Bank of Israel

Research Department

Estimation of a Macroeconomic Model for the Israeli Economy

Yaakov Chen Zion*

Discussion Paper 2021.22
December 2021

Bank of Israel– https://www.boi.org.il

* Bank of Israel, the Research Department – Yaakov.Chen-Zion@boi.org.il
Thanks to Alon Binyamini, Sigal Rivon, Itamar Caspi, Alex Ilek, Jonatan Benchimol and the participants of the seminar of the research division of the Bank of Israel. All remaining errors are my responsibility.

חטיבת המחקר, בנק ישראל ת"ד 780 ירושלים 91007
Research Department, Bank of Israel. POB 780, 91007 Jerusalem, Israel

אמידת מודל מקרו-כלכלי למשק הישראלי

יעקב חן ציון

תקציר

עבודה זו מציגה ניסוח ואמידה של מודל מקרו-כלכלי קטן למשק קטן ופתוח, וכן לכלכלה העולמית המשפיעה עליו. מודל זה, מסוג DSGE, מאפשר הערכה וניתוח כולל של מצב המשק לאורך מחזור העסקים, וכן של המגמות ארוכות הטווח. המודל נאמד באמצעות שילוב של אמידה בייסיאנית וקליברציה. מתוצאות האמידה עולה שהעשור האחרון בכלכלה הישראלית מאופיין בזעזועי היצע חיוביים, המתבטאים בשילוב של פעילות גבוהה לצד אינפלציה נמוכה מיעדה ומדיניות מוניטרית מרחיבה. הייסוף בשער החליפין הריאלי, המאפיין את 15 השנים האחרונות, נובע, על פי המודל בעיקר מתהליכים מבניים ארוכי טווח. במסגרת העבודה נותחה התפתחות יעד האינפלציה המשתמע - אשר משקף את האופן בו נתפס יעד האינפלציה של הבנק המרכזי על ידי הציבור. במסגרת הניתוח, נמצא שלא ניתן לייחס חלק משמעותי מהתפתחות האינפלציה לשינויים ביעד בחמש השנים האחרונות. באמצעות המודל נדונה ההשפעה של זעזוע גדול על פירוש ההתפתחויות ההיסטוריות, ומוצגת הצעה להתערבות שיפוטית (שיקול דעת), שמתבססת על מידע שמחוץ למודל, ומאפשרת ניתוח היסטורי תוך הימנעות מהשפעת יתר של הפתעות. מניתוח כזה עולה שעם פרוץ מגפת קוביד-19 בתחילת שנת 2020, ירדו שיעור הריבית הטבעית ושיעור הצמיחה של התוצר הפוטנציאלי ירידה חדה.

Estimation of a Macroeconomic Model for the Israeli Economy

Yaakov Chen Zion

Abstract

This paper presents a small macroeconomic model for a small and open economy, which allows for an overall assessment and analysis of the state of the economy throughout the business cycle, as well as of its long-term trends. The model is estimated using a combination of Bayesian estimation together with calibration. The results of the estimation reveal that positive supply shocks have characterized the Israeli economy over the last decade, which are reflected in elevated activity, below-target inflation, and an expansionary monetary policy. According to the model, the appreciation in the real exchange rate over the last 15 years stems mainly from long-term structural processes. The development of the implied inflation target was analyzed - which reflects how the central bank's inflation target is perceived by the public. As part of this analysis, it was found that a significant part of inflation development cannot be attributed to changes in the target over the course of the past five years. Using this model, we discuss how a major shock can impact historical developments and present a proposal for judgmental intervention (discretion), which is based on information that exists outside of the model and allows for historical analysis while avoiding the amplified influence of surprises. Such an analysis demonstrates that with the outbreak of Covid-19 epidemic in early 2020, both natural rate of interest and growth rate of potential output decreased sharply

1. Introduction and Literature Review

In this work, a small DSGE (Dynamic Stochastic General Equilibrium) model is estimated for the Israeli economy. QPM (Quarterly Projection Model), is a semi-structural model for a small and open economy and the rest of the world (which is exogenous to the small economy)[1]. The advantages of using this type of semi-structural model include the ability to analyze the significant economic mechanisms while maintaining simplicity and minimalism, which facilitate the analysis. Using this type of model also allows the integration of additional mechanisms in a relatively simple and quick way, without forcing the model to be in line with microeconomic fundamentals. On the other hand, the disadvantage of these models compared to structural models is that the formulation of the equations is not based on explicit optimization which rest on the basis of microeconomic principles, and therefore some difficulty occurs from time to time in providing a structural interpretation of the various parameters and shocks.

An estimation of the development of the economy until the end of 2019 using the presented QPM model shows that the low inflation of the last decade, along with low interest rates and relatively high growth, is mainly due to positive supply shocks, which coincide with the increase in the level of the competition in the economy. Additional factors for low inflation are found in developments in the global economy, which made a negative contribution to local inflation, starting with the great financial crisis in 2008. The model also shows that the output in recent years is higher than its trend, which is reflected in a positive output gap. Added to this is a prolonged process of appreciation of the real exchange rate, which mostly results from a long-term appreciation trend that expresses structural processes that do not arise from the business cycle, while the level of the real exchange rate is for most of the period above its trend.

The breakdown in the model between the trend and the deviation from it is ultimately an estimate. Every estimate has an error, and the potential for error increases as the shocks that hit the economy are stronger and generate noise in the system. Such a shock can be seen in the surprising crisis of Covid-19. In part 6, an illustration of its effect on the estimation of historical trends is presented while using the Kalman smoother, and an intervention of a judgment is proposed that responds to this development, thus enabling a continuous analysis, which also includes the year 2020 without the shocks of that year affecting the depiction of the economic developments in 2019.

[1] Quarterly projection model.

Part 7 examines with the help of the model the "implied inflation target" according to which the interest rate rule aims at a target that is slightly different from the center of the official target. According to the results, there has been in the years 2014-2019 a slight decrease in the implied inflation target, but it did not have a noticeable effect on the development of actual inflation.

The model presented in this work is based on the work of Berg, Karam & Laxton (2006) (hereafter BKL). Similar models based on this work are used for forecasting and policy analysis in various central banks in the world: the Central Bank of India, as presented in the work Benes et al. (2017), the South African Reserve Bank (SARB) as presented in the work Botha, Jager, Ruch & Steinbach (2017) and the Czech National Bank (CNB) as presented in the work Benes et al. (2003). The model, whose equations were formulated ad hoc, is a relatively simple one compared to structural models that were specifically developed from micro-elements, such as the MOISE model[2], which is used to construct the forecast of the research division at the Bank of Israel[3].

When applying a theoretical model in an empirical way, a match between the observed data and the model variables is required. Specifically, business cycle models - both real (RBC) and neo-Keynesian – describe the dynamics of the deviations from the trends. Each of the observed data actually reflects the sum of two unobservable components - a trend and a deviation from it. The deviation embodies the business cycle component, and the trend represents long-term processes. There are different approaches to separate these two unobservable components. In the present work, a multivariate approach was taken: the original observed variables are introduced to the model, and within it the two components are separated, using the information inherent in the joint development of all the data as well as in the model itself. In other words: a filtering based on a model is done here, similar to the works of Argov et al. (2012), and Canova (2008); This is different from the work of BKL, who took a univariate approach and separated the trend from the observed variable while using a filter similar to the Hodrick-Prescott filter[4].

2 Model for the Israeli Economy Argov et al (2012)

3 The structure of the business cycle in the model is based on guidelines similar to those used by the model in the work of Argov & Elkayam (2010), which was used for the forecast of the Bank of Israel until 2010. However, there are also significant differences between the models. One important difference is the method of estimation: in Argov & Elkayam (2010) it was done with the Gaussian Mixture Model (GMM) approach for each equation separately, whereas in this work a set of equations was estimated.

4 Another important difference from the work of Argov & Elkayam (2010) is reflected in the decomposition of the observed variables into trends and deviations from the trends - while Argov & Elkayam (2010) performed a preliminary screening with a univariate approach (as BKL also did), the decomposition here is done within the framework of the model.

The model was estimated using a combination of calibration for the parameters of the equations, describing the long-term trends and Bayesian estimation for the parameters in the equations, which are describing the development of the business cycle.

The analysis in the last part of this work joins additional works that were required to adjust the models and exercise a judgment after the Covid-19 crisis. Lenza & Primiceri (2020) proposed to separate the estimation of the parameters, from which they omitted the abnormal observations, and the forecast, in which the change in volatility during the crisis period was taken into account. Foroni, Marcellino & Stevanović (2020) suggested using a similarity to the forecasts made at the time of the great financial crisis to adapt the forecasts to the current crisis. Primiceri & Tambalotti (2020), by which the proposal in this work is adjacent to their proposal, suggested the basing of the forecast on the assumption that the change in the observed variables at the beginning of 2020 is based on one single shock - the "Covid-19 shock".

The next chapter, Chapter 2, introduces the model. In chapter 3 the estimation is presented. Chapter 4 analyzes the local economic history in the light of the estimated model. Chapter 5 deals with the nature of the model's forecast. Chapter 6 presents a proposal for the intervention of a judgement within the framework of the use of the model for the historical interpretation of economic developments. Chapter 7 presents the development of the implied inflation target in 2020 after the outbreak of the Covid-19 crisis. Chapter 8 concludes.

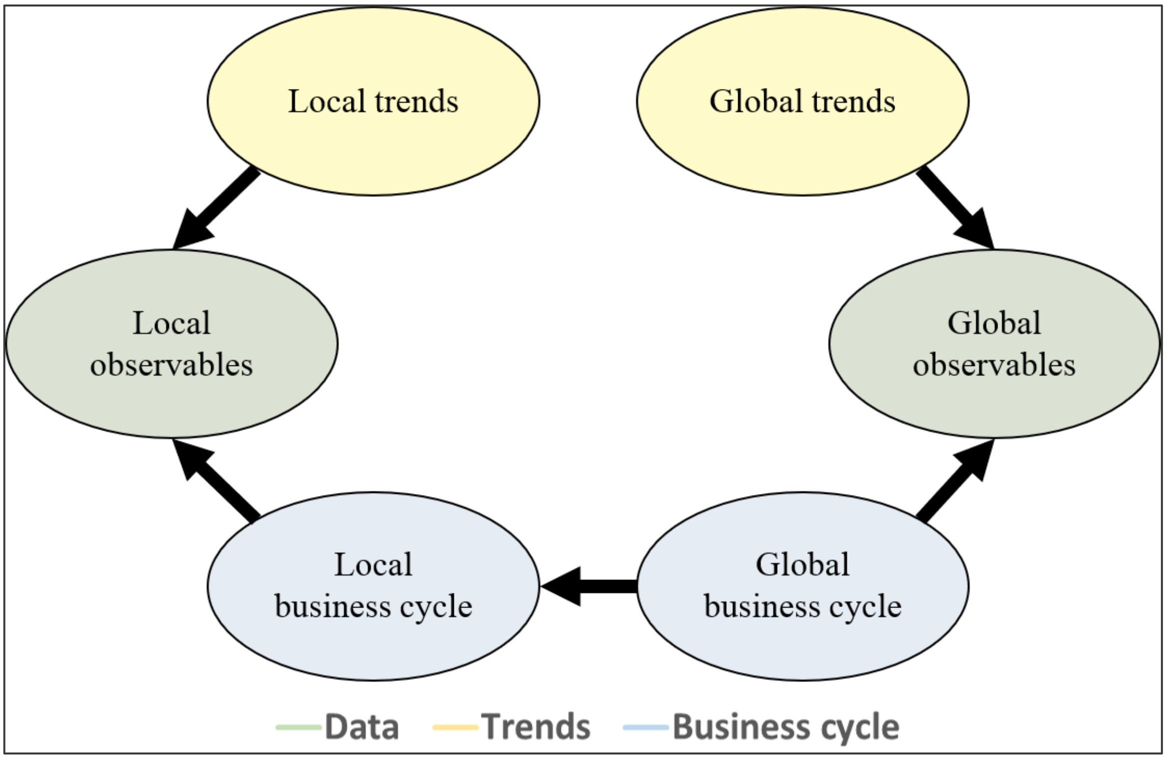
2. The Model

The model is a semi-structural one and has an empirical focus. The structural approach is basing equations on structural microeconomic foundations. But since the model was not developed from such structural elements in an explicit and formal manner, but is satisfied with the formulations of ad hoc equations which draw inspiration from similar structural models, it allows for a flexibility in the adjustments and the expansion of its equations.

The model includes two economies, the local one and the global one, where each of the economies is described in three parts (Figure 1). One part describes structural equations for the dynamics of the business cycle, that is, of the deviation from the trends, another part deals with the trends (which are exogenous in the model) of the observed variables; The last part describes the development of the observed variables in each of the economies as the sum of the two parts - the trend and the gap which deviates from it. As shown in Figure

1, the local economy, being an open economy, is affected by the business cycle in the world, and being small, does not affect them in return.

Figure 1 – The structure of the model[5]



a. The Business Cycle

The dynamics of the business cycle in the local economy is characterized in the model by four (4) key macro variables: the output gap, the inflation, the real exchange rate gap and the real interest rate gap. These gaps are the difference between the actual variables and their long-term trends. Within the framework of the model, these gaps are affected by business cycle shocks. According to the definition, the dynamics of the business cycle, as it is reflected in the structure of the model, leads to the convergence of the various variables to their trends. The business cycle block describes the dynamics of "closing the gaps", that is, of the convergence of the observed variables to their trends. From here on, the denotation of a variable with a star sign will represent the trend of the variable, and the denotation of a variable with a hat sign will represent the gap between the variable and that trend so that:

$$\hat{x} \equiv x - x^*:$$

All parameters express an absolute value (positive), so when the effect of an explanatory

[5] There are several channels in the model where global economic trends have a direct impact on the local business cycle, through the UIP equation, but this work will not focus on them. Please see note no. 9.

variable is negative, a minus sign will appear before the parameter in the equation.

IS equation for the output gap[6]:

$$(1)\quad \hat{y}_t = \beta_{lead} \cdot E_t(\hat{y}_{t+1}) + (1-\beta_{lead})\hat{y}_{t-1} - \beta_{\hat{r}}\hat{r}_{t-1} + \beta_{\hat{z}}\hat{z}_{t-1} + \beta_{\hat{y}^{row}}\hat{y}_t^{row} + \epsilon_t^{\hat{y}}$$

When (while ignoring the time sub-index):

- $\hat{y}$ – The output gap .The output gap ($\hat{y}$) is defined as the gap between the actual output level (y) and the output trend (y^*).
- $\hat{r}$ – The real interest rate gap. The real interest rate gap ($\hat{r}$) is defined as the gap between the actual real interest rate (r) and its trend (r^*). The real interest rate is defined as the difference between the effective nominal interest rate (which includes a margin) and the expected inflation expectation, so that:

$$r_t \equiv i_t^{Boi} + s_t - E_t(\pi_{t+1}).$$

- s – The exogenous and inertial spread between the effective market interest rate and the central bank interest rate.
- $\hat{z}$ – The real exchange rate gap. Defined as the gap between the real exchange rate (z) and its trend (z^*), where the real exchange rate itself is defined as the ratio between the prices in the rest of the world and the local prices.
- $\epsilon^{\hat{y}}$ – A demand shock.

The structure of the equation draws inspiration from the Neo-Keynesian theory: it includes a dynamic relationship between the gaps in output in different periods, as well as a restraining effect of the interest rate, along the side of two expanding effects of an open economy - an expanding effect of the global trade, which contributes to the export demand, and of the real exchange rate, both due to its expanding effect on exports, and due to the shifting of the demand from imports to local products and services ("*expenditure switching effect*")[7].

[6] Here, and later, row indicates the rest of the world (Rest of the world).

[7] The autoregressive factor in the IS equation is aligned with the consumption habits of the individuals and with the costs of the firms to adjust their investment. Please see Smets & Wouters (2003), Argov et al. (2012) and Argov & Elkayam (2010).

1) The Phillips curve for inflation in the consumer price index:

$$(2)\ \pi_t = \alpha_{lead} \cdot E_t(\pi^4_{t+4}) + (1-\alpha_{lead})\pi^4_{t-1} + \alpha_{\hat{y}}\hat{y}_{t-1} + \alpha_{\Delta z}\Delta z_t + \alpha_{\pi^{oil}}\pi^{oil}_t + \epsilon^\pi_t$$

When:

π	–	The local inflation in the current quarter (in annual terms).
π^4	–	The local annual inflation (the cumulative inflation during the last four quarters).
Δz	–	The change in the real exchange rate (in this equation it is a change in the observed rate, i.e., a change that is also affected by the change in the trend of the real exchange rate).
π^{oil}	–	The inflation of oil prices (in local currency terms).
ϵ^y	–	The supply shock ("Cost push shock").

The structure of the equation is consistent with the Phillips curve, obtained according to the Neo-Keynesian theory, and includes price rigidity in the Calvo (1983) approach[8]. The effect of the change in the real exchange rate expresses actually the excess change (beyond the change in local inflation) in the nominal exchange rate and foreign prices.

2) The Uncovered Interest Rate Parity (UIP) equation for the exchange rate:

$$(3)\qquad z_t = \gamma E_t(z_{t+1}) + (1-\gamma)z_{t-1} - (r_t - r^{row}_t - rp_t) + \epsilon^z_t$$

When:

r^{row}	–	The global real interest rate.
rp	–	A "structural" risk premium, which is defined, equivalently, as the sum of two factors: the trend of the real exchange rate and the difference of the trends in the real interest rate (between the local and the global)[9].
ϵ^z	–	A shock to the real exchange rate.

3) Taylor's equation for the interest rule:

$$(4)\ i^{Boi}_t = \delta_{lag} i^{Boi}_{t-1} + \left(1-\delta_{lag}\right) \cdot \left[r^*_t + \pi^4_t - s_t + \delta_\pi E_t(\pi^4_{t+4} - \pi^*) + \delta_{\hat{y}}\hat{y}_t\right] + \epsilon^{ieff}_t$$

---

[8] Unlike a standard Neo-Keynesian Phillips curve, the equation includes an autoregressive factor, which is consistent with the existence of firms that link their prices to the price index. Please see Smets & Wouters (2003), Argov et al. (2012) and Argov & Elkayam (2010).

[9] formally: $rp_t = 4\left((z^*_t - E_t(z^*_{t-1})\right) + (r^*_t - r^{*row}_t)$.

When:

i^{BoI} – The nominal interest rate in the economy

$\epsilon^{i^{BoI}}$ – A policy shock.

The structure of the interest rule is consistent with the accepted literature with regard to the Taylor rule; Taylor (1993). According to the structure of the rule, when $\delta_\pi > 1$ the Taylor principle is fulfilled, which ensures stability of the system around a steady state, which is characterized, among other things, by an inflation rate that corresponds to the target.

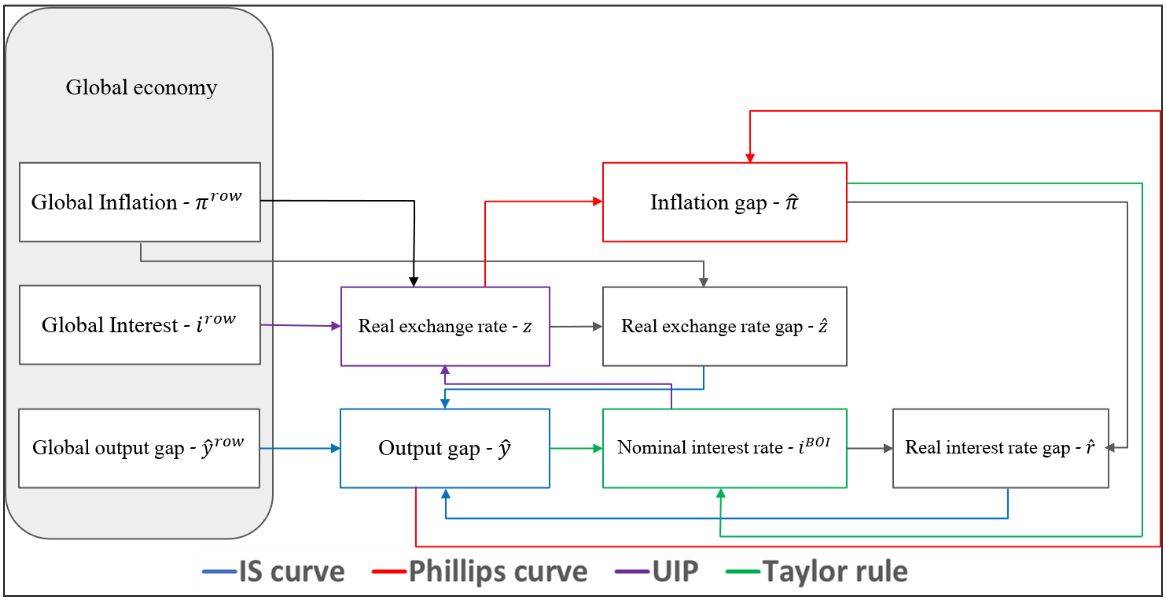
The corresponding equations with regard to the global economy are presented in Appendix A.1.I. Since the local economy is small, it does not influence global developments. Thus, the equations describing the rest of the world are a special case of equations 1, 2 and 4 for a closed economy.

The main policy target is inflation, but the policy does not affect it directly, but only indirectly - through the effect on activity in the economy and the exchange rate. When the central bank lowers the nominal interest rate, and as a result the real interest rate decreases, this has an expanding effect on the demand in the economy, as expressed in the effect on the output gap in the IS equation (Equation no. 1). The output gap directly affects inflation according to the Phillips curve. The cut in the interest rate also affects the real exchange rate through the UIP equation (Equation no. 3). The lowering of the nominal interest rate reduces the nominal exchange rate, and therefore also the real one. Real devaluation has an expanding effect on the inflation - both directly (as expressed in the Phillips equation; Equation no. 2) and indirectly, through the effect of real devaluation on the demand for the output (IS equation; Equation no. 1). According to the interest rate rule (Equation no. 4), the policy does not react directly to the exchange rate (but it does react to it indirectly, through its reaction to inflation).

The mechanisms of influence of the monetary policy, along with other mechanisms, are described in Figure 2, while simplifying some of them[10].

[10] The illustration is based on a similar illustration from the work of Botha, Jager, Ruch & Steinbach (2017).

Figure 2 – The dynamics of the business cycle



b. The block of long-term trends of the observed variables

The long-term trends are formulated in a reduced form (as an *AR1* process). This is because the main purpose of the model is to describe the factors that explain the business cycle, therefore the approach to the long-term processes is a simplified one. The equations of the long-term trends in the local economy are detailed below[11].

The growth trend of the output is as follows:

(5) $$\Delta y_t^* = (1-\rho_{\Delta y^*})\overline{\Delta y} + \rho_{\Delta y^*}\Delta y_{t-1}^* + \epsilon_t^{\Delta y^*}$$

The real interest rate trend:

(6) $$r_t^* = (1-\rho_{r^*})\bar{r} + \rho_{r^*}r_{t-1}^* + \epsilon_t^{r^*}$$

The spread:

(7) $$s_t = (1-\rho_s)\bar{s} + \rho_s s_{t-1} + \epsilon_t^s$$

The real exchange rate change:

(8) $$z_t^* = (1-\rho_{z^*})\bar{z} + \rho_{z^*}z_{t-1}^* + u_t^{z^*}$$

(9) $$u_t^{z^*} = \rho_u u_{t-1}^{z^*} + \epsilon_t^{u^{z^*}}$$

The inflation trend is actually the inflation target[12]. For each of the processes, the size id denoted with an upper line is the value to which the exogenous process converges in the

[11] The equations describing the trends of the global economy are presented in Appendix A.1.II.

[12] In the framework of the model, this target is fixed over time. (The observed variable of inflation is adjusted for this target by subtracting the difference of the inflation target from 2%.) Therefore the block of the trends does not address the dynamics of the inflation trend/target. In part 7, an analysis will be presented that changes this assumption and assumes that the inflation target is also a developing trend.

long run. Thus, for example, the exogenous process for the development of the trend of the real interest rate (r_t^*) converges to a rate of $\bar{r}$ Since the dynamics of the business cycles reflect the convergence of the various variables to their trends (when the Taylor principle is fulfilled as described above), the real interest rate (r) also converges to its trend, and so – also to the parameter $\bar{r}$ which can be considered the natural interest rate of the long term.

The trend of the real exchange rate is modeled slightly differently than the other trends. Thus, from empirical considerations, the trend of the real exchange rate is affected by a shock which is serially correlated.

c. The observable variables block

These equations link the observed variables with their two components according to the model - the long-term trends and the deviations from them (gaps that express the business cycle).

The growth figures are linked to the change in the output trend and the change in the output gap:

(10) $$\Delta y_{obs,t} = \Delta y_t^* + \Delta \hat{y}_t$$

The observed change in the nominal exchange rate is linked to the sum of two changes - in the real exchange rate and the difference in inflation between Israel and the world.

(11) $$\Delta fx_{obs,t} = \Delta z_t + \pi_t - \pi_t^{row}$$

The interest rate and the inflation rate in the model are directly linked to their observed values[13]:

(12) $$i_{obs,t}^{Boi} = i_t^{Boi}$$

(13) $$\pi_{obs,t} = \pi_t$$

3. The estimation

a. The method of the estimation

The model was estimated using a combination of Bayesian estimation and calibration techniques, as is customary in the estimation of DSGE models. The estimation of the parameters in the structural equations of the business cycle (equations nos. 1-4) is Bayesian, and for the purpose of determining an initial prior for the estimation, the structural

[13] As may be recalled, the interest rule includes an exogenous and inertial margin, which can actually be regarded as a trend. Also, seasonality was subtracted from the observed variable of inflation, as well as the difference of the inflation target from 2%, so that the data are "normalized", similar to the situation where the inflation target was 2% throughout the sample period.

equations were estimated using the OLS technique[14]. This combination of Bayesian estimation with a prior from the OLS estimation was chosen to address the shortcomings of each method. The estimation using the OLS method as a collection of individual equations suffers from problems of identification and bias. The estimation as a system of equations provides a certain answer to this problem, but usually requires the use of a Bayesian estimation technique, which includes a prior, to provide convexity of the estimation criterion. A combination of the approaches enables estimation as a system, converging around values that are consistent with the empirical correlations. The parameters in the trend equations (equations nos. 5-9) were calibrated - both for the steady state values of the trends and for the parameters that determine their dynamics. The guiding principles in the calibration were adjustment to the moments of the sample data with regard to the parameters that determine the steady state, and adjustment to the statistical properties of the trend according to the Hodrick-Prescott filter with regard to the parameters that determine the dynamics of the trends[15].

The estimation and calibration were based on the following quarterly data for the years 1992-2019:

1) The change in the domestic output[16];
2) The local inflation[17];
3) The nominal interest rate of the Bank of Israel;
4) The change in the output in the global economy[18];
5) The inflation in the global economy[19];
6) The level of nominal interest in the global economy;
7) The rate of change in the nominal exchange rate;
8) The rate of change in the prices of crude oil.

For the data of the global economy, a weighted average of the US, EU, UK and Japan was used. The weighting was based on Argov et al. (2012)[20].

[14] In some cases, a slightly different prior was used from the OLS preliminary estimation results. Further details of the results of the OLS estimation, of the priors that were used for the estimation and the method of performing the pre-filtering of the data for the purpose of the estimation appear in Appendix A.2.

[15] The adjustment to the Hodrick-Prescott trend was chosen because it is one of the accepted ways to separate the trend from the gap in the series, in particular regarding variables such as the product gap and the real interest gap. Details of the calibration are provided in Appendix A.3.I.

[16] Excluding seasonality, in relation to the size of the working-age population (24-65).

[17] As mentioned, after deducting seasonality, and deducting the difference of the inflation target from 2%, so that the data are "normalized" similar to the situation where the inflation target was 2% throughout the sample period.

[18] excluding seasonality.

[19] excluding seasonality.

[20] The updated weights of the global economy: the USA 43%; the Eurozone 43%; Great Britain 10%; Japan 4%.

b. The results of the estimation[21]

The results of the estimation of the parameters of the structural equations are presented in Table 1.

Table 1: The main results of the estimation[22]

Parameter		**Prior distribution**			**Posterior distribution**		
Description	Symbol	Distribution	Mean	STD	Mode	5%	95%
IS curve							
Future output gap coeffecient	β_{lead}	Betta	0.50	0.06	0.45	0.39	0.51
Real rate effect on output gap	β_r	Betta	0.10	0.05	0.02	0.004	0.04
Real exchange rate effect on output gap	β_z	Betta	0.13	0.08	0.003	0.001	0.01
World outputgap effect on output gap	$\beta_{\hat{y}^{row}}$	Betta	0.03	0.02	0.03	0.01	0.04
Output gap std	$\sigma^{\hat{y}}$	Inv. Gamma	0.89	1.00	0.41	0.35	0.47
Phillips curve							
Calvo parameter for inflation	α_{lead}	Betta	0.54	0.06	0.59	0.51	0.67
Output gap effect on inflation	$\alpha_{\hat{y}}$	Betta	0.10	0.05	0.12	0.05	0.18
Real exchange rate effect on ifnlation	α_z	Betta	0.10	0.05	0.07	0.04	0.11
Oil price inflation effect on inflation	α_{oil_1}	Betta	0.01	0.01	0.01	0.00	0.02
Inflation STD	σ^{π}	Inv. Gamma	0.80	1.00	2.63	2.32	2.94
UIP							
Real exchange rate autoregressice component	γ	Betta	0.51	0.05	0.62	0.57	0.66
Real exchange rate STD	σ^{z}	Inv. Gamma	0.55	1.00	4.28	3.10	5.41
Taylor rule							
Autoregressive component of inflation (inflation smoothing)	δ_{lag}	Betta	0.80	0.10	0.84	0.78	0.92
Inflation effect on interest	δ_{π}	Betta	2.00	0.40	1.83	1.33	2.33
Output gap effect on interest	$\delta_{\hat{y}}$	Betta	0.50	0.10	0.45	0.33	0.56
Interest STD	$\sigma^{i^{eff}}$	Inv. Gamma	1.22	1.00	0.73	0.60	0.86

In relation to the calibration in the work of BKL, the relative flattening of the IS curve stands out.

The direct effects of the real interest rate differential (0.02) and the real exchange rate differential (0.003) are lower than those used by BKL. The Phillips curve is also flatter, and the effect of the output gap on inflation is only 0.12, compared to 0.3 for BKL.

Another notable result includes an estimate of a particularly high standard deviation with regard to the real exchange rate shock (4.28) and the supply shock (2.63); This is along the side of a low standard deviation for the activity shock (0.41). The meaning of the result is that the former will be significant in explaining the development of the business cycle in the model. It should be noted that the standard deviation of the real exchange rate shock decreases considerably when the estimation does not include the years 1992-2000.

c. The dynamics in the model

For an impression of the dynamics in the estimated model, Figures 3-4 show the impulse response (Impulse Response Function - IRF) of the central variables in the model to a policy shock and a shock to the output gap[23]. For comparison, the response of the variables to the corresponding shock for MOISE is shown.

[21] A graphic display of the marginal distribution of the parameters is provided in Appendix A.4.
[22] A breakdown of the estimation results regarding the global economy is provided in Appendix A.6.
[23] Responses to other major shocks are presented in Appendix A.5.

Figure 3 – Response to a shock to monetary policy i^{boi}

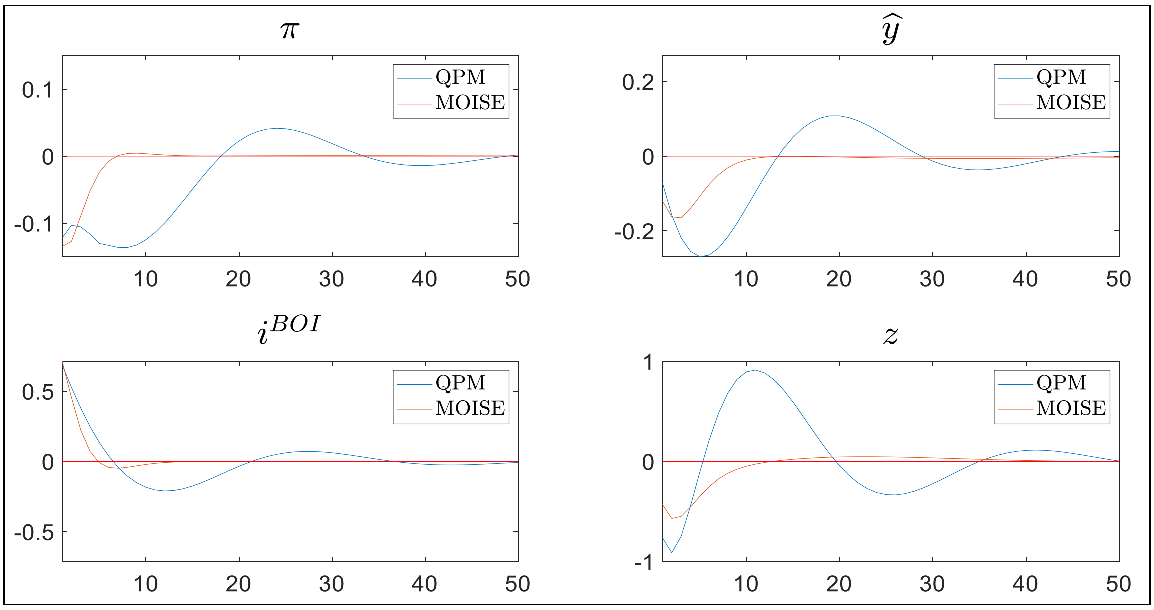
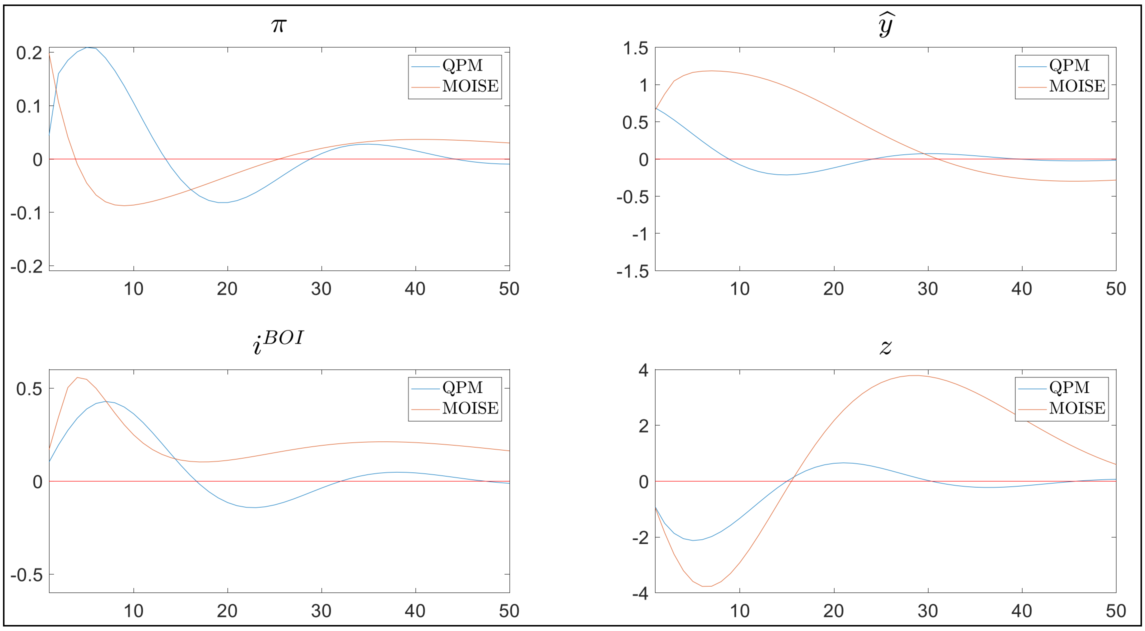


Figure 4 – Response to a shock to the output gap - ŷ



In this response, the nominal interest rate rises in accordance with the direct shock, which restrains the activity through the effect of the real interest rate gap on the output gap in the IS equation (Equation no. 1). The decrease in activity lowers inflation through the Phillips curve (Equation no. 2). Along the side of this, the interest rate increase causes the exchange rate to strengthen (appreciation), through the effect of the real interest rate on the exchange rate in the UIP equation (Equation no. 3). The drop in the exchange rate also lowers prices - both directly, according to the Phillips curve (Equation no. 2), and indirectly, through the effect of the exchange rate on output (IS Equation; no.1).

Compared to MOISE, the response of the various variables is characterized by a stronger inertia, along the side of overshooting, which manifests itself in additional, smaller cycles later on, while the variables converge to the steady state. Thus, QPM is characterized by longer cycles as a response to this shock[24]. However, quantitatively, the magnitudes of the responses in the two models are similar.

4. The examination of the economic history by using the model

a. The smoothing of the data in the model using a Kalman smoother

The model is built, as mentioned, from two parts - the trends and the gaps from them (the business cycle). The observed data are the sums of the two parts, and are affected by the forces which are acting on both. When dealing with the real data, the data is known to us, but it is necessary to break it down into two parts, each of which in itself is not observed. In a model-based estimation framework, the observed data is therefore "decomposed" into trends and gaps from the trends, and both components are estimated simultaneously with the help of a Kalman Smoother algorithm. The principle underlying the shock detection of the Kalman smoother is the maximization of the likelihood of the data. In other words: the algorithm estimates the composition of the shocks in the model - the structural shocks and the trend shocks - which can explain the development of the data in the most reasonable way.

This approach of filtering the data based on the model, which makes use of all the data together, along with the structure of the equations and the parameter values, has many advantages compared to the approach used by BKL; which included the use of filters based on a single variable, such as the Hodrick-Prescott (HP) filter, which performs technical smoothing based on single series data. In contrast, for example, in the multivariate approach, the identification of the output trend (and therefore also of the output gap) does not rely on the output data alone, but makes use of the information contained in the data of the inflation, the interest rate, etc.

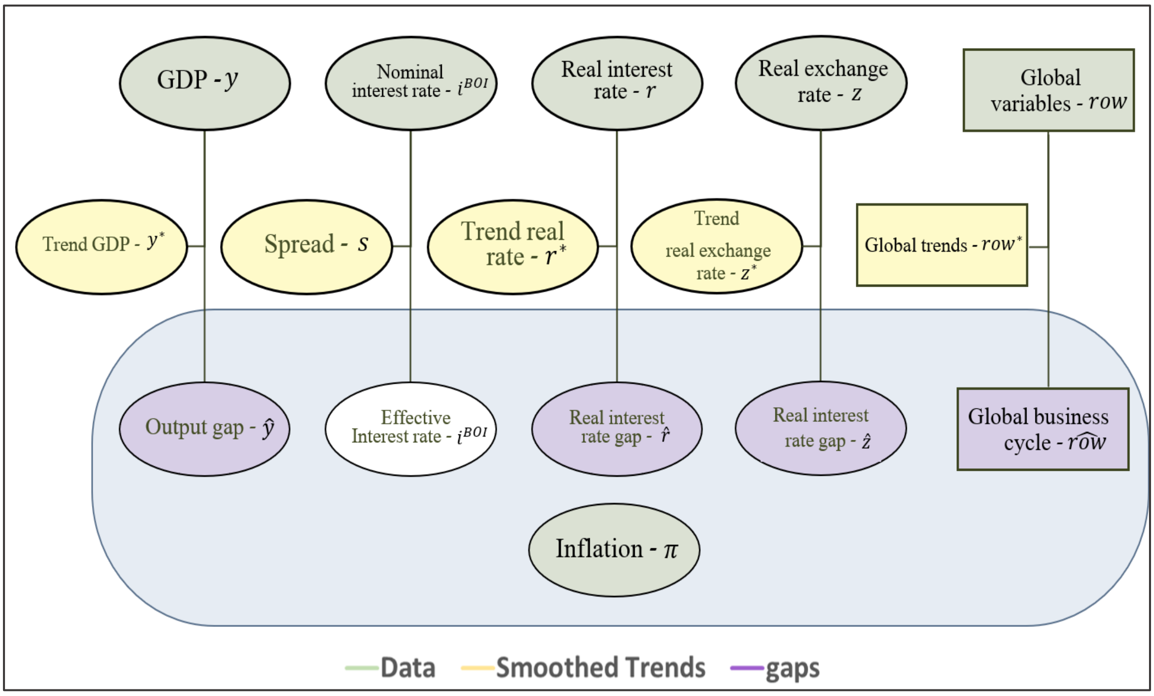
The long-term trends are indeed modeled in a limited way, but since the process of breaking down the data makes use of the structural part of the business cycle, the gap between the actual figure and the business cycle is an estimate of the long-term trend.

---

[24] A similar depiction also emerges from response functions for additional shocks, as detailed in Appendix A.5.

Figure 5 shows the main parts of the model - the observed variables, the trends and the gaps. Each node contains a point where an observed variable must be broken down into a trend and a gap from it that is economically significant in the business cycle. All the decompositions are done simultaneously, so that the decomposition into trends will result in developments that coincide with the business cycle, as it is expressed in the equations of the structural part of the model (2.a.).

Figure 5 – The decomposition of the observed variables into trends and gaps using the Kalman smoother

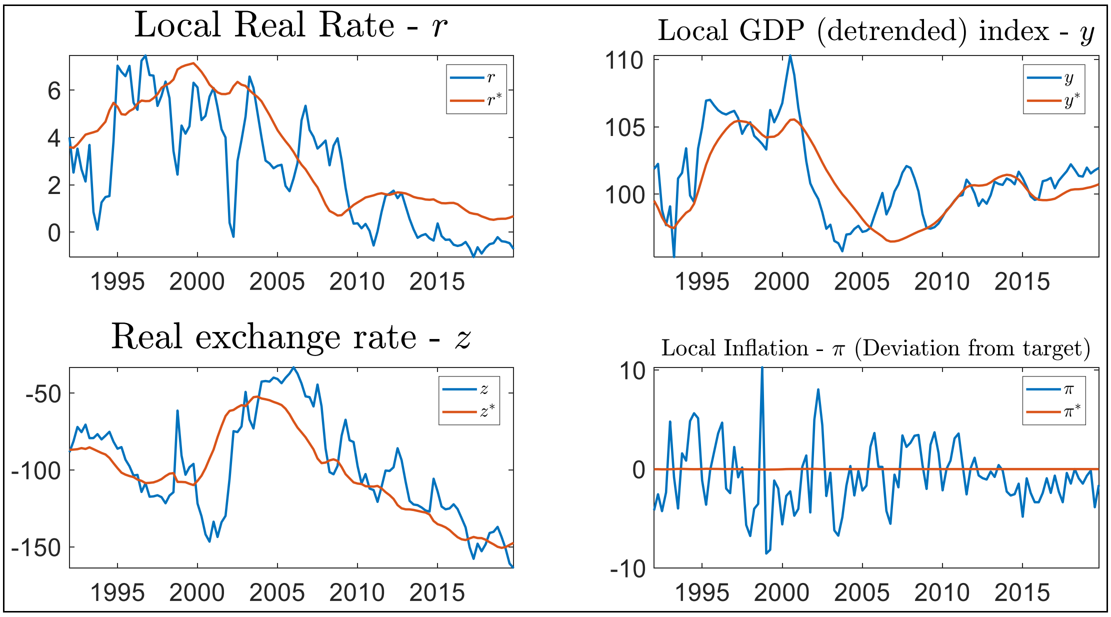


b. The economic interpretation of the development of trends and gaps

Figure 6 shows the long-term trends in the data for the years 1992-2019, as they are obtained from the model while using the Kalman smoother. The choice to limit the sample period at the end of 2019, before the outbreak of the Covid-19 pandemic, was intended to prevent the impact of unusual shocks on the estimation of the historical trends. In part 6, a proposal will be presented that allows for the analysis of a sample that includes 2020, and in the future could also include additional years, while avoiding the unwanted effect of the Covid-19 crisis data on the economic interpretation of developments in earlier years. The trends (in orange) are shown next to the observed variable (in blue). According to the obtained estimate, in recent years (2015-2019) the economy is characterized by a positive output gap, along the side of a negative interest and a negative inflation gap.

Figure 6 – The long-term trends in the model

(1992–2019)

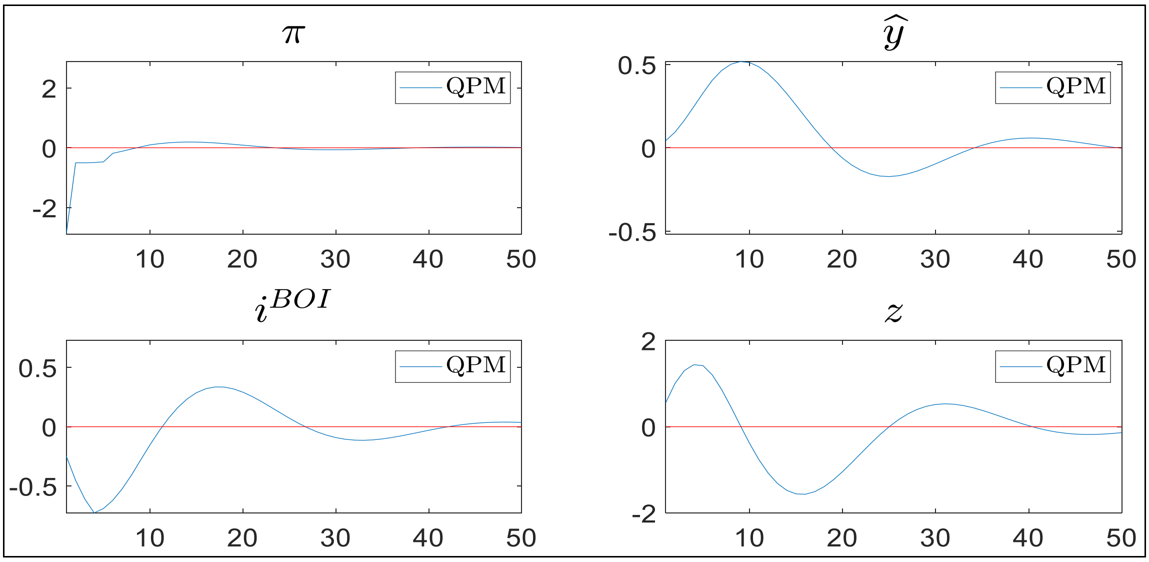


This estimate is consistent with an estimate of positive supply shocks in these years. As can be seen in Figure 7, these shocks act to decrease inflation and interest rates (real and nominal) and to increase activity and the output gap[25]. This development is in line with an interpretation with regard to the increase of competition in the economy, and therefore a decrease in the profitability of the firms in recent years. The aforementioned development emerges from additional analyzes which identified an increase in competition in the Israeli economy in the last decade. This increase is attributed, in part, to the increase in consumer awareness after the social protest of 2011, as well as to a series of reforms, which increased competition in the local economy at the same time as increasing its exposure to online commerce[26]. The increase in competition and its effect on prices arise directly from the work of Nir (2021), who showed, through sectoral Phillips curves, an increase in indicators that measure competition in the Israeli economy at the sectoral level and its contribution to the drop in prices.

[25] The effect on the real exchange rate is positive and works immediately, but produces cyclical waves in the medium term, which can provide an explanation for the cyclicality of the real exchange rate around its trend.

[26] For an analysis of the increase in competition, as reflected in the increase in online purchases, please see Chapter B of the Bank of Israel report for 2017.

Figure 7 – Response to a positive supply shock (a negative shock to the Phillips curve)



This insight emerges directly from a historical breakdown of inflation. In historical decomposition, the variables in the model can be presented as the sum of their response to the historical shocks. With regard to the supply shocks, the effect of each individual shock will look like in Figure 7, and the total effect of the supply shocks on the variable will be the sum of this type of effects at different times. Figure 8 shows the annual inflation (in terms of the deviation from the inflation target), as the sum of the effects of the various historical shocks on it. It can be seen that positive supply shocks do explain a significant part of the negative inflation gap.

Figure 8 – Historical breakdown of the annual inflation
(Deviation from the inflation target)
(1992–2019)

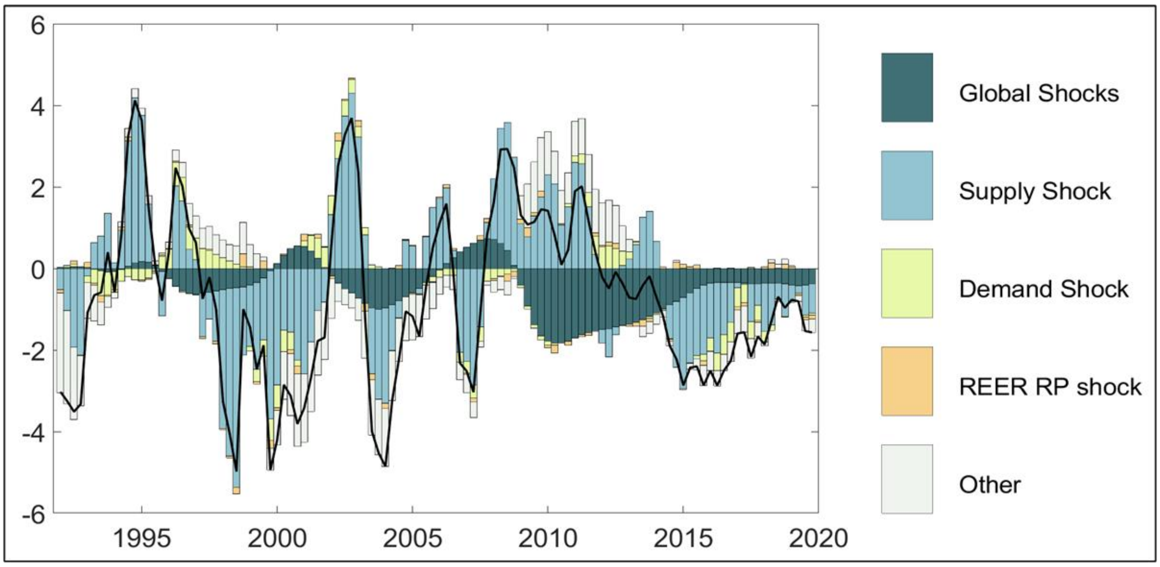


Figure 8 also shows that from the global financial crisis of 2008 until 2019, global shocks took place, which created negative pressures on inflation, and these did not subside completely until 2019. It also seems that demand shocks do not explain a significant part of the development, which could indicate an endogenous monetary policy, absorbing their influence effectively.

The last 15 years have been characterized by a continuous downward process of the real exchange rate. This is against the background of a significant reduction in the deficit in the current account and its transformation into a surplus. From the analysis of the trends in Figure 6, it can be learned that the model interprets this development as a long-term structural process of a decrease in the trend of the real exchange rate[27.]

5. The extent of the suitability of the model

Along with policy analysis, the other key use of a QPM-type model is forecasting. The goal is not just a numerical forecast, but an informed forecast, based on a structural analysis of the state of the economy and the forces acting on it, as expressed in the analysis of the model.

An examination of the quality of the forecast for a period of up to two years, as reflected in the root mean square error (RMSE), shows that the quality of the in sample[28] forecast of QPM with regard to Israel, according to the estimation in this work, is quite similar to the quality of the forecast of naïve models of the AR2 type and a random move (Figure 9). The meaning of this result is that the benefit in the structural analysis provided by the model does not involve an empirically significant cost compared to naïve alternatives. It should be noted that in a review of similar works in the world, such as the one by del Negro & Schorfheide (2013), structural models of the DSGE type achieved a forecast quality similar to that of an AR2 type model.

[27] The trend of the real exchange rate represents long-term developments, as it includes the processes that do not find expression in the business cycle.

[28] The nature of the out-of-sample prediction was not tested in this work, since such a test would have required estimation of the model for a partial sample. The purpose of testing the nature of the forecast within the sample is to illustrate the degree of conformity of the model and the estimation to the actual data.

Figure 9 – The nature of the model forecast according to RMSE for a period of up to 8 quarters (1997–2017)

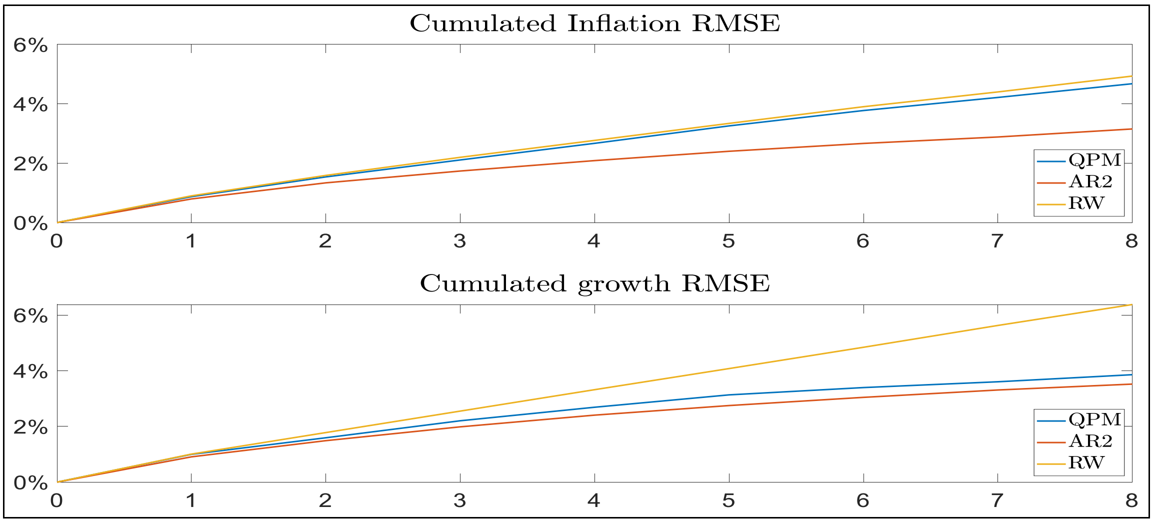
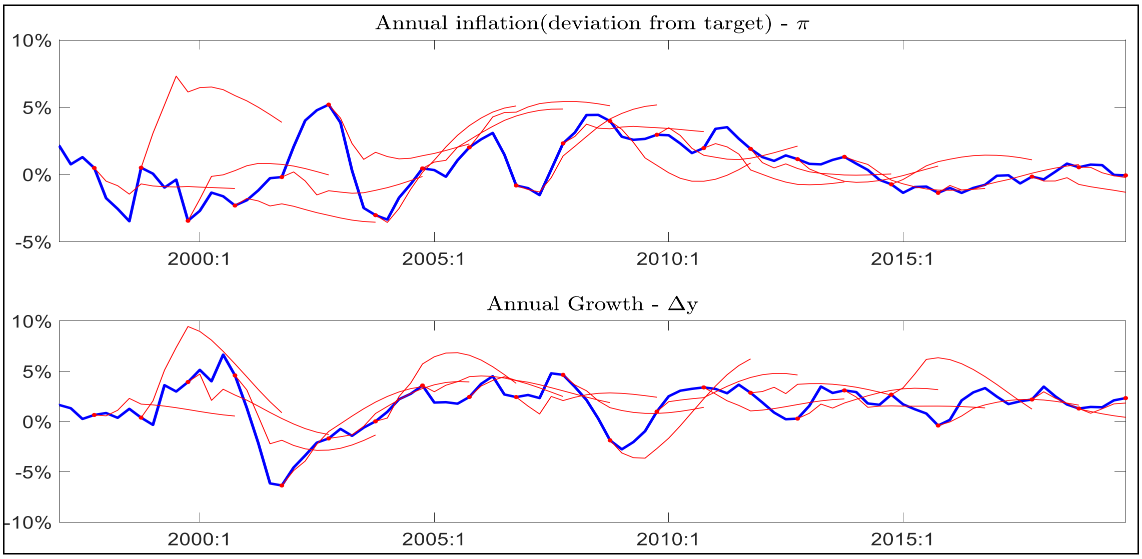


Figure 10 shows the performance of the forecast in an expanding window of the model within the sample, based on the estimation in this work[29], for a period of up to 3 years with regard to annual inflation and growth. It can be seen that the model usually correctly predicts the direction of developments. Accordingly, the bias of the forecast errors is low: in the inflation forecast for the year, the bias is only 0.05, and in the annual growth forecast - the bias is -0.32.

Figure 10 – The model predictions (1997–2019)



The blue line marks the variable, and the red lines are forecasts of the model for a period of three years ahead, while the starting date of the forecast is the end of each calendar year.

[29] The data that was used for the forecast included the data from the beginning of the sample that was used for estimation (1992) until the time of the start of the forecast. The parameters that were used for all predictions were the same, and were based on an estimation of the sample in its entirety.

6. The effect of unusual observations on the estimation of trends and the use of judgement in the Kalman smoother

The estimation and analyzes presented so far are based on a sample that ends in 2019, before the outbreak of the Covid-19 crisis. In the future, when it will be necessary to use data that will also include the year 2020 and other years that will be affected by this crisis, there will be difficulty in providing a historical interpretation based on the model, as was done in chapter 4. As part of the analysis here, I will present a simple way of the intervention of a judgement while using information which is outside of the model, through which by splitting the sample and using the Kalman smoother for each part separately, improves its interpretation of the historical development. In particular, it provides a reasonable estimate for developments in 2019, which were not affected by particularly strong shocks, such as those which occurred in 2020.

a. A challenge in estimating trends and shocks

One of the main advantages of structural models is, as mentioned, the ability to identify structural economic shocks. In this model, the identification of the structural shocks to the business cycles, together with the identification of the shocks of the long-term trends, produce, among other things, an estimate of the trend of the real interest rate (r^*) which can be interpreted as the long-term natural interest rate trend.

In the neo-Keynesian framework, when the actual real interest rate is lower than its "natural" level, monetary policy is expansionary. On the other hand, when the real interest rate is higher than its "natural" rate, the monetary policy is contractionary. Hence the importance in estimating the real interest rate trend, or its "natural" rate. The estimate of the real interest rate trend, like the estimate of other unobservable variables, also significantly affects the model-based forecast.

Similar to the other trends in the model, that of the real interest rate is also characterized by the AR1 process. However, its estimation using a Kalman smoother makes use of the information inherent in the development of all the observed variables, along the side of the information on the structure of the model and the values of the parameters obtained in the estimation.

However, within the framework of estimating such trends with the help of the model, a dilemma is involved: on the one hand, using a Kalman smoother (as opposed to a Kalman filter) allows the use of the information which is inherent in the entire sample. Thus, an interpretation of trends in a certain period is based, among other things, on the known development in later periods. Apparently, this is an advantage - using the information inherent in the entire sample. But the use of such an approach also has a weakness, since based on the model and the statistical assumptions underlying it, it separates the expected developments (which justify the smoothing approach) from the surprising developments. This separation is ultimately an estimate, and as an estimate there is an error in it. The greater the surprises, the greater the potential for error in the estimate. And the bigger this error is, the bigger the error in the interpretation of the historical developments will be. Sometimes the error of the interpretation will not only be quantitative, but also qualitative: for example, sometimes the monetary policy may be interpreted by the model as restraining even though it was clearly expansionary, and vice versa.

Naturally, significant shocks, such as the Covid-19 crisis, will manifest themselves in significant changes in the observed variables. Against this background, the first two quarters of 2020 were characterized by a significant and an exceptional decrease in output. In such a case, the interpretation that would result from using a Kalman smoother that includes 2020 to estimate the trends that prevailed before the crisis, could be very different from the one we would use based on informal judgment. This is because the Kalman smoother makes use of the entire sample; This means that the development of the observed variables in 2020 will affect the derivation of the unobserved variables in earlier years as well. In the next section, a proposal for an intervention of a judgement that makes use of information which lies outside of the model, which provides a response to this type of challenge, will be presented.

b. A responding to the challenge: analysis through the splitting of the sample

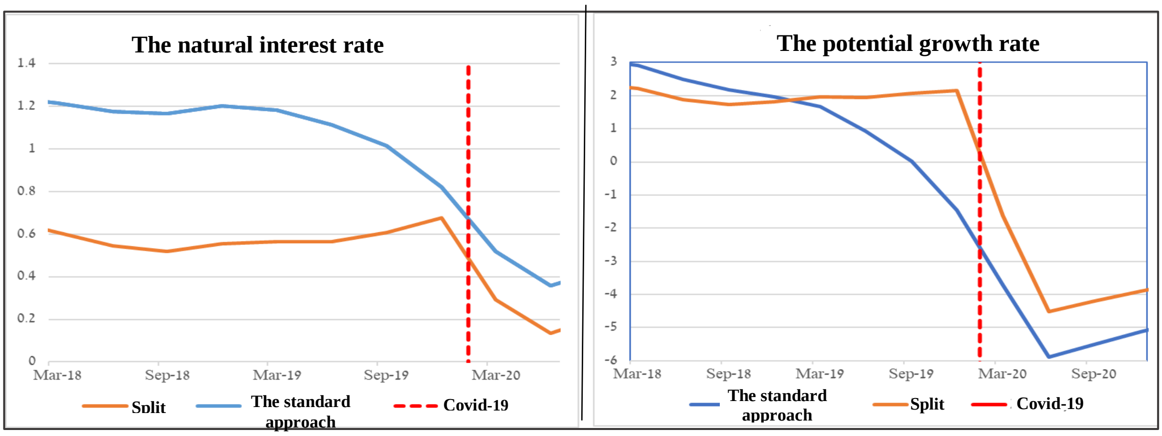
The assumption underlying the proposed intervention is that the strong shocks that took place in 2020 and were reflected in the observed variables were not affected by the developments that occurred earlier, in 2019. Such an assumption is not compatible with the approach underlying the Kalman smoother, which is intended to smooth the shocks and developments over the longest period of time possible, which leads to the assessment that there was an early and gradual decline in the trends of the output and the interest rates back in 2019. Therefore, a judgment is required to deal with this development.

Formally, the smoothing of the Kalman smoother is affected by two assumptions at the base of the model: one is the characterization of the trends as an inertial process; The second one concerns the statistical nature of the shocks in the model - attributing to them a Gaussian distribution which is characterized by relatively thin tails. As a result, a Kalman smoother, based on the principle of maximum likelihood, will tend to "discover" a large number of smaller shocks, rather than "discovering" a small number of large shocks.

Accordingly, an estimate of the trends of the real interest rate and the growth rate while using a standard Kalman smoother for a sample that includes the beginning of 2020, reflects a gradual decline of both of them already in 2019 (Figure 11). The economic meaning of a gradual decline, as obtained from a standard estimation, is that the growth of the production capacity of the economy has slowed down already in 2019. This interpretation does not seem reasonable, and is not consistent with the assumption that 2019 was not characterized by far-reaching changes. A possible intervention of judgment is to "allow" the model to interpret the developments until 2019 with the help of the data until 2019 only, and the developments in the first half of 2020 - with the help of the data of that half. With this split we are preventing the model from inferring from the events of 2020 about the unobserved variables of 2019, as we believe that these events are not beneficial to the explanation of 2019 but damage it.

An estimation of the real interest rate trends and the growth rate while using the split sample method, shows a sudden drop in the trends, as opposed to the gradual decline which is obtained in the analysis according to the standard approach (Figure 11). A sudden drop is more consistent than a gradual decline with the surprising nature of the Covid-19 crisis and its economic consequences.

Figure 11 – the long-term trends in the model - the naïve approach (2018–2020)



The results in the split approach actually align with the assumption that the Covid-19 crisis had a significant impact not only on the business cycle but also on long-term trends. The intervention of the judgement offered in the Kalman filter actually allows this assumption to be expressed.

Although the motivation to analyze by splitting the sample is fundamentally technical, this technique has a dramatic effect on the economic interpretation which is emerging from the model of the historical developments in recent years; This is because a different interpretation of the long-term trends leads to a change in the state of the economy in the model as it is reflected in the gaps.

c. The business cycle in the light of the two approaches

There are therefore two approaches to the examination of the economic developments in the light of the model in recent years:

a) The naïve approach - use of the Kalman smoother for the entire sample, in a way that allows the data of the year 2020 to influence the economic interpretation for previous years.
b) The approach of splitting the sample - the split, as suggested above, so that the economic developments of the year 2020 do not affect the interpretation of the previous years.

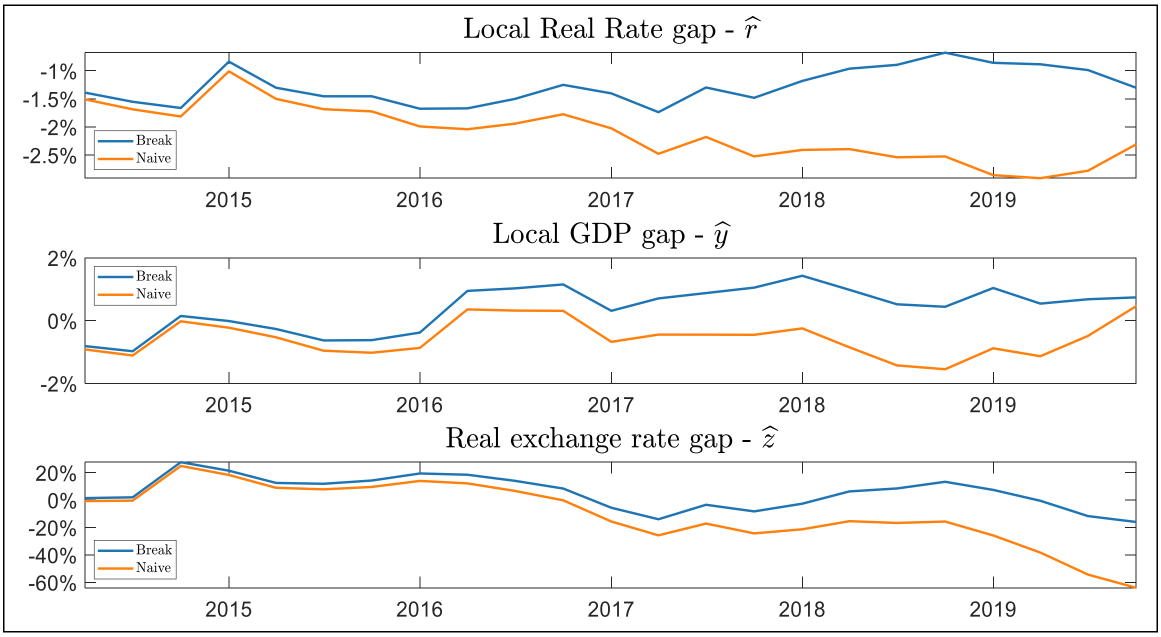
Each of the approaches presents a different depiction of the development of the business cycle in the Israeli economy in the years 2015-2019, and in the splitting approach three positive and larger gaps are obtained than in the naïve approach: the output gap, the real interest rate gap and the real exchange rate gap (Figure 12). In both approaches, the economy in these years is characterized by positive supply shocks, and the naïve approach is also characterized by a significant increase in the trend of the real exchange rate.

According to the naïve approach, the year 2019 is characterized by a significant decrease in the potential growth rate as well as a decrease in the interest rate trend (Figure 11). Such a decrease in the potential growth rate is unlikely, since the year 2019 was not characterized by special developments that were expected to affect the growth potential. Furthermore, this development is not consistent with what is documented in Chapter B of the Bank of Israel report for 2019, according to which the year 2019 was characterized by the continued increase in productivity in the economy. It is worth emphasizing: the fact that such a development emerges from the naïve approach and does not emerge from an analysis in the split approach shows that the data that apparently indicate a significant decrease in the potential growth rate all come from 2020.

Additional reinforcement for the analysis in the split approach emerges from the analysis of the output gap. According to an analysis using the naïve approach, the output gap in the years 2016-2019 was negative, while the split approach indicates a positive output gap in these years. Therefore, it can be said that the estimate of the output gap in the years 2016-2019 has been updated downwards under the influence of the data of 2020. These years are actually known as years of good output growth. This is also evident from the analysis in the Bank of Israel report for 2019[30], as well as from the analysis of the split approach.

In Figure 12 it can be seen that the developments of the real interest rate and the real exchange rate according to the naïve approach are also puzzling: they present a larger gap (in absolute terms) than the trend in the years which, as mentioned, were characterized by relative stability without unusual developments.

Figure 12 – The gaps in the model[31]
(2015–2019)



7. The evolution of the implied inflation target

Figure 12 presented an interpretation of the historical development using the two discussed approaches - the naïve approach and the split approach. The interpretation is reflected in the differences of the real interest rate, the output and the real exchange rate[32]. In contrast to these variables, the development of the "inflation gap" (the deviation of inflation from its target) was not presented, since the inflation target (the "trend" in this case) is an observed variable, and is therefore identical in both approaches.

---

30 According to the estimation of the product gap according to the production function approach.

31 The figure does not include the year 2020 in order to maintain a scale that makes it easier to understand the development. A version that includes the year 2020 is attached in Appendix A.7 (Fig 22).

32 Since the data itself is observed, and are therefore the same in both approaches, it is possible to learn from the development of the gap about the development of the trend.

In the following analysis, we will deviate from this assumption with regard to the inflation target, and assume an "implied inflation target". This target can deviate from the center of the official target range, and it reflects how the inflation target of the central bank is perceived[33]. Within the framework of the model, a change in the implied target means a change in the target used by the interest rule.

Specifically, and for the sake of simplicity, let us assume that the implied inflation target develops as an unobservable randomly process of the AR1 type, similar to the other trends in the model, as described in part 3:

$$(14) \qquad \pi_t^* = (1-\rho_{\pi^*})\bar{\pi} + \rho_{\pi^*}\pi_{t-1}^* + \epsilon_t^{\pi^*}$$

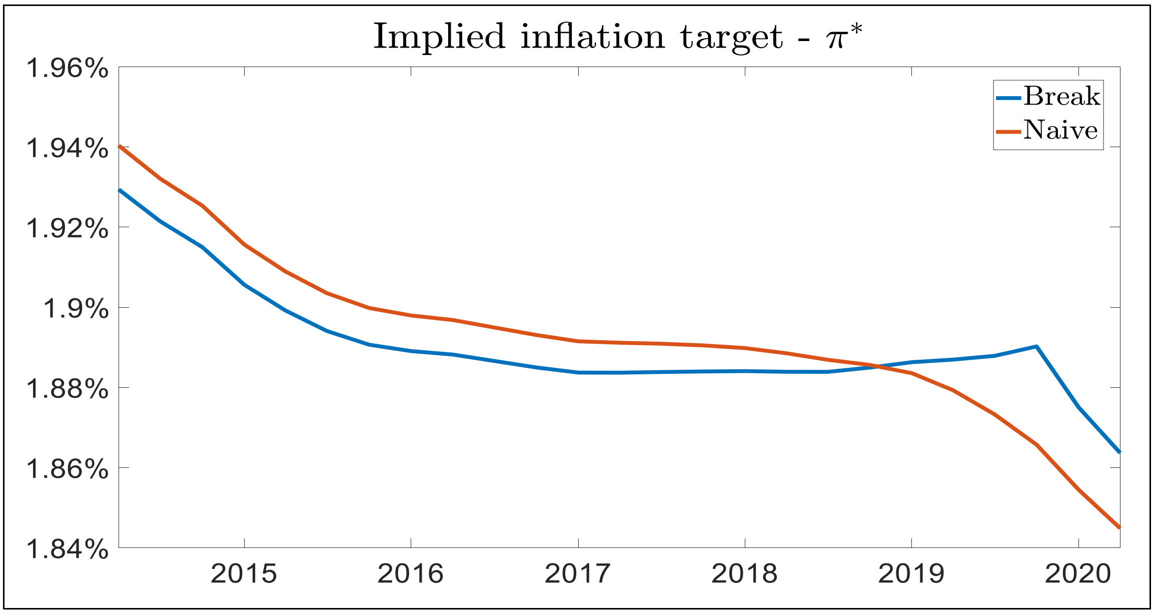
It should be emphasized that this equation does not stand on its own; It is part of the model. Therefore, when inflation deviates from 2%, it can result from a variety of shocks, including a shock to the target whose development is described in equation no. 14. The contribution attributed to this specific shock depends on the joint development of inflation and the other observed variables. Since this equation for the development of the target is, as mentioned, part of the model, the development of the implied inflation target can be estimated using the model in a similar way and in parallel to the estimation of the other trends, as it was done in the previous parts of the article. Now there is another possible explanation for the economic developments: a deviation of inflation from its target, as shown for example in Figure 8, can also be explained by a deviation of the inflation target from the center of the official target. As before, we will explain the historical development while using the entire information on the development of all the variables observed in the model.

Figure 13 shows the development of the implied inflation target in the years 2014-2020 according to the addition of equation 14 according to each of the two approaches, as it was done in part 6[34].

[33] So far in the work, the center of the target has been used as the "official target", which is also a possible and not necessary interpretation of the inflation target range.

[34] Based on the estimation of the model plus equation 14 according to the data set which is detailed in part 3, without the observed variable of the inflation target. The estimation process included the new parameters, and the values of the other parameters are in accordance with the results, as presented in the rest of the work. The prior and the posterior for estimating the parameters:
ρ_{π^*}: Prior average: 0.98. Standard deviation of the prior: 0.01. The posterior: 0.978
$\sigma^{\epsilon^{\pi^*}}$: The average: 0.1. Standard deviation of the prior: 0.05. The posterior: 0.094.

Figure 13 – The gaps in the model
(2014–2020)

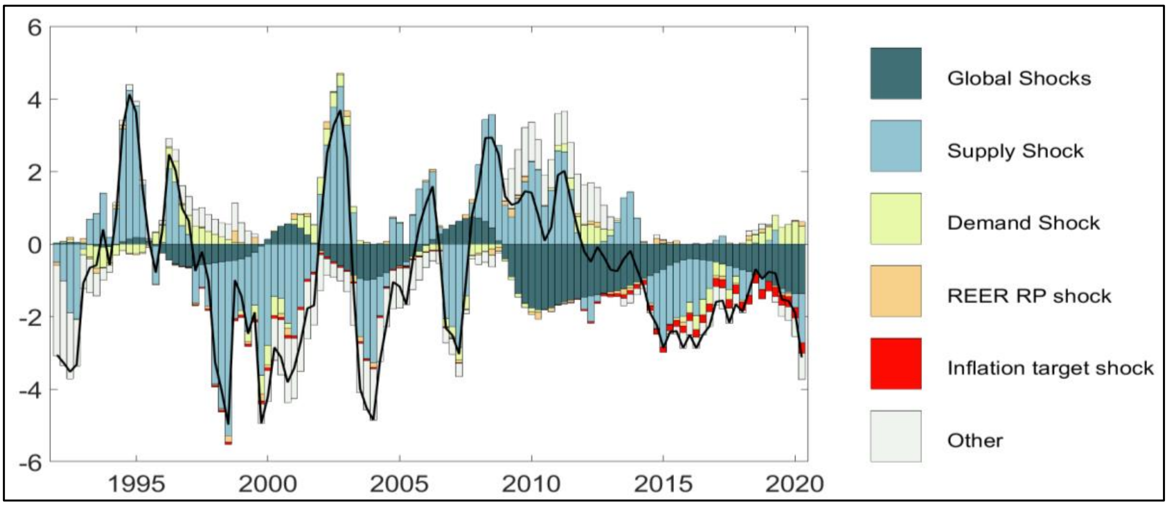


The figure shows that according to both approaches there was a slight decrease in the inflation target, to a level that is only slightly lower than the center of the official target (which is known to be 1%-3%). Such an interpretation with regard to a decrease in the target to which the monetary policy aims, can be attributed to a significant decrease both in the rate of the output growth and in the actual inflation, while the interest rate has not changed significantly[35]. However, even after this decline, the implied target is above the 1.8% level, still relatively close to the center of the target range. Therefore, even in the light of the model, the commitment of the Bank of Israel to strive to reach the center of the target area remains relatively high, and the development of inflation can, according to the model, be attributed to other economic shocks (and not to shocks to the target), as can be seen in Figure 14, which presents the contributions of the various shocks to the development of the inflation according to the split approach[36]. The figure shows that according to the model, the deviation of the target explains only a negligible part of the development of inflation.

[35] The Bank of Israel interest rate decreased during the second quarter from 0.25% to 0.1% 2021. It did not decrease to a lower level due to the approach to the zero barrier.

[36] A similar depiction also emerges from analysis using the naïve approach.

Figure 14 – Historical decomposition of the annual inflation (deviation from the inflation target) with an implied inflation target (according to the split approach) (1992–2020)



8. Summary

The model presented in this work makes it possible to understand the interrelationships between the main macro variables in the economy, as well as the influence of the global economy on them. The model can be used for the purpose of evaluating the macroeconomic situation, and especially for the detection of the location of the economy in the business cycle. From this assessment it is possible to derive a forecast for economic developments, relevant policy recommendations, scenario analyses, retrospective policy evaluation, and more.

Since the model is a semi-structural one and a small one, it greatly facilitates the rapid execution of various extensions and their theoretical and empirical examination, as a preliminary step for extensions which are based on micro-foundations. Possible extensions of this type include the effect of asset prices, of credit, macroprudential policy, intervention in the foreign exchange market, and more.

The analysis of the Israeli economy while using the model indicates a relatively flat Phillips curve, which means a low effect of economic activity on inflation. According to the model, the dominant factor that explains most of these developments is the occurrence of global shocks along the side of positive supply shocks, which can be interpreted as an increase in competition in the economy in these years, as it is also stemming out from other studies. Conversely, demand shocks are the cause of a relatively small part of the inflation development, which points to a monetary policy that is able to effectively absorbs demand shocks.

With regard to the examination of the economic development including the developments after 2019, during the covid-19 crisis, by smoothing the data on the base of the model - two approaches are proposed in this work: the naïve approach and the split approach, which uses information which is outside of the model to obtain an estimate of the state of the economy. It seems that the split approach is preferable, thanks to a more reasonable and acceptable estimate of the development of the output gap and the potential output in recent years. According to the naïve approach, the output gap in the last five (5) years was negative, and the potential output shrank significantly in 2019. On the other hand, according to the split approach, the output gap in the last five (5) years was positive, and the potential output trend did not change significantly in 2019.

According to the analysis in both approaches, the last decade was characterized by a boom in economic activity, which was manifested in a positive output gap, along the side of inflation that was below its target, an interest rate which was below its trend (a negative real interest rate gap, which expresses an expansionary monetary policy), and a downward trend in the real exchange rate along the side of its positive gap.

According to both approaches, the economic developments in the years 2014-2020 coincide with a slight decrease in the implied inflation target below the center of the target, but still close to it, and this decrease is attributed a relatively negligible effect on the development of the actual inflation.

The analysis of trends is important in itself, as they contribute to the understanding of the state of the economy in the present and in the past. But they have an additional importance, since making a forecast with the help of the model will be significantly affected by the model's interpretation of developments, and in particular, the way it separates trends from business cycles. When the analysis was conducted by splitting the sample, it was found that after the outbreak of the Covid-19 crisis, the potential growth fell by a steep drop of about 6%, and the real interest rate trend declined by about 0.5%.

Bibliography

Argov, E., Barnea, E., Binyamini, A., Borenstein, E., Elkayam, D., & Rozenshtrom, I. (2012). MOISE: A DSGE model for the Israeli economy. *Bank of Israel Discussion Paper*.

Argov, E., & Elkayam, D. (2010). An Estimated New Keynesian Model for Israel. *Israel Economic Review*, *7*(2), 1-40.

Bank of Israel (2018). *Annual Report 2017*Jerusalem: Bank of Israel. Bank of Israel (2020). Annual Report 2019. Jerusalem: Bank of Israel. Benes, M. J., Clinton, K., George, A., Gupta, P., John, J., Kamenik, O. & Nadhanael, G. V. (2017). *Quarterly projection model for India: Key elements and properties*. International Monetary Fund.

Benes, J., Hledik, T., Vavra, D., & Vlcek, J. (2003). The Quarterly Projection Model and its Properties. *The Czech National Bank's Forecasting and Policy Analysis System. Prague, CNB*.

Botha, B., de Jager, S., Ruch, F., & Steinbach, R. (2017). *The quarterly projection model of the sarb* (No. 8000).

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of monetary Economics*, *12*(3), 383-398.

Canova, F. (2009). Bridging cyclical DSGE models and the raw data. *Manuscript, Centre de Recerca en Economia Internacional*.

Del Negro, M., & Schorfheide, F. (2013). DSGE model-based forecasting. In *Handbook of economic forecasting* (Vol. 2, pp. 57-140). Elsevier.

Foroni, C., Marcellino, M. G., & Stevanović, D. (2020). Forecasting the Covid-19 recession and recovery: Lessons from the financial crisis.

Laubach, T., & Williams, J. C. (2003). Measuring the natural rate of interest. *Review of Economics and Statistics*, *85*(4), 1063-1070.

Lenza, M., & Primiceri, G. E. (2020). *How to Estimate a VAR after March 2020* (No. w27771). National Bureau of Economic Research.

Nir, S. (2021). *Competition in the Israeli Economy and its Effect on Prices: A Sector-Based Phillips Curve Analysis* (No. 2021.01). Bank of Israel.

Primiceri, G. E., & Tambalotti, A. (2020). Macroeconomic Forecasting in the Time of COVID-19. *Manuscript, Northwestern University*.

Smets, F., & Wouters, R. (2003). An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European economic association*, *1*(5), 1123-1175.

Taylor, J. B. (1993, December). Discretion versus policy rules in practice. In *Carnegie-Rochester conference series on public policy* (Vol. 39, pp. 195-214). North-Holland.

A. Appendices

A.1. The equations of the global economy

i. The block of the business cycle

The IS equation for the output gap:

$$(15) \qquad \hat{y}_t^{row} = \beta_{lead}^{row} \cdot E_t(\hat{y}_{t+1}^{row}) + (1 - \beta_{lead}^{row})\hat{y}_{t-1}^{row} - \beta_{\hat{r}}^{row}\hat{r}_{t-1}^{row} + \epsilon_t^{\hat{y}^{row}}$$

The Phillips curve for the inflation in the Consumer Price Index:

$$(16) \qquad \pi_t^{row} = \alpha_{lead}^{row} \cdot E_t\left(\pi_{t+4}^{4,row}\right) + (1 - \alpha_{lead}^{row})\pi_{t-1}^{4,row} + \alpha_{\hat{y}}^{row}\hat{y}_{t-1}^{row} + \epsilon_t^{\pi^{row}}$$

The Taylor equation for the interest rate rule:

$$(17) \qquad i_t^{row} = \delta_{lag}^{row} i_{t-1}^{row} + \left(1 - \delta_{lag}^{row}\right) \cdot \left[r_t^{*,row} + \pi_t^{4,row} + \delta_{\pi}^{row} E_t\left(\pi_{t+4}^{4,row} - \pi^{*,row}\right) + \delta_{\hat{y}}^{row}\hat{y}_t^{row}\right] + \epsilon_t^{i^{row}}$$

ii. The block of the long-term trends of the observed variables

The growth trend of the output:

$$(18) \qquad \Delta y_t^{*,row} = (1 - \rho_{\Delta y^*}^{row})\overline{\Delta y}^{row} + \rho_{\Delta y^*}^{row}\Delta y_{t-1}^{*,row} + \epsilon_t^{\Delta y^{*,row}}$$

The trend of the real interest rate:

$$(19) \qquad r_t^{*,row} = (1 - \rho_{r^*}^{row})\bar{r}^{row} + \rho_{r^*}^{row} r_{t-1}^{*,row} + \epsilon_t^{r^{*,row}}$$

The spread:

$$(20) \qquad s_t^{row} = (1 - \rho_s^{row})\bar{s}^{row} + \rho_s^{row} s_{t-1}^{row} + \epsilon_t^{s^{row}}$$

A.2. OLS preliminary estimation for the prior

In order to obtain a prior, the business cycle equations (2.A.I) were estimated while using the OLS technique. However, in many cases using the results of the OLS estimation as they were, did not allow the convergence of the visibility function, and in order to allow this, a manual modification of the prior was introduced. This usually shows that the value that allows convergence is the one that is more consistent with the Bayesian estimation that was done as a system. This is in contrast to the endogenous estimation in OLS.

Since the separation of the observed variables into trend and gap is based on the model and the use of the Kalman smoother, the Hodrick-Prescott filter was used in order to produce a trend and a gap in relation to it. With regard to the parameters of the coefficients of the variables in the equations, the coefficient from the OLS equation was used, or a value of 0.1 - if the estimate was lower than this value[37]. With regard to the standard deviation of the prior, the standard error from the OLS estimation is used. However, in some cases, in order to ensure the convergence of the visibility function, a standard deviation greater or less than the standard error of the coefficients was chosen.

Also with regard to the shocks for the equations, the standard deviation of the standard error in the OLS estimation was chosen as the expectancy of the standard deviation of the shock. With regard to the standard deviation of the prior to the standard deviation parameter of the shock[38], the value 1 was chosen in order to produce relative freedom for the Bayesian estimation in identifying the significant shocks in the economy.

With regard to the Taylor rule, the estimation of the variables in OLS provided problematic results due to endogeneity, therefore with regard to the coefficients of the output gap and the inflation gap, the coefficients used by BKL were used as a prior.

The priors that were actually used for estimation are presented in Table 1.

1) The IS equation for the output gap:

$$(1)\ \hat{y}_t = \beta_{lead} \cdot E_t(\hat{y}_{t+1}) + (1 - \beta_{lead})\hat{y}_{t-1} - \beta_{\hat{r}}\hat{r}_{t-1} + \beta_{\hat{z}}\hat{z}_{t-1} + \beta_{\hat{y}^{row}}\hat{y}_t^{row} + \epsilon_t^{\hat{y}}$$

	Coefficients	Standard Error	t Stat	P-value
β_{lead}	0.50	0.06	8.87	0.00
$\beta_{\hat{r}}$	0.01	0.03	0.23	0.82
$\beta_{\hat{z}}$	0.27	1.04	0.26	0.80
$\beta_{\hat{y}^{row}}$	0.03	0.12	0.27	0.78

2) The Phillips curve for the inflation in the Consumer Price Index

$$(2)\ \pi_t = \alpha_{lead} \cdot E_t(\pi_{t+4}^4) + (1 - \alpha_{lead})\pi_{t-1}^4 + \alpha_{\hat{y}}\hat{y}_{t-1} + \alpha_{\Delta z}\Delta z_t + \alpha_{\pi^{oil}}\pi_t^{oil} + \epsilon_t^{\pi}$$

	Coefficients	Standard Error	t Stat	P-value
α_{lead}	0.54	0.12	4.55	0.00
$\alpha_{\hat{y}}$	0.01	0.05	0.27	0.79
$\alpha_{\Delta z}$	0.04	2.88	1.35	0.18
$\alpha_{\pi^{oil}}$	0.01	0.01	2.30	0.02

[37] The prior for the coefficients was derived from a beta distribution between 0 and 1, so that in order for the actual expectation to be higher than 0, a value of 0.1 was chosen as the minimum value for the expectation.
[38] Which represents the degree of confidence of the prior in the average intensity of the shock.

3) The UIP equation for the exchange rate

(3) $$z_t = \gamma E_t(z_{t+1}) + (1-\gamma)z_{t-1} - (r_t - r_t^{row} - rp_t) + \epsilon_t^z$$

	Coefficients	Standard Error	t Stat	P-value
γ	0.50	0.05	10.64	0.00

4) The Taylor equation for the interest rate rule:

	Coefficients	Standard Error	t Stat	P-value
δ_{lag}	0.86	0.02	37.33	0.00
δ_{π}	-1.04	0.19	-5.40	0.00
$\delta_{\tilde{y}}$	-0.06	0.40	-0.16	0.87

A.3. Calibrations for the trend equations

i. The performance of the calibration

The equations for which calibrations were performed:

(5) $$\Delta y_t^* = (1-\rho_{\Delta y^*})\overline{\Delta y} + \rho_{\Delta y^*}\Delta y_{t-1}^* + \epsilon_t^{\Delta y^*}$$

(6) $$r_t^* = (1-\rho_{r^*})\bar{r} + \rho_{r^*}r_{t-1}^* + \epsilon_t^{r^*}$$

(7) $$s_t = (1-\rho_s)\bar{s} + \rho_y s_{t-1} + \epsilon_t^s$$

(8) $$z_t^* = (1-\rho_{z^*})\bar{z} + \rho_{z^*}z_{t-1}^* + u_t^{z^*}$$

(9) $$u_t^{z^*} = \rho_u u_{t-1}^{z^*} + \epsilon_t^{u^{z^*}}$$

The trend equations include three types of parameters:

1) Parameters that determine the durable state of the trend and are denoted with a hat.
2) Parameters that determine the dynamics of the trend are: the inertia of the trend, which is denoted by the letter ρ, and the standard deviation of the shock to the trend, denoted as σ^{ϵ}) these do not appear explicitly in the equations, since they are properties of the shocks which are denoted by the letter ϵ).

With regard to the parameters of the durable state of the trends, calibration was performed with the help of the sample average, or in a manner of applying a judgement.

With regard to the trend of output growth, the average growth in the sample was used. With regard to the trend of the real exchange rate, a convergence toward a constant was chosen. With regard to the real interest rate trend, the value of 3% was chosen as a judgement[39].

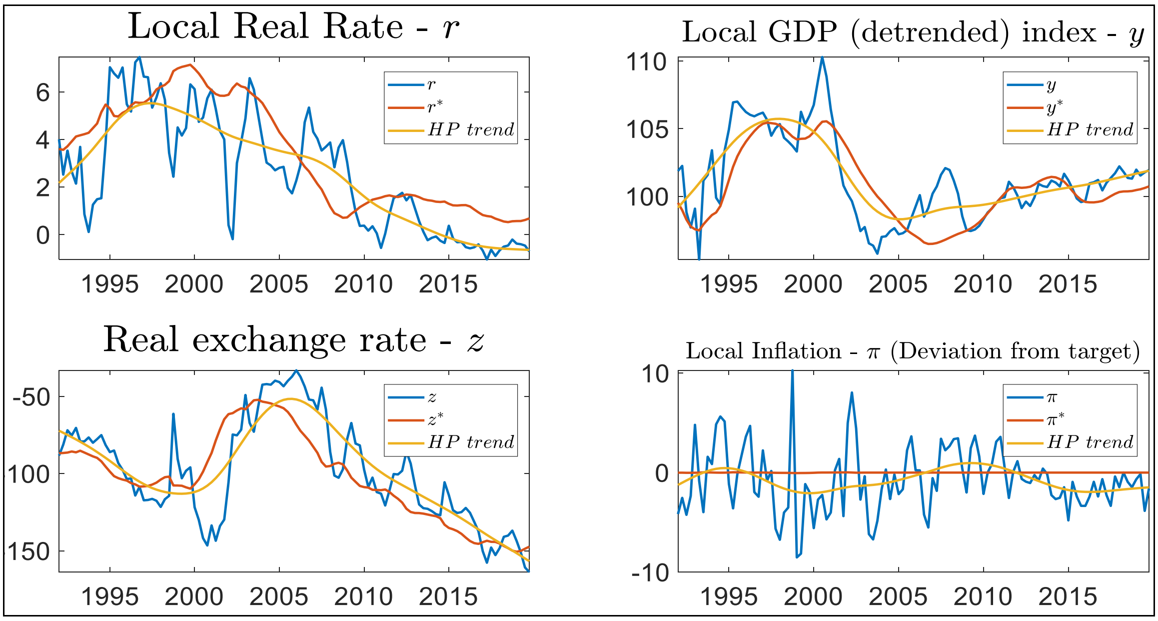
With regard to the parameters that determine the dynamics of the trend, a calibration was made, the purpose of which is to match the statistical properties of the trends to the statistical properties of the trends by using the Hodrick-Prescott filter method. The Hodrick-Prescott trend adjustment was chosen because it is one of the accepted ways of decomposing a series into a trend and a gap, in particular with regard to variables such as the output gap and the real interest gap.

From an informal point of view of the sensitivity of the trends to the parameter values, it seems that the behavior of the trends is not particularly sensitive to the selected parameter values. However, a complete sensitivity test of the parameters should also include the sensitivity of the estimation of the structural parameters to the calibration, and this was not tested in the present work.

Figure 15 shows the trends obtained for the final calibration values (orange), next to the original variables (blue) and the trend in the Hodrick-Prescott filter (yellow).

Figure 15 - <u>The long-term trends in the model</u>

(1992–2019)



The values of all the parameters of the trends are detailed in Table 2.

39 An average level of 3% for the real interest rate is obtained from the average of the sample in the years 1992-2015. This value is also estimated in various studies around the world. Please see Williams & Laubach

Table 2 - The calibration values for the trend equations

Description	Notation	Value
The output trend		
Resistant state of the trend		1.8%
Inertia of the trend		0.95
A shock to the trend		0.89
The real interest rate trend		
Resistant state of the trend		3%
Inertia of the trend		0.97
A shock to the trend		0.61
The trend of the real exchange rate		
Resistant state of the trend		0
Inertia of the trend		0.85
Inertia of the trend		1.999
A shock to the trend		2.19
The spread		
Resistant state of the trend		0
Inertia of the trend		0.8
A shock to the trend		1.34

A.2. The marginal distribution of the parameters

Figure 16 - The marginal distribution of the parameters – A

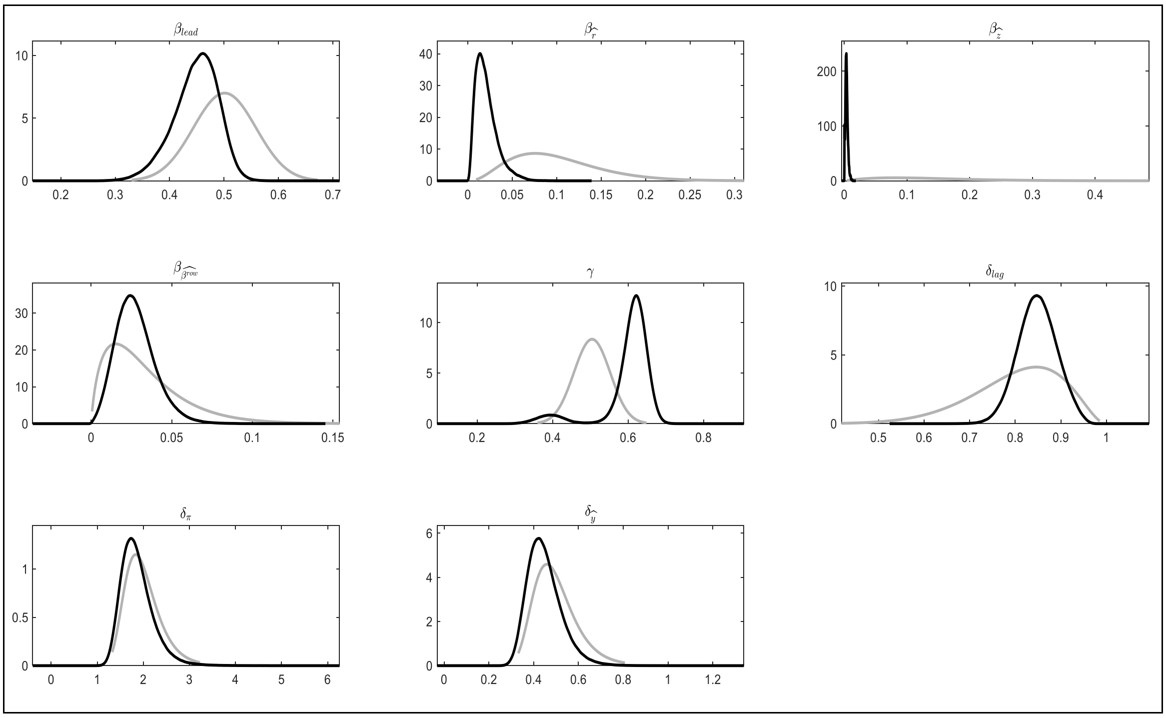
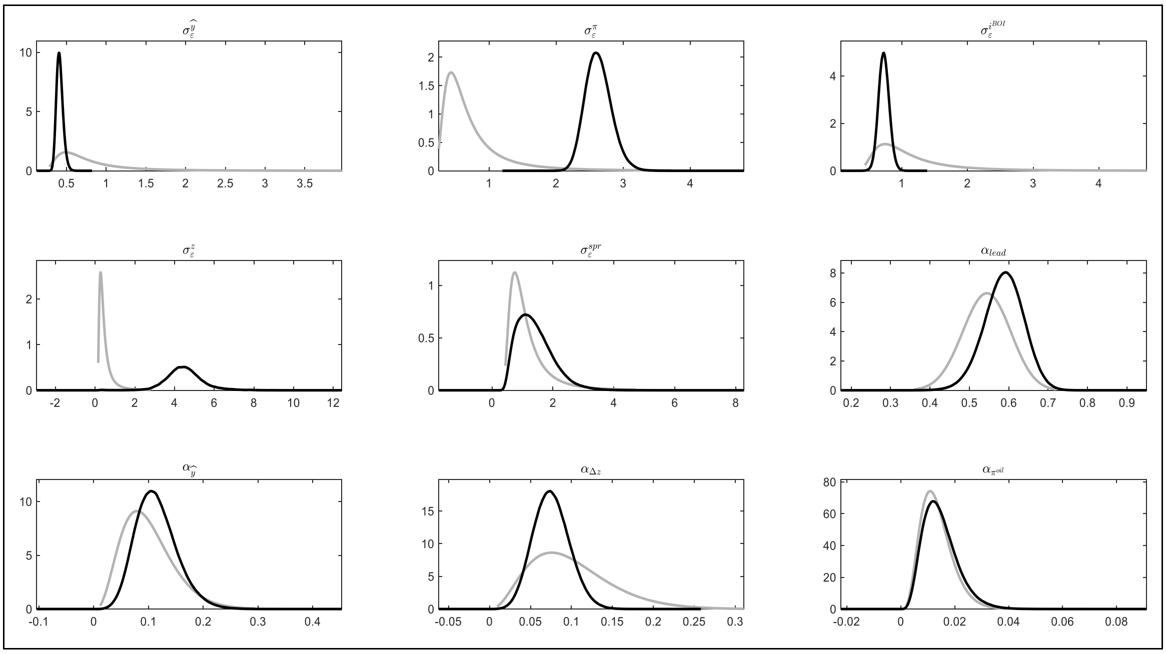


Figure 17 - The marginal distribution of the parameters – B



The shock response functions

Figure 18 - The response to the inflation shock $-\pi$

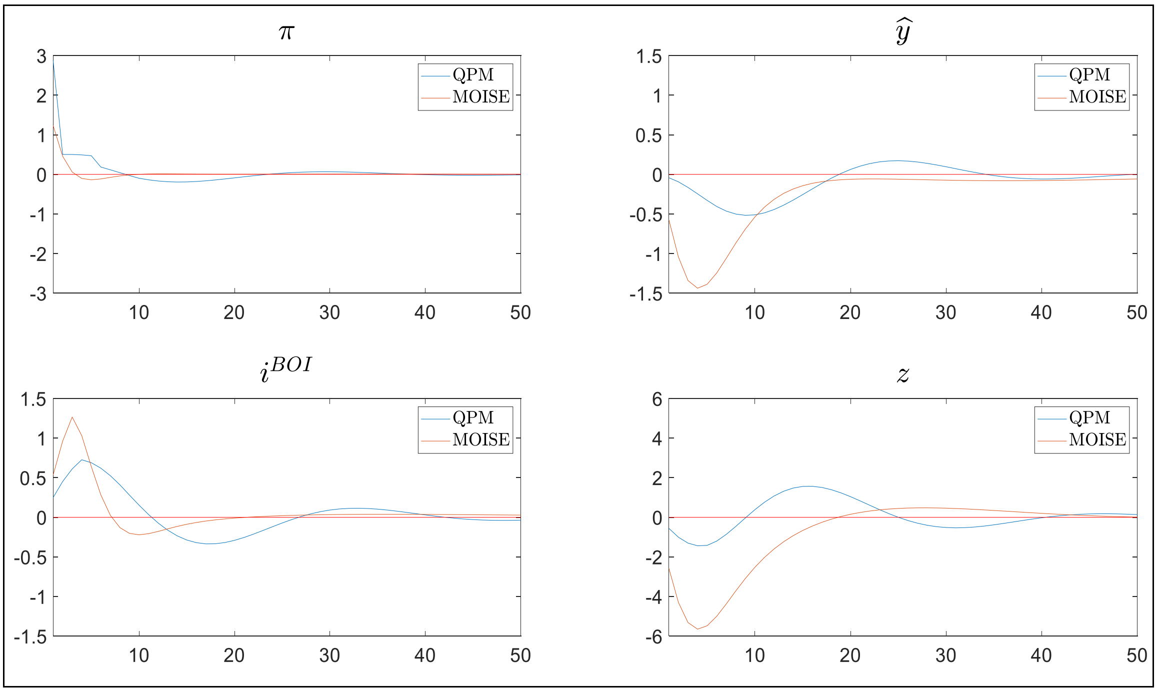
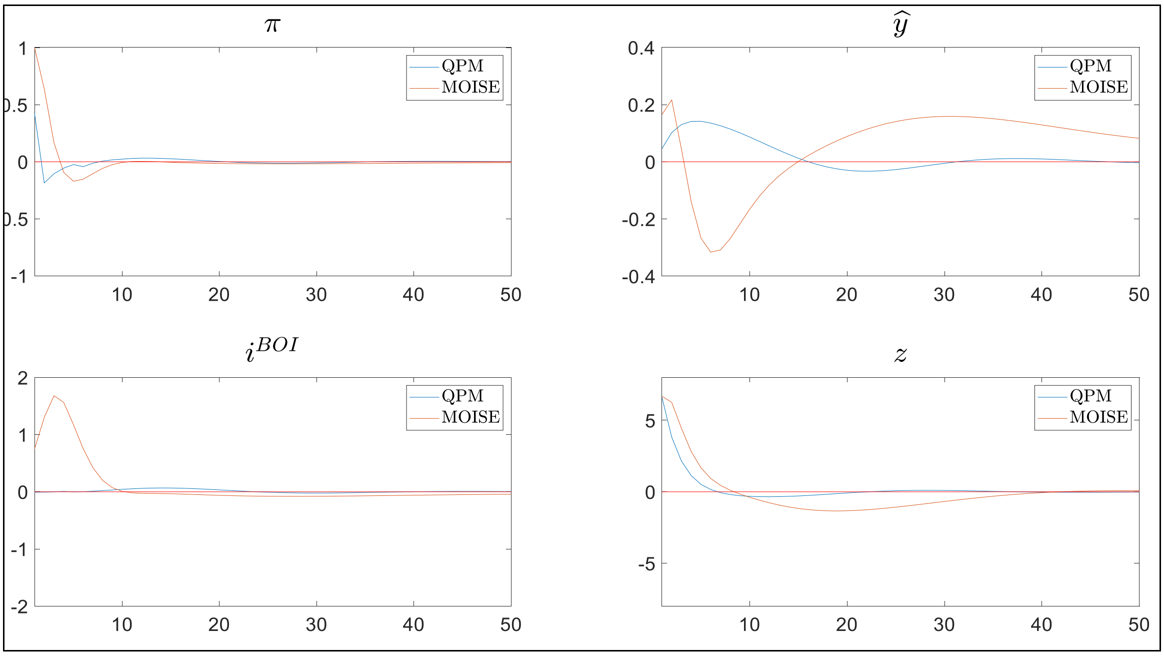


Figure 19 - The response to the exchange rate shock – z



A.6. Estimation and calibration for the global economy

i. Preliminary OLS estimation for the prior

The IS equation for the output gap:

$$(15) \qquad \hat{y}_t^{row} = \beta_{lead}^{row} \cdot E_t(\hat{y}_{t+1}^{row}) + (1-\beta_{lead}^{row})\hat{y}_{t-1}^{row} - \beta_{\hat{r}}^{row}\hat{r}_{t-1}^{row} + \epsilon_t^{\hat{y}^{row}}$$

	Coefficients	*Standard Error*	*t Stat*	*P-value*
β_{lead}^{row}	0.49	0.03	17.82	0.00
$\beta_{\hat{r}}^{row}$	0.01	0.01	1.21	0.23

The Phillips curve for the inflation in the Consumer Price Index:

$$(16) \qquad \pi_t^{row} = \alpha_{lead}^{row} \cdot E_t\left(\pi_{t+4}^{4,row}\right) + (1-\alpha_{lead}^{row})\pi_{t-1}^{4,row} + \alpha_{\hat{y}}^{row}\hat{y}_{t-1}^{row} + \epsilon_t^{\pi^{row}}$$

	Coefficients	*Standard Error*	*t Stat*	*P-value*
α_{lead}^{row}	0.55	0.15	3.58	0.00
$\alpha_{\hat{y}}^{row}$	0.04	0.05	0.86	0.39

The Taylor equation for the interest rate rule:

$$(17) \qquad i_t^{row} = \delta_{lag}^{row} i_{t-1}^{row} + \left(1-\delta_{lag}^{row}\right) \cdot \left[r_t^{*,row} + \pi_t^{4,row} + \delta_{\pi}^{row} E_t\left(\pi_{t+4}^{4,row} - \pi^{*,row}\right) + \delta_{\hat{y}}^{row}\hat{y}_t^{row}\right] + \epsilon_t^{i^{row}}$$

	Coefficients	*Standard Error*	*t Stat*	*P-value*
δ_{lag}^{row}	0.96	0.02	60.50	0.00
δ_{π}^{row}	1.15	0.66	1.74	0.09
$\delta_{\hat{y}}^{row}$	0.59	1.06	0.56	0.58

ii. The results of the Bayesian estimations

Table 3 - Key estimation results for the global economy

Parameter		Prior distribution			Prior distribution		
Description	Symbol	Distribution	Mean	STD	Mode	5%	95%
IS curve							
Future output gap coeffecient	β^{row}_{lead}	Betta	0.49	0.03	0.39	0.36	0.41
Real rate effect on output gap	$\beta^{row}_{\hat{r}}$	Betta	0.10	0.01	0.10	0.086	0.11
Output gap std	$\sigma^{\hat{y}^{row}}$	Inv. Gamma	0.19	1.00	0.17	0.14	0.20
Phillips curve							
Calvo parameter for inflation	α^{row}_{lead}	Betta	0.55	0.15	0.71	0.61	0.80
Output gap effect on inflation	$\alpha^{row}_{\hat{y}}$	Betta	0.10	0.01	0.10	0.09	0.11
Inflation STD	$\sigma^{\pi^{row}}$	Inv. Gamma	0.45	1.00	1.62	1.43	1.81
Taylor rule							
Autoregressive component of inflation (inflation smoothing)	δ^{row}_{lag}	Betta	0.96	0.02	0.90	0.88	0.92
Inflation effect on interest	δ^{row}_{π}	Betta	1.15	0.13	1.50	1.15	1.84
Output gap effect on interest	$\delta^{row}_{\hat{y}}$	Betta	0.59	0.11	0.30	0.15	0.45
Interest STD	$\sigma^{i^{row}}$	Inv. Gamma	0.32	1.00	0.22	0.19	0.25

Figure 20 - The marginal distribution of the parameters

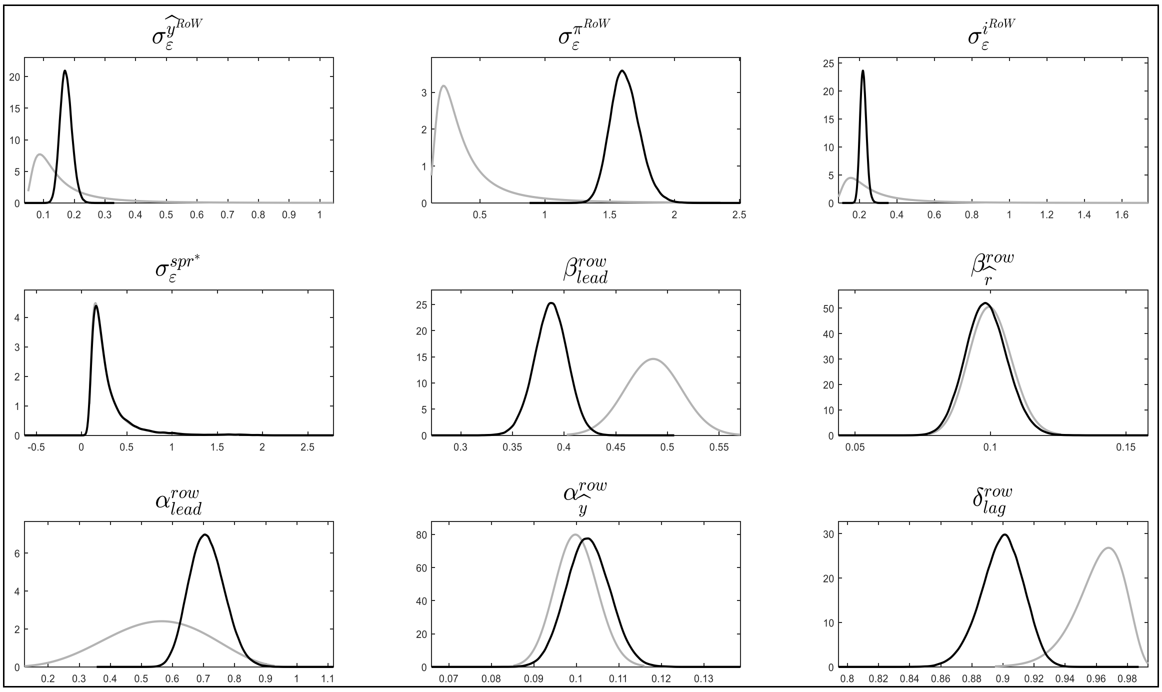
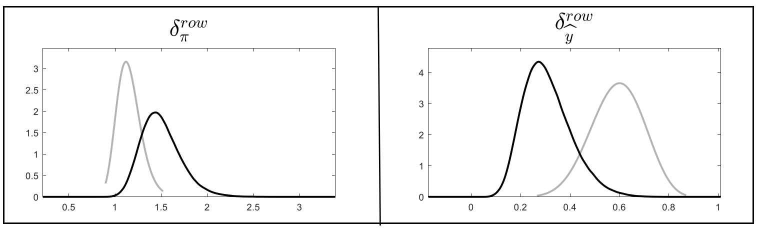


Figure 21 - The marginal distribution of the parameters

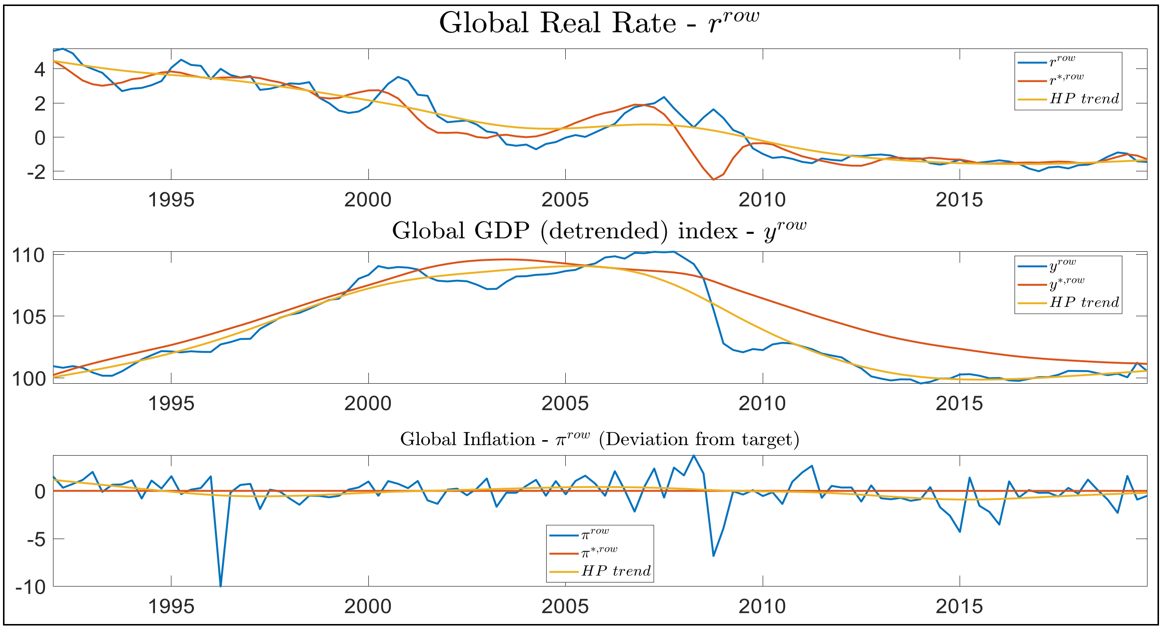


iii. Calibrations for the trend equations

Table 4 - The calibration values for the trend equations in the global economy

Description	Symbol	Value
GDP trend		
Trend steady state	$\Delta\bar{y}^{row}$	1.8%
Trend inertia	$\rho^{row}_{\Delta y^*}$	0.95
Trend shock	$\sigma_{\epsilon^{row}_{\Delta y^*}}$	0.19
Real interest rate trend		
Trend steady state	$\bar{r}^{row}$	3%
Trend inertia	$\rho^{row}_{r^*}$	0.96
Trend shock	$\sigma_{\epsilon^{row}_{r^*}}$	0.32
Spread		
Trend steady state	$\bar{s}^{row}$	0
Trend inertia	ρ^{row}_{s}	0.85
Trend shock	$\sigma_{\epsilon^{row}_{s}}$	1.10

Figure 22 - The long-term trends in the global economy (2019–1992)



A.7. The differences in the model according to the two approaches

The gaps in the model

Figure 23 - (2020–2015)