC, and rose again after 2009. The estimated NRI in all three countries rose significantly in the second half of 2009 and in 2010, but dropped again in 2012 after the euro area was hit by the sovereign debt crisis (into mildly negative territory in Israel and Sweden and into low positive territory in Canada), rose mildly until 2018, and dropped again toward zero in 2019.

Figures 2–4 show the estimated NRI and the corresponding MS in the three countries.[19] In the midst of the GFC the MS jumped upward for a relatively short period due to the sharp and unanticipated decline in the NRI. The latter was a result of the drop in both expected domestic (potential) output and foreign (actual) output. But since then, in most periods, monetary policy in all three countries has been accommodative (negative MS), reflecting the CBs' actions to strengthen real activity and to stabilize inflation. A similar evolution of the MS during and after the GFC is found by Arena et al. (2020) (see Figure 7 there), who estimate the NRI for several countries in the euro area, using an extended model of Laubach and Williams (2003), where they add the growth of trade partners as one of the NRI's components.[20] Carter et al. (2019) evaluated the NRI in Canada using several models. Among these models, the "reduced-form

[19] We were able to calculate the MS for Canada only up to 2013:Q4, due to the limited availability of data of inflation expectations.

[20] Arena et al. (2020) modeled the (unobserved) wedge between the NRI and the growth of potential output in Laubach and Williams (2003) using an AR(2) process and several economic variables. However, their approach is ad hoc and does not rely on economic theory.

model" is similar to our model in the sense that in both models the NRI reacts to growth of domestic potential output. But in contrast to our model, the second component of the NRI in Carter et al. (2019) is the NRI abroad. Looking at the range of NRI estimates in 2018 and 2019 (Table 1 in Carter et al. (2019)), we find that our estimates of 3.3 and 2.4 percent for those years fall within their range.[21] A comparison of our NRI estimates for Israel with those of Zhang et al. (2021) indicates that they are similar, with a correlation of 0.8. Our NRI estimates for Canada are also similar to theirs, with a similar correlation of 0.8, although the estimates in Zhang et al. (2021) are lower than our estimates after the GFC. However, our NRI estimates for Sweden are significantly different from theirs, especially in the GFC and afterwards – with a correlation of 0.5 throughout the tested sample. Specifically, the drop in the GFC in our estimates is significantly sharper – to about -18% at its nadir- - while Zhang et al.'s (2021) estimates drop to only 0%. Another difference is in the dynamics of the estimates after the GFC: from 2011 estimates rose monotonically, while Zhang et al.'s (2021) estimates declined slightly in the second half of 2015. Nevertheless, in all three countries, the implied MS from our estimators and from Zhang et al.'s (2021) estimators is similar in sign at least 85% of the time, including in Sweden.

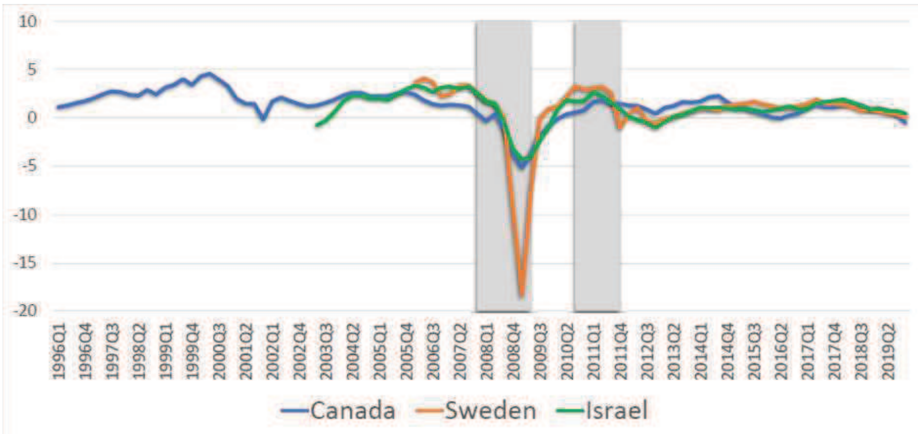

Figure 1 - Estimated NRI in Israel, Sweden and Canada.

* The shaded gray area relates to the GFC and the Europe sovereign debt crisis, respectively.

[21] For the years 2018 and 2019 we obtain an average real NRI of 1.3 and 0.4 percent, respectively. To convert the NRI to nominal terms as in Table 1, we add an inflation target of 2 percent.

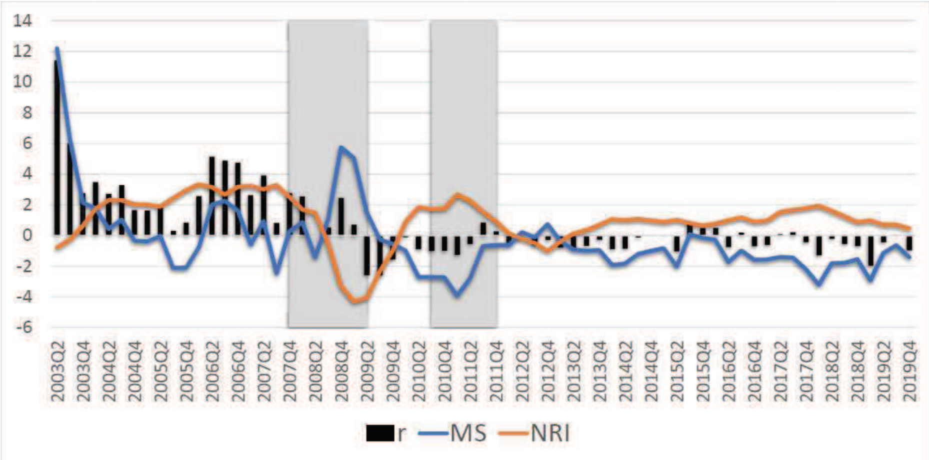

Figure 2 - CB real interest rate, estimated NRI and the implied monetary stance in Israel, 2003:Q2-2019:Q4.

* The shaded gray area relates to the GFC and the Europe sovereign debt crisis, respectively.

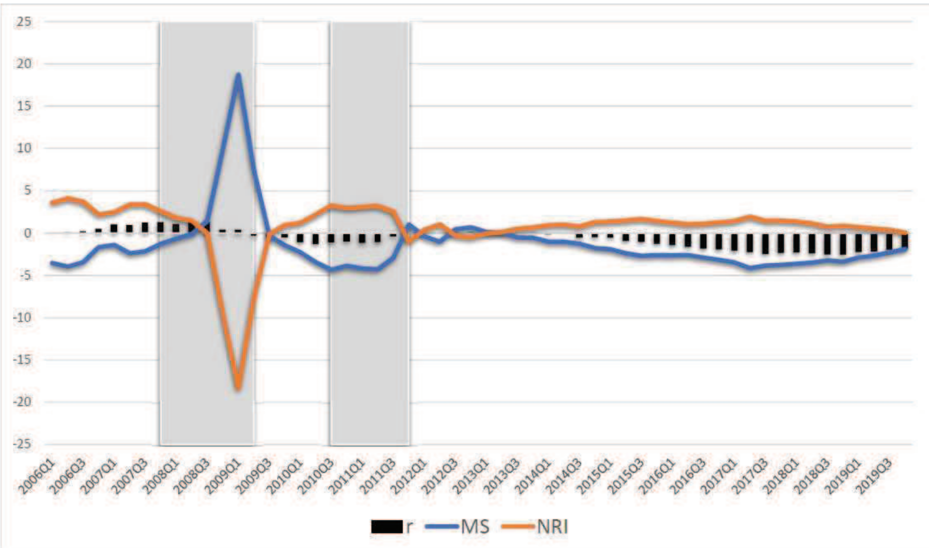

Figure 3 - CB real interest rate, estimated NRI and the implied monetary stance in Sweden, 2006:Q1-2019:Q4.

* The shaded gray area relates to the GFC and the Europe sovereign debt crisis, respectively.

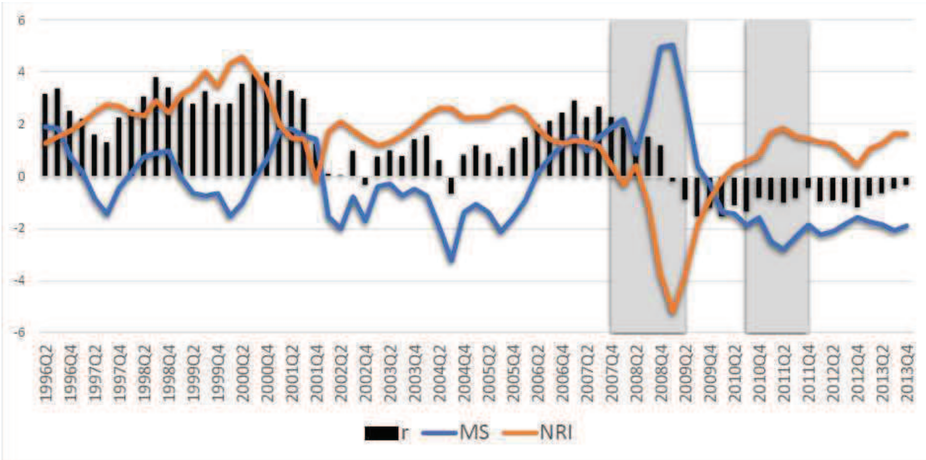

Figure 4- CB real interest rate, estimated NRI and the implied monetary stance in Canada, 1996:Q2-2013:Q4.

* The shaded gray area relates to the GFC and the Europe sovereign debt crisis, respectively.

7 Testing for plausibility of the NRI estimators

The significant responses of the CB interest rates to the expected domestic potential output growth and to the expected growth of the economy's trade partners (Section 5) do not necessarily imply that these variables, combined with the estimated elasticities, yield a good estimate of the NRI in the economy. The NRI estimator that is derived from the policy rule reflects an assessed CB-perceived NRI, which may deviate significantly from the "true" NRI in the economy. Alternatively, it may reflect the demand and/or supply sides of the economy that are taken into account by the CB, which have nothing in common with the true NRI. To test whether the estimated NRI is a good proxy for the NRI in the economy, some criteria should be satisfied. Clarida et al. (2002) show that in an open economy, the output gap is affected by the next-period expected output gap and by the monetary stance (the IS equation in the NK model). In practice, in many models the real exchange rate gap has an impact on real activity as well. For example, Laxton et al. (2006) propose a practical model for monetary policy analysis within the NK framework, where both monetary stance and (real) exchange rate gap affect the output gap dynamics captured in their IS equation. The exchange rate gap is absent from Clarida et al.'s (2002) IS equation, because they assume the law of one price, implying that the CPI-based real exchange rate is always equal to one. We adopt Laxton et al.'s (2006) IS equation to test the plausibility of the NRI estimates. For robustness, we consider two other IS-type equations. Due to the data limitations in the countries considered (see Section 4), the tests presented below for the NRI estimates are based on the sample 2003:Q3–2019:Q4 for Israel, 2006:Q2–2019:Q4 for Sweden, and, 1996:Q3–2014:Q1 for Canada.

We start with the original IS equation in Laxton et al. (2006):

$$\widetilde{y}_t = \delta E_t \widetilde{y}_{t+1} + \eta \widetilde{y}_{t-1} + \alpha MS_{t-1} + \beta \ \widetilde{q}_{t-1} + \varepsilon_t, \tag{4}$$

where the first two expressions on the RHS of Eq. (4) present the expected and lagged output gap, respectively. The third term is the monetary stance, $MS_t \equiv ri_t - \widehat{r}_t^n$, which is calculated using the estimator of the NRI ($\widehat{r}_t^n$). $\widetilde{q}_{t-1}$ is the lagged real exchange rate gap. For simplicity, in Eq. (4) we estimate only the parameters η, α, and β; the parameter δ is calibrated to values in the range of 0.05 to 0.2, as proposed by Laxton et al. (2006).

A second version of the IS equation is the same as Eq. (4) but with $\delta = 0$. This specification is similar to Holston et al. (2017) and Armelius et al. (2018), in the sense that the expected output gap is omitted, i.e.,

$$\widetilde{y}_t = c + \eta \widetilde{y}_{t-1} + \alpha MS_{t-1} + \beta \ \widetilde{q}_{t-1} + \varepsilon_t. \tag{5}$$

A third version of the IS equation relates the output gap ($\widetilde{y}_t$) to a moving average of the monetary stance and of the real exchange rate gaps in the past:

$$\widetilde{y}_t = c + \alpha \frac{\sum_{j=1}^{J} MS_{t-j}}{J} + \beta \frac{\sum_{p=1}^{P} \widetilde{q}_{t-p}}{P} + \varepsilon_t. \tag{6}$$

The lags J and P were chosen to maximize the (adjusted) $\overline{R}^2$ of Eq. (6).[22] Here the parameters α and β capture the average cumulative effect of MS and $\widetilde{q}$ on the output gap.

To test the NRI estimator according to Eq. (4) we should use the best estimate of the actual output gap, rather than the real-time (or quasi-real time) estimate, which is used in the estimation of the CB policy rules. Therefore, in this section we use the most updated data on the output gap and on the real exchange rate gap time series constructed within each CB, where such series are available (see Appendix B).

Table 2 presents the estimation results of Eqs. (4)–(6) for Israel, Sweden, and Canada. Our main interest is the sign (and significance) of the monetary stance, as implied by the NRI estimate. From a bird's-eye view, for all countries and for all specifications of the IS equation the sign of the MS is negative (in line with the NK theory) and highly significant. In particular, the size of the estimated parameter of the MS as well as other parameters in Eq. (4) are similar to the parameters in Laxton et al. (2006). However, one could reasonably suspect that the negative sign obtained for the MS reflects a negative effect of the CB real interest rate only on the output gap, and that the MS was constructed using a poor estimator of the NRI. To refute this possibility, we reestimate Eqs.

[22] For Israel $J = 10, P = 8$, for Sweden $J = 8, P = 8$, for Canada $J = 14, P = 6$.

(4)–(6), but now we split the MS into its two components, r and NRI, each having separate parameters. If the NRI estimator is poor, its parameter will be insignificant. The estimation results show that for all specifications of the IS equation and for all three countries considered, the parameter of the NRI is positive and highly significant, as expected, and in most cases is of the same order of magnitude as the (negative) parameter of r. These findings provide strong support that the NRI estimate for each country in Section (5) is a good estimator of the true NRI.

Equations (4) and (5) also show that the effect of the exchange rate gap is positive, as expected, but small (and significant in most versions of the IS equation) once we control for the effect of the lagged output gap. When the lagged output gap is omitted (see Eq. (6)), the average cumulative effect of the exchange rate gap and of the monetary stance becomes much bigger, as expected.

Table 2 - Estimated effect of monetary stance and real exchange rate gap on the output gap in Israel, Sweden and Canada.

Parameter	$Eq.(4)$			$Eq.(5)$			$Eq.(6)$		
	IS	SW	CA	IS	SW	CA	IS	SW	CA
δ[23]	0.05	0.05	0.05						
c				0.17 (1.45)	-0.24^{*} (-1.95)	-0.03 (-0.34)	1.31^{**} (3.36)	-1.16^{**} (-3.36)	0.05 (-2.54)
α	-0.09^{**} (-3.08)	-0.10^{**} (-3.42)	-0.11^{**} (-2.64)	-0.10^{**} (-3.40)	-0.15^{**} (-4.20)	-0.12^{**} (-2.56)	-0.59^{**} (-3.23)	-0.56^{**} (-4.13)	$-0.49^{\dagger}$ (-2.54)
β	$0.01^{\dagger}$ (1.42)	$0.03^{\dagger}$ (1.47)	$0.03^{\dagger}$ (1.54)	0.03^{*} (2.20)	0.02 (1.19)	$0.03^{\dagger}$ (1.59)	0.29^{**} (3.12)	0.19^{**} (2.77)	0.10✠ (1.66)
η	0.75^{**} (11.32)	0.82^{**} (16.92)	0.83^{**} (17.65)	0.79^{**} (11.75)	0.82^{**} (15.60)	0.86^{**} (16.13)			
$\overline{R}^2$	0.88	0.89	0.86	0.87	0.88	0.84	0.41	0.56	0.29
DW[24]	1.70	1.74	1.35	1.72	1.75	1.27	0.50	0.43	0.25

$**$ and $*$ indicate statistical significance at the 1 percent and 5 percent levels, respectively. ✠ and † indicate statistical significance at the 10 percent and 15 percent levels, respectively. The value in parentheses represents the *t-value* of the estimated parameter (corrected using the Newey-West methodology). DW presents the Durbin-Watson statistic for serial correlation in residuals.

[23] Similar parameters are obtained for various values of δ in the range of 0.05–0.2, as proposed by Laxton et al. (2006). We also estimated Eq. (4) for Canada with the CB's staff forecast expectations for the output gap. We obtained $\delta = 0.16$ (*pvalue*=0.01). The rest of the parameters are similar to Table 2. The parameter β in Eqs. (4) and (5) for Israel refers to $\widetilde{q}_{t-2}$, whereas for Sweden and Canada it refers to $\widetilde{q}_{t-1}$.

[24] The low values of DW in Eq. (6) stem from overlapping of the explanatory variables due MA specification, and do not reflect serial correlation.

8 Conclusions

We propose a simple method to estimate the short-run NRI for small open economies and we believe that our approach can be complementary to existing methods. Instead of building and estimating a multi-equational model to derive the NRI, we propose to estimate it from a single equation: the central bank policy rule in which the NRI is included. We rely on the new-Keynesian theory, where the NRI in small open economies is compounded by the expected domestic potential output and the expected growth of the economy's trade partners. Hence, we exploit a simple proxy for the former and observable data for the latter. We apply our methodology to Israel, Sweden, and Canada, and find strong comovement between the NRI estimates in these countries. The source of the comovement comes primarily from the mutual dependence of these countries on the expected growth of their trade partners (large economies).

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Appendix A - Estimating a policy rule and NRI for Sweden if the ELB was binding

In Section 3, we asserted that there is no evidence that the interest rate of -0.5% in Sweden from 2016:Q2 to 2018:Q4 was at its ELB. Nevertheless, one can claim that the ELB was indeed binding. Here we examine how the estimated policy rule and the NRI in Sweden are changed if we assume that -0.5% was the ELB. We estimate the policy rule by IV-Tobit exactly the same way as for Israel (Section 5). In the first stage, we derive the "exogenous" component of the endogenous variables–the expected inflation expectations and the lead of potential output growth–by regressing them on the instrumental variables listed in Table 1. In the second stage, we estimate the policy rule by a standard Tobit, where the explanatory endogenous variables are replaced by the fitted values from the first stage.

Initially, we tried to estimate the following equation (by Tobit) as it appears in Table 1:

$$\begin{aligned} i_t &= \widehat{\rho}_1 i_{t-1} + \widehat{\rho}_2 i_{t-2} + (1-\widehat{\rho}_1-\widehat{\rho}_2)[\overline{\pi} + \hat{\gamma} + \widehat{\gamma}_y \hat{E}_t \left\{\Delta\overline{y}_{t+1}\right\} + \widehat{\tau}_{y*} E_t \left\{\Delta y^*_{t+1}\right\} \\ &\quad + \widehat{\tau}_\pi \left\{ \left[\widehat{\omega}\widetilde{\pi}_{t-1} + (1-\widehat{\omega})\hat{E}_t \left\{\widetilde{\pi}_{t+4}\right\} \right] \right\}], \end{aligned} \tag{7}$$

where $\hat{E}_t \left\{\Delta\overline{y}_{t+1}\right\}$ and $\hat{E}_t \left\{\widetilde{\pi}_{t+4}\right\}$ are the fitted values from the first stage. We obtained $\widehat{\rho}_1$ very close to one and $\widehat{\rho}_2$ close to null (and insignificant). This means that the main component of the policy rule is eliminated because $1-\widehat{\rho}_1-\widehat{\rho}_2$ is close to null, and therefore the parameters in the "Taylor-part" of the policy rule cannot by identified. This indicates that an alternative policy rule should be considered, a first-difference rule. The first-difference rule in the spirit of Cochrane et al. (2019) (Table 1 there), where the NRI is also included, is a good candidate.

$$i_t = i_{t-1} + r^n_t + \widehat{\tau}_\pi \left\{ \left[\widehat{\omega}\widetilde{\pi}_{t-1} + (1-\widehat{\omega})\hat{E}_t \left\{\widetilde{\pi}_{t+4}\right\} \right] \right\}], \tag{8}$$

where $r_t^n = \gamma + \gamma_y E_t \left\{ \Delta \overline{y}_{t+1} \right\} + \tau_{y*} E_t \left\{ \Delta y^*_{t+1} \right\}$. In Eq. (8) the parameter on the lagged interest rate is equal to one , the "first-difference" part, and there is no restriction on the "Taylor-part" of the policy rule. The estimation results of Eq. (8) are shown in Table A1. All parameters are highly significant and the parameters of the NRI are much smaller than in Table 1 in Section 5. The parameter of the inflation environment gap is 0.21, while it is close to 0.5 in Cochrane et al. (2019) for the FED.

Table A1- The estimated first-difference Taylor Rule for Sweden (IV-Tobit)

Parameter/Country	SW
$\widehat{\tau}_\pi$	0.205^* (6.08)
$\widehat{\omega}$	0.28^* (7.24)
$\hat{\gamma}$	-0.12^* (-3.96)
$\widehat{\gamma}_y$	0.033^* (2.67)
$\widehat{\tau}_{y*}$	0.196^* (13.91)
R^2	0.99
$D.W.$	1.99

* indicates statistical significance at the 1 percent levels. The value in parentheses represents the *t-value* of the estimated parameter. DW presents the Durbin-Watson statistic for serial correlation in residuals.

Figure A1 shows the NRI and the implied MS for Sweden based on Eq. (8). We can see that the NRI is much more stable compared to the estimated NRI in Figure 3: during the GFC the NRI fell to -2% and after that was close to null. According to this estimate, the monetary policy was accommodative since the GFC as reflected in the negative MS.

The next step is to test the plausibility of the NRI estimate by testing the implied MS along the real exchange rate gap in Eqs. (4)–(6). The estimation results are shown in Table A2. While the MS is always negative and significant as expected, the real exchange rate gap is negative in the first two specifications of the IS equation and positive only in the third specification, but it is highly insignificant in all three specifications. Therefore, it seems that the IS equation with the alternative NRI estimator (derived from the first-difference Taylor rule, following the assumption that the ELB of -0.5% was binding) completely eliminates the effect of the real exchange rate gap on the output gap, a result that sheds considerable doubt on the plausibility of the alternative NRI estimator.

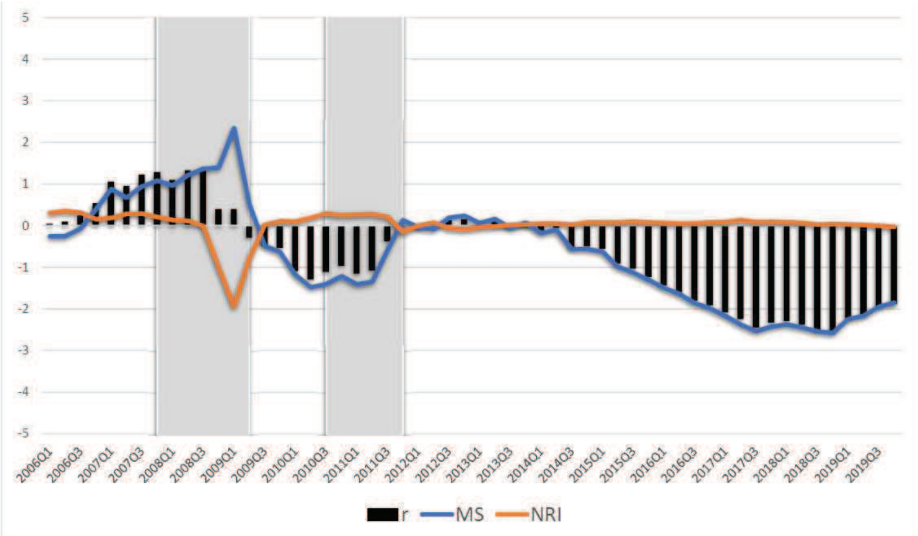

Figure A1 - CB real interest rate, estimated NRI and the implied monetary stance in Sweden based on the alternative rule, 2006:Q1-2019:Q4.

Table A2 - Estimated effect of monetary stance and real exchange rate gap on the output gap in Sweden.

Parameter	$Eq.(4)$	$Eq.(5)$	$Eq.(6)$
	SW	SW	SW
δ	0.05		
c		-0.28 (-1.12)	-1.49^{**} (-2.60)
α	$-0.191^{\dagger}$ (-1.85)	-0.325^{*} (-2.04)	-1.21^{**} (-2.85)
β	-0.007 (-0.38)	-0.022 (-1.31)	0.02 (0.22)
η	0.87^{**} (16.64)	0.89^{**} (11.83)	
$\overline{R}^2$	0.88	0.87	0.39
DW[25]	1.41	1.40	0.31

$**$ and $*$ indicate statistical significance at the 1 percent and 5 percent levels, respectively. † indicates statistical significance at the 10 percent level. The value in parentheses represents the *t-value* of the estimated parameter (corrected using the Newey-West methodology). DW presents the Durbin-Watson statistic for serial correlation in residuals.

[25] The low values of DW in Eq. (6) stem from overlapping of the explanatory variables due MA specification, and do not reflect serial correlation.