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A DSGE-Based Analysis

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Abstract

The unemployment rate in Israel declined from 10 percent in 2009, following the Global Financial Crisis, to 3.5 percent in 2019, just before the global COVID-19 crisis. Using an empirically oriented DSGE model, with a focus on the labor market, I estimate and analyze the main contributions to that decline. The model departs from the neoclassical approach to the labor market by including search and matching frictions, endogenous participation, nominal wage rigidity, salaried employees, efficient bargaining over hours worked and coexistence of both margins—extensive (employment) and intensive (hours worked). The model is estimated, based on the Bayesian approach, using quarterly data of the Israeli economy from 1992 to 2019. It generates labor-share dynamics which, although supported by robust empirical evidence, are not replicated by standard models. A model-based analysis sheds light on a positive trend in productivity, and a negative one in employees' bargaining power, as two dominant contributions to the boom in the Israeli labor market—a boom that was interrupted by the outbreak of the global COVID-19 crisis in 2020. Finally, accounting for possible reallocation effect of the COVID-19 crisis, the model is employed to discuss policy considerations related to unemployment benefits.

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הדינמיקה של שוק העבודה בישראל: ניתוח מבוסס מודל מאקרו

אלון בנימיני

תקציר

שיעור האבטלה בישראל ירד מ-10% ב-2009, על רקע המשבר הפיננסי הגלובלי, ל-3.5% ב-2019, טרם משבר הקורונה. התרומות העיקריות לירידה נאמדות בעזרת מודל מאקרו אמפירי המתמקד בשוק העבודה. המודל סוטה מהגישה הניאו-קלאסית לשוק העבודה, באמצעות התייחסות לחיכוכים בחיפוש ובהתאמת עובדים ומשרות, השתתפות אנדוגנית, קשיחות בשכר הנומינלי, שכר גלובאלי, מקח על השעות והתייחסות, בו-זמנית, הן למועסקים והן לשעות עבודה. המודל נאמד בגישה הבייסייאנית, על נתונים רבעוניים של המשק הישראלי לשנים 1992-2019. ניתוח המבוסס על המודל שופך אור על פריון עבודה, שעלה בקצב מהיר מהשכר, כהסבר הדומיננטי מאחורי התרחבות התעסוקה במשק – התרחבות שהופרעה על ידי משבר הקורונה שפרץ ב-2020. לסיום, על רקע השפעות אפשריות של משבר הקורונה על תהליכים מבניים ארוכי-טווח בשוק העבודה, המודל משמש כבסיס לדיון בשיקולי מדיניות תמיכה במובטלים.

1 Introduction

This work develops a Dynamic Stochastic General Equilibrium (DSGE) model with labor market imperfections, and estimates the model using Israeli data for the period 1992-2019. The model is then utilized to shed light, both qualitatively and quantitatively, on the developments of Israeli unemployment from 2009, following the Global Financial Crisis, to 2019, just before the global COVID-19 crisis.

The results shed light on two main forces that drove the Israeli unemployment rate down during those years: households' declining bargaining power over wages and increasing productivity. Both factors seem to boost demand for labor due to a declining labor share, among other channels. When it comes to households' declining bargaining power, a lower labor share seems to be a straightforward result. But it seems to be a less straightforward result of a technology boost. Under the labor market frictions assumed in the model of this work, technology contributes both to labor productivity and to labor income, but asymmetrically, such that the latter doesn't catch up with the former. This implies that increasing labor productivity can explain declining labor share and, therefore, increasing demand for labor services.

This work follows the branch that has been developed within the new-Keynesian (NK) literature in recent years, which departs from the neoclassical approach to the labor market. This is done by allowing for unemployment and rigid wage contracting. Unemployment is integrated into the NK models by merging the Diamond-Mortensen-Pissarides search and matching setup into them.[1] This implies that labor input is adjusted on the extensive margins (employment), in addition to or instead of adjustment on the intensive margins (hours worked) alone. Wage rigidity builds on later contributions that add real wage rigidity, first into RBC models, such as Hall (2005) and Shimer (2005). Models with real wage rigidity seem to have a better empirical fit, compared with flexible-wage models, as they generate lower wage fluctuations that result in higher unemployment fluctuations.

Integration of wage rigidity into the NK literature was gradual, initially using *ad hoc*

[1] Diamond (1982) and Mortensen and Pissarides (1999). See the reviews by Yashiv (2007) and Mortensen (2011).

real wage rigidity, such as Faia (2008) or Blanchard and Galí (2010), and later using nominal wage rigidity, such as Gertler and Trigari (2009), Gertler et al. (2008), de Walque et al. (2009) and Christiano et al. (2011). Compared with real wage rigidity, nominal wage rigidity opens room for an additional mechanism that affects business cycle dynamics. Under nominal wage rigidity, inflation erodes the real wage, thus generating a real wage that is not only rigid, but can also be countercyclical. In turn, a countercyclical wage can contribute as an automatic stabilizer. The consequences of this mechanism, and their implications for optimal monetary policy, are analyzed by Thomas (2008), among others.

At the same time, considering efficient bargaining over hours worked (the intensive margins), wage setting does not directly affect either margin of existing employment relationships. Accordingly, as in Gertler et al. (2008), where only the extensive margins are endogenously adjusted, or in Christiano et al. (2011), with both margins being adjusted, the model here is immune to the Barro (1977) critique, which argues that wages are not allocational in *ongoing* employment relationships. Instead, wage in the above-mentioned models, as well as in the present one, affects existing employment relationships only indirectly, through the rate of new hirings.

As for the motivation to integrate endogenous participation into a business-cycle analysis of the labor market—Veracierto (2008) argues that labor-force participation is (weakly) procyclical. Erceg and Levin (2014) supply some empirical evidence, as well as theoretical analysis, suggesting that although labor-force participation is practically acyclical during "normal times", it may nevertheless be procyclical following a large and persistent demand shock. They also mention some old studies revealing that although labor supply in the prime working age has been largely acyclical, that of teenagers has been moderately procyclical.

An interesting property of the model presented here is the labor share dynamics it generates. Cantore et al. (2020) find robust evidence suggesting that, conditional on monetary policy shocks, the labor share is countercyclical. They further draw attention to the sharp contrast between this robust empirical evidence and the predictions of the NK model—in both its basic and more extended forms. In contrast, the labor share dynamic in the model of this work is consistent with the empirical evidence discussed by Cantore et al.

(2020). This is achieved due to the interactions assumed between the intensive and extensive margins of labor input, and between them and the negotiated wage. The implications, being relevant to the labor share dynamics, are inevitably relevant to the dynamics of the markup, as well as to that of labor productivity.

As noted, this work contributes a quantitative and qualitative analysis of the macro developments in the Israeli labor market, using an estimation of a DSGE model on Israeli data. To that end, it integrates, in a single DSGE model, various labor market features and imperfections: search and matching frictions, endogenous participation, combination of both intensive and extensive margins (that is, both hours worked and employment), nominal wage rigidity, salaried employees, and efficient bargaining over hours worked. The model essentially integrates two other models, and includes some additional ingredients. The integrated models include endogenous participation decision (Binyamini and Larom, 2012) and search and matching unemployment with nominal wage rigidity (Binyamini, 2013). The former is closely related to the works of Haefke and Reiter (2006) and Campolmi and Gnocchi (2011), whereas the latter expands the model of Blanchard and Galí (2010). Finally, the model in this work is closely related to the model presented by Galí (2010), and can be considered as a generalization of it.

The rest of the paper is organized as follows: Section 2 derives the model, which is then calibrated and estimated in Section 3. The model's properties are presented and analyzed by Section 4. This is followed by empirical and theoretical implementations in Section 5, quantifying the historical contributions to the evolution of the Israeli labor market and discussing the mechanisms involved. Section 6 extends the discussion by presenting policy dilemmas related to possible future changes in the labor market, brought about by the global COVID-19 pandemic. Finally, Section 7 offers concluding remarks.

2 The Model

The model is built on the standard search and matching setup with an exogenous job-destruction probability, referred to as the Diamond-Mortensen-Pissarides model.

The model economy consists of representative households that consume goods and supply labor. Nonemployed members of these households are allocated into two pools—unemployed and nonparticipants. Apart from their labor market status, family members are otherwise homogenous. Thus, all matches are equally productive and, therefore, continuation values and surpluses considered by firms and workers are the same across the economy.

The production sector consists of two types of producers: competitive producers of intermediate goods, and retailers who differentiate these intermediate goods, selling them under monopolistic competition with sticky prices. Households and producers of intermediate goods are involved in Nash bargaining over wages (which are set only once in a while) and over hours worked (which are set flexibly in an efficient manner).

Finally, there are official authorities that run fiscal and monetary policies.

In what follows, trending variables are denoted by capital letters, whereas stationary variables are denoted by small ones.

2.1 Labor Market Frictions

2.1.1 Search and Matching Frictions

Jobs matches, m_t, are formed as a standard positive and homogenous function of vacancies and unemployment,

$$m_t = \eta^m \varepsilon_t^m v_t^\phi u_t^{1-\phi}, \tag{1}$$

where v_t and u_t are the rates of posted vacancies and unemployment, respectively, and ε_t^m is an exogenous shock to the matching technology. η^m is a scaling parameter and $\phi \in (0, 1)$. All quantities here—m_t, v_t and u_t— are expressed in terms of their ratio over the *entire* population.

Due to the homogeneity of the matching technology, matching probabilities are related to the degree of labor market tightness, defined as

$$\theta_t \equiv v_t / u_t. \tag{2}$$

Thus, the job-finding rate,

$$x_t \equiv m_t / u_t = \eta^m \varepsilon_t^m \theta_t^\phi, \tag{3}$$

is a positive function of θ, whereas the vacancy-filling rate,

$$q_t \equiv m_t/v_t = \eta^m \varepsilon_t^m \theta_t^{\phi-1}, \tag{4}$$

is a negative function of θ.

The scaling parameter μ^m keeps the probabilities, x_t and q_t, bounded between zero and one.

Matches formed become productive only in the following period, such that

$$l_t = (1-\delta)\, l_{t-1} + m_{t-1}, \tag{5}$$

where l_t is the labor rate and $\delta \in (0,1)$ denotes an exogenous job-destruction rate. Being constant, the job destruction rate greatly simplifies the analysis. This simplifying assumption is supported by some empirical evidence (Shimer, 2005, 2012), suggesting that variations in separations explain only a small fraction of the unemployment volatility over the business cycle.

The assumption that matches become productive only in the following period is also a simplifying one, which enables treating employment as a predetermined state. One implication of such assumption is the effect of participation on the unemployment rate—on impact, an increase in participation causes an increase in unemployment. This mechanism and its implications are considered by Yakhin and Presman (2015), Binyamini and Larom (2012), Veracierto (2008), Shimer (2004), and Tripier (2004), among others.

Since the extensive margin, i.e. employment, is a predetermined state variable, firms also have flexible intensive margins, i.e. hours worked, enabling an on-impact adjustment of a demand-determined output to unexpected shocks. Such a setup is in line with the empirical results of Brender and Gallo (2009) for Israel, documenting that much of the change in labor demand across the business cycle is satisfied by the intensive margins.

Finally, let $h_t \in [0,1]$ denote the nonparticipation rate. Normalizing the entire population to 1, we get

$$h_t = 1 - u_t - l_t. \tag{6}$$

2.1.2 Nominal Wage Rigidity

Adopting the Calvo (1983) setup for nominal wage rigidity, it is assumed that wage contracts are Nash bargained only upon receiving an idiosyncratic signal, which happens with probability $(1-\zeta_w) \in (0,1)$. In periods where a job match is not subject to such a wage renegotiation signal, a distinction is made between wage contracts of old and newly-hired workers.

Wages of existing workers, if not renegotiated, are adjusted to the gross inflation-target, Π_t^{tar}, and to the gross productivity growth-rate, $\overline{gr}^z$. That is, defining $W_{n,t}$ as the wage of contract n in period t, the adjustment rule takes the following form:

$$W_{n,t} = gr_t^W \cdot W_{n,t-1}, \tag{7}$$

where

$$gr_t^W \equiv \Pi_t^{tar} \cdot \overline{gr}^Z.$$

Such an indexation mechanism assures a steady state without wage dispersion. Treating the inflation target as a unit root process, it will be assumed in the paper that $E_t \Pi_{t+s}^{tar} = \Pi_t^{tar}$; $\forall s > 0$.

Wages of newly hired workers, if not Nash bargained, are simply set equal to $gr_t^W \cdot W_{t-1}$, where W_t denotes the aggregate wage level that, based on the law of large numbers and accounting for the indexation schemes just described, evolves as

$$W_t = \zeta_w \cdot gr_t^W \cdot W_{t-1} + (1-\zeta_w)\widetilde{W}_t. \tag{8}$$

Here $\widetilde{W}_t$ is the wage set by all contracts negotiated in period t, which is solved in Subsection 2.6.2 below.[2,3]

[2]Modeling wage rigidity at the contract level, rather than at the firm level, reflects a simplifying assumption that enables treating all firms symmetrically. In contrast, Thomas (2008) and Galí (2010) choose to model that rigidity at the firm level. The modeling choice, however, does not seem to have implications for the aggregate dynamic of the linearized model.

[3]Such wage behavior of newly hired workers essentially implies that those workers get the “going wage”. Gertler et al. (2008), Gertler and Trigari (2009), and Christiano et al. (2011) support such an assumption based on both empirical and theoretical literature.

2.2 Households

2.2.1 Structure

The economy is populated by identical (representative), infinitely lived households. Consider each household to be a big family, which consists of a continuum $[0, 1]$ of family members with complete risk sharing within each household.

With big enough households, it is assumed that the distribution of labor market states within each household reflects the distribution across the economy. Thus, all three labor market pools—namely h_t, l_t, and u_t—present the fractions of nonparticipants, employed, and unemployed agents, respectively, both across the economy and within each household.

Reflecting the idea of diminishing marginal productivity from hours dedicated to home activity, disutility from time dedicated to labor market activity is associated with the term

$$\widetilde{l}_t \equiv \chi^u \varepsilon_t^u u_t + \frac{n_t^{1+\sigma_n}}{1+\sigma_n} l_t + \frac{1}{2} \kappa^h \left(h_t - h_{t-1}^{exo} \right)^2 . \tag{9}$$

Here, the parameter χ^u reflects the time that has to be dedicated to search effort in the steady state. Outside the steady state, this number may vary due to the exogenous shock ε_t^u. Similarly, the term $\frac{n_t^{1+\sigma_n}}{1+\sigma_n} l_t$ is associated with disutility from employment, where n_t, the amount of hours worked, ends up being the same for all working family members, as will be shown by Subsection 2.6.1 below. The choice of that particular expression reflects the assumption that disutility, on the margins, is more sensitive to hours worked than to employment. This assumption helps to provide a stable equilibrium in a setup where the allocation of each margin, extensive and intensive, interacts with that of the other. Finally, the last term is introduced in order to smooth out variations in participation (or, more precisely, in its first difference), where the parameter κ^h denotes the participation adjustment cost, and the variable h_t^{exo} is the aggregate level of nonparticipation, taken as exogenous by the individual household. It is easy to get a private case of constant participation by taking the adjustment cost parameter to $\kappa^h \to \infty$.

Each individual can be in one out of two predetermined states—employed or not employed. If employed, the individual is included in the pool l_t. Otherwise, the individual is allocated to either u_t or h_t.

Eq. (9) shows that utility from nonparticipation is essentially normalized to zero, and that disutility from search activity grows with the time dedicated to the search effort, $\chi^u \varepsilon_t^u$. Eliminating this disutility, the cost-benefit ratio of the search is essentially zeroed out as well. Therefore, we can get full participation as a private case by setting $\chi^u = 0 \rightarrow h_t = 0;\ \forall t$.

Accordingly, the representative household maximizes the following utility function[4]:

$$\max\ E_s \sum_{t=s}^{\infty} \varepsilon_t^{\beta} \beta^{t-s} \left(\varepsilon_t^C \frac{\left(\frac{C_t}{Z_t} - \kappa^c \frac{C_{t-1}^{exo}}{Z_{t-1}} \right)^{1-\sigma_c}}{1-\sigma_c} - \varepsilon_t^l \chi^l \frac{\widetilde{l}_t^{(1+\sigma_l)}}{1+\sigma_l} \right), \tag{10}$$

where E_s is the conditional expectation operator (conditional on information available until period s); $\beta \in (0,1)$ denotes the time discount factor; and $C_t \equiv \left[\int_0^1 \left(C_{j,t}^{\frac{1}{\mu_t}} \right) \cdot dj \right]^{\mu_t}$ is the Dixit and Stiglitz (1977) composite of final goods, where $j \in [0,1]$ is a goods index and $\mu_t > 1$, $\forall t$, is negatively related to the elasticity of substitution across differentiated goods. It is driven by an exogenous process, so that $\mu_t = \varepsilon_t^{\mu}$. The parameter κ^c denotes the degree of external habit persistence in consumption, where C_t^{exo} is the average level of C_t across households, taken as exogenously given by the individual household. The parameters σ_c and σ_l denote, respectively, the constant relative risk aversion (CRRA) and the inverse of the Frisch elasticity of labor supply.

The exogenous process ε_t^{β} is an impatient shock; ε_t^C is a goods demand shock; ε_t^l is a labor supply shock; and Z_t is the productivity level, characterized by a unit root with drift, which is included in order to neutralize the effect of long-run growth on participation and hours worked. It means that the utility associated with consumption is driven by consumption relative to trend, C_t/Z_t. The (gross) growth rate of the technological trend is

$$Z_t = gr_t^Z \cdot Z_{t-1},$$

where

$$gr_t^Z = \left(\overline{gr}^Z \right)^{\left(1-\rho^Z\right)} \left(gr_{t-1}^Z \right)^{\rho^Z} \cdot \varepsilon_t^Z$$

(ε_t^Z is an exogenous shock process).

[4] Some other papers (Campolmi and Gnocchi, 2011; Galí, 2010) take a similar approach for modeling endogenous participation, though they use a different formalization.

Workers get paid a salary, not an hourly wage, in line with an interpretation of hours worked as unobserved, endogenous labor intensity.[5] Brender and Gallo (2009) document that, for the Israeli economy, hours are inelastic to wages, which also motivates a modeling choice of a salary rather than of an hourly wage. There is a wage dispersion, caused by nominal wage rigidity *à la* Calvo (1983). Nevertheless, with big enough families, the distribution of wages is the same across families, with the average wage being denoted by W_t.

Whenever not employed, whether they search or not, family members get direct transfers, $\tau_t^N P_t Z_t$, where $\tau_t^N \in [0,1]$ is an exogenous policy choice and P_t is the Dixit and Stiglitz (1977) price index. Nonparticipants are entitled to these benefits as well as unemployed, which is justified by the assumption that the search effort is unobservable to the policy-maker.[6]

The household's budget constraint is therefore

$$P_t C_t + \frac{1}{1+i_t} B_t = (h_t + u_t)\,\tau_t^N P_t Z_t + l_t \left(1 - \tau_t^W\right) W_t + B_{t-1} + \Xi_t.$$

Here, i_t is a risk-free nominal interest rate; B_t denotes risk-free bonds holdings; the variable $\tau_t^W \in [0,1]$ is the exogenous income tax rate; and Ξ_t collects lump sum taxes, transfers and dividends.

2.2.2 Consumption and Saving

Let λ_t / Z_t denote the marginal utility from consumption, obtained by the first order condition with respect to C_t, so that

$$\frac{\lambda_t}{Z_t} \equiv \frac{\varepsilon_t^{\beta} \varepsilon_t^{C}}{Z_t} \left(\frac{C_t}{Z_t} - \kappa^c \frac{C_{t-1}}{Z_{t-1}} \right)^{-\sigma_c}, \tag{11}$$

where we use $C_t = C_t^{exo}$, based on the representative household assumption.

[5] Treating working hours as unobservable is partly motivated by a seemingly poor measurement of this variable.

[6] Blondal and Pearson (1995) stress that, "In half of OECD countries the number of recipients of these benefits exceeds the official numbers of unemployment". Some of those papers considering optimal schemes for unemployment systems are concerned with the moral hazards issue related to the search effort that cannot be monitored. See, for instance, Pavoni (2007), Hopenhayn and Nicolini (1997), and Coles (2006).

Now, defining $\Lambda_{t,t+k} \equiv E_t \frac{\lambda_{t+k}/Z_{t+k}}{\lambda_t/Z_t}$, the standard Euler condition is obtained by a first order condition with respect to B_t, so that

$$\beta E_t \Lambda_{t,t+1} \frac{(1+i_t)}{\Pi_{t+1}} = 1, \tag{12}$$

where Π_t denotes the gross inflation rate.

Optimal allocation across goods leads to[7]

$$C_{j,t} = C_t \left(\frac{p_{j,t}}{P_t} \right)^{-\frac{\mu_t}{\mu_t - 1}}, \tag{13}$$

with the corresponding CPI being

$$P_t = \left[\int_0^1 p_{j,t}^{\frac{1}{1-\mu_t}} \cdot dj \right]^{1-\mu_t}. \tag{14}$$

2.2.3 Labor Supply

Based on the utility function (10), the marginal disutility from labor market activity, in terms of the consumption composite, is

$$MRS_t^{\widetilde{l}} = Z_t \frac{\varepsilon_t^{\beta} \varepsilon_t^{l} \chi^{l} \widetilde{l}_t^{\sigma_l}}{\lambda_t}.$$

This is essentially the marginal rate of substitution (MRS) between home activity and consumption.

Further, using $h_t = h_t^{exo}$ (based on the large-number assumption made earlier), and substituting the population constraint (6) into the labor market activity (9), the MRS between search effort and consumption is

$$MRS_t^{u} = MRS_t^{\widetilde{l}} \cdot \frac{\partial \widetilde{l}_t}{\partial u_t} = MRS_t^{\widetilde{l}} \cdot \chi^{u} \left\{ \varepsilon_t^{u} - \kappa^{h} \left(h_t - h_{t-1} \right) \right\},$$

and the MRS between employment and consumption is

$$MRS_t^{l} = MRS_t^{\widetilde{l}} \cdot \frac{\partial \widetilde{l}_t}{\partial l_t} = MRS_t^{\widetilde{l}} \cdot \left\{ \frac{n_t^{1+\sigma_n}}{1+\sigma_n} - \kappa^{h} \left(h_t - h_{t-1} \right) \right\}.$$

[7] Thus, period $t's$ elasticity of substitution across differentiated goods is $\frac{\mu_t}{\mu_t - 1}$.

The last one is a function of the actual number of hours worked, n_t. In turn, the latter, as already noted, is identical for all employees, as discussed in Subsection 2.6.1 below.

$\chi^u < 1$ assures that $MRS_t^l > MRS_t^u$. Since this condition is satisfied in the steady state, under small enough shocks it is satisfied outside the steady state as well.

Finally, the MRS between hours worked and consumption,

$$MRS_t^n = MRS_t^{\widetilde{l}} \cdot \frac{\partial \widetilde{l}_t}{\partial n_t} = MRS_t^{\widetilde{l}} \cdot \chi^l l_t n_t^{\sigma_n},$$

is proportional to the employment level of the household.

These MRSs are only *direct* contributions (of market activity to the households' utility). As such, they do not reflect the values of the employment states. The latter also account for the *indirect* contributions, which include state-dependent incomes and continuation values.

As stated, wages are negotiated in a Calvo (1983) fashion. But only after a job match is formed it is exogenously determined whether the wage contract is negotiated in a Nash bargaining process, or simply determined based on the wage norm from the previous period, adjusted as in Eq. (7).

We now continue with computing two relevant value functions, for each one of the two predetermined states. As noted, $W_{n,t}$ denotes the wage level of contract n, where $n \in [0,1]$ is a contract index.[8] Thus, in terms of the consumption good, the value for the household from the marginal working family member is

$$\begin{aligned} V_t^E(W_{n,t}) &= \left(1-\tau_t^W\right)\frac{W_{n,t}}{P_t} - MRS_t^l + \\ &\beta E_t\left\{\Lambda_{t,t+1}\left[(1-\delta)\left(\begin{array}{c}\zeta_w V_{t+1}^E\left(gr_{t+1}^W W_{n,t}\right)+\\ (1-\zeta_w)V_{t+1}^E\left(\widetilde{W}_{t+1}\right)\end{array}\right)+\delta V_{t+1}^N\right]\right\}, \end{aligned} \tag{15}$$

and that from a nonworking family member is:

$$\begin{aligned} V_t^N &= \tau_t^N Z_t + \beta E_t\left\{\Lambda_{t,t+1}\cdot V_{t+1}^N\right\} - MRS_t^u + \\ &\max\left[MRS_t^u,\ \beta x_t E_t\left\{\Lambda_{t,t+1}\left[\left(\begin{array}{c}\zeta_w V_{t+1}^E\left(gr_{t+1}^W W_t\right)+\\ (1-\zeta_w)V_{t+1}^E\left(\widetilde{W}_{t+1}\right)\end{array}\right)-V_{t+1}^N\right]\right\}\right]. \end{aligned} \tag{16}$$

It is then straightforward that there is a cutoff value for MRS_t^u, for which the household is indifferent between the two choices reflected by the maximization operator of the value of

[8] Not to be confused with n_t, which denotes hours worked.

nonworking family member (16). The equilibrium result would therefore be this threshold level,

$$\begin{aligned} MRS_t^u &\leq x_t \beta E_t \left[\Lambda_{t,t+1} \cdot S_{t+1}^E\right] = \\ & x_t \beta E_t \left[\Lambda_{t,t+1} \left(\begin{array}{c} \zeta_w S_{t+1}^E \left(gr_{t+1}^W W_t\right) + \\ \left(1-\zeta_w\right) S_{t+1}^E \left(\widetilde{W}_{t+1}\right) \end{array} \right)\right], \end{aligned} \tag{17}$$

where $S_t^E(W_{n,t}) \equiv V_t^E(W_{n,t}) - V_t^N$ is the surplus from employment and $S_t^E \equiv \int_0^1 S_t^E(W_{n,t}) \cdot dn$. Thus, in equilibrium, households choose the participation level such that the utility loss involved is equal to the discounted value of the expected surplus from participation. Due to the search and matching frictions outlined in Subsection 2.1, the threshold (17) reflects a forward-looking decision.[9]

Thus, all individuals who do not belong to the *predetermined* employment pool, l_t, have to be allocated between the two other pools, h_t and u_t. Unless $\chi^u = 0$, we get $\partial MRS_t^u / \partial u_t > 0$. Therefore, an individual moved from h_t to u_t increases the left side of the threshold condition (17). Although taken as exogenous by each household, the right hand side of (17) is reduced with each individual moving from from h_t to u_t, due to externalities in the labor market. Thus, as long as $\chi^u > 0$, the optimality condition (17) holds with equality. The private case of full participation is $\chi^u = 0 \rightarrow MRS_t^u = 0, \forall t$, which means that condition (17) holds with inequality. In either case, whether (17) holds with equality or not, the value of a nonemployed family member (16) can be reduced to

$$\begin{aligned} V_t^N &= \tau_t^N Z_t - MRS_t^u \\ & +\beta E_t \left\{\Lambda_{t,t+1} \left[x_t \left(\begin{array}{c} \zeta_w V_{t+1}^E \left(gr_{t+1}^W W_t\right) + \\ \left(1-\zeta_w\right) V_{t+1}^E \left(\widetilde{W}_{t+1}\right) \end{array} \right) + \left(1-x_t\right) V_{t+1}^N \right]\right\}. \end{aligned} \tag{18}$$

Due to the homogeneity *across* households, each one chooses the same threshold and, since households are big enough, there are similar distributions of states and choices across households. However, there is heterogeneity in the state of members *within* each household. After the threshold is determined, the household essentially decides what share of its non-working members would search and what share would be engaged in home activity. Once

[9] See Binyamini and Larom (2012) for a discussion.

this decision is made, the ascription of each individual to one of the two pools (participants, u_t, and nonparticipants, h_t) is arbitrary.

Substitute the value functions, (15) and (18), into the definition of the surplus from employment. Then, using the participation decision (17 with equality), the surplus from employment can be rewritten as

$$S_t^E(W_{n,t}) = \frac{\left(1-\tau_t^W\right)W_{n,t}}{P_t} - \tau_t^N Z_t - MRS_t^l + \beta E_t\left[\Lambda_{t,t+1}\left\{(1-\delta)\left(\begin{array}{c}(1-\zeta_w)\,S_{t+1}^E\left(\widetilde{W}_{t+1}\right)+\\ \zeta_w S_{t+1}^E\left(gr_{t+1}^W W_{n,t}\right)\end{array}\right)\right\}\right]. \tag{19}$$

The last expression is going to be used by the Nash bargaining over wages and hours worked (§2.6). However, for an interior solution under the cases $\chi^u > 0 \rightarrow MRS_t^u > 0 \rightarrow h_t > 0$, we shall integrate (19) over $n \in [0,1]$ and use the cutoff rule (17) to substitute for S_{t+1}^E:

$$S_t^E = \frac{\left(1-\tau_t^W\right)W_t}{P_t} - \tau_t^N Z_t - MRS_t^l + \frac{(1-\delta)}{x_t}MRS_t^u.$$

Moving one period ahead and substituting back into (17), we get the forward-looking participation decision, without any explicit reference to surpluses[10]:

$$MRS_t^u = x_t\beta E_t\left\{\Lambda_{t,t+1}\cdot\left[\frac{\left(1-\tau_{t+1}^W\right)W_{t+1}}{P_{t+1}} - \tau_{t+1}^N Z_{t+1} - MRS_{t+1}^l + \frac{(1-\delta)}{x_{t+1}}MRS_{t+1}^u\right]\right\}. \tag{20}$$

This is relevant only if (17) is satisfied with equality, in which case we have an interior solution for h_t. Otherwise, for the private case $\chi^u = 0 \rightarrow MRS_t^u = 0$, we get a corner solution and the forward-looking condition (20) is replaced by $h_t = 0$ (in which case, u_t and l_t become predetermined).

2.3 Producers

2.3.1 Structure

An intermediate good is produced competitively. Each producer is assumed to be big enough so that the distribution of wage contracts within each producing firm is the same,

[10] This condition, in its steady state version, is also going to be useful for calibrating some of the utility function parameters.

and therefore identical to the distribution in the economy as a whole.

Facing the same technology, each producer also employs the same number of workers. Therefore, the technology shared by all producers in this competitive sector can be expressed as a function of l_t and n_t, the aggregate employment rate and number of hours worked:

$$Y_t^I = Z_t \varepsilon_t^Y \eta^Y \left(l_t n_t\right)^\alpha . \tag{21}$$

Here, the log of the exogenous shock ε_t^Y is a stationary process. While l_t is predetermined, and the extensive margins are therefore adjusted only with a lag, n_t is not and therefore enables an on-impact adjustment of the intensive margins.

As discussed in Subsection 2.6.1 below, each producer has the same amount of hours worked per employee, n_t.

Thus, the marginal productivity of the extensive margin is

$$MPL_t = \alpha \frac{Y_t^I}{l_t},$$

whereas that of the intensive margin is

$$MPN_t = \alpha \frac{Y_t^I}{n_t}.$$

Considering the symmetry across producers, they all post the same number of vacancies each period, meaning that the rate of vacancies, v_t, is the same across producers.

Hiring workers involves a cost. Following others, such as Thomas (2008), this is referred to as a managers-utility cost, which is therefore absent from the clearing condition in the goods market presented below.[11] Expressed in terms of C_t, the composite good, this cost is

$$VC_t = Z_t \varepsilon_t^v \chi^v \frac{\left(\frac{\varphi^v v_t + (1-\varphi^v) q_t \cdot v_t}{l_t}\right)^{1+\kappa^v}}{1+\kappa^v}, \tag{22}$$

where ε_t^v is a hiring cost shock, the log of which follows a stationary process. The functional form (22) is justified by Yashiv (2006).

[11] Alternatively, but similarly motivated, Christoffel et al. (2009) treat this cost as a tax that is rebated lump-sum to the households.

Here, q_t and l_t are taken as exogenously given by the individual firm. It follows that the marginal cost involved in vacancy posting, in real terms, is

$$MVC_t = Z_t \varepsilon_t^v \chi^v \left(\frac{\varphi^v + (1 - \varphi^v) q_t}{l_t} \right)^{1+\kappa^v} v_t^{\kappa^v}. \tag{23}$$

2.3.2 Vacancy Postings

Let P_t^I be the competitive price of the intermediate good. The value of a filled job for the firm is therefore

$$\begin{aligned} J_t^F (W_{n,t}) &= \frac{P_t^I}{P_t} MPL_t - \frac{W_{n,t}}{P_t} + \\ & \beta E_t \left\{ \Lambda_{t,t+1} \left[(1-\delta) \left(\begin{array}{c} \zeta_w J_{t+1}^F \left(gr_{t+1}^W W_{n,t} \right) + \\ (1-\zeta_w) J_{t+1}^F \left(\widetilde{W}_{t+1} \right) \end{array} \right) + \delta J_{t+1}^V \right] \right\}, \end{aligned} \tag{24}$$

where

$$J_t^V = -MVC_t + \beta E_t \left\{ \Lambda_{t,t+1} \left[q_t \left(\begin{array}{c} \zeta_w J_{t+1}^F \left(gr_{t+1}^W W_t \right) + \\ (1-\zeta_w) J_{t+1}^F \left(\widetilde{W}_{t+1} \right) \end{array} \right) + (1-q_t) J_{t+1}^V \right] \right\} \tag{25}$$

is the value of a posted vacancy.

Since vacancy postings involve no frictions, and since there is a free entry of firms, the value of an open vacancy is pushed toward zero, so that $J_t^V = 0, \ \forall t$. Hence, the firm's surplus from an existing employment relationship is $S_t^F (W_{n,t}) \equiv J_t^F (W_{n,t}) - J_t^V = J_t^F (W_{n,t})$. Substituting into Eq. (25), we get the standard equilibrium condition related to posted vacancies. That is, at the margin, the cost of a vacancy equals its expected benefit:

$$\begin{aligned} MVC_t &= q_t \beta E_t \left[\Lambda_{t,t+1} \cdot S_{t+1}^F \right] \\ &= q_t \beta E_t \left[\Lambda_{t,t+1} \left(\begin{array}{c} \zeta_w J_{t+1}^F \left(gr_{t+1}^W W_t \right) + \\ (1-\zeta_w) J_{t+1}^F \left(\widetilde{W}_{t+1} \right) \end{array} \right) \right], \end{aligned} \tag{26}$$

where $S_t^F \equiv \int_0^1 S_t^F (W_{n,t}) \cdot dn$. This is similar to the equivalent optimal threshold for the household participation decision (17).

Now, from the value functions (24-25) we can see that the firm's surplus from an existing employment relationship is given by

$$S_t^F(W_{n,t}) = \frac{P_t^I}{P_t}MPL_t - \frac{W_{n,t}}{P_t} + MVC_t + \beta E_t \left\{ \Lambda_{t,t+1} \left[\begin{array}{c} (1-\delta)\left(\begin{array}{c} (1-\zeta_w) S_{t+1}^F\left(\widetilde{W}_{t+1}\right) + \\ \zeta_w S_{t+1}^F\left(gr_{t+1}^W W_{n,t}\right) \end{array} \right) \\ - \\ q_t \underbrace{\left[(1-\zeta_w) S_{t+1}^F\left(\widetilde{W}_{t+1}\right) + \zeta_w S_{t+1}^F\left(gr_{t+1}^W W_t\right)\right]}_{E_t S_{t+1}^F} \end{array} \right] \right\}.$$

That is, the firm's surplus from an established employment relationship consists of two components. First, the worker's direct contribution to the firm, that is, his productivity net of his wage. Second, the continuation value from saving in vacancy cost this period. The latter, in turn, consists of two components: saving of MVC_t in the present period, and the surplus conditional on job survival for the next period (which has the probability $1-\delta$) net of that surplus conditional on a match in the following period (which has the probability q_t).

As in the case of households, this can be simplified by substituting the vacancy posting decision (26) in, so that

$$S_t^F(W_{n,t}) = \frac{P_t^I}{P_t}MPL_t - \frac{W_{n,t}}{P_t} + \beta E_t \left\{ \Lambda_{t,t+1}(1-\delta) \left(\begin{array}{c} (1-\zeta_w) S_{t+1}^F\left(\widetilde{W}_{t+1}\right) + \\ \zeta_w S_{t+1}^F\left(gr_{t+1}^W W_{n,t}\right) \end{array} \right) \right\}. \quad (27)$$

Integrating over all possible values of $n \in [0,1]$, and substituting (26) in, we get

$$S_t^F = \frac{P_t^I}{P_t}MPL_t - \frac{W_{n,t}}{P_t} + \frac{(1-\delta)}{q_t}MVC_t.$$

Then, taking one period forward and substituting into (26), we can express the optimal vacancy posting condition without any explicit reference to S_t^F:

$$MVC_t = q_t \beta E_t \left[\Lambda_{t,t+1} \left(\frac{P_{t+1}^I}{P_{t+1}} MPL_{t+1} - \frac{W_{t+1}}{P_{t+1}} + \frac{(1-\delta)}{q_{t+1}} MVC_{t+1} \right) \right]. \quad (28)$$

Here, again, this is similar to the equivalent participation decision made by the household (20).

2.4 Retailers

Retailers are treated here as in the standard NK literature. They buy homogenous intermediate goods, differentiate them costlessly, and supply them in a monopolistic competitive market. Each differentiated good, and the associated retailer, are indexed by $j \in [0,1]$. The demand for $c_{j,t}$, the differentiated good supplied by retailer j, is given by Eq. (13).

Using the Calvo (1983) setup to model price rigidity, each retailer faces an exogenous probability, $(1-\zeta_p) \in (0,1)$, of receiving an idiosyncratic price update signal. This probability depends neither upon the price level, nor upon the time elapsed since the last update. Whenever receiving the price update signal, a retailer reoptimizes its price level. With probability ζ_p, a signal is not received and the retailer adjusts its price level using the following rule of thumb:

$$p_{j,t}^{ind} = \Pi_t^{tar(1-\gamma)} \cdot \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma} \cdot p_{j,t-1}, \tag{29}$$

where $\gamma \in [0,1]$.

Thus, using the law of large numbers together with the optimal CPI given by Eq. (14), the aggregate price level is

$$P_t = \left\{ \zeta_p \left[\Pi_t^{tar(1-\gamma)} \cdot \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma} \cdot P_{t-1} \right]^{\frac{1}{1-\mu_t}} + \left(1-\zeta_p\right) \widetilde{P}_t^{\frac{1}{1-\mu_t}} \right\}^{1-\mu_t},$$

where $\widetilde{P}_t$ is the optimal price level set by all retailers that receive idiosyncratic price update signals at date t.

Thus, whenever reoptimizing, all reoptimizing retailers choose the same price level, $\widetilde{P}_t$, so as to maximize the part of the expected profits flow that is influenced from period's t decision,

$$\underset{\widetilde{p}_t}{Max}\ E_t \sum_{s=0}^{\infty} \frac{\lambda_{t+s}/Z_{t+s}}{\lambda_t/Z_t} \cdot \beta^s \cdot \frac{P_t}{P_{t+s}} \cdot \zeta_p^s \left[\underbrace{\left(\Pi_t^{tar^s}\right)^{(1-\gamma)} \cdot \left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\gamma} \cdot \widetilde{P}_t \cdot C_{j,t+s}}_{total\ revenue\ at\ period\ t+s} - \underbrace{P_{t+s}^I \cdot C_{j,t+s}}_{total\ \cos t} \right].$$

In the last expression we used the Euler condition (12) to substitute for the discount factor and the rule of thumb (29). The discount factor also consists of ζ_p^s, the probability that the price survives to period $t+s$.

Using Eq. (13) to substitute for $C_{j,t+s}$, and optimizing with respect to $\widetilde{p}_t$, we get the following standard NK pricing rule[12]:

$$E_t \sum_{s=0}^{\infty} \frac{\lambda_{t+s}}{\lambda_t} \cdot \beta^s \cdot \zeta_p^s \cdot \frac{C_{j,t+s}/Z_{t+s}}{\mu_{t+s}-1} \left[\frac{\left(\Pi_t^{tar^s}\right)^{(1-\gamma)} \cdot \left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\gamma} \cdot \widetilde{P}_t}{P_{t+s}} - \mu_{t+s} \cdot \frac{P^I_{t+s}}{P_{t+s}} \right] = 0, \quad (30)$$

from which it can be seen that μ_t ends up being a (time varying) optimal price markup.

2.5 Authorities

2.5.1 Fiscal Policy

Essentially, exogenous transfers and tax rates follow a unit root process. However, in order to avoid unit root issues, their inertia is set to $0.99 < 1$, so that

$$\tau_t^W = \left(\tau^W\right)^{(1-0.99)} \left(\tau_{t-1}^W\right)^{0.99} \varepsilon_t^{\tau^W} \quad (31)$$

and

$$\tau_t^N = \left(\tau^N\right)^{(1-0.99)} \left(\tau_{t-1}^N\right)^{0.99} \varepsilon_t^{\tau^N}. \quad (32)$$

For simplicity, but without loss of generality, we assume that the government runs a balanced budget, so that

$$(h_t + u_t)\, \tau_t^N Z_t P_t = \tau_t^W W_t l_t + LST_t,$$

where LST_t denotes lump sum taxes.[13]

2.5.2 Monetary Policy

Define the benchmark interest rate as

$$1 + i_t^{Benchmark} = E_t \left\{ \Pi_t^{tar} \frac{1}{\beta} gr_{t+1}^Z \frac{\varepsilon_t^\beta}{\varepsilon_{t+1}^\beta} \frac{\varepsilon_t^C}{\varepsilon_{t+1}^C} \left(\frac{\varepsilon_t^Y}{\varepsilon_{t+1}^Y} \frac{\varepsilon_{t+1}^{NA}}{\varepsilon_t^{NA}} \right)^{-\sigma_c} \right\} \quad (33)$$

(ε_t^{NA} is introduced below, and Appendix A explains the motivation for this definition).

[12] See, for instance, Binyamini (2007) for the familiar interpretation.

[13] It is possible to have the private case $LST_t < 0$, which implies positive net transfers.

It is then assumed that the central banks follows a monetary policy rule of the following form,

$$(1+i_t) = \varepsilon_t^i \left(1+i_{t-1}\right)^{\rho^i} \left[\left(1+i_t^{Benchmark}\right) \left(\frac{\Pi_{t+1}^4}{\Pi_t^{tar}}\right)^{\varphi^\pi} \left(\frac{Y_t^I/Z_t}{\left(\overline{Y^I/Z}\right)}\right)^{\varphi^y} \right]^{1-\rho^i}, \tag{34}$$

where Π_t^{tar} denotes the (gross) inflation target, $\Pi_t^4 \equiv \left(\Pi_t \cdot \Pi_{t-1} \cdot \Pi_{t-2} \cdot \Pi_{t-3}\right)^{0.25}$, and ε_t^i is an exogenous monetary policy shock.

Finally, the gross inflation target follows a unit root process,

$$\Pi_t^{tar} = \overline{\Pi}^{(1-0.99)} \cdot \left(\Pi_{t-1}^{tar}\right)^{0.99} \cdot \varepsilon_t^{tar}. \tag{35}$$

2.6 Equilibrium

2.6.1 Hours Worked and Pricing Decision

Following the Barro (1977) critique, and similar to Thomas (2008) and Trigari (2006), hours worked are determined in a privately efficient way, so as to maximize the joint surplus of the worker and the firm.[14] This is done on a period-by-period basis, which leads to the following Pareto efficient allocation of hours worked, where

$$\frac{P_t^I}{P_t} \cdot MPN_t = MRS_t^n. \tag{36}$$

Efficient bargaining therefore implies that the marginal contribution to the profit of the firm, the left side in Eq. 36, equals the subjective cost of the marginal hour to the worker, the right side. As discussed by Christoffel et al. (2009), we can see here that the wage is not directly allocative for hours, as the condition (36) determines hours worked without any direct reference to the wage level.

As such, in addition to addressing the Barro (1977) critique, efficient bargaining is also helpful in simplifying the model—the wage dispersion, characterizing the Calvo (1983) style staggered wages assumed here, is not mapped into a dispersion of hours worked. As a

[14] Actually, the same condition is also obtained as a result of a period-by-period Nash bargaining over hours.

result, thanks to efficient bargaining, despite the wage dispersion, all firms can be treated symmetrically.

Under general equilibrium, the condition (36) reflects bidirectional relationships between two endogenous variables that simultaneously drive each other—price and hours (the latter being determined by the ratio MRS_t^n/MPN_t). Nevertheless, a distinction between micro and macro perspectives can be made here.

From a micro perspective, the equilibrium condition (36) adjusts the intensive margins in an intermediate firm active in a competitive market. This micro perspective is based on the firm's point of view, for which the price P_t^I/P_t is exogenously determined. Thus, from a micro perspective, the intermediate-good firms view the condition (36) as determining the intensive margins for an exogenously given price level.

From a macro perspective, however, it is the other way around. At the macro level, output is demand-determined (due to price rigidity of the final good). Thus, from a macro perspective, *hours worked* are demand-determined. This means that the condition (36) can be also viewed as determining the relative price of the intermediate good, P_t^I/P_t, for a given level of hours worked. Now, P_t^I/P_t is actually the marginal cost that drives inflation in the Phillips curve of the final goods (30). Accordingly, the efficient-bargaining condition (36) determines the economy's marginal cost. Christoffel et al. (2009) discuss that point, and accordingly emphasize that instead of the wage level, it is MRS_t^n that directly drives the economy's marginal cost.[15] That is, under efficient bargaining it is not the firm's, but rather the worker's, subjective time-value that directly drives inflation. Efficient bargaining therefore removes the direct 'wage channel' through which activity drives the marginal cost.

2.6.2 Wage Bargaining

Nash Bargaining

Workers and firms Nash bargain the wage level so as to share the total economic rent from

[15] In a general equilibrium under classical assumptions, the wage and the MRS_t^n essentially end up being one and the same. Classical models include the well-established result of labor demand that differs from the efficient-bargaining condition (36) only by the wage level that substitutes for MRS_t^n. Similarly, as shown by Christoffel et al. (2009), when the bargaining process is based on 'right to manage' (where the firm is free to choose hours worked at a previously bargained wage), the demand for hours would also follow the classical model, with wage replacing MRS_t^n.

a match, $S_t^E(W_{n,t}) + S_t^F(W_{n,t})$. The Nash bargaining maximizes

$$\max_{W_{n,t}} \left[S_t^E(W_{n,t})\right]^{\varepsilon_t^\psi \cdot \psi} \left[S_t^F(W_{n,t})\right]^{1-\varepsilon_t^\psi \cdot \psi},$$

where $\psi \in (0,1)$ denotes the bargaining power of the household, and ε_t^ψ is an exogenous shock. The implied first-order condition is

$$\varepsilon_t^\psi \cdot \psi \cdot \varpi_t^E \cdot S_t^F(W_{n,t}) = \left(1 - \varepsilon_t^\psi \cdot \psi\right) \cdot \varpi_t^F \cdot S_t^E(W_{n,t}),$$

where $\varpi_t^E \equiv \frac{\partial S_t^E(W_{n,t})}{\partial W_{n,t}} = \frac{\left(1-\tau_t^W\right)}{P_t} + \beta(1-\delta)\zeta_W \cdot gr_{t+1}^W \cdot E_t\left[\Lambda_{t,t+1}\varpi_{t+1}^E\right]$ is the derivative of the household surplus with respect to the bargained wage, $W_{n,t}$, and $\varpi_t^F \equiv -\frac{\partial S_t^F(W_{n,t})}{\partial W_{n,t}} = \frac{1}{P_t} + \beta(1-\delta)\zeta_W \cdot gr_{t+1}^W \cdot E_t\left[\Lambda_{t,t+1}\varpi_{t+1}^F\right]$ is (minus) that of the firm. For simplicity, the last condition can be rewritten as

$$\widetilde{\psi}_t S_t^F(W_{n,t}) = \left(1 - \widetilde{\psi}_t\right) S_t^E(W_{n,t}),$$

where $\widetilde{\psi}_t \equiv \frac{\varepsilon_t^\psi \cdot \psi}{\varepsilon_t^\psi \cdot \psi + \left(1-\varepsilon_t^\psi \cdot \psi\right)\varpi_t^F/\varpi_t^E}$.

Under the private case without income tax, we see that $\tau_t^W = 0 \Longrightarrow \varpi_t^F = \varpi_t^E \Longrightarrow \widetilde{\psi}_t = \varepsilon_t^\psi \cdot \psi$.

In addition, under a constant tax rate, we get $\varpi_t^F/\varpi_t^E \rightarrow 1/\left(1-\tau^W\right) \Longrightarrow \widetilde{\psi}_t \rightarrow \frac{\varepsilon_t^\psi \cdot \psi}{\varepsilon_t^\psi \cdot \psi + \left(1-\varepsilon_t^\psi \cdot \psi\right)/(1-\tau^W)} \in (0,1)$; $\forall t$. Thus, if we assume that both the tax rate and the (log of the) shock to bargaining power follow a unit root process[16], such that $E_t\tau_{t+k}^W = \tau_t^W$ and $E_t\varepsilon_{t+k}^\psi = \varepsilon_t^\psi$, we get $E_t\widetilde{\psi}_{t+k} = \widetilde{\psi}_t$.

Thus, the result of the Nash bargaining, which is expected to take place k periods ahead, satisfies

$$\varepsilon_t^\psi \cdot \psi \cdot E_t S_{t+k}^F(W_{n,t+k}) = \frac{\left(1 - \varepsilon_t^\psi \cdot \psi\right)}{\left(1-\tau_t^W\right)} E_t S_{t+k}^E(W_{n,t+k}). \tag{37}$$

This is also the result in the case of flexible wages, even if τ_t^W and ε_t^ψ do not satisfy a unit root process, since in that case we have $\zeta_W = 0 \Longrightarrow \widetilde{\psi}_t = \frac{\varepsilon_t^\psi \cdot \psi}{\varepsilon_t^\psi \cdot \psi + \left(1-\varepsilon_t^\psi \cdot \psi\right)/\left(1-\tau_t^W\right)}$.

[16] Here, a unit root process for the shock to bargaining power is assumed for simplicity. But it can also be justified if we view a shift in the bargaining power as a slowly evolving, structural process. Among other things, such a process can reflect a change in the labor market composition.

Flexible wage benchmark

Before approaching the specification of sticky wages, it is useful to begin with a benchmark of flexible ones, that is, the equilibrium wage under the case in which wages are negotiated on a period-by-period basis. Define

$$\frac{\underline{W}_t^{FlexNet}}{P_t} \equiv \tau_t^N Z_t + MRS_t^l \tag{38}$$

and

$$\frac{\overline{W}_t^{Flex}}{P_t} \equiv \frac{P_t^I}{P_t} MPL_t. \tag{39}$$

Now, under flexible wages, where $\zeta_W = 0$, the surpluses (19) and (27) are simplified to the private cases

$$S_t^E\left(W_t^{Flex}\right) = \frac{\left(1-\tau_t^W\right) W_t^{Flex}}{P_t} - \frac{\underline{W}_t^{FlexNet}}{P_t} + \beta E_t\left[\Lambda_{t,t+1}\left(1-\delta\right) S_{t+1}^E\left(W_{t+1}^{Flex}\right)\right]$$

and

$$S_t^F\left(W_t^{Flex}\right) = \frac{\overline{W}_t^{Flex}}{P_t} - \frac{W_t^{Flex}}{P_t} + \beta E_t\left[\Lambda_{t,t+1}\left(1-\delta\right) S_{t+1}^F\left(W_{t+1}^{Flex}\right)\right].$$

These are substituted into the Nash bargaining result (37), treating $W_{n,t} = W_t^{Flex}$ for every n, to finally obtain the equilibrium wage under this case:

$$W_t^{Flex} = \varepsilon_t^{\psi} \cdot \psi \cdot \overline{W}_t^{Flex} + \left(1 - \varepsilon_t^{\psi} \cdot \psi\right) \underline{W}_t^{Flex}, \tag{40}$$

where

$$\underline{W}_t^{Flex} \equiv \frac{1}{\left(1-\tau_t^W\right)} \underline{W}_t^{FlexNet}.$$

Nominal wage rigidity

We now turn to the solution of the Nash bargaining problem under rigidity of nominal wages. With large enough firms and households, all contracts that are Nash bargained at period t end up setting the same wage level, $\widetilde{W}_t$.

It is then useful to use recursive substitution for manipulating the surpluses of the household and the firm, Eqs. (19) and (27) respectively. We thus obtain

$$
\begin{aligned}
S_t^E\left(W_{n,t}\right) &= E_t \sum_{k=0}^{\infty} \left\{ \left[\beta\left(1-\delta\right)\zeta_w\right]^k \frac{\Lambda_{t,t+k}}{P_{t+k}} \left[\left(1-\tau_{t+k}^W\right)\left(gr_{t+1}^W\right)^k W_{n,t} - \underline{W}_{t+k}^{FlexNet} \right] \right\} \\
&+ \frac{\left(1-\zeta_w\right)}{\zeta_w} E_t \sum_{k=1}^{\infty} \left\{ \left[\beta\left(1-\delta\right)\zeta_w\right]^k \Lambda_{t,t+k} S_{t+k}^E\left(\widetilde{W}_{t+k}\right) \right\}
\end{aligned}
$$

and

$$
\begin{aligned}
S_t^F\left(W_{n,t}\right) &= E_t \sum_{k=0}^{\infty} \left\{ \left[\beta\left(1-\delta\right)\zeta_w\right]^k \frac{\Lambda_{t,t+k}}{P_{t+k}} \left[\overline{W}_{t+k}^{Flex} - \left(gr_{t+1}^W\right)^k W_{n,t} \right] \right\} \\
&+ \frac{\left(1-\zeta_w\right)}{\zeta_w} E_t \sum_{k=1}^{\infty} \left\{ \left[\beta\left(1-\delta\right)\zeta_w\right]^k \Lambda_{t,t+k} S_{t+k}^F\left(\widetilde{W}_{t+k}\right) \right\}.
\end{aligned}
$$

Substituting these expressions for $S_t^E\left(W_{n,t}\right)$ and $S_t^F\left(W_{n,t}\right)$ into the solution for the Nash bargaining problem under the general case of nominal wage rigidity (37), we get

$$
E_t \sum_{k=0}^{\infty} \left\{ \left[\beta \Lambda_{t,t+k}\left(1-\delta\right)\zeta_w\right]^k \left[\frac{\left(gr_{t+1}^W\right)^k \widetilde{W}_t}{P_{t+k}} - \frac{W_{t+k}^{Flex}}{P_{t+k}} \right] \right\} = 0. \tag{41}
$$

That is, whenever Nash-bargained, the equilibrium wage agreed upon is a weighted average of future target wages—the equilibrium wages that would have prevailed under the case of flexible wages. In addition to the standard stochastic discount factor $\beta^k \Lambda_{t,t+k}$, the weight of each future target-wage also includes the survival probability of the wage contract, $\left[\zeta_w\left(1-\delta\right)\right]^k$.

A subtle distinction is made by Galí (2010), which should be made here as well: The equilibrium solution of all endogenous variables would be different under flexible wages than under wage rigidity. Therefore, we have to treat the above equations of flexible wages as describing wages that would be observed if set flexibly, but conditional on all other endogenous variables being generated in a rigid-wages environment. That is, the term W_{t+k}^{Flex} in the last expression is a *conditional* flexible wage.

The bargaining set

In order to assure the existence of a bargaining set, it must be verified that $\overline{W}_t \geqslant \underline{W}_t$ for every t. Substituting the calibrated values from Subsection 3.3.1 below shows that the model's steady state indeed satisfies this condition. In addition, shocks have to be small enough so that this condition is not violated outside the steady state either.

In addition, Hall (2005) and Galí (2010), among others, state that employment relationships have to be privately efficient, such that neither party has an incentive to terminate them. This means that shocks also have to be small enough to assure that actual wages don't deviate too often or too persistently from the bargaining set.

2.6.3 Clearing Conditions

Eq. (6) sets an equilibrium condition in the labor market.

In the goods market we have

$$Y_t = \varepsilon_t^{NA} \cdot C_t, \tag{42}$$

where the shock ε_t^{NA} takes the place of National Accounts components that are assumed away for simplicity (investment, government spending, and net export). This shock enables an analysis of a demand shock that is not generated by domestic households.

Finally, although not required for closing the model, in the analysis below it will also be useful to follow the dynamics of the labor share[17],

$$ls_t \equiv \frac{W_t l_t}{P_t Y_t} = \alpha \frac{W_t / P_t}{MPL_t}. \tag{43}$$

2.7 Exogenous Processes and Model Solution

The exogenous shocks in the model follow the process[18]

$$\ln\left(\varepsilon_t^{var}\right) = \rho^{\varepsilon^{var}} \cdot \ln\left(\varepsilon_{t-1}^{var}\right) + \ln\left(\zeta_t^{var}\right); \qquad \ln\left(\zeta_t^{var}\right) \overset{i.i.d.}{\sim} N\left(0, \sigma_{var}^2\right),$$

where $var \in \left\{m,\ \beta,\ C,\ l,\ Z,\ Y,\ v,\ \psi,\ i,\ NA,\ \tau^W,\ \tau^N,\ tar,\ \mu,\ u\right\}$.

Appendix C stationarizes the system around its deterministic trends, based on those of Z_t and P_t; Appendix D solves for its steady state; and Appendix E presents a log-linear

[17] For simplicity, this definition of the labor share refers to the income tax as part of the labor income.

[18] The only exception is the labor supply shock that is assumed to satisfy $\ln\left(\varepsilon_t^l / \varepsilon_{t-1}^l\right) = \rho^{\varepsilon^l} \cdot \ln\left(\varepsilon_{t-1}^l / \varepsilon_{t-2}^l\right) + \ln\left(\zeta_t^l\right)$. Such generalization of the exogenous process was helpful in generating stationarity of the smoothed shocks.

approximation of the dynamic system around this steady state. The reduced form of this log-linearized system is then solved[19], and the solution yields the law of motion for the model's variables. These, using small letters to denote a deviation from the trend, are:

$$\left\{\begin{array}{l} m_{pp,t},\ v_{pp,t},\ \theta_{pp,t},\ x_{pp,t},\ q_{pp,t}, \\ l_{pp,t},\ \widetilde{l}_{pp,t},\ h_{pp,t},\ u_{pp,t},\ \widehat{n}_t, \\ \widehat{gr}_t^Z,\ \widehat{\lambda}_t,\ \widehat{c}_t,\ \widehat{y}_t^I,\ \widehat{y}_t, \\ \widehat{mrs}_t^n,\ \widehat{mrs}_t^{\widetilde{l}},\ \widehat{mrs}_t^u,\ \widehat{mrs}_t^l,\ \widehat{mpl}_t,\ \widehat{mpn}_t, \\ \tau_{pp,t}^W,\ \tau_{pp,t}^N,\ \widehat{\mu}_t,\ \widehat{\pi}_t,\ \widehat{\pi}_t^{tar}, \\ \widehat{vc}_t,\ \widehat{mvc}_t,\ \widehat{ls}_t \\ \widehat{i}_t,\ \widehat{\widetilde{p}}_t,\ \widehat{p}_t^I, \\ \widehat{\widetilde{w}}_t,\ \widehat{w}_t^{Flex},\ \widehat{\overline{w}}_t^{Flex},\ \widehat{\underline{w}}_t^{Flex},\ \widehat{\underline{w}}_t^{FlexNet},\ \widehat{w}_t, \\ \widehat{\varepsilon}_t^m,\ \widehat{\varepsilon}_t^\beta,\ \widehat{\varepsilon}_t^c,\ \widehat{\varepsilon}_t^l,\ \widehat{\varepsilon}_t^Y,\ \widehat{\varepsilon}_t^v,\ \widehat{\varepsilon}_t^\psi,\ \widehat{\varepsilon}_t^i,\ \widehat{\varepsilon}_t^{NA}, \\ \widehat{\varepsilon}_t^{\tau^W},\ \widehat{\varepsilon}_t^{\tau^N},\ \widehat{\varepsilon}_t^{tar},\ \widehat{\varepsilon}_t^\mu,\ \widehat{\varepsilon}_t^z,\ \widehat{\varepsilon}_t^u. \end{array}\right\}.$$

Here, a hat denotes logarithmic deviation, and the subscript *pp* denotes deviation in terms of percentage points. We thus end up having a linear system of 52 variables: 37 endogenous variables and 15 $AR(1)$ shocks (all detailed in Appendix F). The latter, in turn, are driven by the 15 linearized innovations

$$\left\{\widehat{\zeta}_t^m,\ \widehat{\zeta}_t^\beta,\ \widehat{\zeta}_t^c,\ \widehat{\zeta}_t^l,\ \widehat{\zeta}_t^Y,\ \widehat{\zeta}_t^v,\ \widehat{\zeta}_t^\psi,\ \widehat{\zeta}_t^i,\ \widehat{\zeta}_t^{NA},\widehat{\zeta}_t^{\tau^W},\ \widehat{\zeta}_t^{\tau^N},\ \widehat{\zeta}_t^{tar},\ \widehat{\zeta}_t^\mu,\ \widehat{\zeta}_t^z,\ \widehat{\zeta}_t^u.\right\}.$$

3 Estimation

Model parameters that govern the steady state were calibrated using observed first moments. Parameters driving the dynamics of the model were estimated using Bayesian technique. The model was expanded so as to cope with structural trends characterizing the observed data. This section discusses all these issues, and concludes with model evaluation.

3.1 The Data

The calibration and the estimation are both based on quarterly Israeli data for the period 1992:Q1-2019:Q4. The sources of the data are the Israeli Ministry of Industry Trade and

[19] To this end we employ the Dynare toolbox for Matlab. See Adjemian et al. (2011).

Labor (MoITaL),[20] the Israeli Central Bureau of Statistics (CBS), and the Bank of Israel.[21]

Figure 1: The data (1992:Q1-2019:Q4, percents)

The data. Vacancies, untill 1998, are based on a model-consistnent Kalman smoothing.

Figure 1 presents the data, which include 11 observed variables: the growth rate of consumption ΔC^{obs} and output ΔY^{obs} (both in per working-age-population terms), quarterly inflation π^{obs} and its annual target π^{obs}_{tar}, the average monthly nominal wage ΔW^{obs} (rate of change), the interest rate i^{obs}, the rates of income tax τ^{obs}_{W} and of unemployment benefits τ^{obs}_{N}, the percentage-point changes in the rates of participation $\Delta participation^{obs} \equiv 1 - \Delta h$, and of unemployment Δu^{obs}, and, finally, the vacancies rate v^{obs}.

The unemployment rate is provided to the model in terms of its percentage-point change, not as the unemployment level per se. This is so because, during the sample period, the

[20] The Ministry's name until 2013. In 2013, the Ministry's responsibilities were modified, along with its name.

[21] Data on vacancies are only available since 1998 (first, based on the MoITaL survey, and later based on the CBS). As such, for the years 1992-1997, vacancies are treated as an unobserved variable.

Israeli unemployment rate was not stationary (as can be inferred from the series of changes in it, in Figure 1).

Employment l_t is not treated as an observed variable since, with h_t and u_t being essentially observed, l_t is identified based on the equilibrium condition (6). Similarly, with l_t at hand, matchings m_t are identified based on employment's law of motion (5). Thus, m_t is treated as an unobserved variable as well.

Finally, hours worked n_t are treated as an *unobserved* working intensity, which is why they are not included in the estimation data. This is due to the poor quality of the officially published data. However, more importantly, even if accurately measured, hours worked do not properly reflect the true (and endogenously determined) working effort.

3.2 Observation Block

In order to estimate the model's parameters (standard deviations of the shocks, their inertias and the noncalibrated structural parameters) an observation block was added to the linearized model. This observation block includes 11 equations connecting the 11 observed variables (Subsection 3.1 and Figure 1) to their model counterparts. Some of the observed variables are stationary

$$\left\{\pi^{obs},\ \pi_{tar}^{obs},\ v^{obs},\ i^{obs},\ \tau_W^{obs},\ \tau_N^{obs}\ \right\}.$$

However, some are trending and are therefore provided to the model after stationarization

$$\left\{\Delta Y^{obs},\ \Delta C^{obs},\ \Delta W^{obs},\ \Delta participation^{obs},\ \Delta u^{obs}\right\}.$$

The estimation essentially decomposes each observation of these variables into two groups of driving forces—one that consists of the model-based shocks discussed thus far, and another that consists of structural trends yet to be described. The model is therefore included in the set of identifying restrictions based on which unobserved trends are extracted. Those unobserved "out-of-model trends" are estimated as part of the model as a whole, so that those trends are estimated jointly with all other parameters and unobserved shocks. Thus, the model gets the chance to extract relevant information from the comovement of different observed variables. Under prefiltering, in contrast, at least some components of this comovement may potentially be washed out, together with possibly relevant information for the estimation of model parameters and historical shocks.

The structural trends follow an estimated stochastic process of the generic form[22]

$$\Gamma_t^{var} = \left(1 - \rho^{\gamma^{var}}\right)\overline{\Gamma^{var}} + \rho^{\gamma^{var}} \cdot \Gamma_{t-1}^{var} + \widehat{\zeta}^{\gamma^{var}},$$

where Γ_t^{var} represents the trend of the variable var.

These trends are estimated under restrictions that essentially enable treating them as structural trends, rather than as merely idiosyncratic ones. Thus, the model is actually taken to the data after being expanded to cope, at the same time, with structural and cyclical movements of the data. One such restriction is actually a trivial equilibrium condition for the trends of all labor market pools, such that

$$\Gamma_t^u + \Gamma_t^l + \Gamma_t^h = 0. \tag{44}$$

Another restriction connects those trends to output and consumption, as discussed by the following.

A generic structure of an observation equation, to an observed variable var, takes the form

$$\Delta \ln\left(var_t^{obs}\right) = \widehat{var}_t - \widehat{var}_{t-1} + \Gamma_t^{var} + \mathbb{I}_{C,Y} \cdot \left(gr_t^Z - 1 + \Gamma_t^l\right) + \mathbb{I}_{Y,W,\tau^W,\tau^N} \cdot \widehat{\zeta}^{mERR^{var}}. \tag{45}$$

The first indicator function in the measurement equation (45) is

$$\mathbb{I}_{C,Y} = \begin{cases} 1 \; if \;\; var \in \{C,\ Y\} \\ 0 \qquad\quad otherwise, \end{cases}$$

such that the observed consumption and output are also driven by the structural trends of technology and employment, gr_t^Z and Γ_t^l respectively.

The second indicator function in the measurement equation (45) is

$$\mathbb{I}_{Y,W,\tau^W,\tau^N} \begin{cases} 1 \; if \;\; var \in \left\{Y, W, \tau^W, \tau^N\right\} \\ 0 \qquad\qquad\qquad otherwise, \end{cases}$$

[22] For a private case of a stationary observed variable, the long-run value is assumed to be $\overline{\Gamma^{var}} = 0$ and the standard error of the shock $\widehat{\zeta}^{\gamma^{var}}$ is simply turned off.

such that a measurement error is assumed to characterize some of the observed variables. The measurement errors follow an estimated $AR(1)$ process, with the degree of inertia denoted as $\rho^{mERRvar}$. Applied to the wage, rather than a measurement error per se, Eq. (45) can also be interpreted as reflecting a compositional shift related to the business cycle. Such a compositional shift is motivated by empirical evidence, suggesting that labor skills are not uniformly represented by business-cycle-related flows of the labor market.

3.3 Calibration

3.3.1 Calibrated Parameters

The calibration is based on sample averages of observed variables, which are referred to as steady-state values.[23] First, the model is stationarized, using normalization of required equations by the technology and price trends, Z_t and P_t respectively (see also Subsection 2.7. Appendix C presents a stationarized version of the model, and Appendix D solves for its steady state). In what follows, a steady-state value is denoted by a small letter with a bar. In line with the model's definitions, and in order to assure consistency across variables, labor market pools are expressed as ratios of the population, where "population" refers to the prime working ages (ages 25-64). The steady-state values of variables calibrated based on observable first moments are reported by Table 1. The results for the calibrated parameters are reported in Table 2.

The average rate of nonparticipation within the prime working-age population in Israel during the sample period is $\overline{h} = 24.5\%$. Substituting this value into the population normalization (6), together with the average unemployment rate $\overline{u} = 5.9\%$, yields the share of the working population, $\overline{l} = 69.6\%$.

Based on separations reported by the MoITaL data, the separation rate is calibrated at $\delta = 7\%$. Substituting those values, of $\overline{l}$ and δ, into a steady-state version of the employment law of motion (5), we obtain the ratio of matches to working-age population, $\overline{m} = 4.9\%$. It then follows that $\overline{x} = 82.6\%$, using the definition of the job-finding rate (3). Since the model does not allow for on-the-job search, this probability is likely to have an upward

[23] The calibration is based on quarterly observations from 1992 to 2017.

Table 1: Sample average ratios

Variable	Description	Value*
$\overline{h}$	Nonparticipation	24.5%
$\overline{u}$	Unemployment	5.9%
$\overline{l}$	Working population	69.6%
$\overline{v}$	Vacancies	6.6%
$\overline{m}$	Matching	4.9%
$\overline{x}$	Job finding rate	82.6%
$\overline{q}$	Vacancy filling rate	73.8%
$\overline{\theta}$	Labor market tightness	1.12
$\overline{w}$	Labor share	62.5%
$\overline{n}$	Working hours (normalization)	1
$\overline{c}$	Consumption (normalization)	1
$\overline{y}$	Final output (normalization)	1
$\overline{y}^I$	Intermediate goods (normalization)	1

* Ratios with respect to prime working age population.

biased.

The long-run growth of the nonparticipation and unemployment rates are $\overline{\Gamma}^h = -0.001$ and $\overline{\Gamma}^u = -0.0002$, respectively, based on filtered trends of their observed time series.

The average vacancies rate is $\overline{v} = 6.6\%$, which reflects the sum of hiring *during the quarter* and vacancies *in replies* to the MoITaL survey. Thus, from the definition of labor market tightness (2), we obtain the steady-state labor market tightness $\overline{\theta} = 1.12$ and the vacancy-filling rate (4) is $\overline{q} = 73.8\%$.

The calibration of the matching elasticity with respect to vacancies is $\phi = 0.40$, based on a separate estimation of the static matching function (1), and within the range of values typically found in the literature. It follows that the scaling parameter is $\eta^m = 0.79$. Since the steady-state vacancy and unemployment rates are very similar, and considering the homogeneity of the matching technology (1), the calibration of the scaling parameter η^m is not sensitive to the calibration of the matching elasticity ϕ, meaning that the latter can be freely estimated.

Technology- and pricing-related parameters were calibrated as follows. The long-run quarterly growth rate, $\overline{gr}^z = 1.0043$, is based on a log linear trend of real output, whose

Table 2: Calibrated values of structural parameters

Parameter	Description	Value
Preferences		
χ^u	Search effort	0.50
χ^l	Consumption-leisure MRS scaling	31.53
κ^c	External habit persistence in consumption	0.85
σ_n	Leisure elasticity with respect to hours worked	2.00
σ_l	Inverse of labor supply elasticity	1.00
σ_c	CRRA	2.00
β	Time discount factor (quarterly)	0.997
Labor market frictions		
η^m	Matching scaling	0.79
ϕ	Matching elasticity with respect to vacancies	0.40
δ	Separation rate	0.07
ψ	Bargaining power of households	0.11
χ^v	Vacancies cost scaling	11.60
κ^v	Vacancies cost elasticity	1.00
φ^v	Hiring weight of vacancies cost	0.30
Technology and pricing		
$\overline{gr}^z$	Long-run productivity growth rate	1.0043
α	Output elasticity with respect to labor input	0.66
η^Y	Output scaling	1.27
$\overline{\mu}$	Long-run markup	1.30
Policy		
τ^N	Benefits	0.20
τ^W	Income tax rate	0.37
$\overline{\Pi}$	Long-run inflation target (quarterly)	1.005
Labor market trends		
$\overline{\Gamma}^u$	Long-run growth of unemployment	−0.0002
$\overline{\Gamma}^h$	Long-run growth of nonparticipation	−0.0010

elasticity with respect to labor effort is standard, $\alpha = 0.66$ (Argov et al., 2012). This is used while calibrating the scaling parameter of the production function, $\eta^Y = 1.27$, based on a steady-state version of the (stationarized) production function (21), normalizing $\overline{y} = \overline{y}^I = \overline{n} = 1$, and using the steady-state value of $\overline{l}$. Neglecting the low aggregate level of the vacancies cost, which is justified below, we also get $\overline{c} \simeq \overline{y} = 1$. Based on Argov et al. (2012), as well as on expensive literature, the long-run price markup is set to $\overline{\mu} = 1.3$.

Preference parameters are calibrated using standard values from the literature. Using the intertemporal consumption decision (12), the time discount factor is calibrated to satisfy a steady-state real interest rate of 3%, in annual terms, while also accounting for the long-run growth-rate $\overline{gr}^z$, so that $\beta = 0.997$. For the CRRA, we use a value within the range of standard calibrations in the RBC and NK literature, so that $\sigma_c = 2$. The habit persistence in consumption is $\kappa^c = 0.85$, so as to generate the desired, hump-shaped impulse response of output to selected shocks. Based on Argov et al. (2012), the inverse of the Frisch elasticity of labor supply is $\sigma_l = 1$. Leisure elasticity with respect to hours worked is $\sigma_n = 2$. Since this parameter does not govern the steady state, its calibration is based on the impulse response it generates. We assume that unemployed family members dedicate $\chi^u = 0.5$ of the time to search effort, in line with the estimation results of Yashiv (2000). This implies that in the steady state, the effective time dedicated to labor market activity is $\overline{\widetilde{l}} = 0.26$.

The average income tax rate was calibrated to $\tau^W = 0.37$, and the ratio of benefit to the average wage was calibrated to $\tau^N = 0.2$. Calibration of the first is based on Argov et al. 2012, and that of the second on processing data published by the National Insurance Institute of Israel.[24] With the inflation target in the range of 1–3% since 2003, its long-run rate is calibrated to $\overline{\Pi} = 1.02^{0.25}$ (quarterly terms).

The wage to output ratio is calibrated as $\overline{w} = 0.625$, based on the average labor-share in the Israeli business sector since the late 1990s.[25] It then follows that the scaling of the consumption-leisure MRS is $\chi^l = 31.53$.[26]

[24] In practice, dividing the average unemployment benefit by the average wage, based on data for 2012, gives ratio of 0.4. However, since only about half of the unemployed are entitled to unemployment benefits, the number chosen is $\tau^N = 0.2$.

[25] Bank of Israel Annual Report for 2018 (Chapter 2, Table 2.9).

[26] The following steady-state ratios are defined: $\overline{mrs}^u \equiv MRS^u_t/Z_t$, $\overline{mrs}^l \equiv MRS^l_t/Z_t$, and $\overline{mrs}^{\widetilde{l}} \equiv$

The vacancy cost function (22) is assumed to be quadratic, following others such as Gertler et al. (2008), Gertler and Trigari (2009), Thomas (2008), and Christiano et al. (2011). This implies $\kappa^v = 1$. Calibrating $\varphi^v = 0.3$, based on Yashiv (2006), enables calibrating the scaling parameter of the vacancy function, $\chi^v = 11.6$.[27] Substituting into the vacancy cost (22), it follows that the total vacancy cost, in terms of output percentage, is $\overline{vc} = 3.5\%$.[28,29]

Finally, with all the above calibrated values in hand, we can use the stationarized version of the wage equations to calibrate the bargaining power so as to obtain the above-mentioned value for the steady-state wage. Thus, the household's bargaining power is $\psi = 0.11$, much lower than values typically found in the literature for other economies (usually higher than 0.5) and implying that the calibration is far from satisfying the Hosios (1990) condition for maximizing social welfare.

3.3.2 Calibrated Inertias

Table 3 shows the calibration values of parameters denoting the inertia of shocks. The inertia of the markup process is $\rho^{\varepsilon^\mu} = 0.2$, based on the estimation results of Argov et al.

$MRS_t^{\widetilde{l}}/Z_t$. After stationarizing the participation threshold (20) by Z_t, its steady state satisfies $\overline{mrs}^u = \frac{\overline{x}\beta\left[\left(1-\overline{\tau}^W\right)\overline{w}-\overline{\tau}^N-\overline{mrs}^l\right]}{1-\beta(1-\delta)}$. From the steady-state solution of the stationarized model we also know that $\overline{mrs}^u = \chi^u \cdot \overline{mrs}^{\widetilde{l}}$ and $\overline{mrs}^l = \overline{mrs}^{\widetilde{l}}$. Substituting these into the expression for $\overline{mrs}^u$ just described, we can rearrange to get $\overline{mrs}^{\widetilde{l}} = \frac{\overline{x}\beta\left[\left(1-\overline{\tau}^W\right)\overline{w}-\overline{\tau}^N\right]}{\overline{x}\beta+\chi^u[1-\beta(1-\delta)]} = 0.1855$. Now, from the steady-state solution of the stationarized model we also get $\overline{mrs}^{\widetilde{l}} = \chi^l \overline{\widetilde{l}}^{\sigma_l}/\overline{\lambda}$. With the values for $\overline{\widetilde{l}}$ and σ_l already in hand, and with the steady-state solution $\overline{\lambda} = (1-\kappa^c)^{-\sigma_c}$, we can put all of this together, to get $\chi^l = 31.53$.

[27] This is done by computing the steady-state version of the (stationarized) optimal vacancy-posting decision (28), $\overline{mvc} = \frac{\overline{q}\beta\left(\overline{p}^I\,\overline{mpl}-\overline{w}\right)}{1-\beta(1-\delta)} = 1.0546$, (for which we use the calibration $\overline{p}^I = 1/\overline{\mu} = 0.7629$, and $\overline{mpl} = \alpha/\overline{l} = 0.9483$). Using this value in the equivalent version of the marginal vacancy cost (23), $\overline{mvc} = \chi^v \left(\frac{\varphi^v+(1-\varphi^v)\overline{q}}{\overline{l}}\right)^{1+\kappa^v} \cdot \overline{v}^{\kappa^v}$, we finally get $\chi^v = 11.6$.

[28] More than 1.5% of the jobs in Israel are related to human-resources and manpower management (based on Labor Force Surveys of the Israeli Central Bureau of Statistics for the years 2010-11). In addition, there are of course efforts and resources utilized by other managers and workers in recruiting and training new workers.

[29] While such hiring costs may seem high, the calibration in Hagedorn and Manovskii (2008) for the United States is based on surveys implying that as much as 3.0–4.5% of the labor cost is spent on hiring! Other works assume hiring cost that are much lower, but still within the environment of the one suggested by this work: A hiring cost of 1% from GDP is assumed by Thomas (2008), Gertler and Trigari (2009), Blanchard and Galí (2010), and others.

Table 3: Calibrated values of parameters denoting inertias

Parameter	Description	Value
Based on theoretical considerations		
$\rho^{\varepsilon^{\mu}}$	markup shock	0.20
$\rho^{\varepsilon^{i}}$	monetary policy shock	0.00
$\rho^{\varepsilon^{tar}}$	inflation target shock	0.00
$\rho^{\varepsilon^{\tau^W}}$	income tax shocks	0.00
$\rho^{\varepsilon^{\tau^N}}$	unemployment benefit shocks	0.00
$\rho^{\varepsilon^{\psi}}$	bargaining power shock	0.99
Corner solution in estimation		
ρ^{z}	tech trend	0.00
$\rho^{\varepsilon^{m}}$	matching technology shock	0.99
$\rho^{\varepsilon^{Y}}$	TFP shock	0.99
$\rho^{\gamma^{u}}$	unemployment trend	0.00
$\rho^{\gamma^{h}}$	participation trend	0.00
Inertias of measurement errors		
$\rho^{mERR\Delta W}$	nominal wage	-0.20
$\rho^{mERR\tau^W}$	income tax	0.55
$\rho^{mERR\tau^N}$	unemployment benefits	0.75
$\rho^{mERR\Delta Y}$	ΔY	-0.14

(2012). Inertia in the monetary policy shock results in a counter-intuitive impulse response. As such, that shock is assumed to satisfy a white noise process, such that $\rho^{\varepsilon^{i}} = 0$. Moreover, the Taylor rule (34) already includes an interest rate smoothing, represented by the smoothness parameter ρ^{i}.

The other policy-related variables—the inflation target (35), and the fiscal policy tools (31, 32)—are structured as unit root processes in the first place. Accordingly, their shocks are designed as white noises, such that $\rho^{\varepsilon^{tar}} = \rho^{\varepsilon^{\tau^W}} = \rho^{\varepsilon^{\tau^N}} = 0$.

The first group in the table concludes with the bargaining power shock, assumed to follow a unit root process—in line with the assumption made in the wage bargaining process (Subsection 2.6.2). This is treated by calibrating $\rho^{\varepsilon^{\psi}} = 0.99$.

The second group in Table 3 reports the values for parameters that were "pushed" by the estimation to the corner of the posterior distributions, and were therefore calibrated accordingly so as to end up with well-behaved posterior distributions for all estimated parameters. Finally, the third group in the table reports the inertias of the measurement errors. These are not characterized by well-behaved posterior distributions, and therefore destabilize the entire estimation results. They were therefore calibrated based on the peaks of their posterior distributions.

3.4 Estimated Parameters

Parameters that drive the model's dynamics were estimated using the Bayesian technique. Table 4 presents the prior and posterior distributions of the estimated parameters. Following the common practice, parameters that are restricted to the unit interval $[0, 1]$ were assigned a β distribution, positive parameters were assigned a Γ distribution, and the standard deviations of shocks were assigned an inverse Γ distribution.

Prior distributions were formed in an iterative manner, in the spirit of the discussion by Del Negro and Schorfheide (2008) and of the related approach ('endogenous priors' approach) taken by Christiano et al. (2011).[30] Accordingly, prior distributions were initially formed based on common practice, and then—for some of the parameters—they were fine-tuned by iteratively shifting and tightening them, closer to or around the posterior mode, so as to stabilize the estimation results by adding curvature to the likelihood.[31] This was particularly the case with the estimation of parameters driving exogenous processes, in line with common practice, as described by An and Schorfheide (2007). Such a somewhat judgmental (rather than formal) approach to prior elicitation can be justified by An and Schorfheide (2007), who view DSGE models as merely an approximation of the law of

[30] Del Negro and Schorfheide (2008) and Christiano et al. (2011) offer two distinct but related formal methods for adjusting prior distributions based on relevant moments. These formal methods essentially imply that the priors are formed based on a quasi-likelihood criterion. Adopting the spirit of their approaches while using judgement, rather than a strict formal criterion, is useful in avoiding an issue of over-fitting such as the one reported by Copaciu (2012).

[31] One motivation for such an approach for prior elicitation is a point made by Del Negro and Schorfheide (2008)—that the common simplifying assumption of independent priors results in joint distribution with a non-negligible probability mass in unreasonable regions. Tightened prior distributions are useful in addressing such a problem. However, while indeed helpful in identification, such an "endogenous" approach for prior elicitation also enhances an existing problem associated with the Bayesian method: As discussed by Del Negro and Schorfheide (2008) and Schorfheide (2008), some posterior distributions may end up being too concentrated and, therefore, may understate associated uncertainties.

Table 4: Prior and posterior dsitributions of the estimated parameters

Parameter		Prior distribution			Posterior distribution			
Notation	Description	Dist.	Mean	Std	Mean	Std	5%	95%
Structural parameters								
γ	Inflation indexation	β	0.35	0.08	0.312	0.063	0.209	0.416
ζ_p	Goods Calvo	β	0.74	0.01	0.741	0.005	0.733	0.749
ζ_w	Wage Calvo	β	0.82	0.01	0.820	0.009	0.805	0.836
κ^h	Participation adjustment cost	Γ	100.00	10.00	100.110	9.961	84.696	116.749
ρ^i	TR inertia	β	0.80	0.02	0.796	0.013	0.775	0.819
φ^π	MP π elasticity	Γ	1.70	0.04	1.708	0.040	1.646	1.771
φ^y	MP y elasticity	Γ	0.70	0.03	0.703	0.030	0.654	0.749
Inertias of shocks								
ρ^{ϵ^l}	to home activity utility	β	0.72	0.02	0.721	0.015	0.698	0.745
ρ^{ϵ^z}	to productivity trend	β	0.80	0.05	0.785	0.045	0.710	0.861
Std of structural shocks								
$\widehat{\zeta}^Y$	Stationary TFP	inv-Γ	0.01	0.00	0.008	0.001	0.006	0.009
$\widehat{\zeta}^i$	Monetary Policy	inv-Γ	0.00	0.00	0.002	0.000	0.001	0.002
$\widehat{\zeta}^m$	Matching technology	inv-Γ	0.05	0.02	0.044	0.006	0.034	0.054
$\widehat{\zeta}^l$	Labor supply	inv-Γ	0.02	Inf	0.009	0.001	0.007	0.011
$\widehat{\zeta}^{\tau^W}$	Income tax	inv-Γ	0.00	0.00	0.004	0.000	0.003	0.004
$\widehat{\zeta}^{\tau^N}$	Unemployment benefits	inv-Γ	0.00	0.00	0.003	0.000	0.001	0.005
$\widehat{\zeta}^{tar}$	π target	inv-Γ	0.00	0.00	0.001	0.000	0.001	0.001
$\widehat{\zeta}^\psi$	Bargaining power	inv-Γ	0.07	0.00	0.066	0.006	0.055	0.077
$\widehat{\zeta}^z$	Technology trend	inv-Γ	0.00	0.00	0.003	0.000	0.002	0.004
$\widehat{\zeta}^\mu$	Markup	inv-Γ	0.08	0.01	0.072	0.006	0.062	0.082
Std of shocks to structural trends								
$\widehat{\zeta}^{\gamma^{lf}}$	Labor-force trend	inv-Γ	0.00	Inf	0.004	0.000	0.003	0.004
$\widehat{\zeta}^{\gamma^c}$	Consumption trend	inv-Γ	0.01	0.00	0.013	0.001	0.011	0.014
$\widehat{\zeta}^{\gamma^u}$	Unemployment trend	inv-Γ	0.00	0.00	0.003	0.000	0.002	0.003
Std of shocks to measurement errors								
$\widehat{\zeta}^{mERR^W}$	Wage meas. err.	inv-Γ	0.01	Inf	0.010	0.001	0.009	0.011
$\widehat{\zeta}^{mERR^{\tau^W}}$	Income tax rate meas. err.	inv-Γ	0.00	0.00	0.001	0.000	0.000	0.001
$\widehat{\zeta}^{mERR^{\tau^N}}$	Unemp. benefits meas. err.	inv-Γ	0.00	0.00	0.002	0.000	0.001	0.004
$\widehat{\zeta}^{mERR^{\Delta Y}}$	ΔY meas. err.	inv-Γ	0.00	Inf	0.007	0.001	0.006	0.008

Based on Metropolis-Hasting simulations with 2 blocks of 200,000 draws, after a burn in of the first 50%.

motion of time series. Accordingly, so they argue, there need not exist a single parameter vector that delivers a "true" data generating process. The prior distributions of the final iteration are those reported by Table 4.

The estimation indicated that five of the structural shocks are empirically irrelevant: the vacancies shock $\widehat{\zeta}^{v}$, the search disutility shock $\widehat{\zeta}^{u}$, and the demand shocks $\widehat{\zeta}^{\beta}$, $\widehat{\zeta}^{C}$, and $\widehat{\zeta}^{NA}$. These shocks and their associated inertias are therefore absent from Table 4.

The posterior mean of inflation indexation, $\gamma = 0.3$, is in line with the estimation results of Argov et al. (2012). The posterior mean of the Calvo parameters in goods prices and wage setting are $\zeta_p = 0.74$ and $\zeta_w = 0.82$, respectively. These values suggest that wages are more rigid than prices, as they imply an expected wage duration of 5.5 quarters, compared with a price duration of only 4 quarters. Although intuitive, and contributing to the rigidity of *real* wages, this is not always the result found by other DSGE models.[32]

The posterior mean of the Taylor rule parameters are in the environment of those from Argov et al. (2012). In particular, the smoothness parameter $\rho^i = 0.8$ is very similar in both works. But, compared with Argov et al. (2012), this work suggests that the Bank of Israel is less aggressive with respect to inflation ($\varphi^{\pi} = 1.7$ here, compared with 2.5 there), and more aggressive with respect to activity ($\varphi^{y} = 0.7$ here, compared with 0.2 there). However, the sample period in Argov et al. (2012) ends in 2009, whereas the sample in this work is extended by a whole decade, until 2019. As Figure 1 indicates, that additional decade was characterized by a very low inflation rate and, nevertheless, by an interest rate that was generally not reduced since it was up against (what at least at the time seemed to be) its lower bound. These characteristics of the last decade probably explain the empirical evidence in this work of a more liberal monetary policy, compared with evidence of a more conservative policy in Argov et al. (2012).

3.5 Model Evaluation

An empirical evaluation of the model is focused on six observed variables: the BoI interest rate i^{obs}, vacancies v^{obs}, and the growth rates of GDP ΔY^{obs}, nominal wages ΔW^{obs}, the CPI π^{obs} and unemployment Δu^{obs}. As opposed to the estimation sample that begins in 1992, the evaluation is based on a subsample of only 84 quarterly observations (1999:Q1 to 2019:Q4). There are two reasons for that. First, vacancies are observed only since 1998. Second, the economic environment, with an emphasis on nominal variables, is far more stable in the second part of the estimation sample, thus making it more relevant as an evaluation benchmark. The evaluation consists of two fit tests—of dynamic moments and of forecast. Neither was utilized as an estimation criterion.

Figure 2 compares observed moments with their associated model-based confidence intervals. The moments compared are cross-correlations—that is, the correlations between one variable in period t and another variable in period $t-k$, where $k \in \{0, ..., 5\}$. In order to compute the model-based confidence intervals, 1,000 simulations of 84 periods each (the same as the length of the evaluation sample) were generated. Next, cross-correlations were calculated for each simulated sample.[33] We thus obtain a distribution of 1,000 estimators for each moment. The confidence intervals presented in Figure 2 represent the middle 90% of each such distribution.[34]

Over all, the results presented in Figure 2 seem satisfactory—most observed moments are within their respective model-based confidence intervals. In addition, the paths of the moments are generally followed.

However, the cross-correlations associated with the interest rate, particularly its dy-

[32] For instance, for the US economy, Gertler et al. (2008) estimate a price rigidity (Calvo parameter) of 0.85, higher than the wage rigidity they estimate—0.72 (even though their prior means reflect the opposite). Similarly, for the Israeli economy, Argov et al. (2012) estimate a price rigidity of 0.6, higher than the wage rigidity of 0.46 (with a similar prior mean for both rigidities—0.6). For the eurozone, however, Christoffel et al. (2009) report very similar results to those found here.

[33] All the simulations used the estimated model with parameter values at their posterior means. Thus, the source of uncertainty represented by the confidence intervals is the realization of the shocks, and not parameter uncertainty.

[34] The confidence intervals are based on simulations that included shocks to the structural trends of observed variables (Subsection 3.2). Thus, the simulated moments are consistent with the model's description of business-cycle properties but, at the same time, also with the statistical properties of structural trends inherent in the data.

namic correlations with vacancies, do not seem impressively fit. In this context, it should be noted that the interest rate in Israel remained largely unchanged for a long period of time during the latest part of the relevant subsample, as it has been essentially up against (what was considered to be) the zero lower bound. As such, its observed correlations with other variables do not properly reveal their full interactions.[35]

Figure 3 compares in-sample forecast errors of the model, with a horizon of up to 8 quarters, with those of three naive alternatives: Bayesian VAR (BVAR) with one lag[36], a Random Walk (RW) process, and a constant based on model-based steady-state values. Root mean square errors (RMSEs) are computed and presented for the *cumulative* forecasts (that is, the forecasts of the *levels*), with the exception of the interest rate and vacancies forecasts (which are originally in levels). The RMSEs in the figure refer to the entire range, from one-step to eight-period ahead forecasts.[37]

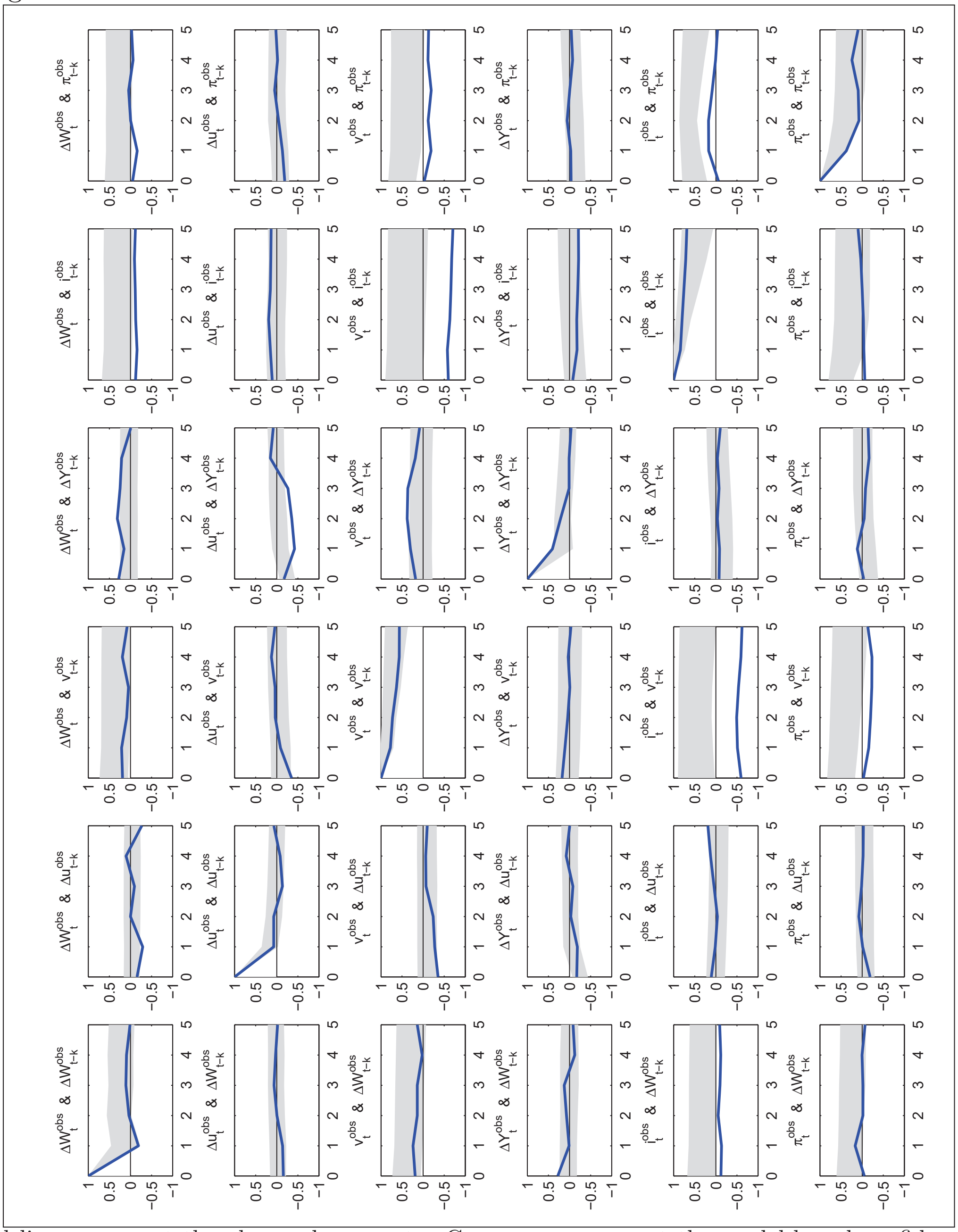
Based on the forecast RMSEs presented in Figure 3, the empirical fit of the model seems to be in line with the naive benchmarks. In some cases, the model even seems to preform better, implying that the benefits of a structural model are not costly in empirical terms. There are, however, two disappointing exceptions: the interest rate and vacancies forecasts for horizons longer than three quarters. The interest rate forecast was, as mentioned, quite constant during a very large portion of the evaluation subsample (while it was essentially up against the zero lower bound). An interest rate that is constant for such a long period, gives an edge to the highly inertial naive forecast alternatives—the RW and the BVAR-based forecast. Over all, the general forecast performances then seem to legitimize the model as an analytical tool for analysis of mechanisms at work and estimation of structural shocks.

[35] The confidence intervals of the dynamic correlations between the interest rate and vacancies match the observed correlation if the simulations are repeated without using two of the shocks—the matching-technology shock $\widehat{\zeta}^{m}$ and the labor supply shock $\widehat{\zeta}^{l}$.

[36] A higher order BVAR preformed very poorly in the medium-term forecasts.

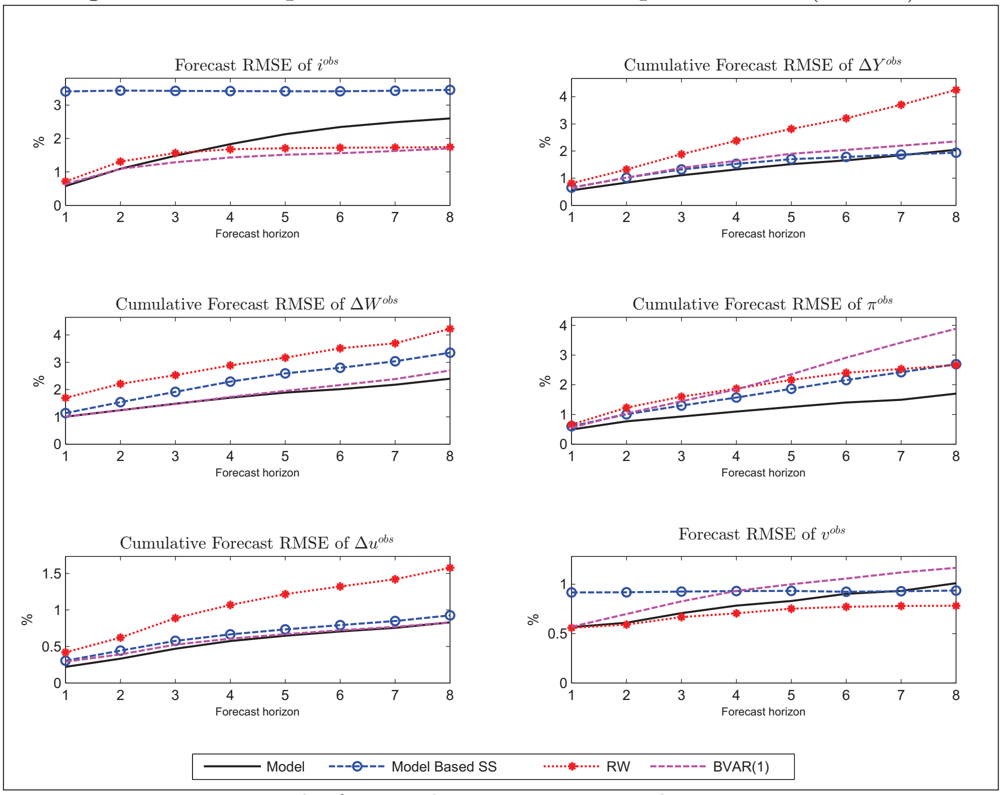
[37] Here, the forecasts are generated based on the most recent data release. Similarly, the forecast errors are computed with respect to that same data vintage. As a common argument in favor of the most recent data vintage, as opposed to the first data release, Del Negro and Schorfheide (2013) mention that it is likely to be closer to the "true" actual data.

Figure 2: Cross-correlations: Observed vs. Model-Based Confidence Intervals



Solid lines represent the observed moments. Gray areas represent the model-based confidence intervals (90%). The order of the cross correlation (k) appears in the x-axis. The shock to the inflation target, $\widehat{\zeta}^{tar}$, was de-activated.

Figure 3: In-Sample Forecast Root Mean Square Errors (RMSE)



Note: The forecast horizon appears in the x-axis.

4 Model Properties

4.1 Variance Decomposition

Table 5 presents the forecast-errors variance decomposition for an eight-quarter horizon. Interestingly, 51% of the unemployment forecast errors are associated with structural changes in the labor market (Subsection 3.2). About half of the rest is driven by labor supply shocks, which contribute 27%. The matching technology shock comes only third, with a contribution of 11%, as opposed to contributing near half the variance in the case of vacancy postings.

Table 5: Forecast error variance decompostion
(percent, eight-quarter horizon)

Shock	Description	u^{obs}	v^{obs}	ΔW^{obs}	ΔY^{obs}	π^{obs}	i^{obs}
$\widehat{\zeta}^{Y}$	Transitory TFP	3	5	1	9	14	34
$\widehat{\zeta}^{Z}$	Productivity trend	4	4	1	22	4	1
$\widehat{\zeta}^{i}$	Monetary policy	1	1	0	0	2	5
$\widehat{\zeta}^{m}$	Matching technology	11	46	1	0	10	13
$\widehat{\zeta}^{l}$	Labor supply	27	39	6	0	32	41
$\widehat{\zeta}^{\tau^W}$	Income tax	0	0	0	0	0	0
$\widehat{\zeta}^{\mu}$	Markup	0	1	0	0	36	5
$\widehat{\zeta}^{\psi}$	Bargaining power	2	3	2	0	1	1
$\widehat{\zeta}^{\gamma^{lf}}$	Labor-force trend	0	0	0	12	0	0
$\widehat{\zeta}^{\gamma^{u}}$	Unemployment trend	51	0	0	6	0	0
$\widehat{\zeta}^{mERR^{\Delta W}}$	ΔW measurement error	0	0	87	0	0	0
$\widehat{\zeta}^{mERR^{\Delta Y}}$	ΔY measurerment error	0	0	0	50	0	0

Vacancy postings are almost entirely driven by two shocks. Matching shock, which affects the vacancy filling rate, contributes 46% (although, as noted, it contributes just 11% to unemployment). Labor supply shocks contribute another 39%.

The nominal wage forecast errors are almost entirely driven by the wage-measurement errors, with a contribution of 87%. It is therefore possible that most of the volatility in the aggregate wage is driven by compositional changes in the labor market, rather than by individual wage changes per se.

Fifty percent of the variance of ΔY forecast errors is explained by measurement errors, and 31% by both productivity shocks together (the productivity trend contributes 22% and the transitory TFP shock another 9%). Trends aside, shocks related to the labor market seem irrelevant in explaining ΔY. This result is shared by Christiano et al. (2011), using Swedish data, who attribute it to efficient bargaining (over the intensive margins, Eq. 36 in this paper) which disconnects the tight relationships between wages and activity found in other models (those characterized by labor that is allocated in a neoclassical spot market). Similarly, Chang and Schorfheide (2003) argue that, based on postwar US data, where labor supply shocks are important drivers of labor input, they have little contribution to output. They assume that an increase in labor input, due to labor supply shocks, reduces labor productivity.

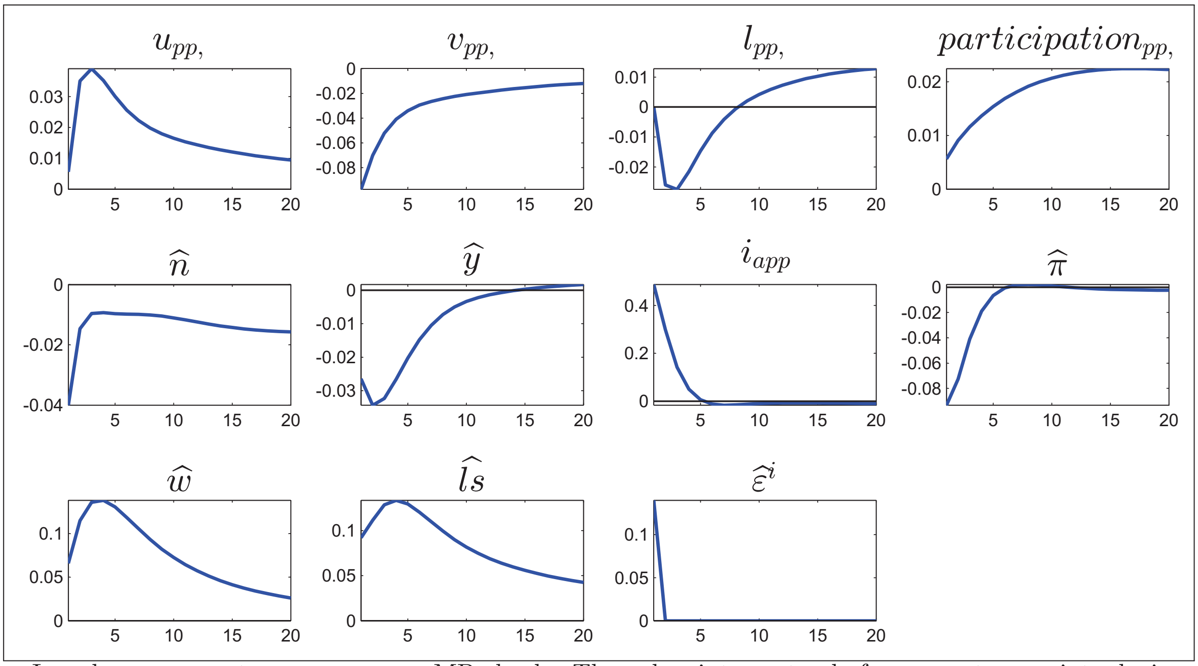
While markup shocks, not surprisingly, seem to be an important driver of inflation forecast errors (36% of their variance), they do not seem to make an important contribution to the interest rate (to the variance of which it contributes only 5%). This is probably related to the short-lived effect of this shock, characterized by inertia of only $\rho^{\varepsilon^{\mu}} = 0.2$, combined with the highly inertial interest rate (with a smoothness parameter of $\rho^{i} = 0.8$). Interest rate surprises are mostly driven by *real* shock—labor supply shocks contribute 41% to the variance of the interest rate forecast errors, and TFP shocks contribute 34%.

4.2 Impulse Response

Figure 4 presents the model's impulse response (IR) to a monetary policy shock ε_t^i. In contrast to the standard result of NK models, but in line with the robust empirical evidence discussed by Cantore et al. (2020), the figure shows how the labor share increases following a contractionary shock to monetary policy.[38]

[38] Preliminary experiments with VAR on Israeli data (1998:Q1-2017:Q4) yield an impulse response to a monetary policy shock that is consistent, in directions and magnitudes, with the one presented in Figure 4 here. That VAR(2) consists of four variables: labor share, labor market tightness, interest rate, and inflation. A monetary policy shock in that VAR system is identified using the Cholesky decomposition, where the interest rate shock has a simultaneous effect only on inflation.

Figure 4: Impulse response to a monetary policy shock

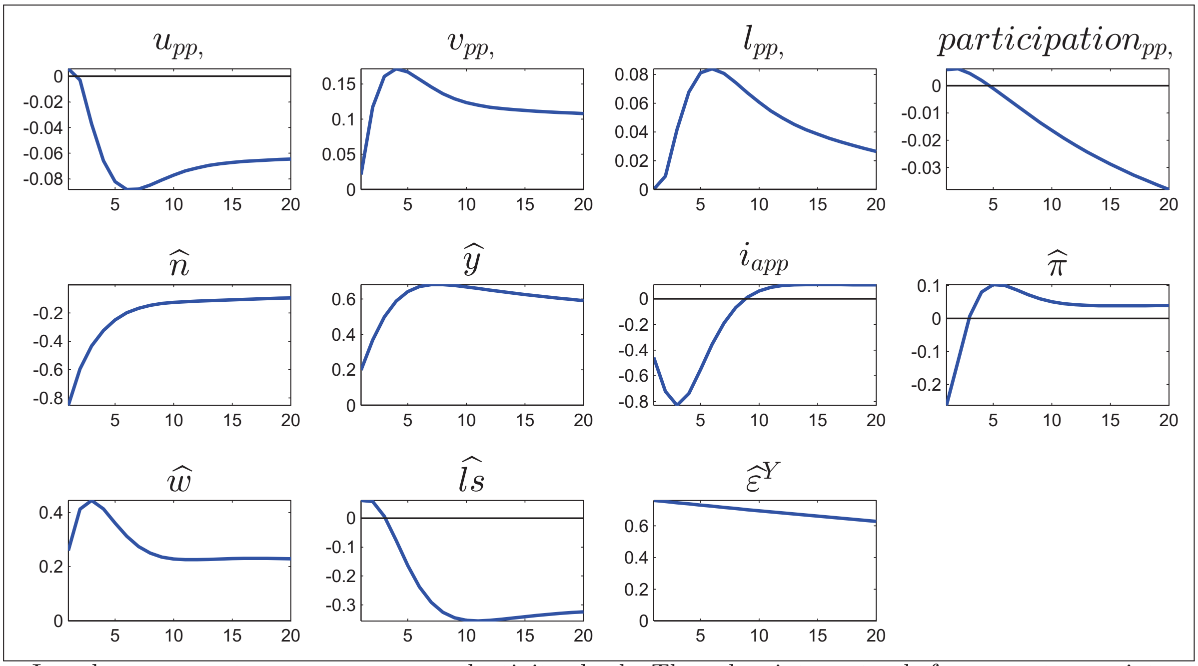


Impulse response to an exogenous MP shock. The subscript *pp* stands for percentage points deviation from (the stationarized) steady-state values. Similarly, the subscript *app* denotes *annualized* percentage points. The other variables are expressed in terms of percentage deviation (quarterly for π) from their steady state. w and ls denote the *real* wage and labor share, respectively.

This response of the labor share (43) is driven by its two components—the real wage and labor productivity. The real wage increases due to the combination of falling inflation and nominal-wage rigidity. Labor productivity falls, since firms respond to lower demand by reducing hours (effort), rather than employment (at least on impact). This last point, suggesting that firms tend to absorb demand fluctuations using their intensive margins, is consistent with the empirical evidence that Brender and Gallo (2009) find for the Israeli economy. It also implies that, conditional on monetary policy shocks, labor productivity is procyclical (since $\partial MPL_t/\partial n_t > 0$). Such comovement of the real wage and labor productivity implies the discussed labor share dynamics. That nonstandard result is therefore due to the nature and degree of wage rigidity (more than the degree of price rigidity) and due to the dominant role of the intensive margins in this model. Finally, the increased labor share de-incentivizes vacancy postings. Vacancies therefore fall and, in turn, increase unemployment as a response to a contractionary shock to monetary policy.

Next, Figure 5 presents the IR to a TFP shock, ε_t^Y, and Figure 6 presents that of the households' bargaining power, ε_t^ψ. These particular two shocks are discussed here, as they appear to dominate the historical decomposition of unemployment up until 2019, presented in Section 5 below. That is, the following discussion is motivated by a backward looking perspective. Taking a forward-looking perspective, related to the world following COVID-19, an additional IR is discussed by Section 6 below—an IR to the matching technology, ε_t^m. Appendix B presents figures of two additional IRs—for a markup shock, ε_t^μ, and for a labor supply shock, ε_t^l.

Figure 5: Impulse response to a TFP shock



Impulse response to an exogenous productivity shock. The subscript *pp* stands for percentage points deviation from (the stationarized) steady-state values. Similarly, the subscript *app* denotes *annualized* percentage points. The other variables are expressed in terms of percentage deviation (quarterly for π) from their steady state. w and ls denote the *real* wage and labor share, respectively.

The estimation results indicate that the TFP shock ε_t^Y is highly inertial (Figure 5). Considering the production technology (21), such a shock directly affects output, which increases on impact. But, with output being demand-determined, the production technology

(21) also implies that a TFP improvement leads to lower work intensity. With predetermined extensive margins (employment), the intensive margins (hours worked) absorb the lower requirement for work intensity and, therefore, fall on impact.[39]

The competitive price (36) is driven by two forces. Neither are presented in the figure, but both push prices down: With lower hours worked, workers face lower marginal disutility from each and, at the same time, producers experience higher marginal productivity from each working hour. The latter is driven directly by the TFP shock itself, but also indirectly, due to the lower hours worked under a diminishing marginal-productivity technology. So, considering the efficient bargaining condition (36), all forces unambiguously reduce prices of intermediate goods. As a result, considering the NK pricing rule of retailers (30), inflation falls on impact. Led by the extended Taylor rule (34), the central bank then cuts the interest rate, responding not only to the falling inflation, but also to a lower benchmark interest rate (33). In turn, the lower interest rate contributes to an increase in demand, in addition to the contribution of the shock's income effect, such that demand meets the increased output capacity.

These macro-level developments, of course, interact with the labor market. The general equilibrium interactions involve both prices (wages) and quantities (vacancies and labor market pools). Starting with wages, Figure (5) suggests that it increases in real terms. While in the medium term this is an unambiguous result of both labor supply and demand, that is not the case for the short term. In the short term there are two forces with conflicting effects on the real wage. The dominating one is the positive, caused by a reduced labor supply—households' reservation wage increase as the income effect of the shock reduces labor supply. This is evident from an increased reservation wage (38) that, in spite of the lower working effort, is dominated by MRS, which increases with consumption. The other, wage-reducing, force in the short term is the falling price of the intermediate goods, which reduces the producers' reservation wage (39). But, as mentioned, this is dominated by the increased reservation wage of the households—among other considerations, due to the forward-looking nature of the wage bargaining process, and therefore of wage itself (41),

[39] The predeterminacy of the extensive margins, employment, is formalized by its law of motion (5).

brought about by wage rigidity.

Even though the real wage increases, the figure shows how the labor share is expected to be persistently low. This is the case because, as mentioned earlier, wages are pushed up mostly by labor supply considerations. They are pushed up by demand only to some degree, as, at least in the short term, the effect of lower competitive prices dominates that of increased labor productivity. That is, although wages increase, they do not catch up with marginal productivity. This is how we end up with a falling labor share, although with some delay.

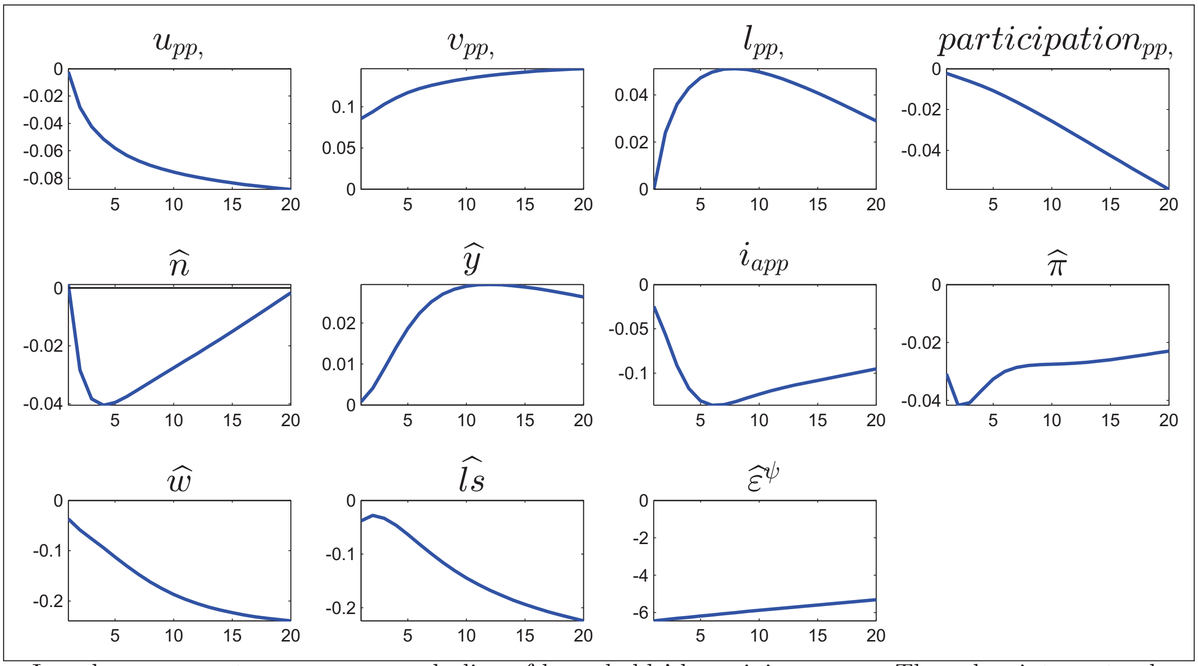
Expectations of a lower labor share increase producers' surplus from employment relationships and, therefore, incentivize vacancy postings. As a result, unemployment falls and employment increases. Interestingly, even though participation is somewhat lower, due to the income effect discussed earlier, employment is higher. This seems to be the case as the effect of more vacancies (higher demand) dominates that of lower participation (lower supply).

Down the road, as employment picks up, the MRS of hours worked increases. This pushes the competitive real price (36), which translates to some inflation.

To conclude, a shock to productivity, which is highly inertial but stationary nevertheless, is reflected by a lower labor share, which increases labor demand. As a result, employment increases and unemployment falls. This is the case in the model, even though the shock induces an income effect that reduces labor supply. In addition, productivity shock drives labor market tightness and labor share in opposite directions, which is consistent with causality running from the latter to the former.

Next, Figure (6) depicts the IR to declining household bargaining power. The shock directly affects the Nash bargaining result (40 for the flexible wages benchmark), which reduces real wages. This is translated into a lower labor share, namely, a higher firm surplus from established employment relationships, which motivates vacancy postings (28). In turn, the increased vacancy postings contribute to employment and reduce unemployment. The production technology (21) translates higher employment to both increased output and reduced working hours. Thus, the extensive margins are expanded, somewhat crowding out the intensive ones.

Figure 6: Impulse response to a negative shock to households' bargaining power



Impulse response to an exogenous decline of households' bargaining power. The subscript *pp* stands for percentage points deviation from (the stationarized) steady-state values. Similarly, the subscript *app* denotes *annualized* percentage points. The other variables are expressed in terms of percentage deviation (quarterly for π) from their steady state. w and ls denote the *real* wage and labor share, respectively.

As a result, inflation falls. The specific channel is the increased marginal productivity of hours worked (not shown in the figure), driven by both reduced working hours and increased employment. This reduces inflation as it affects the producers' competitive price (36). But the big picture is that a shock that directly reduces the cost of a production input reduces inflation. In the model it also reduces the share of that input, namely, the labor share.

As a side note, a lower wage reduces the households' surplus from employment, which demotivates participation (20). In addition, just as with the IR to a TFP shock, higher demand (vacancies) dominates the effect of lower supply (participation) on equilibrium employment.

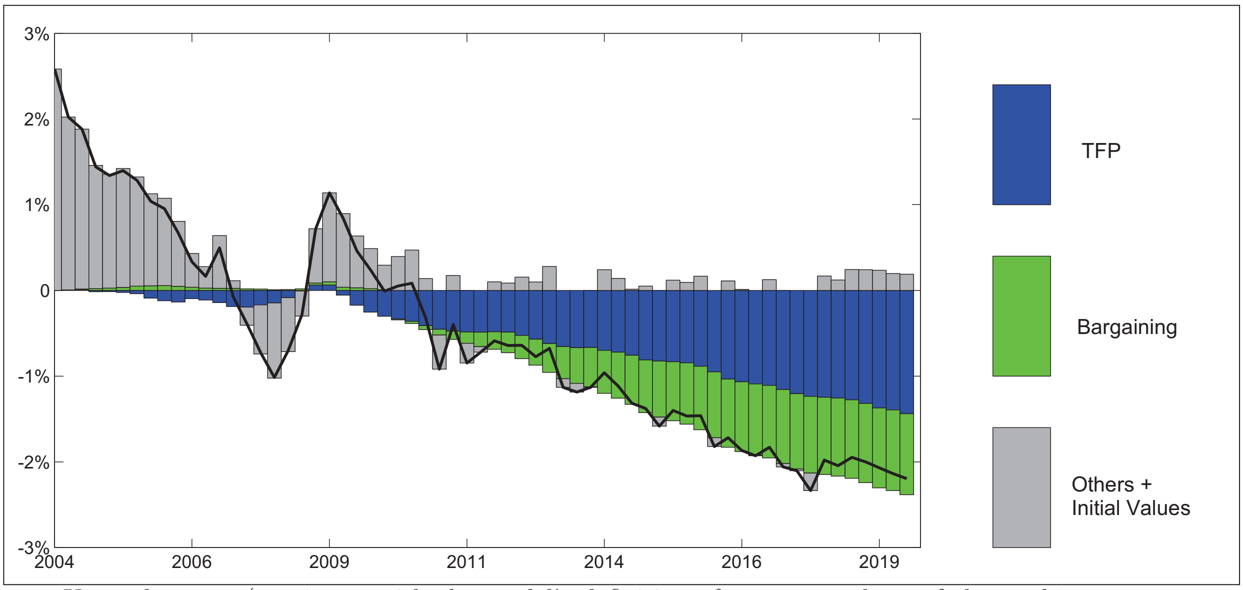
To conclude, just like the positive TFP shock discussed earlier, a negative shock to households' bargaining power increases employment and output. In both cases, the transmission channels of the shocks involved an endogenously driven labor-share reduction. Following either shock, based on the equilibrium condition (6), the combination of lower participation with higher employment inevitably implies lower unemployment. However, while a TFP shock induces some inflation as it fades out down the road, households' lower bargaining power is deflationary. That is, whereas one shock generates a negative correlation between labor share and inflation, the other generates a positive correlation between the two.

5 Historical Decomposition

Figure 7 depicts historical shock decomposition of unemployment[40], for the period 2004:Q1-2019:Q4. The two shocks, whose contribution to unemployment is explicit in the figure, are those whose IRs are discussed in Subsection 4.2 above—a TFP shock ε_t^Y, and a shock to households' bargaining power, ε_t^ψ. With the exception of these two shocks, all other contributions to unemployment in the figure are grouped together, as "others + initial values". Interestingly, although these two shocks, ε_t^Y and ε_t^ψ, seem very dominant in the historical decomposition of unemployment during the last few years, they have only a negligible theoretical contribution, based on the variance decomposition analysis (Subsection 4.1 and Table 5). One explanation of this alleged contradiction is related to the way the "other" variables are grouped together. The small contribution of that group as a whole reflects few shocks—some with a contribution that increases unemployment, some with a contribution that decreases it. The contributions thus cancel each other out, such that the total contribution of the shocks in the "other" group ends up being very small. Still, during the later years of the reviewed period, the individual contributions of the TFP and bargaining shocks are striking, even compared with each and every other contribution in the "other" group.

[40] Unemployment here is expressed in terms of deviations from a constant. Unemployment in the model is defined as the ratio of unemployed over the entire (prime) working age population, and not over participants only. That is, the model's definition of unemployment obviously yields a smaller unemployment rate than the official published one.

Figure 7: Historical Shock Decomposition of Unemployment (2004:Q1-2019:Q4)



Unemployment (consistent with the model's definition of u_{pp}—as a share of the working age population—not of the labor force). Deviations from steady state (percentage points).

An examination of the observed developments (Figure 1) highlights interesting similarities to the dynamics associated with the two IRs discussed in Subsection 4.2—TFP and bargaining power (Figures 5 and 6). Understanding these similarities helps to understand why the two shocks end up accounting for most of the explanation of the development of unemployment between 2009 and 2019. One obvious similarity is the negative rates of change in unemployment that characterized most of the last decade of data in the sample (Figure 1), as well as both IRs. Another interesting similarity is the increase in vacancies and employment[41], combined with the falling interest rate and labor share.[42]

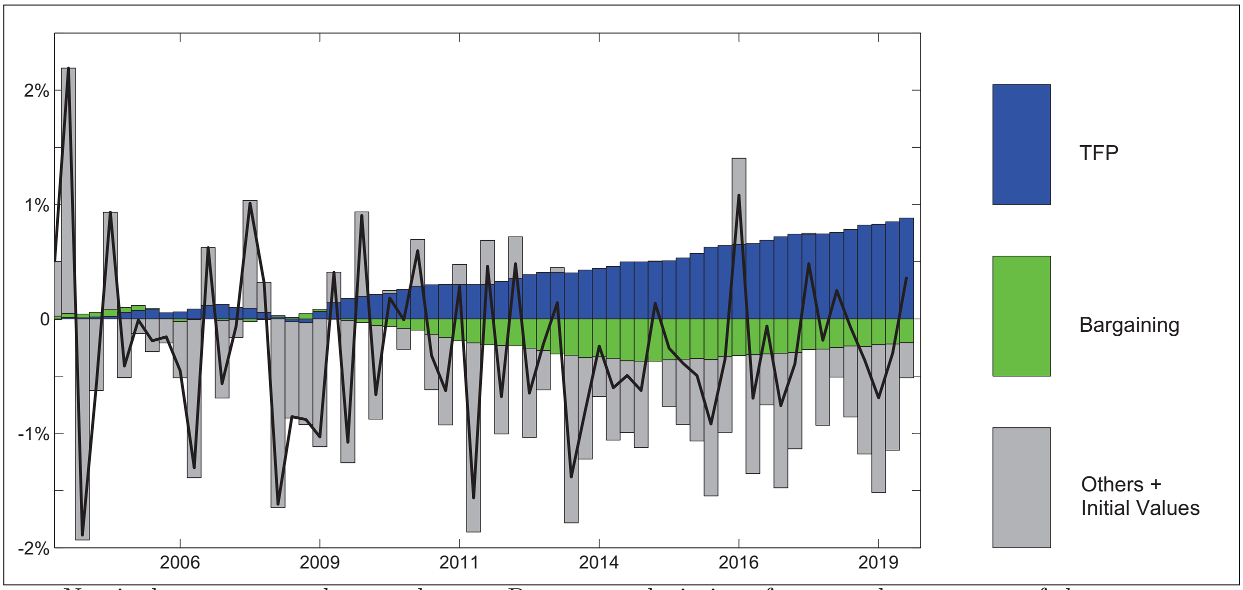
In light of the real wage[43], the two IRs present conflicting dynamics—while increasing

[41] Yakhin and Presman (2015) attribute at least half of the unemployment decline during the period 2004:Q1-2011:Q4 to a "noncyclical factor" (essentially mapped to a matching technology shock in the present model). However, the IR to the matching technology shock generates a positive correlation of unemployment and vacancies (see Figure 10 below). Thus, considering the positive trend of vacancies during the period analyzed here (2009:Q1-2019:Q4), a very low contribution is attributed to matching technology shocks in the historical decomposition presented here.

[42] The labor share is not directly observed by the model (and is thus absent from Figure 1), but it is exposed indirectly (see Figure 9 and discussion below).

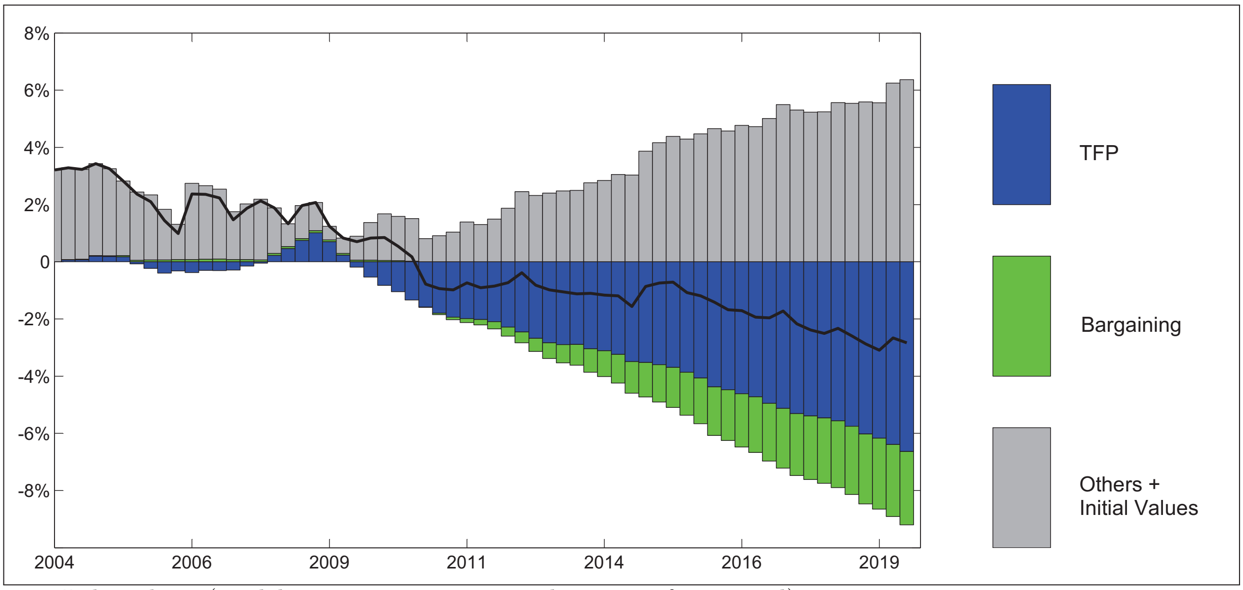
[43] The exact interpretation of this variable, $\widehat{w}$, in the IR figures 5-6 is the deviation of the real wage,

Figure 8: Historical Shock Decomposition of ΔW^{obs} (2004:Q2-2019:Q4)



Nominal wage, quarterly growth rate. Percnetage deviations from steady-state rate-of-change. Starting in 2004:Q2, due to an outlier in Q1.

Figure 9: Historical Shock Decomposition of the Labor Share (2004:Q1-2019:Q4)



Labor share (model consistent percentage deviations from trend).

following a positive TFP shock (Figure 5), it naturally decreases following a shift of bargaining power from households to firms (Figure 6). In reality, however, although the real wage generally increases over time, it is a matter of interpretation whether it grows faster or slower than its long-term trend. In other words, the gap between the real wage and its trend is not observed. Thus, each of the IRs can be consistent with the actual evolution of the real-wage gap. In addition, and perhaps more importantly, even if the real wage increases beyond its long-term trend, which characterizes the model economy following a positive TFP shock, it may not fully catch up with the increased labor productivity (directly driven by that same TFP shock). Thus, even though in the model economy the real wage increases following the TFP, the labor share nevertheless falls. A falling labor share indeed characterizes the IRs of both shocks discussed here, as well as the actual developments in Israel between the two global crises.[44]

This is all reflected by the historical decomposition of two additional variables—the observed nominal-wage growth (Figure 8) and the model-based estimation of the labor share (Figure 9). Indeed, the figures show that declining household bargaining power makes a negative contribution to wage and, by extension, to labor share.[45,46,47] The case

W_t/P_t, from its stationarized trend, Z_t (see Appendix C).

[44] The model-based estimation of the labor share may not necessarily coincide with the officially published one. Some differences between the two may emerge due to structural trends, as well as due to wage and output measurement errors.

[45] Mundlak (2020) documents and describes a long-term structural decline of labor unions' membership and coverage rate in Israel, starting at least in the 1980s. Such a structural trend is one possible explanation to an equivalent trend in the labor share.

[46] The IMF (2018) documents and analyzes changes in labor force participation rates during the years 2008-16. Based on that report, among advanced economies, Israel has the greatest increase in the participation rate of women. In addition, the OECD (2017) reports that the gender wage gap in Israel is among the highest in the OECD. Finally, Cortés and Pan (2020) find a strong link between the number of children and labor market gender gaps. Thus, the relatively high fertility rate in Israel may explain some of its relatively high gender gap. Taken together, these findings suggest that a compositional shift of the Israeli labor force (toward an increased share of women) is yet another possible development behind the model-based interpretation of declining bargaining power. Of course, bargaining power is only one possible explanation of the gender wage gap. Mazar and Michelson (2014) find evidence of an alterative explanation.

[47] Figura and Ratner (2015) offer a similar interpretation of US labor market developments during the years 2005-12. They refer to the negative correlation between tightness ($\theta_t = v_t/u_t$) and labor share within the US during those years. Connecting this with predictions based on the Diamond-Mortensen-Pissarides model, they argue that the data are driven by a reduction in the workers' bargaining power.

with the TFP shock is, however, different. Although the shock contributes to wage (Figure 8), that contribution does not catch up with a greater contribution to labor productivity. As a result, the total effect on the labor share is negative (Figure 9).

Obviously, there are also variables whose actual evolution is not in line with the two IRs discussed (Figures 5 and 6). In addition, there are some variables for which the two IRs are not consistent with each other, implying that the two IRs are distinguishable and that, in a sense, they complete each other. Based on these views, we should be referring to the dynamics of participation and inflation.

Starting with participation—for both IRs the participation rate is declining, which is in stark contrast to its actual development in Israel during the reviewed period (Figure 1). However, the model-based explanation of the evolution of participation consists of other shocks, the effect of which on unemployment is relatively small and dominated by that of TFP and bargaining power. One such explanation is a shock to the structural trend of participation which, as it is matched by a compatible trend of employment (due to the equilibrium condition 44), has little effect on unemployment.

Finally, one explanation of the actual evolution of inflation being weakly related to the discussed IRs would have a similar reasoning to the explanation offered for participation, namely that it had been driven by other shocks with little relevance for unemployment. However, there are also other, more appealing, ways to line up the inflation dynamics characterizing the two IRs and reality. As demonstrated by Figures 5-6, following a positive TFP shock, inflation increases in the *medium-term*. However, according to the historical decomposition, that positive contribution to inflation seems to be dominated by two other effects: First, inflation declines following a bargaining power shift to firms; second, its *short-term* response to a TFP shock is negative.[48]

Taken together, two developments seem to dominate unemployment in Israel between the years 2004-19: a TFP improvement and a shift of bargaining power from workers to firms. Both developments probably capture a compositional shift, among other factors,

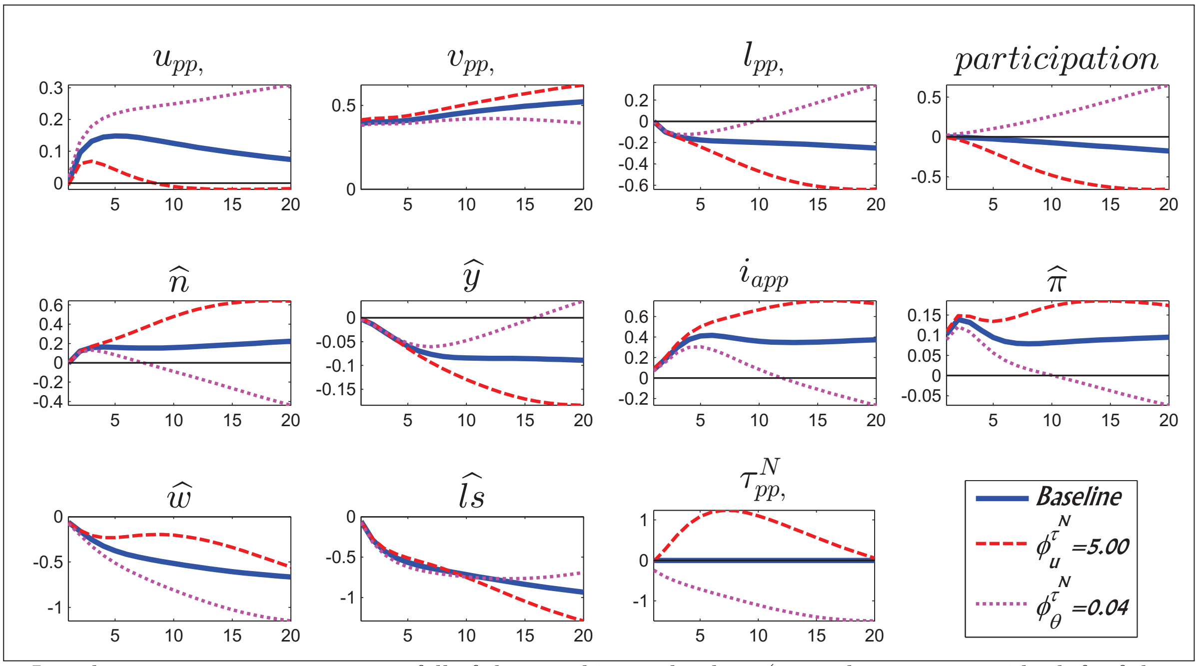
[48] Recall that the contribution of each shock to the historical decomposition, at each point in time, is actually a mixture. It includes the contribution of this shock hitting at that point in time, together with contributions of the same shock hitting earlier. Thus, every point in time includes a mixture of immediate and delayed effects of this shock.

and both appear to tighten the Israeli labor market during this period. Following both developments, a reduced labor share seems to function as a transmission mechanism, through which the exogenous developments tighten the labor market (formally speaking—increasing θ_t, the vacancies-unemployment ratio). In the spirit of the model, these two developments are treated here as exogenous and independent, although they probably capture a more internally connected reality. In keeping with the model's spirit, however, increasing TFP is followed by a declining labor share because the increased real wage doesn't catch up with labor productivity, as discussed in great detail above. In light of the model, apparently, such a development is indeed successful in explaining some of the decline in labor share and unemployment. But, as noted, it appears to explain just some, not all, of that decline. As such, a story of bargaining-power shift is required to complete the explanation. And so, the massive decline in the Israeli unemployment between 2004 and 2019 (Figure 1) is attributed to both developments. In both, however, a declining labor share functioned as an endogenous mechanism that contributed to tightening the labor market.

6 Policy Following the Reallocating Shock of the COVID-19 Crisis

The historical decomposition presented and discussed in Section 5 took a backward-looking perspective. In particular, its focus is the period prior to the COVID-19 crisis. It suggested that the dominating forces behind Israeli unemployment, and possibly elsewhere as well, were such that they would move vacancies and unemployment in opposite directions along the Beveridge curve. Taking a forward-looking perspective, however, it makes sense that a once-in-a-century shock, such as the COVID-19 shock, would be reflected and followed by a different "structural shocks" composition. One such shock, which should be carefully considered from a policy perspective, is an increase in structural unemployment. If the skills required in the labor market shift following the crisis and it takes time until such skills evolve in practice, economies may face a period of mismatch between labor market supply and demand. Barrero et al. (2020) refer to such likely implication of the COVID-19 crisis as a "reallocation shock". In the spirit of the model, a reduced form of such a development is a negative shock to the matching technology ε_t^m of the matching process (1).

Figure 10: Impulse response to a negative matching-technology shock.



Impulse response to an exogenous fall of the matching technology (equivalent to an outside shift of the Beveridge curve). The subscript pp stands for percentage points deviation from (the stationarized) steady-state values. Similarly, the subscript app denotes *annualized* percentage points. The other variables are expressed in terms of percentage deviation (quarterly for π) from their steady state. w and ls denote the *real* wage and labor-share, respectively.

Figure 10 describes the IR to such a negative shock to the matching technology. The IR suggests that such a shock boosts both vacancies and unemployment. That is, such a shock drives the Beveridge curve outward.[49] If the period following COVID-19 is indeed to be usefully described as a period with reduced matching technology, some of the expected increase of unemployment should be described as "structural". The corresponding IR depicted by Figure 10 suggests that, based on the baseline model, such a development is

[49] In the model, this is the only shock that has such an effect—some shocks are observationally equivalent to a slope change of the Beveridge curve, but none are reflected by a location shift of the curve. In contrast, during the years preceding the COVID-19 crisis, the observed data indicated an improvement of the matching technology and, as a result, some shift of the Beveridge curve inward.

to be followed by a falling employment and participation as well.

However, the figure also depicts two alternative IRs, corresponding to two policy alternatives. While in the baseline, unemployment benefits (UB) τ_t^N are exogenously determined, under the policy alternatives they are endogenized. One policy alternative is to increase UB with unemployment, such that $\phi_u^{\tau^N} \equiv \partial\tau_t^N/\partial u_t = 5.00$.[50] Another policy alternative is to reduce them with labor market tightness (2), such that $\phi_\theta^{\tau^N} \equiv -\partial\tau_t^N/\partial\theta_t = 0.04$.[51]

Under the policy alternative that increases UB with unemployment, the figure suggests that UB are increased following an outward shift of the Beveridge curve, brought about by declining matching technology. As a result, unemployment converges more rapidly than it does under the baseline model. But, this is a result of lower participation than in the baseline, caused by the higher UB.[52] In contrast, under the policy alternative that reduces UB as the labor market tightens, the figure shows that UB fall, leading to increased unemployment. However, focusing on unemployment alone is misleading in this case, because under such a policy alternative, employment and output are actually higher than in the baseline! This is obviously so due to the UB effect on the search effort (or participation in the model).

There is a practice of setting the UB rate endogenously to the business cycle. Such a practice is motivated by both economic and political considerations, and there are periods were, indeed, it makes perfect sense. For instance, following the COVID-19 outbreak, during early-mid 2020 there seemed to be about 10 unemployed people competing for each vacancy in Israel. The picture in many other economies was not that different. Under such circumstances, it is easy to argue in favor of more generous UB. However, it is important to closely monitor the evolution of vacancies down the road,[53] and to design optimal incentives

[50] Under this alternative, UB increase by 5 percentage points with any increase of 1 percentage point in unemployment. Such formalization is made here only for convenience. In practice, variations of statutory UB along the business cycle often take the form of changes in eligibility criteria. The formalization here can, for instance, be thought of as a reduced form of extension of the UB eligibility period by 50% (say from 6 to 9 months) following a 10 percentage point increase in unemployment.

[51] The different magnitudes of the parameters $\phi_u^{\tau^N}$ and $\phi_\theta^{\tau^N}$ reflect nothing but the different magnitudes of the coresponding variables, u_t and θ_t.

[52] In the model, nonparticipants are also entitled to UB. Such model effect of UB on participation can be thought of as a simplified formalization aimed at capturing *endogenous* search intensity.

[53] Increasing vacancies, in a period of low employment, may signal that policy measures have a dominating

for the unemployed to acquire and adjust the time varying skills required by potential employers.

A model-based discussion restricts our attention to UB as the single relevant policy instrument. More broadly, in addition to UB levels that exceed workers' earnings, Barrero et al. (2020) discuss policies that subsidize employee retention, occupational licensing restrictions, and regulatory barriers to business formation impeding reallocation responses to the COVID-19 shock.

That said, it is of course important to remember that the analysis presented here is based on a macro model. As such, it ignores the enormous labor-market heterogeneities, and the heterogeneity therefore required in policy design. Professional training programs should be a central policy tool in addressing such heterogeneity.

7 Concluding Remarks

The paper presents a DSGE model with an enriched labor market, thus extending the set of variables typically analyzed by more standard models. The model is estimated based on quarterly Israeli data from 1992 to 2019, and evaluated based on two perspectives—dynamics correlations and forecast errors—neither of which was employed to fit the model parameters in the first place. Although there is a risk of improper use of prior assumptions, in the sense of Canova and Sala (2009) who highlight the identification problems characterizing DSGE models, the model evaluation nevertheless suggests that, at least in some dimensions, it preforms better than some alternative benchmarks. The model is used to shed light on developments in the Israeli labor market between 2009 and 2019. Based on the perspective represented by the model, past developments in the Israeli labor market were analyzed, and some considerations for future labor market policy were discussed.

In analyzing past developments, a model-based analysis highlights the importance of two forces in reducing the Israeli unemployment until the COVID-19 crisis: employees' declining bargaining power, and a transitory TFP acceleration. Compositional changes in the Israeli labor market provide one possible explanation behind both macro-level developments.

de-incentivizing effect.

With regard to future policy considerations following the COVID-19 crisis, the paper discusses a possible mismatch of labor supply and demand, even if only temporary, due to a possible structural change. Under such possible developments, the implications of alternative approaches to unemployment benefits were discussed. In particular, it seems important that policy makers base related decisions not only on the evolution of unemployment per se, but on the joint evolution of unemployment and vacancies, that is, on labor market tightness.

Being both theoretically and empirically oriented, the model is useful for policy assessment in general, and for monetary policy formulation in particular. In that arena, the model is an important complement to existing ones, not only because it formalizes and enriches the labor market analysis, but also due to its predictions related to labor share dynamics. As discussed by Cantore et al. (2020), the labor share dynamics in most standard central banks models stand in sharp contrast to the very robust empirical evidence and, by extension, so do the associated dynamics of labor productivity and of price markups. The model, due to its labor market structure, offers labor share dynamics that are consistent with the empirical evidence. This, in addition to the formalization of labor market dynamics per se, makes the model an important tool for policy-related macroeconomic analysis.

At the same time, however, while being relatively rich in terms of labor market variables and mechanisms, the model excludes many other macroeconomic ingredients that are important for forecasting, historical analysis, and policy assessment. Such ingredients include, for instance, investment, government expenditures, import, export, capital flows, and real and financial assets. Each one of these components is characterized by a market of its own, which requires a comprehensive specification. Additional policy tools—both fiscal and monetary—are yet another area that requires attention. Moreover, under general equilibrium, each one of these ingredients interacts with all the others. Put differently, a comprehensive macroeconomic analysis requires a reference to many more variables and interactions than just those included in the model. The model nevertheless was kept relatively limited in its scope, so as to enable a pragmatic and tractable model-based analysis. Thus, for some policy and research applications, such a model should be accompanied by other models, typically less comprehensive in terms of the labor market, but more so in other macroeconomic aspects.

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Appendices

Appendix A Benchmark Interest Rate

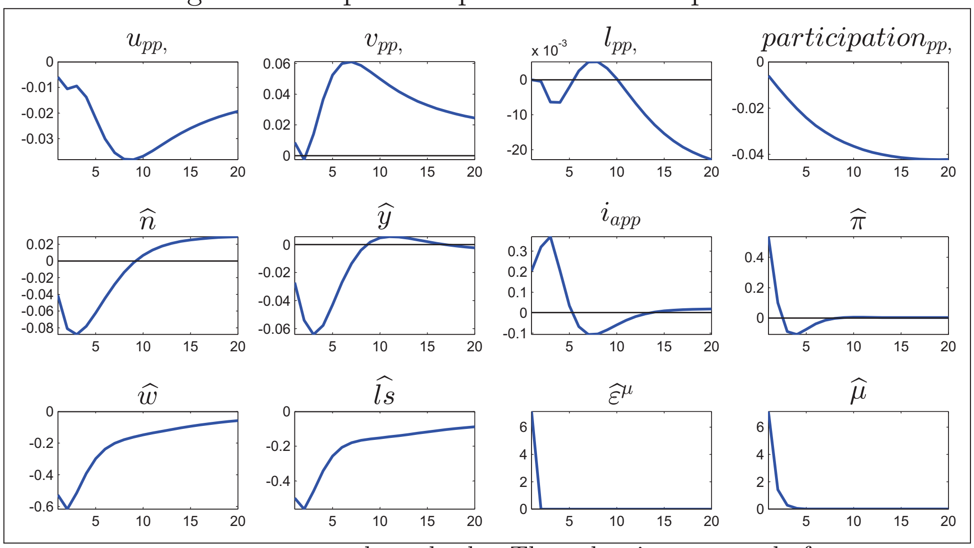
In the saving-consumption decision (12), we assume away the external habit formation and substitute in the clearing condition (42). Next, we substitute the technology (21), to get

$$1 + i_t \approx E_t \left\{ \Pi_{t+1} \frac{1}{\beta} \frac{Z_{t+1}}{Z_t} \frac{\varepsilon_t^{\beta} \varepsilon_t^{C}}{\varepsilon_{t+1}^{\beta} \varepsilon_{t+1}^{C}} \frac{\left(\frac{\varepsilon_t^{Y} (l_t n_t)^{\alpha}}{\varepsilon_t^{NA}} \right)^{-\sigma_c}}{\left(\frac{\varepsilon_{t+1}^{Y} (l_{t+1} n_{t+1})^{\alpha}}{\varepsilon_{t+1}^{NA}} \right)^{-\sigma_c}} \right\}.$$

The benchmark interest rate (33) is defined as the one consistent with the above equation with $\Pi_t = \Pi_t^{tar}$, and with the labor input $(l_t n_t)$ being in its steady state, even if the exogenous variables are not.

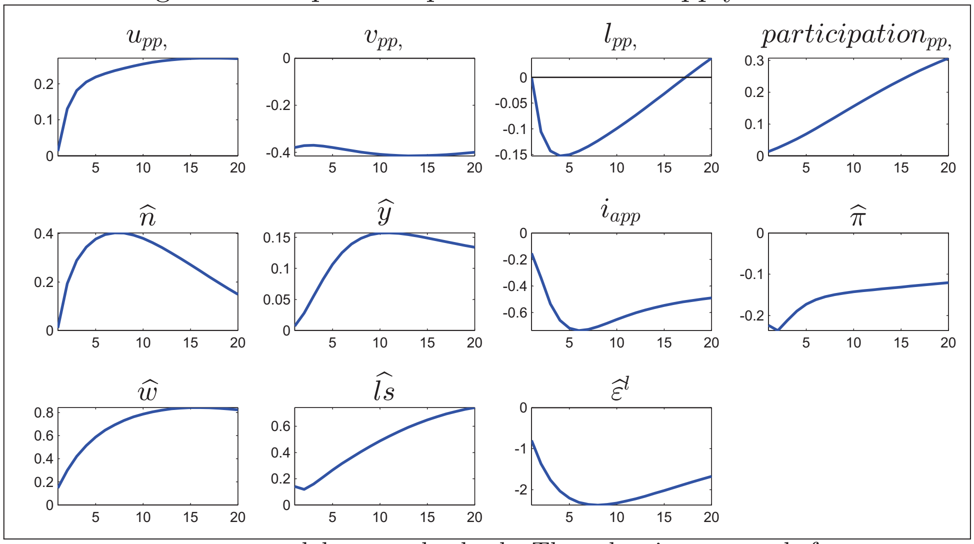
Appendix B Additional Impulse Response Figures

Figure 11: Impulse response to a markup shock.



Impulse response to an exogenous markup shock. The subscript *pp* stands for percentage points deviation from (the stationarized) steady state values. Similarly, the subscript *app* denotes *annualized* percentage points. The other variables are expressed in terms of percentage deviation (quarterly for π) from their steady state. w and ls denote the *real* wage and labor share, respectively.

Figure 12: Impulse response to a labor supply shock.



Impulse response to an exogenous labor supply shock. The subscript *pp* stands for percentage points deviation from (the stationarized) steady state values. Similarly, the subscript *app* denotes *annualized* percentage points. The other variables are expressed in terms of percentage deviation (quarterly for π) from their steady state. w and ls denote the *real* wage and labor share, respectively.

Appendix C   Stationarization

In order to stationarize trending variables, real ones are divided by Z_t, and nominal ones by P_t. The wage level is stationarized by both. A stationarized variable is denoted by a small letter, so that, for example, $c_t \equiv \frac{C_t}{Z_t}$, $p_t^I \equiv \frac{P_t^I}{P_t}$ and $\widetilde{w}_t \equiv \frac{\widetilde{W}_t}{P_t Z_t}$. Variables that are inherently stationary were denoted by small letter in the first place.

A steady state value is denoted by a bar over the variable name (with the exception of two cases—$\overline{w}$ and $\underline{w}$, which denote the steady state values of $\overline{w}_t$ and $\underline{w}_t$).

C.1   Stationary Equations

$$m_t = \eta^m \varepsilon_t^m v_t^\phi u_t^{1-\phi}, \tag{C.1}$$

$$\theta_t = v_t / u_t, \tag{C.2}$$

$$x_t = m_t / u_t, \tag{C.3}$$

$$q_t = m_t / v_t, \tag{C.4}$$

$$l_t = (1-\delta)\, l_{t-1} + m_{t-1}, \tag{C.5}$$

$$\widetilde{l}_t = \chi^u \varepsilon_t^u u_t + \frac{n_t^{1+\sigma_n}}{1+\sigma_n} l_t + \frac{1}{2}\kappa^h \left(h_t - h_{t-1}\right)^2, \tag{C.6}$$

$$gr_t^Z = \left(\overline{gr}^Z\right)^{\left(1-\rho^Z\right)} \left(gr_{t-1}^Z\right)^{\rho^Z} \varepsilon_t^Z, \tag{C.7}$$

$$\lambda_t = \varepsilon_t^\beta \varepsilon_t^C \left(c_t - \kappa^c \cdot c_{t-1}\right)^{-\sigma_c}, \tag{C.8}$$

$$E_t\left(\frac{\lambda_t}{\lambda_{t+1}}\right) = \beta E_t \frac{(1+i_t)}{gr_{t+1}^Z \Pi_{t+1}}, \tag{C.9}$$

$$p_t^I \equiv \frac{P_t^I}{P_t} = \frac{mrs_t^n}{mpn_t}, \tag{C.10}$$

$$\tau_t^W = \left(\overline{\tau}^W\right)^{(1-0.99)} \left(\tau_{t-1}^W\right)^{0.99} \varepsilon_t^{\tau^W}, \tag{C.11}$$

$$\tau_t^N = \left(\overline{\tau}^N\right)^{(1-0.99)} \left(\tau_{t-1}^N\right)^{0.99} \varepsilon_t^{\tau^N}, \tag{C.12}$$

$$1 + i_t^{Benchmark} = E_t\left\{\Pi_t^{tar} \beta^{-1} gr_{t+1}^Z \frac{\varepsilon_t^\beta}{\varepsilon_{t+1}^\beta} \frac{\varepsilon_t^C}{\varepsilon_{t+1}^C} \left(\frac{\varepsilon_t^Y}{\varepsilon_{t+1}^Y} \frac{\varepsilon_{t+1}^{NA}}{\varepsilon_t^{NA}}\right)^{-\sigma_c}\right\}, \tag{C.13}$$

$$(1+i_t) = \varepsilon_t^i \left(1+i_{t-1}\right)^{\rho^i} \left[\left(1+i_t^{Benchmark}\right) \left(\frac{\Pi_{t+1}^4}{\Pi_t^{tar}}\right)^{\varphi^\pi} \left(\frac{Y_t^I/Z_t}{\left(\overline{Y^I/Z}\right)}\right)^{\varphi^y}\right]^{1-\rho^i}, \tag{C.14}$$

$$ls_t = \alpha \frac{w_t}{mpl_t}, \tag{C.15}$$

$$\Pi_t^{tar} = \overline{\Pi}^{(1-0.99)} \left(\Pi_{t-1}^{tar}\right)^{0.99} \varepsilon_t^{tar}, \tag{C.16}$$

$$u_t = 1 - l_t - h_t, \tag{C.17}$$

$$\mu_t = \overline{\mu}^{(1-\rho^\mu)} \mu_{t-1}^{\rho^\mu} \varepsilon_t^\mu, \tag{C.18}$$

$$\begin{aligned} \ln\left(\varepsilon_t^{var}\right) &= \rho^{\varepsilon^{var}} \cdot \ln\left(\varepsilon_{t-1}^{var}\right) + \ln\left(\zeta_t^{var}\right); \\ var &\in \left\{m,\ \beta,\ C,\ h,\ Y,\ v,\ i,\ NA,\ \tau^W,\ \tau^N,\ tar,\ \mu,\ z,\ u\right\} \end{aligned} \tag{C.19}$$

C.2 Stationarization by Technological Level

$$y_t^I \equiv \frac{Y_t^I}{Z_t} = \varepsilon_t^Y \eta^Y \left(l_t n_t\right)^{\alpha}, \tag{C.20}$$

$$y_t^I = y_t, \tag{C.21}$$

$$mrs_t^{\widetilde{l}} \equiv \frac{MRS_t^{\widetilde{l}}}{Z_t} = \frac{\chi^l \varepsilon_t^{\beta} \varepsilon_t^l \widetilde{l}_t^{\sigma_l}}{\lambda_t}, \tag{C.22}$$

$$mrs_t^u \equiv \frac{MRS_t^h}{Z_t} = \chi^u \cdot mrs_t^{\widetilde{l}} \cdot \left\{\varepsilon_t^u - \kappa^h \left(h_t - h_{t-1}\right)\right\}, \tag{C.23}$$

$$mrs_t^l \equiv \frac{MRS_t^l}{Z_t} = mrs_t^{\widetilde{l}} \left\{\frac{n_t^{1+\sigma_n}}{1+\sigma_n} - \kappa^h \left(h_t - h_{t-1}\right)\right\}, \tag{C.24}$$

$$mrs_t^n \equiv \frac{MRS_t^n}{Z_t} = mrs_t^{\widetilde{l}} \cdot l_t \cdot n_t^{\sigma_n}, \tag{C.25}$$

$$mpl_t \equiv \frac{MPL_t}{Z_t} = \alpha \frac{y_t^I}{l_t}, \tag{C.26}$$

$$mpn_t \equiv \frac{MPN_t}{Z_t} = \alpha \frac{y_t^I}{n_t}, \tag{C.27}$$

$$vc_t \equiv \frac{VC_t}{Z_t} = \varepsilon_t^v \chi^v \frac{\left(\frac{\varphi^v v_t + (1-\varphi^v) q_t \cdot v_t}{l_t}\right)^{1+\kappa^v}}{1+\kappa^v} \tag{C.28}$$

$$mvc_t \equiv \frac{MVC_t}{Z_t} = \varepsilon_t^v \chi^v \left(\frac{\varphi^v + (1-\varphi^v) q_t}{l_t}\right)^{1+\kappa^v} v_t^{\kappa^v}, \tag{C.29}$$

$$y_t = \varepsilon_t^{NA} \cdot c_t, \tag{C.30}$$

$$\frac{mrs_t^u}{x_t \beta} = E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(\left(1 - \tau_{t+1}^W\right) w_{t+1} - \tau_{t+1}^N - mrs_{t+1}^l + \frac{(1-\delta)}{x_{t+1}} mrs_{t+1}^u\right)\right]$$

for an interior solution, when (17) holds with equality, or (C.31)

just $h_t = 0$ for a corner solution.

$$\frac{mvc_t}{q_t \beta} = E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(p_{t+1}^I mpl_{t+1} - w_{t+1} + \frac{(1-\delta)}{q_{t+1}} mvc_{t+1}\right)\right]. \tag{C.32}$$

C.3 Stationarization by the CPI

$$1 = \zeta_p \left[\Pi_t^{tar(1-\gamma)} \Pi_{t-1}^{\gamma} \frac{1}{\Pi_t} \right]^{\frac{1}{1-\mu_t}} + \left(1-\zeta_p\right) \widetilde{p}_t^{\frac{1}{1-\mu_t}}, \tag{C.33}$$

$$E_t \sum_{s=0}^{\infty} \frac{\lambda_{t+s}}{\lambda_t} \beta^s \zeta_p^s \frac{C_{j,t+s}/Z_{t+s}}{\mu_{t+s}-1} \left[\frac{\left(\Pi_t^{tar(1-\gamma)}\right)^s \left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\gamma} \widetilde{p}_t}{P_{t+s}/P_t} - \mu_{t+s} p_{t+s}^I \right] = 0. \tag{C.34}$$

The ratio $C_{j,t+s}/Z_{t+s}$ is stationary.

C.4 Stationarization by both Technology and CPI

$$w_t^{Flex} \equiv \frac{W_t^{Flex}}{P_t Z_t} = \varepsilon_t^{\psi} \psi \overline{w}_t^{Flex} + \left(1 - \varepsilon_t^{\psi} \psi\right) \underline{w}_t^{Flex}, \tag{C.35}$$

$$\overline{w}_t^{Flex} \equiv \frac{\overline{W}_t^{Flex}}{P_t Z_t} = p_t^I \cdot mpl_t, \tag{C.36}$$

$$\underline{w}_t^{Flex} \equiv \frac{\underline{W}_t^{FlexNet}}{P_t Z_t} = \frac{1}{(1-\tau_t^W)} \underline{w}_t^{FlexNet}, \tag{C.37}$$

$$\underline{w}_t^{FlexNet} \equiv \frac{\underline{W}_t^{Flex}}{P_t Z_t} = \tau_t^N + mrs_t^l, \tag{C.38}$$

$$w_t \equiv \frac{W_t}{P_t Z_t} = \zeta_w \frac{gr_t^W}{\Pi_t \cdot gr_t^z} w_{t-1} + (1-\zeta_w) \cdot \widetilde{w}_t, \tag{C.39}$$

$$\sum_{k=0}^{\infty} \left\{ \left[(1-\delta)\beta\zeta_w\right]^k \frac{\lambda_{t+k}/\lambda_t}{P_{t+k}/P_t} \frac{Z_t}{Z_{t+k}} \left[\left(gr_t^W\right)^k \cdot \widetilde{w}_t - \frac{P_{t+k} Z_{t+k}}{P_t Z_t} w_{t+k}^{Flex} \right] \right\} = 0. \tag{C.40}$$

Appendix D The Steady State

This appendix presents the steady state of the stationarized, yet nonlinear, model.

D.1 SS of Originally Stationary Variables

Given calibrated values for $\overline{v}$ and $\overline{u}$, we get the following solution for the steady state values of originally stationary variables:

$$\overline{m} = \eta^m \overline{v}^{\phi} \overline{u}^{1-\phi}, \tag{D.1}$$

$$\overline{\theta} = \overline{v}/\overline{u}, \tag{D.2}$$

$$\overline{x} = \frac{\overline{m}}{\overline{u}}, \tag{D.3}$$

$$\overline{q} = \frac{\overline{m}}{\overline{v}}, \tag{D.4}$$

$$\bar{l} = \frac{1}{\delta}\overline{m}, \overset{(using\ the\ next\ Subsection)}{\Longrightarrow} \overline{m} = \delta\left(\frac{1}{\eta^{Y}}\right)^{\frac{1}{\alpha}} \tag{D.5}$$

$$\bar{\tilde{l}} = \chi^{u}\overline{u} + \frac{\overline{n}^{1+\sigma_n}}{1+\sigma_n}\bar{l} \overset{normalizing\ \overline{n}=1}{=} \chi^{u}\overline{u} + \bar{l}, \tag{D.6}$$

$$\overline{gr}^{Z} = calibrated\ based\ on\ observed\ data, \tag{D.7}$$

$$\overline{\lambda} = \left[(1-\kappa^{c})\,\overline{c}\right]^{-\sigma_c} \overset{(using\ the\ next\ Subsection)}{=} (1-\kappa^{c})^{-\sigma_c}, \tag{D.8}$$

$$\bar{i} = \frac{\overline{gr}^{z}}{\beta}\overline{\Pi} - 1,\ Based\ on\ Euler \tag{D.9}$$

$$\overline{p}^{I} = \frac{\overline{mrs}^{n}}{\overline{mpn}} = 1/\overline{\mu}, \tag{D.10}$$

$$\overline{\tau}^{W} = calibrated\ based\ on\ observed\ data, \tag{D.11}$$

$$\overline{\tau}^{N} = calibrated\ based\ on\ observed\ data, \tag{D.12}$$

$$\bar{i}^{Benchmark} = \overline{\Pi}\frac{\overline{gr}^{z}}{\beta} - 1, \tag{D.13}$$

$$\begin{aligned} \bar{i} &= \frac{\overline{gr}^{z}}{\beta}\overline{\Pi} - 1,\ based\ on\ Taylor. \\ & \quad The\ same\ as\ the\ one\ based\ on\ Euler, \\ & \quad so\ we'll\ need\ another\ equation - \\ \overline{\Pi} &= \overline{\Pi}^{tar}, \end{aligned} \tag{D.14}$$

$$\overline{\Pi}^{tar} = calibrated\ based\ on\ observed\ data, \tag{D.15}$$

$$\overline{ls} = \alpha\frac{\overline{w}}{\overline{\overline{mpl}}} = \overline{w}\left(\eta^{Y}\right)^{-\frac{1}{\alpha}}, \tag{D.16}$$

$$\overline{h} = 1 - \bar{l} - \overline{u} \tag{D.17}$$

$$\overline{\mu} = calibrated\ , \tag{D.18}$$

$$\overline{\varepsilon}^{var} = 1;\ var \in \left\{m,\ \beta,\ C,\ h,\ Y,\ v,\ i,\ NA,\ \tau^{W},\ \tau^{N},\ tar,\ \mu,\ z,\ u\right\} \tag{D.19}$$

D.2 SS of Stationarized Variables other then Prices and Wages

Without loss of generality, the following assumes that the steady state value of hours worked is $\overline{n} = 1$, and that of the output is $\overline{y}^I = \overline{y} = 1$. It also assumes that χ^v is small enough so that $\overline{vc}/\overline{y} \to 0$, which means that $\overline{y} \approx \overline{c}$.

Thus, based on the previous subsection's solution, we have the following:

$$\overline{y}^I = \eta^Y \left(\bar{l} \cdot \overline{n}\right)^{\alpha} = 1$$

$$\Longrightarrow$$

$$\bar{l} = \left(\frac{1}{\eta^Y}\right)^{\frac{1}{\alpha}}, \tag{D.20}$$

$$\overline{y}^I = \overline{y} = 1, \tag{D.21}$$

$$\overline{mrs}^{\tilde{l}} = \frac{\chi^{l}\tilde{\bar{l}}^{-\sigma_l}}{\overline{\lambda}}, \tag{D.22}$$

$$\overline{mrs}^u = \chi^u \overline{mrs}^{\tilde{l}}, \tag{D.23}$$

$$\overline{mrs}^l = \overline{mrs}^{\tilde{l}}, \tag{D.24}$$

$$\overline{mrs}^n = \overline{mrs}^{\tilde{l}}\bar{l}, \tag{D.25}$$

$$\overline{mpl} = \alpha \frac{\overline{y}^I}{\bar{l}} = \alpha \left(\eta^Y\right)^{\frac{1}{\alpha}}, \tag{D.26}$$

$$\overline{mpn} = \alpha \frac{\overline{y}^I}{\overline{n}} = \alpha, \tag{D.27}$$

$$\overline{vc} = \chi^v \frac{\left(\frac{\varphi^v \overline{v} + (1-\varphi^v)\overline{q}\cdot\overline{v}}{\bar{l}}\right)^{1+\kappa^v}}{1+\kappa^v} \tag{D.28}$$

$$\overline{mvc} = \chi^v \left(\frac{\varphi^v + (1-\varphi^v)\,\overline{q}}{\bar{l}}\right)^{1+\kappa^v} \cdot \overline{v}^{\kappa^v} \tag{D.29}$$

$$\overline{y} = \overline{c} = 1, \tag{D.30}$$

$$\overline{mrs}^u = \frac{\overline{x}\beta\left[\left(1-\overline{\tau}^W\right)\overline{w} - \overline{\tau}^N - \overline{mrs}^l\right]}{1-\beta\left(1-\delta\right)}$$

for an interior solution, when (17) holds with equality, or (D.31)

just $\overline{h} = 0$ for a corner solution.

$$\overline{mvc} = \frac{\overline{q}\beta\left(\overline{p}^I\overline{mpl} - \overline{w}\right)}{1-\beta\left(1-\delta\right)}. \tag{D.32}$$

D.3 SS of Prices and Wages

Based on the previous subsection, we can solve for the steady state of the prices and wages:

$$\overline{\widetilde{p}} = 1, \tag{D.33}$$

$$\overline{p}^I = \frac{\overline{\widetilde{p}}}{\overline{\mu}} = \frac{1}{\overline{\mu}}, \tag{D.34}$$

$$\overline{w^{Flex}} = \psi\overline{\overline{w}^{Flex}} + (1-\psi)\,\overline{\underline{w}^{Flex}}, \tag{D.35}$$

$$\overline{\overline{w}^{Flex}} = \overline{p}^I \cdot \overline{mpl}, \tag{D.36}$$

$$\overline{\underline{w}^{Flex}} = \frac{\overline{\underline{w}^{FlexNet}}}{\left(1-\overline{\tau}^W\right)}, \tag{D.37}$$

$$\overline{\underline{w}^{FlexNet}} = \overline{\tau}^N + \overline{mrs}^l, \tag{D.38}$$

$$\overline{w} = \overline{\widetilde{w}}, \tag{D.39}$$

$$\overline{\widetilde{w}} = \overline{w^{Flex}}. \tag{D.40}$$

Appendix E Log Linearization

This appendix presents the log linearized system of equations. Ultimately, only the numbered equation in this appendix should be considered.

Variables with a hat denote logarithmic deviation from the steady state, whereas the subscript *pp* denotes deviation in terms of percentage points. For variables whose steady state value is close to one, such as the gross inflation and the gross interest rate, logarithmic and *pp* deviations are approximately the same.

E.1 Of Originally Stationary Equations

$$\frac{m_{pp,t}}{\overline{m}} = \phi\frac{v_{pp,t}}{\overline{v}} + (1-\phi)\,\frac{u_{pp,t}}{\overline{u}} + \widehat{\varepsilon}_t^m, \tag{E.1}$$

$$\frac{\theta_{pp,t}}{\overline{\theta}} = \frac{v_{pp,t}}{\overline{v}} - \frac{u_{pp,t}}{\overline{u}}, \tag{E.2}$$

$$\frac{x_{pp,t}}{\overline{x}} = \frac{m_{pp,t}}{\overline{m}} - \frac{u_{pp,t}}{\overline{u}}, \tag{E.3}$$

$$\frac{q_{pp,t}}{\overline{q}} = \frac{m_{pp,t}}{\overline{m}} - \frac{v_{pp,t}}{\overline{v}}, \tag{E.4}$$

$$l_{pp,t} = (1-\delta)\, l_{pp,t-1} + m_{pp,t-1}, \tag{E.5}$$

$$\widetilde{l}_{pp,t} = \chi^u \left(u_{pp,t} + \overline{u}\widehat{\varepsilon}^u_t\right) + l_{pp,t} + \overline{l}\widehat{n}_t, \tag{E.6}$$

$$\widehat{gr}^z_t = \rho^z \cdot \widehat{gr}^z_{t-1} + \widehat{\varepsilon}^z_t, \tag{E.7}$$

$$\widehat{\lambda}_t = -\frac{\sigma_c}{(1-\kappa^c)} \left(\widehat{c}_t - \kappa^c \widehat{c}_{t-1}\right) + \widehat{\varepsilon}^\beta_t + \widehat{\varepsilon}^C_t, \tag{E.8}$$

$$\widehat{\lambda}_t = E_t\widehat{\lambda}_{t+1} + \widehat{i}_t - E_t\widehat{\pi}_{t+1} - E_t\widehat{gr}^z_{t+1}, \tag{E.9}$$

$$\widehat{p}^I_t = \widehat{mrs}^n_t - \widehat{mpn}_t, \tag{E.10}$$

$$\tau^W_{pp,t} = 0.99 \cdot \tau^W_{pp,t-1} + \overline{\tau}^W \cdot \widehat{\varepsilon}^{\tau^W}_t, \tag{E.11}$$

$$\tau^N_{pp,t} = 0.99 \cdot \tau^N_{pp,t-1} + \overline{\tau}^N \widehat{\varepsilon}^{\tau^N}_t, \tag{E.12}$$

$$\widehat{i}^{Benchmark}_t = \widehat{\pi}^{tar}_t + \widehat{gr}^z_{t+1} + \widehat{\varepsilon}^\beta_t - \widehat{\varepsilon}^\beta_{t+1} + \widehat{\varepsilon}^C_t - \widehat{\varepsilon}^C_{t-1} - \sigma_c \left(\widehat{\varepsilon}^Y_t - \widehat{\varepsilon}^Y_{t+1} + \widehat{\varepsilon}^{NA}_{t+1} - \widehat{\varepsilon}^{NA}_t\right) \tag{E.13}$$

$$\begin{aligned} \widehat{i}_t &= \rho^i \cdot \widehat{i}_{t-1} + \\ &\left(1-\rho^i\right) \left\{ \begin{array}{c} \widehat{i}^{Benchmark}_t + \varphi^y \cdot \widehat{y}^I_t + \\ \varphi^\pi \left[0.25 \left(E_t\widehat{\pi}_{t+1} + \widehat{\pi}_t + \widehat{\pi}_{t-1} + \widehat{\pi}_{t-2}\right) - \widehat{\pi}^{tar}_t\right] \end{array} \right\} + \widehat{\varepsilon}^i_t, \end{aligned} \tag{E.14}$$

$$\widehat{ls}_t = \widehat{w}_t - \widehat{mpl}_t,$$

$$\widehat{\pi}^{tar}_t = 0.99 \cdot \widehat{\pi}^{tar}_{t-1} + \widehat{\varepsilon}^{tar}_t, \tag{E.15}$$

$$u_{pp,t} = -l_{pp,t} - h_{pp,t}, \tag{E.16}$$

$$\widehat{\mu}_t = \rho^\mu \ln \widehat{\mu}_{t-1} + \widehat{\varepsilon}^\mu_t, \tag{E.17}$$

$$\widehat{\varepsilon}^{var}_t = \rho^{\varepsilon^{var}} \cdot \widehat{\varepsilon}^{var}_{t-1} + \widehat{\zeta}^{var}_t;\ var \in \left\{m,\ \beta,\ C,\ h,\ Y,\ v,\ \psi,\ i,\ NA,\ \tau^W,\ \tau^N,\ tar,\ \mu,\ z,\ u\right\} \tag{E.18}$$

E.2 Of Equations Stationarized by the Technological Trend

$$\widehat{y}^I_t = \alpha \left(\frac{l_{pp,t}}{\bar{l}} + \widehat{n}_t \right) + \widehat{\varepsilon}^Y_t, \tag{E.19}$$

$$\widehat{y}^I_t = \widehat{y}_t \tag{E.20}$$

$$\widehat{mrs}^{\tilde{l}}_t = \sigma_l \frac{\tilde{l}_{pp,t}}{\bar{\tilde{l}}} - \widehat{\lambda}_t + \widehat{\varepsilon}^\beta_t + \widehat{\varepsilon}^l_t, \tag{E.21}$$

$$\widehat{mrs}^u_t = \begin{cases} \widehat{mrs}^{\tilde{l}}_t + \widehat{\varepsilon}^u_t - \kappa^h \left(h_{pp,t} - h_{pp,t-1} \right) & if \quad \chi^u > 0 \\ 0 & if \quad \chi^u = 0, \end{cases} \tag{E.22}$$

$$\widehat{mrs}^l_t = \widehat{mrs}^{\tilde{l}}_t + \widehat{n}_t - \kappa^h \left(\left(h_{pp,t} - h_{pp,t-1} \right) \right), \tag{E.23}$$

$$\widehat{mrs}^n_t = \widehat{mrs}^{\tilde{l}}_t + \frac{l_{pp,t}}{\bar{l}} + \sigma_n \widehat{n}_t, \tag{E.24}$$

$$\widehat{mpl}_t = \widehat{y}^I_t - \frac{l_{pp,t}}{\bar{l}}, \tag{E.25}$$

$$\widehat{mpn}_t = \widehat{y}^I_t - \widehat{n}_t, \tag{E.26}$$

$$\widehat{vc}_t = (1+\kappa^v) \left(\frac{v_{pp,t}}{\bar{v}} - \frac{l_{pp,t}}{\bar{l}} + \frac{(1-\varphi^v)}{\varphi^v/\bar{q} + (1-\varphi^v)} \cdot \frac{q_{qq,t}}{\bar{q}} \right) + \widehat{\varepsilon}^v_t, \tag{E.27}$$

$$\widehat{mvc}_t = \kappa^v \frac{v_{pp,t}}{\bar{v}} + (1+\kappa^v) \left[\bar{l} \left(\frac{(1-\varphi^v)}{\varphi^v/\bar{q} + (1-\varphi^v)} \right) \frac{q_{pp,t}}{\bar{q}} - \frac{l_{pp,t}}{\bar{l}} \right] + \varepsilon^v_t, \tag{E.28}$$

$$\widehat{y}_t = \widehat{c}_t + \widehat{\varepsilon}^{NA}_t, \tag{E.29}$$

$$\widehat{mrs}^u_t - \frac{x_{pp,t}}{\bar{x}} = E_t \widehat{\lambda}_{t+1} - \widehat{\lambda}_t +$$

$$\frac{\bar{x} \cdot \beta}{\overline{mrs}^u} E_t \left[\bar{\tilde{w}} \left(\left(1 - \bar{\tau}^W\right) \widehat{w}_{t+1} - \tau^W_{pp,t+1} \right) - \tau^N_{pp,t+1} - \overline{mrs}^l \cdot \widehat{mrs}^l_{t+1} \right] +$$

$$\beta (1-\delta) E_t \left[\widehat{mrs}^u_{t+1} - \frac{x_{pp,t+1}}{\bar{x}} \right],$$

or

$$h_t = 0, \quad (if \ \chi^u = 0), \tag{E.30}$$

$$\widehat{mvc}_t - \frac{q_{pp,t}}{\bar{q}} = E_t \widehat{\lambda}_{t+1} - \widehat{\lambda}_t +$$

$$\frac{\bar{q}\beta}{\overline{mvc}} \left[\bar{p}^I \cdot \overline{mpl} \left(\widehat{p}^I_{t+1} + \widehat{mpl}_{t+1} \right) - \widehat{w}_{t+1} \right] +$$

$$\beta (1-\delta) \left(\widehat{mvc}_{t+1} - \frac{q_{pp,t+1}}{\bar{q}} \right). \tag{E.31}$$

E.3 Of Equations Stationarized by the CPI

$$0 = \zeta_p \left[(1+\gamma)\,\widehat{\pi}_t^{tar} + \gamma \widehat{\pi}_{t-1} - \widehat{\pi}_t \right] + \left(1-\zeta_p\right) \widehat{\widetilde{p}}_t$$

$$\Longrightarrow$$

$$\widehat{\widetilde{p}}_t = \frac{\zeta_p}{\left(1-\zeta_p\right)} \left[\widehat{\pi}_t - \gamma \widehat{\pi}_{t-1} - (1+\gamma)\,\widehat{\pi}_t^{tar} \right]. \tag{E.32}$$

$$E_t \sum_{s=0}^{\infty} \left(\beta \cdot \zeta_p\right)^s \left[(1-\gamma) \sum_{j=1}^{s} \widehat{\pi}_t^{tar} + \gamma \sum_{j=1}^{s} \widehat{\pi}_{t+j-1} + \widehat{\widetilde{p}}_t - \sum_{j=1}^{s} \widehat{\pi}_{t+j} - \widehat{\mu}_{t+s} - \widehat{p}_{t+s}^I \right] = 0,$$

The last equation can be rearranged and expressed as

$$\begin{aligned}
\widehat{\widetilde{p}}_t &= \left(1-\beta\cdot\zeta_p\right) E_t \sum_{s=0}^{\infty} \left(\beta\cdot\zeta_p\right)^s \left[\widehat{\mu}_{t+s} + \widehat{p}_{t+s}^I + \sum_{j=1}^{s} \widehat{\pi}_{t+j} - \gamma \sum_{j=1}^{s} \widehat{\pi}_{t+j-1} - (1-\gamma) \sum_{j=1}^{s} \widehat{\pi}_t^{tar} \right] \\
&= \left(1-\beta\cdot\zeta_p\right) \left(\widehat{p}_t^I + \widehat{\mu}_t\right) + \beta\cdot\zeta_p \left(E_t \widehat{\pi}_{t+1} - \gamma \widehat{\pi}_t - (1-\gamma)\,\widehat{\pi}_t^{tar}\right) + \beta\cdot\zeta_p E_t \widehat{\widetilde{p}}_{t+1}.
\end{aligned}$$

Substituting in the optimal price (E.32), and rearranging, we get the NK Phillips curve:

$$\begin{aligned}
&\left[\widehat{\pi}_t - \gamma \widehat{\pi}_{t-1} - (1-\gamma)\,\widehat{\pi}_t^{tar} \right] \\
&= \beta \left[E_t \widehat{\pi}_{t+1} - \gamma \widehat{\pi}_t - (1-\gamma)\,\widehat{\pi}_t^{tar} \right] + \frac{\left(1-\zeta_p\right)\left(1-\beta\cdot\zeta_p\right)}{\zeta_p} \left(\widehat{p}_t^I + \mu_t\right).
\end{aligned} \tag{E.33}$$

E.4 Of Equations Stationarized by both

$$\widehat{w}_t^{Flex} = \psi \cdot \frac{\overline{\overline{w}^{Flex}}}{\overline{\overline{w}^{Flex}}} \cdot \widehat{\overline{w}}_t^{Flex} + (1-\psi) \cdot \frac{\overline{\underline{w}^{Flex}}}{\overline{\overline{w}^{Flex}}} \cdot \widehat{\underline{w}}_t^{Flex} + \psi \frac{\left(\overline{\overline{w}^{Flex}} - \overline{\underline{w}^{Flex}}\right)}{\overline{\overline{w}^{Flex}}} \widehat{\varepsilon}_t^{\psi}, \tag{E.34}$$

$$\widehat{\overline{w}}_t^{Flex} = \frac{\overline{p}^I \cdot \overline{mpl}}{\overline{\overline{w}^{Flex}}} \left(\widehat{p}_t^I + \widehat{mpl}_t\right), \tag{E.35}$$

$$\widehat{\underline{w}}_t^{Flex} = \widehat{\underline{w}}_t^{FlexNet} + \frac{1}{\left(1-\overline{\tau}^w\right)} \tau_{pp,t}^W, \tag{E.36}$$

$$\widehat{\underline{w}}_t^{FlexNet} = \frac{1}{\overline{\underline{w}^{FlexNet}}} \left(\tau_{pp,t}^N + \overline{mrs}^l \cdot \widehat{mrs}_t^l\right), \tag{E.37}$$

$$\widehat{w}_t = \zeta_w \left(\widehat{w}_{t-1} - \pi_t + \widehat{\pi}_t^{tar} - \widehat{gr}_t^Z\right) + \left(1-\zeta_w\right) \cdot \widehat{\widetilde{w}}_t,$$

which can be rearranged:

$$\widehat{\widetilde{w}}_t = \frac{1}{\left(1-\zeta_w\right)} \widehat{w}_t - \frac{\zeta_w}{\left(1-\zeta_w\right)} \left(\widehat{w}_{t-1} - \widehat{\pi}_t + \widehat{\pi}_t^{tar} - \widehat{gr}_t^Z\right), \tag{E.38}$$

$$\sum_{k=0}^{\infty}\left\{\left[\beta\left(1-\delta\right)\zeta_{w}\right]^{k}\left[\begin{array}{c}\widehat{\pi}_{t,t+k}^{tar}+\widehat{\widetilde{w}}_{t}-\\ \left(\widehat{\pi}_{t,t+k}+\widehat{gr}_{t,t+k}^{z}+\widehat{w}_{t+k}^{Flex}\right)\end{array}\right]\right\}=0.$$

Solving for $\widehat{\widetilde{w}}_t$, we get

$$\begin{aligned}\widehat{\widetilde{w}}_{t} &= \left(1-\beta\Upsilon_{w}\right)\sum_{k=0}^{\infty}\left[\left(\beta\Upsilon_{w}\right)^{k}\left(\widehat{\pi}_{t,t+k}+\widehat{gr}_{t,t+k}^{z}+\widehat{w}_{t+k}^{Flex}-\widehat{\pi}_{t,t+k}^{tar}\right)\right]\\ &= \left(1-\beta\Upsilon_{w}\right)\widehat{w}_{t}^{Flex}+\beta\Upsilon_{w}E_{t}\left[\widehat{\widetilde{w}}_{t+1}+\widehat{\pi}_{t+1}+\widehat{gr}_{t+1}^{z}-\widehat{\pi}_{t}^{tar}\right],\end{aligned}$$

where $\Upsilon_w \equiv (1-\delta)\,\zeta_w$.

Finally, substituting for $\widehat{\widetilde{w}}_t$, based on equation (E.38), and rearranging, we get a quasi-NK wage Phillips curve,

$$\widehat{w}_{t}=\frac{\Upsilon_{w}}{1+\Upsilon_{w}\zeta_{w}\beta}\left[\begin{array}{c}\frac{\zeta_{w}}{\Upsilon_{w}}\left(\widehat{w}_{t-1}+\widehat{\pi}_{t}^{tar}-\widehat{\pi}_{t}-\widehat{gr}_{t}^{Z}\right)+\\ \frac{(1-\zeta_{w})(1-\beta\Upsilon_{w})}{\Upsilon_{w}}\cdot\widehat{w}_{t}^{Flex}+\\ \beta E_{t}\left[\widehat{w}_{t+1}+\widehat{\pi}_{t+1}-\widehat{\pi}_{t}^{tar}+\widehat{gr}_{t+1}^{z}\right]\end{array}\right]. \tag{E.39}$$

Under flexible wages we have $\Upsilon_w = \zeta_w = 0$. In this case, the last equation is reduced to $\widehat{w}_t = \widehat{w}_t^{Flex}$.

Equations (E.1-E.39) comprise the complete log linearized system of equations.

Appendix F List of Variables

F.1 Search and matching

$m_{pp,t}$ — matchings

$v_{pp,t}$ — open vacancies

$\theta_{pp,t}$ — labor market tightness

$x_{pp,t}$ — job finding rate

$q_{pp,t}$ — vacancy filling rate

$\widehat{vc}_t$ — vacancy cost

$\widehat{mvc}_t$ — marginal vacancy cost

F.2 Labor market pools and flows

$\widetilde{l}_{pp,t}$ — labor market activity

$l_{pp,t}$ — employment rate

$h_{pp,t}$ — nonparticipation rate

$u_{pp,t}$ — unemployment rate

$\widehat{n}_t$ — hours worked

F.3 Goods market

$\widehat{c}_t$ — consumption

$\widehat{y}_t^I$ — intermediate goods

$\widehat{y}_t$ — final goods

F.4 Marginal contributions

$\widehat{mrs}_t^n$ — MRS of hours worked

$\widehat{mrs}_t^l$ — MRS of employment

$\widehat{mrs}_t^{\widetilde{l}}$ — MRS of labor market activity

$\widehat{mrs}_t^u$ — MRS of search effort

$\widehat{mpl}_t$ — marginal employee productivity

$\widehat{mpn}_t$ — marginal hour productivity

$\widehat{\lambda}_t$ — marginal consumption utility

$\widehat{ls}_t$ — labor share

F.5 Prices

$\widehat{\widetilde{w}}_t$ — Nash bargained wage

$\widehat{w}_t^{Flex}$ — flexible equilibrium wage

$\widehat{\overline{w}}_t^{Flex}$ — firm reservation wage

$\widehat{\underline{w}}_t^{Flex}$ — household reservation wage

$\widehat{\underline{w}}_t^{FlexNet}$ — net $\widehat{\underline{w}}_t^{Flex}$

$\widehat{w}_t$ — aggregate wage

$\widehat{i}_t$ — interest rate

$\widehat{\widetilde{p}}_t$ — final goods price

$\widehat{p}_t^I$ — intermediate good price

$\widehat{\pi}_t$ — inflation

F.6 AR(1) processes

$\widehat{gr}_t^Z$ — technological trend rate

$\tau_{pp,t}^W$ — income tax rate

$\tau_{pp,t}^N$ — unemployment benefits rate

$\widehat{\mu}_t$ — price markup

$\widehat{\pi}_t^{tar}$ — inflation target

F.7 AR(1) Shocks

$\widehat{\varepsilon}_t^m$ — matching technology shock

$\widehat{\varepsilon}_t^\beta$ — time discount shock

$\widehat{\varepsilon}_t^c$ — consumption utility shock

$\widehat{\varepsilon}_t^l$ — labor supply shock

$\widehat{\varepsilon}_t^Y$ — TFP shock

$\widehat{\varepsilon}_t^v$ — vacancies cost shock

$\widehat{\varepsilon}_t^\psi$ — bargaining power shock

$\widehat{\varepsilon}_t^i$ — monetary policy shock

$\widehat{\varepsilon}_t^{NA}$ — aggregate demand shock

$\widehat{\varepsilon}_t^{\tau^W}$ — income tax shock

$\widehat{\varepsilon}_t^{\tau^N}$ — unemployment benefits shock

$\widehat{\varepsilon}_t^{tar}$ — inflation target shock

$\widehat{\varepsilon}_t^\mu$ — price markup shock

$\widehat{\varepsilon}_t^z$ — technology trend shock

$\widehat{\varepsilon}_t^u$ — search disutility shock