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A Long-Run Growth Model for Israel

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Abstract

This paper describes the project of developing a long-term growth model to be used by the staff of the Bank of Israel. The purpose of the model is to forecast GDP growth over a horizon of approximately 50 years given various assumptions, and to evaluate how different exogenous developments, or policy steps, are expected to affect the long-run growth rate. The model is composed of five distinct blocks, each focused on a different factor of production or productivity. The blocks draw on different modeling approaches along the tradeoff between theoretical, detailed and empirical advantages. The baseline forecast indicates that the future growth rate of GDP and GDP per capita are expected to be lower than historical averages, mainly due to future demographic developments and the exhaustion of significant growth drivers that operated in the past.

מודל צמיחה ארוכת טווח למשק הישראלי

איל ארגוב ושי צור

תקציר

מאמר זה מתאר את הפיתוח של מודל צמיחה ארוכת טווח שמיועד לשמש את בנק ישראל. ייעודו של המודל לנפק תחזית לצמיחת התוצר באופק של 50 שנה קדימה בהינתן הנחות שונות, ולכמת את השפעתן של התפתחויות חיצוניות או צעדי מדיניות שונים על הצמיחה באופק זמן זה. המודל מורכב מחמישה חלקים נפרדים שכל אחד מהם מוכוון לחזוי גורם ייצור מסוים או הפריון. החלקים השונים משתמשים בגישות מתודולוגיות הנבדלות בממדי השימוש בתיאוריה, בפירוט המרובה ובממצאים אמפיריים. הסימולציה הבסיסית מצביעה על כך שצמיחת התוצר והתוצר לנפש צפויים להיות נמוכים מאשר הממוצעים ההיסטוריים, בעיקר בגלל שינויים דמוגרפים שעוברים על המשק ומיצויים של מנועי צמיחה משמעותיים שפעלו בעבר.

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1 Introduction

What is the long-run rate of growth and what are the main factors that affect it? Finding the answers to these questions, or even just a few of them, is of great importance to policy makers. Understanding and quantifying factors affecting long-run growth is important for analyzing policy measures and structural reforms; estimating the expected growth rate for long horizons is important for planning fiscal policy strategies; and forecasts for the long-run growth rate help short-term policy makers, such as central banks, to assess the stage of the business cycle by placing numerical benchmarks for the actual growth rate.

The goal of the project outlined in this paper is to develop a long-run growth simulation model to be used by the staff of the Bank of Israel. The main purpose of the model is to predict economic growth in Israel over a horizon of approximately 50 years, and to help the Bank's staff evaluate how different exogenous developments or policy steps are expected to affect Israel's long-run growth rate.

There are several ways to tackle this task. One approach would be to develop a theoretical growth model that would rest on clean theoretical derivations such as utility optimization, micro foundation, etc. This surely introduces legitimacy for policy analysis, but on the expense of the flexibility of the model to answer a wide range of questions. Alternatively, one could collect data on the most disaggregated level possible, forecast the micro units' development and reaggregate all the units assuming their future weight. This approach would allow the testing of policies that affect only some units of the economy at the cost of specifying many behavioral equations. A third approach would be to econometrically estimate the effect of factors that affect long-run growth. Given that we have, at most, only a few observations of long-run growth per country, this approach must depend on cross country estimations that typically assume homogeneity in elasticities between countries. Given the prons and cons of each approach, we decided to allow the use of a different approach for each part of the model and then unify them. In doing so, we exploit the relative advantage of each approach, rather than trying to build one integrated model in which all the parts are perfectly consistent.

The suggested model includes five main model-blocks. (1) The unifying model block combines, through an assumed production function, the forecasts for aggregate physical capital, human capital and productivity in order to forecast GDP. This part of the model also includes the dynamic physical capital production function. (2) The demographic investment rate model block describes how main demographic factors affect the long-run investment rate, where elasticities are derived from cross-country panel estimations. (3) The general equilibrium model block allows us to quantify how some structural changes effect the long-run investment rate based on a static version of a general equilibrium micro-founded open economy model. (4) The effective human capital model block determines the effective human capital by aggregating the human capital of 84 population groups divided by gender, 5-year-age-group and religion: non-ultra-Orthodox Jews (hereinafter, Jews or Jewish), Ultra-orthodox Jews (hereinafter, Orthodox) and Arabs. Effective human capital of each group is defined by its labor input (group size, participation rate, unemployment rate and hours per worker), and human capital from effective education years and from work experience. (5) The Total Factor Productivity model block simulates the future growth rate of TFP based on a "conditional convergence" concept.

The parameters of models (1),(3) and (4) are calibrated based on several approaches. In some cases we used data based estimates (for example, we used the labor compensation share for the production function elastically with respect to human capital input); in some cases we used elasticities estimated from concrete empirical Israeli or international research (such as the effect of experience on human capital); and finally, some parameters are calibrated ad hoc, based, as much as possible, on historical experience (for example, the speed of convergence in cell-specific labor force characteristics was chosen to approximately match previous years' dynamics). The parameters of models (4) and (5) are based on cross-section and panel data estimations adopted from other papers.

The baseline forecast points to an expected drop in the average annual growth rate of GDP and GDP per capita (to 2.4% and 0.7%, respectively,) compared to the historical averages. The expected growth rate is driven down mainly by expected demographic developments: average expected growth rate of the prime working age population is expected to slow by 0.9 percentage points (p.p.) compared to the historical average.

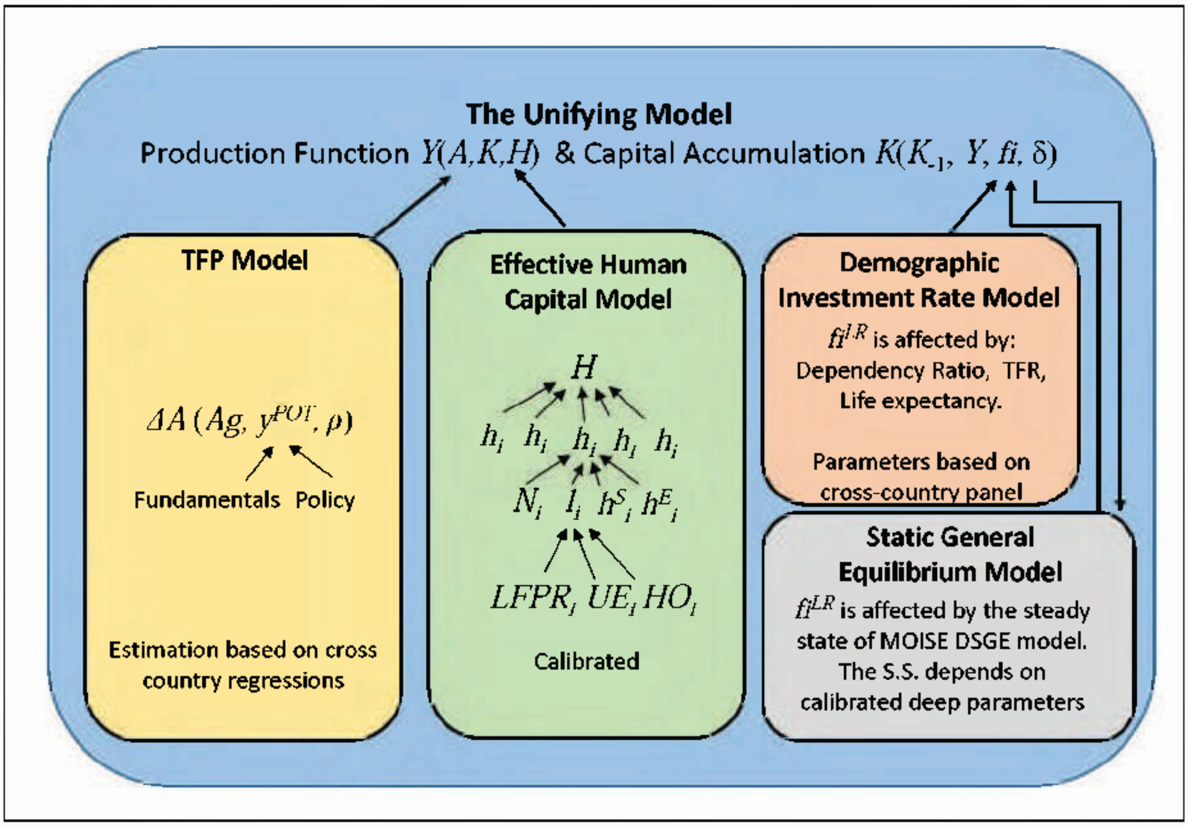
In the next section we give an overview of the structure of the model. In Section 3 we provide a full technical description of the model's equations and parametrization (some by calibration and some by estimation). Section 4 will describe the baseline forecast. In Section 5 we offer a measure of the uncertainty regarding the baseline forecast. In Section 6 we will test the capability of the model to forecast past developments. Section 7 will describe future scenarios given alternative assumptions. Section 8 will summarize.

2 Overview of the model

The model outlined in this paper was built to forecast average GDP growth over a horizon of approximately 50 years. The basic time period in the model is one year while running on five year intervals (that is, we forecast annual GDP in five year jumps - 2015, 2020, 2025...2065). The model is a combination of five building blocks, each focused on a different factor of production or productivity (Figure 1). Each block has a different modeling approach which varies on the space of the theoretical-detailed-empirical modeling tradeoff as discussed in the introduction above. This section provides a nontechnical overview of the model, while the next section (3) is a complete technical description of the model and its parametrization.

The unifying model block (detailed in Section 3.1) includes two main economic relations: a production function and a dynamic process for physical capital. The assumed constant returns to scale translog production function determines how much output (GDP) may be produced from a combination of physical capital, aggregate effective human capital and total factor productivity (TFP). Some elements of the physical capital will be determined in other blocks of the model, and the last two will be totally determined in other

Figure 1: Scematic Diagram of the Long-Run Growth Model



blocks of the model. Net physical capital evolves over time according to the perpetual inventory method; The stock of physical capital depends on the previous period stock, net of depreciation, and gross fixed investment. Gross fixed investment is a (time varying) fraction of GDP. While in a closed economy the investment rate is identical to the national saving rate, in an open economy some of the saving may be invested in foreign assets through a current account surplus. Theoretically, over very long horizons a small country may not hold a current account surplus or deficit as foreign assets or liabilities will explode. However, empirical stylized facts show that this horizon is beyond our long-run forecast. For example, until 2003, Israel almost continuously carried a current account deficit for more than 20 years. Therefore, we will allow the investment rate (which is the relevant measure for our domestic output concept) to be different from the saving rate. However, given our long-term horizon forecast we will assume that factors, such as demographic structure, that effect the average saving rate will affect the investment rate: it will depend on demographic factors through the demographic investment rate model block. In addition one may use the general equilibrium model block to evaluate the effects of structural economic changes on the long-run investment rate.

The demographic investment rate model block (detailed in Section 3.4) describes the empirical long-run relation between the aggregate investment rate and three demographic variables: the economy-wide Total Fertility Rate, life expectancy at birth and the old (aged 65+) to middle-age (15–64) dependency ratio (the forecast of the demographic variables is constructed within the effective human capital model block). Demographic developments effect the investment rate through two main channels. First, the longevity channel reflects the increase in life expectancy which, given the stickiness in actual retirement ages, operates to increase the saving rate during the working years. Second, the dependency channel reflects the greater weight of elderly or children in the population reducing the average saving rate because there is no income in those ages. In addition, a high weight of dependent groups may reduce the saving rate of the working ages that need to economically provide for the dependent ages. The choices of the explanatory demographic variables, as well as the calibration of the coefficients, are based on the empirical cross-country fixed effect panel estimation in Li, Zhang, and Zhang (2007) that covers 149 countries over the period 1963 to 2003.

The general equilibrium model block (detailed in Section 3.5) solves the steady state of the economy (one variable of which is the investment rate) based on a micro-founded general equilibrium (GE) model.[1] Originally, the GE model was developed for the purpose of short-term business cycle frequency analysis, with the aim of supporting monetary policy decision making and understanding the monetary transmission mechanism. However, the

[1] Bank of Israel's general equilibrium model (MOISE). See Argov et al. (2012).

dynamic model was formed around the steady-state solution of the model which depends on its fundamental parameter values. Among the parameters that determine the steady state of the GE model (which may be changed in order to affect our model through the investment rate) are: the physical capital depreciation rate; the home bias and the elasticity of substitution in final goods; the terms of trade; the long-run share of exports in world trade; government consumption and various tax rates; various markups; and more. The baseline calibration of these parameters (mainly taken from Argov et al. (2012)) was set to replicate main macro ratios of the Israeli economy during the sample period of 1995–2009.

In the context of our long-term growth model, we employ the ability of the GE model to solve for the expected long run economic ratios of the economy, and how they are expected to change in response to modifications in the model's structural parameters. For the time being we exploit the GE model's prediction of the effect of macro changes in the economy (for example, a permanent increase in the capital depreciation rate) on the long run value of the investment to GDP ratio (investment rate). However, potentially one may use the steady state solution to quantify how macro changes affect additional variables such as the labor input.

The effective human capital model block (detailed in Section 3.2) aggregates the human capital of 84 population groups divided by gender, 5-year-age-group and religion: non-ultra-Orthodox Jews (thereafter Jews or Jewish), ultra-Orthodox Jews (thereafter Orthodox) and Arabs.[2] Effective human capital of each group is defined by its labor input, human capital from effective education years (with an assumption on the return to schooling) and from work experience. Labor input depends on the population size of the group, its labor force participation rate, its unemployment rate and its average hours per worker.

To forecast the aggregate effective human capital we make detailed forecasts for each of the components in each population group. The forecast for each group specific component is

[2]The model includes an additional cell of foreign workers which in Israel make up approximately 10% of the human capital in 2010.

based on simple, partly ad-hoc, long-run reverting or between-group assimilation equations with a few structural economic relations. The forecast for the population size of each group uses standard demographic forecast methods with exogenous assumptions on the fertility rate, the survival rate and net immigration. For the forecasts of the components of labor input and average years of effective schooling we normally assume for the Jewish group gradual convergence to assumed long run values. For the Orthodox and Arab groups we normally assume gradual convergence to the economic characteristics of the parallel Jewish group. The speed of convergence is calibrated within the model. However these convergence processes are subject to a few economic relations. The participation rate and hours per worker of women negatively depends on the groups' age-specific fertility rate as there is a labor supply cost of childbearing. The effective average years of schooling of each cohort negatively depends on the total fertility rate at the time that the cohort was born. This captures the quality-quantity tradeoff in raising children. The term *effective* years of schooling comes from our practice of correcting measured actual years of schooling for schooling years that do not contribute to economic income from labor (mainly we reduce religious study years of the Orthodox men). Finally, the human capital from experience is an inverted U-shaped function of the calculated average experience of each group. It depends on the group's age net of average years of actual schooling and years of army service. In the forecast we also negatively correct the experience for the entrance of new participants in older than normal ages.

This block of the model was built on some previous works described in the literature. The concept of aggregating forecasts for the mentioned 84 Israeli population groups in order to produce long forecasts for GDP was already conducted in Geva (2013) and Braude (2013). While Braude (2013) concentrated only on labor input, Geva (2013) accounted for effective human capital through different average real wage of each group. Argov (2018) aggregated the forecasts of the average years of schooling of each group in order to assess the future contribution of human capital from schooling to long-run growth.[3] As in our model, in all three of the mentioned papers the population forecasts are taken exogenously. But while they took the population structure directly from Paltiel et al. (2012), we use the underlying assumptions of an updated version of the CBS population forecast to compute our own demographic forecast. This leaves us the flexibility of using alternative assumptions on the parameters determining the demographic forecast. In addition, Geva (2013), Argov (2018) and Braude (2013) do not allow for the economic effects discussed above. We take the structure of the economic effects from Ashraf, Weil, and Wilde (2013), who built a long-run simulation model calibrated for Nigeria. The heart of their model is a disaggregated model for human capital with demographic effects on participation, schooling and experience. Their model disaggregates the population only by age and gender. An additional similar simulation model, SEDIM, is described in Sanderson (2004). An added value in their model is that the saving rate depends on the age structure of the population. We introduce this feature from a different setting as discussed in the demographic investment rate model block. The effective human capital block is also related to the recent work in Cavalleri and Guillemette (2017) who forecast the employment rate for OECD countries based on disaggregated groups while taking the demographics as exogenous. The main differences in their approach are that they concentrate on the cyclically adjusted employment rate while we divided it into participation and unemployment, and that their group specific dynamics are cohort-based while in our case it is age-based. We find the age-base to be more convenient when thinking of between groups- or international-convergence in participation rates. In addition, data breaks in the Israeli Labor Force Survey makes it hard, temporarily, to calculate recent cohort-based entry or exit rates from employment. Finally, Cavalleri and Guillemette (2017), also include macro policy effects on employment calibrated based on recent OECD research.[4] We leave the addition of such effects to future versions of the

[3] Our correction for effective years of schooling is taken directly from Argov (2018).

[4] Mainly Gal and Theising (2015) and Égert and Gal (2017).

model.

The Total Factor Productivity (TFP) model block (detailed in Section 3.3) simulates the future growth rate of TFP based on the "conditional convergence" concept. It includes a world growth component and a component that reflects the potential of each economy to grow faster (or slower). The model draws on empirical estimates based on cross country data including both developing and advanced economies. The model includes two main equations. The first equation estimates the initial potential level of labor productivity for each country, based on a cross country regression of labor productivity in 2010 on a group of fundamental variables and additional "policy variables". Among the fundamental independent variables, we include geography, genetic diversity, and ethnic characteristics . The "policy variables" belong to three groups: Institutions ("Doing Business" index, "Economic Freedom" index), Infrastructure (quality of roads, communications infrastructure) and Education (international tests, inequality in years of schooling). The use of this equation also allows us to quantify how the potential level of labor productivity may change in the forecast horizon due to changes in the "policy variables". The second equation determines the long-term growth rate of the global TFP path and the speed of convergence toward the country specific potential labor productivity path through TFP growth. The parameter values of this equation were estimated in a panel regression of 5-year growth of TFP on the initial potential labor productivity level (as estimated in the first regression) over the sample period of 1960 to 2010.

3 A complete description of the model

In this section we detail the concept and equations of each of the five blocks of the model (the subsections are organized according to the different blocks). In each subsection, following the description of the equations, we also provide the source of the parameter values to be used in the baseline simulation.

3.1 The unifying model

3.1.1 Description of the model

The aggregate production function is given by:

$$Y_t = A_t \cdot F(K_t, H_t) \tag{1}$$

Where:

Y_t is total GDP.

A_t is Total Factor Productivity (described in Section 3.3).

K_t is physical capital.

H_t is total human capital (described in Section 3.2).

We assume a constant return to scale translog form which allows for substitution elasticities (governed by α) to differ over time

$$\ln\left(\frac{Y_t}{Y_{t-1}}\right) = \ln\left(\frac{A_t}{A_{t-1}}\right) + \left(\frac{\alpha_t + \alpha_{t-1}}{2}\right)\ln\left(\frac{K_t}{K_{t-1}}\right) + \left[1 - \left(\frac{\alpha_t + \alpha_{t-1}}{2}\right)\right]\ln\left(\frac{H_t}{H_{t-1}}\right) \tag{2}$$

where t indicates time period of 1-year. Unless otherwise stated the time interval between t and $t-1$ is 5 years.

To approximate the elasticities, we calibrate α_t as the share of GDP that is not earned by labor.[5]

Physical capital dynamically evolves according to:

$$K_t = fi_t Y_t + (1 - \delta_t) K_{t-1} \tag{3}$$

[5]This production function is used in Penn World Table to calculate the TFP. It is equivalent to the regular Cobb-Douglas production function when α is constant. See Feenstra, Inklaar, and Timmer (2015).

Because equation (3) involves dynamic processes containing both stock and flow variables, it is evaluated on a 1-year time interval: the interval between t and $t-1$ is only 1 year and δ_t is the (exogenous, time-varying) annual capital depreciation rate.[6] fi_t is the fixed investment rate out of GDP. Since our model is for an open economy it does not necessarily equal the saving rate (as the current account serves as a buffer between the saving and investment rates). However, given our long-term horizon forecast we will assume that factors, like demographic structure, that effect the average saving rate will effect the investment rate. fi_t is assumed to evolve according to:

$$fi_t = \left(1 - \rho_{fi}\right) fi_{t-1} + \rho_{fi} fi_t^{LR} \tag{4}$$

where ρ_{fi} is the 5-year convergence rate. fi_t^{LR} is the endogenous long-run fixed investment rate to which the economy evolves. It is composed of three components: an exogenous constant level ($\overline{fi^{LR}}$) which is based on historical average levels, accumulated demographic effects ($\widehat{fi}_t^{DEM}$) such as the dependency ratio and longevity that will be detailed in Section 3.4, and, the effects of changes in structural parameters (as the depreciation rate) as derived from the GE steady-state model (see Section 3.5):

$$fi_t^{LR} = \overline{fi^{LR}} + \widehat{fi}_t^{DEM} + \widehat{fi}^{GE} \tag{5}$$

3.1.2 Baseline calibration

The parameterization of the unifying model for the baseline forecast is described in Table 1. We set the production function initial elasticity of GDP to physical capital (α) to 0.45, consistent with the current labor share data of 0.55. We assume the historical decreasing trend in the labor share will continue, and reach 0.525 in 2065 (Panel A in figure 2), meaning capital elasticity will increase to 0.475. We set the initial depreciation rate (δ) to 0.07, as

[6] For running the equation within the model-wide 5-year interval, we log-linearly interpolate the development of Y_t within the 5-year period.

implicit in aggregate BOI net-capital stock data. We assume the historical upward trend will continue and reach 0.082 in 2065 (Panel B in figure 2). For the case of the depreciation rate, we will use the steady-state GE model to quantify the offsetting effect: a depreciation rate increase will lead to a parallel increase in the investment rate.[7]

We set the exogenous fixed component of the investment rate ($\overline{fi^{LR}}$) according to the average level of investment to GDP - 19%. The projected level of the investment rate may change due to changes in the long-run rate with an assumed convergence rate (ρ_{fi}) of 0.2.

Table 1: Parameter Values for Unifying Model

Parameter		Value
Capital elasticity int.	α_0	0.450
Capital elasticity LR	α_T	0.475
Depreciation rate int.	δ_0	0.070
Depreciation rate LR	δ_T	0.082
Save rate convergence	ρ_{fi}	0.200
Fixed inv. rate int.	fi_0	0.190
Fixed inv. rate ex. LR	$\overline{fi^{LR}}$	0.190

3.2 The model for aggregate human capital

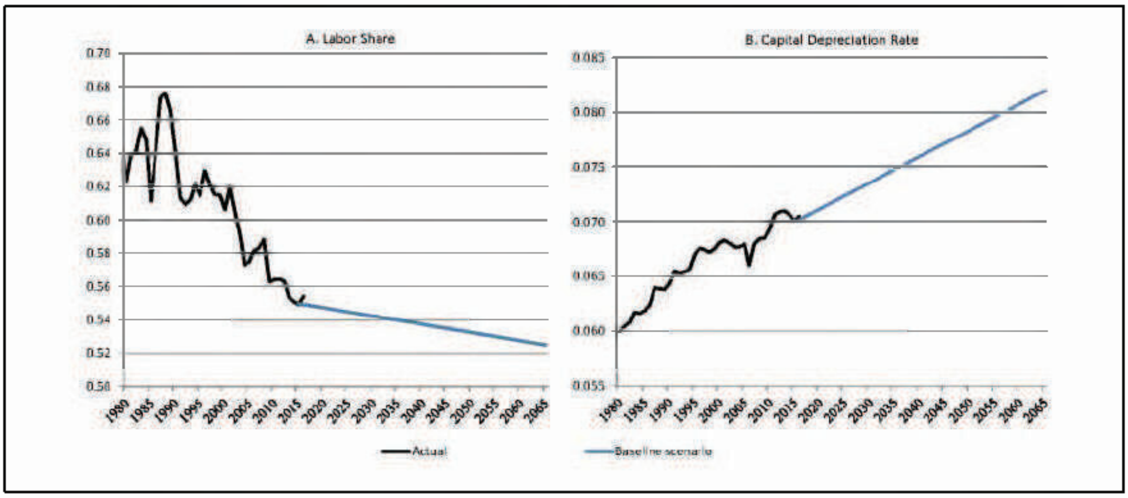
Total human capital, H_t, is composed of Israelis (H_t^{ISR}), and non-Israelis (H_t^{FRN}) which includes both foreign workers and Palestinians working in Israel:

$$H_t = H_t^{ISR} + H_t^{FRN} \tag{6}$$

The aggregate human capital of Israelis, H_t^{ISR}, is the sum over the human capital of various groups in the economy indexed by (a, r, s). We divide the population according to: 5-year age groups, $a \in (0-4, 5-9, \cdots, 75-79, 80+)$; gender, $s \in (M, W)$; and religion groups – non-ultra-Orthodox Jews (thereafter Jews or Jewish), ultra-Orthodox Jews (thereafter Orthodox) and Arabs – $r \in (J, O, A)$. In total, we have 84 working-age

[7] We will not quantify the effect of the capital elasticity increase, as the Cobb-Douglas production function in the steady state GE model is not fit for quantifying changes in the capital elasticity parameter.

Figure 2: Parameters of the Unifying Model - Actual and Baseline Assumption



(15+) population groups and 18 additional under working-age population groups. The effective human capital per-capita of group (a, r, s) in year t is defined by their human capital from effective years of schooling, $h^S_{(a,r,s),t}$, human capital from work experience, $h^E_{(a,r,s),t}$ and their per-capita labor input $(l_{(a,r,s),t})$:

$$H_t^{ISR} = \sum_i \left(h^S_{(a,r,s),t} \cdot h^E_{(a,r,s),t} \cdot l_{(a,r,s),t} \right) N_{(a,r,s),t} \tag{7}$$

where:

$N_{(a,r,s)}$ is the population size of group (a, r, s). The matrix $N_{(a,r,s),0}$ is an (exogenous) initial condition given by the estimated actual population size in the point of departure. The matrix $N_{(a,r,s),t}$ is forecasted using standard demographic forecasts methods discussed in Section 3.2.1.

Each groups' per-capita labor input is composed of the labor force participation rate $(LFPR_{(a,r,s),t})$, the unemployment rate $(UE_{(a,r,s),t})$, and hours per employed person $(HO_{(a,r,s),t})$:

$$l_{(a,r,s),t} = LFPR_{(a,r,s),t} \cdot \left(1 - UE_{(a,r,s),t}\right) \cdot HO_{(a,r,s),t} \tag{8}$$

The matrices $LFPR_{(a,r,s),0}$, $UE_{(a,r,s),0}$ and $HO_{(a,r,s),0}$ are exogenous initial conditions given by the data for the point of departure. Their forecast is discussed in Sections 3.2.2, 3.2.3 and 3.2.4.

Human capital from schooling of group (a, r, s) is determined by its average effective years of schooling $(YSC_{(a,r,s),t})$:

$$h^S_{(a,r,s),t} = e^{\theta_{(a,r,s)} YSC_{(a,r,s),t}} \tag{9}$$

where $\theta_{(a,r,s)}$ is a parameter representing the group-specific macroeconomic return to schooling. The reference to "effective" years expresses our practice of downward correcting the reported years of schooling of ultra-Orthodox men due to their choice of religious Yeshiva studies that do not contribute to their labor market ability. We replace their reported years of schooling, which was approximately 16 years, on average, in 2011, by up to 10 years according to the calibration detailed in Argov (2018). The forecast model for $YSC_{(a,r,s),t}$ is discussed in Section 3.2.5.

Human capital from experience is a quadratic equation on years of labor experience:

$$h^E_t = e^{\phi EXP_{(a,r,s),t} + \psi \left(EXP_{(a,r,s),t}\right)^2} \tag{10}$$

where $EXP_{(a,r,s),t}$ is the calculated average experience in group (a, r, s), and the parameters $\phi > 0$ and $\psi < 0$ determine the returns to labor market experience.[8] The calculation and forecast of experience are discussed in Section 3.2.6.

For non-Israelis we assume a similar structure as for each Israeli group:

$$H^{FRN}_t = e^{\theta^{FRN} YSC^{FRN}} \cdot e^{\phi EXP^{FRN} + \psi \left(EXP^{FRN}\right)^2} \cdot EM^{FRN}_t \cdot HO^{FRN}_t \tag{11}$$

The first two components of (11) are human capital from schooling and from experience, taking the same structure as in equations (9) and (10). EM^{FRN}_t and HO^{FRN}_t are the number of non-Israeli workers and their hours per worker, respectively.

Israel is characterized by a relatively large, by international standards, share of non-Israelis in the labor force: in 2010 they made up 9% of the workers in the economy and 7%

[8] For calculating the experience we use the actual, rather than the effective, years of schooling.

of total human capital (given our baseline parameterization for the return to schooling and experience). Nevertheless, our main motivation for introducing non-Israelis in our model is due to their historical increase of share, mainly in the 1990s (before 1980 they made less than 6% of the workers). Disregarding them would upwardly bias our historical measures of Total Factor Productivity growth.

3.2.1 Forecasting population

In order to forecast population we use standard demographic forecasting methods.[9] The growth of population is determined by four basic exogenous variables: the initial level ($N_{(a,r,s),t-1}$), age- and religion-specific womens' fertility, group-specific survival rate and net-migration.

Step I: Population alive at year t:

The population of groups aged 5-79, that is $a \in \{5-9, 10-14, ..., 75-79;\}$, alive at year t is given by:

$$N_{(a,r,s),t} = N_{(a-1,r,s),t-1} \cdot Sur_{(a-1,r,s),t} + Mig_{(a-1,r,s),t}\left(1 + Sur_{(a-1,r,s),t}\right)/2 \quad \text{for} \quad a \in \{5-79\} \tag{12}$$

where $Sur_{(a,r,s),t}$ is the 5-year survival rate of group (a, r, s) which is a years old, and $Mig_{(a,r,s),t}$ is the net-balance of migration during the 5-years proceeding t.

The population of groups aged 80+, that is $a \in \{80+;\}$, alive at t:

$$\begin{aligned} N_{(80+,r,s),t} &= N_{(75-79,r,s),t-1} \cdot Sur_{(75-79,r,s),t} + N_{(80+,r,s),t-1} \cdot Sur_{(80+,r,s),t} \\ &\quad + Mig_{(75+,r,s),t}\left(1 + Sur_{(75+,r,s),t}\right)/2 \end{aligned} \tag{13}$$

Step II: Population born between $t-1$ and t:

[9] See chapter 3 in Shorter, Sendek, and Bayoumy (1995)

Define:

$$\overline{W}_{(a,r),t} = \left(N_{(a,r,W),t-1} + N_{(a,r,W),t}\right)/2 \quad \text{for} \quad a \in \{15-19, ..., 45-49\}$$

as the average, group-specific, women at potential childbearing age between $t-1$ and t. There are seven age-groups in the fertility-age: 15-19,...,45-49. Then the number of births during the 5-year cycle, for some religion-group r, is given by:

$$B_{(r),t} = 5 \cdot \sum_{a=15-19}^{45-49} \left(F_{(a,r),t} \cdot \overline{W}_{(a,r),t}\right) \tag{14}$$

where $F_{(a,r),t}$ is the age- and time-specific fertility rate (number of yearly births per woman). Finally, the number of 0-4 age population by sex is given by:

$$N_{(0-4,r,s),t} = \begin{cases} (1-w_g) \cdot B_{(r),t} \cdot Sur_{(0,r,s),t} + Mig_{(0-4,r,s),t}\left(0.67 + 0.33 \cdot Sur_{(0,r,s),t}\right) & \text{for} \quad s = M \\ w_g \cdot B_{(r),t} \cdot Sur_{(0,r,s),t} + Mig_{(0-4,r,s),t}\left(0.67 + 0.33 \cdot Sur_{(0,r,s),t}\right) & \text{for} \quad s = W \end{cases} \tag{15}$$

where w_g is the exogenous rate of girl birth and $Sur_{(0,r,s),t}$ is the baby survival rate.

Let us define the Total Fertility Rate ($TFR_{(r),t}$) of religion group r in year t as the number of expected children per woman (of religion r) given the current fertility rates in year t. That is:

$$TFR_{(r),t} = \sum_{a=15-19}^{45-49} F_{(a,r),t} * 5/1000 \tag{16}$$

3.2.2 Forecasting labor force participation rate

The general structure of the dynamic evolvement of the labor force participation rate, $LFPR_{(a,r,s),t}$, for working-age groups ($a \in \{15-19, ..., 80+\}, r, s$) in time t is:

$$LFPR_{(a,r,s),t} = \begin{cases} (1-\tilde{\rho}^{LFPR}_{(a,r,s),t})LFPR_{(a,r,s),t-1} + \tilde{\rho}^{LFPR}_{(a,r,s),t}LFPR^{LR}_{(a,r,s),t} & \text{for} \quad s = M \\ (1-\tilde{\rho}^{LFPR}_{(a,r,s),t})\left[LFPR_{(a,r,s),t-1} + cc^{LF}_{(r)}\left(F_{(a,r),t} - F_{(a,r),t-1}\right)\right] & \text{for} \quad s = W \\ +\tilde{\rho}^{LFPR}_{(a,r,s),t}LFPR^{LR}_{(a,r,s),t} & \end{cases} \tag{17}$$

Equation (17) is a simple AR(1) process toward a group-specific long-run benchmark value ($LFPR^{LR}_{(a,r,s),t}$) to be discussed below. The 5-year speed of convergence parameter ($\tilde{\rho}^{LFPR}_{(a,r,s),t}$) is age-, religion-, gender- and cohort-specific:

$$\tilde{\rho}^{LFPR}_{(a,r,s),t} = \rho^{LFPR}_{(a,r,s)} ind^{LFPR}_{(c,r,s)} \tag{18}$$

where $\rho^{LFPR}_{(a,r,s)} \in (0,1)$ is a matrix of age-, religion- and gender-specific parameters of convergence, and $ind^{LFPR}_{(c,r,s)}$ is a matrix of fixed 5-year cohort indicators, exogenously taking the value of 1 if cohort c is expected to experience convergence in the participation-rate and 0 otherwise. A cohort, indexed by c is defined by its age (in 5 years intervals) in the forecast's point of departure ($t = 0$), rather than its time varying age (a). To understand this structure let us look at the following example: we generally assume that the speed of convergence for Orthodox men ($\rho^{LFPR}_{(a,O,M)}$) is 0.15 for all ages. However, we also assume that this speed of convergence will apply only for cohorts that are aged below 35 at the forecast's point of departure ($t = 0$). Older cohorts will not experience convergence ($ind^{LFPR}_{(c>35,A,M)} = 0 \rightarrow \tilde{\rho}^{LFPR}_{(a,r,s),t} = 0$ for $a \geq 35$ in $t = 0$).

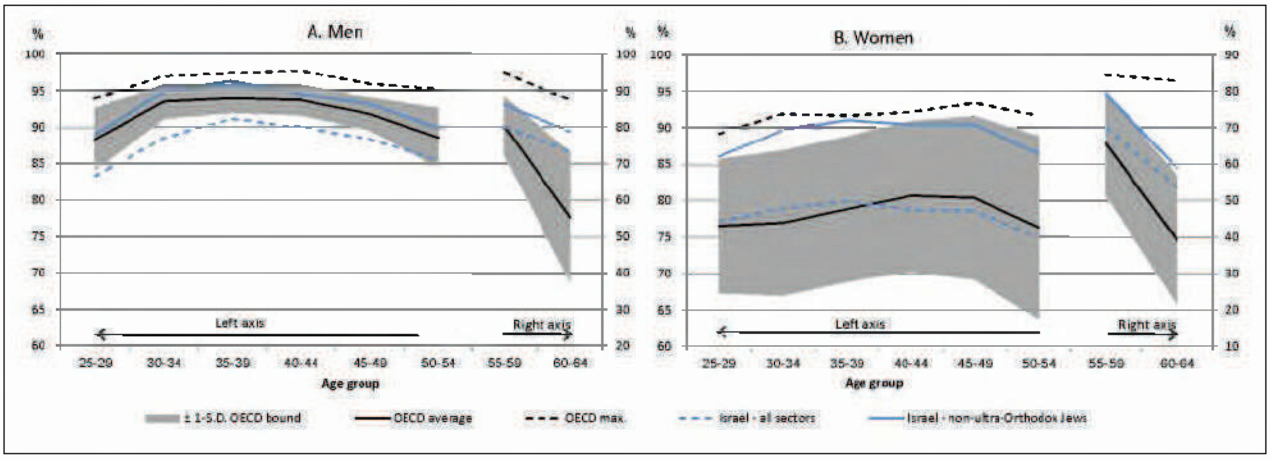
For women ($s = W$), equation (17) also includes an effect of changes in age-specific fertility ($F_{(a,r),t}$) in order to capture the trade-off between raising children and participating in the labor force. The religion-specific parameter $cc^{LF}_{(r)}$ determines the child cost in terms of labor force participation.

The dynamic process for the benchmark value of participation rate ($LFPR^{LR}_{(a,r,s),t}$) of the Jewish group ($r = J$) is:

$$LFPR^{LR}_{(a,J,s),t} = \begin{cases} LFPR^{LR}_{(a,J,s),t-1} & \text{for} \quad s = M \\ LFPR^{LR}_{(a,J,s),t-1} + cc^{LF}_{(J)} \left(F_{(a,J),t} - F_{(a,J),t-1} \right) & \text{for} \quad s = W \end{cases} \tag{19}$$

In (19), we assume a fixed level for the Jewish men, while for women we assume the benchmark level of participation changes according to developments in age-specific fertility ($F_{(a,r),t}$). However, we limit their benchmark rate to that of men in the same age.

Figure 3: Labor Force Participation Rate, Israel vs. OECD, Ages 25-64, 2015



In 2015, the participation rates of most Jewish age groups were high compared to OECD averages (Figure 3). Therefore, we set the initial level of benchmark rate, $LFPR^{LR}_{(a,J,s),t=0}$ for most Jewish groups according to actual participation rates at the point of departure. However, as the existing gaps in participation rates between Jewish men and women have been narrowing in the past decades,[10] for age groups between 30–59 we use the mens' actual rate for the womens' benchmark as well, to facilitate future convergence between gender groups (for other age groups, below 30 and above 59, we use the own group's actual rate at the point of departure). Formally:

[10] See Figure 13.

$$LFPR^{LR}_{(a,J,s),t=0} = \begin{cases} LFPR_{(a,J,M),t=0} & \text{for} \quad a \in \{30-34, ..., 55-59\} \\ LFPR_{(a,J,s),t=0} & \text{for} \qquad otherwise \end{cases} \tag{20}$$

For Orthodox and Arab groups we set the benchmark rate as the actual participation rate of the parallel (age) Jewish group, corrected for levels of fertility in the case of women:

$$LFPR^{LR}_{(a,r,s),t} = \begin{cases} LFPR_{(a,J,s),t-1} & \text{for} \quad s = M; r \in \{O, A\} \\ LFPR_{(a,J,s),t-1} + cc^{LF}_{(r)} \left(F_{(a,r),t} - F_{(a,J),t}\right) & \text{for} \quad s = W; r \in \{O, A\} \end{cases} \tag{21}$$

3.2.3 Forecasting unemployment

The general structure of the dynamic evolvement of the unemployment rate, $UE_{(a,r,s),t}$, for working-age groups $(a \in \{15-19, ..., 80+\}, r, s)$ in time t is:

$$UE_{(a,r,s),t} = (1 - \widetilde{\rho}^{UE}_{(a,r,s),t})UE_{(a,r,s),t-1} + \widetilde{\rho}^{UE}_{(a,r,s),t}UE^{LR}_{(a,r,s),t} \tag{22}$$

Equation (22) is a simple AR(1) process toward a group-specific long-run benchmark value $(UE^{LR}_{(a,r,s),t})$ to be discussed below. The 5-year speed of convergence parameter $(\widetilde{\rho}^{UE}_{(a,r,s),t})$ is age-, religion-, gender- and cohort-specific:

$$\widetilde{\rho}^{UE}_{(a,r,s),t} = \rho^{UE}_{(a,r,s)} ind^{UE}_{(c,r,s)}$$

where $\rho^{UE}_{(a,r,s)} \in (0,1)$ is a matrix of age-, religion- and gender-specific parameters of convergence, and $ind^{UE}_{(c,r,s)}$ is a matrix of fixed 5-year cohort indicators, exogenously taking the value of 1 if cohort c is expected to experience convergence in the unemployment rate and 0 otherwise (see discussion following equation 17 for an example). In general we will assume that the speed of convergence is proportional to that of the Labor Force Participation block, and the cohort indicator is identical.

The dynamic process for the benchmark value of participation rate $(UE^{LR}_{(a,r,s),t})$ follows:

$$UE^{LR}_{(a,r,s),t} = \begin{cases} UE^{LR}_{(a,r,s),t-1} & \text{for} \quad r \in \{J\} \\ UE_{(a,J,s),t-1} & \text{for} \quad r \in \{O, A\} \end{cases} \tag{23}$$

In (23), we assume a fixed level for the benchmark unemployment rate of the Jewish group ($r \in \{J\}$). We set the initial level of the benchmark rate of the Jewish group, $UE^{LR}_{(a,J,s),t=0}$ to the actual unemployment rate at the point of departure ($UE_{(a,J,s),t=0}$). For the Orthodox and Arab groups we set the benchmark rate as the actual unemployment rate among the parallel (age) Jewish group.

3.2.4 Forecasting hours per worker

The general structure of the dynamic evolvement of hours per worker, $HO_{(a,r,s),t}$, for working-age groups ($a \in \{15-19, ..., 80+\}, r, s$) in time t is:

$$HO_{(a,r,s),t} = \begin{cases} (1-\widetilde{\rho}^{HO}_{(a,r,s),t})HO_{(a,r,s),t-1} + \widetilde{\rho}HO^{LR}_{(a,r,s),t} & \text{for} \quad s = M \\ (1-\widetilde{\rho}^{HO}_{(a,r,s),t})\left[HO_{(a,r,s),t-1} + cc^{HO}_{(r)}\left(F_{(a,r),t} - F_{(a,r),t-1}\right)\right] & \text{for} \quad s = W \\ +\widetilde{\rho}HO^{LR}_{(a,r,s),t} & \end{cases} \tag{24}$$

Equation (24) is a simple AR(1) process toward a group-specific long-run benchmark value ($HO^{LR}_{(a,r,s),t}$) to be discussed below. The 5-year speed of convergence parameter ($\widetilde{\rho}^{HO}_{(a,r,s),t}$) is age-, religion-, gender- and cohort-specific:

$$\widetilde{\rho}^{HO}_{(a,r,s),t} = \rho^{HO}_{(a,r,s)} ind^{HO}_{(c,r,s)}$$

where $\rho^{HO}_{(a,r,s)} \in (0,1)$ is a matrix of age-, religion- and gender-specific parameters of convergence, and $ind^{HO}_{(c,r,s)}$ is a matrix of fixed 5-year cohort indicators, exogenously taking the value of 1 if cohort c is expected to experience convergence in the unemployment rate and 0 otherwise (see discussion following equation 17 for an example). In general we will assume that the speed of convergence is proportional to that of the Labor Force Participation block, and the cohort indicator is identical.

For women ($s = W$), equation (24) also includes an effect of changes in age-specific fertility ($F_{(a,r),t}$) in order to capture the tradeoff between raising children and putting in hours at work. The religion specific parameter $cc_{(r)}^{HO}$ determines the child cost in terms of hours per worker.

The dynamic process for the benchmark value of hours per worker $(HO_{(a,r,s),t}^{LR})$ of the Jewish group $(r = J)$ is:

$$HO_{(a,J,s),t}^{LR} = \begin{cases} HO_{(a,J,s),t-1}^{LR} & \text{for} \quad s = M \\ HO_{(a,J,s),t-1}^{LR} + cc_{(J)}^{HO} \left(F_{(a,J),t} - F_{(a,J),t-1}\right) & \text{for} \quad s = W \end{cases} \tag{25}$$

In (25), we assume a fixed level for Jewish men, while for women we assume the benchmark level of hours changes according to developments in age-specific fertility $(F_{(a,r),t})$. However, we limit their benchmark level of hours to that of men in the same age. We set the initial level of benchmark rate, $HO_{(a,r,s),t=0}^{LR}$ to the actual hours per worker at the point of departure $(HO_{(a,r,s),t=0})$.

For Orthodox and Arab groups we set the benchmark rate as the actual hours per worker of the parallel (age) Jewish group , corrected for levels of fertility in the case of women:

$$HO_{(a,r,s),t}^{LR} = \begin{cases} HO_{(a,J,s),t-1} & \text{for} \quad s = M; r \in \{O, A\} \\ HO_{(a,J,s),t-1} + cc_{(r)}^{HO} \left(F_{(a,r),t} - F_{(a,J),t}\right) & \text{for} \quad s = W; r \in \{O, A\} \end{cases} \tag{26}$$

3.2.5 Forecasting effective years of schooling

In general we assume that most of schooling occurs up to the age of 34, while the decision of how many years to study is taken between the ages of 25 and 34. The dynamic equation for average years of schooling, $YSC_{(a,r,s)}$, of the youngest working-age groups is: $(a \in \{15-19, 20-24\})$ is:

$$YSC_{(a,r,s),t} = (1 - \tilde{\rho}_{(a,r,s),t}^{YSC}) YSC_{(a,r,s),t-1} + \tilde{\rho}_{(a,r,s),t}^{YSC} YSC_{(a,r,s),t}^{LR} \quad \text{for} \quad a \in \{15-24\} \tag{27}$$

Equation (27) describes a simple AR(1) process toward a group specific benchmark value ($YSC^{LR}_{(a,r,s),t}$) to be discussed below. The 5-year speed of convergence parameter ($\widetilde{\rho}^{YSC}_{(a,r,s),t}$) is age-, religion-, gender- and cohort-specific:

$$\widetilde{\rho}^{YSC}_{(a,r,s),t} = \rho^{YSC}_{(a,r,s)} ind^{YSC}_{(c,r,s)}$$

where $\rho^{YSC}_{(a,r,s)} \in (0,1)$ is a matrix of age-, religion- and gender-specific parameters of convergence, and $ind^{YSC}_{(c,r,s)}$ is a matrix of fixed 5-year cohort indicators, exogenously taking the value of 1 if cohort c is expected to experience convergence in years of schooling and 0 otherwise (see discussion following equation 17 for an example).

The equation for the "decision making ages" ($a \in \{25-29, 30-34\}$) includes a cohort specific Joshi and Schultz (2007) quantity-quality effect ($JS_{(c,r),t}$).

$$YSC_{(a,r,s),t} = \begin{array}{c} (1-\widetilde{\rho}^{YSC}_{(a,r,s),t}) \left[YSC_{(a,r,s),t-1} + \left(JS_{(a,r),t} - JS_{(a,r),t-1}\right)\right] \\ +\widetilde{\rho}^{YSC}_{(a,r,s),t} YSC^{LR}_{(a,r,s),t} \end{array} \quad \text{for} \quad a \in \{25-34\} \tag{28}$$

where the cohort specific Joshi-Schultz effect is determined by the religion groups' Total Fertility Rate (TFR) at the time that the cohort was born ($a \in \{0-4\}$):

$$JS_{(a,r),t} = \begin{cases} js \cdot \ln\left(\frac{\overline{JS}}{TFR_{(r),t}}\right) & \text{for} \quad a \in \{0-4\} \\ JS_{(a-1,r),t-1} & \text{for} \quad a \in \{5-9, ..., 80+\} \end{cases} \tag{29}$$

In equation (29) the parameter $\overline{JS}$ technically normalizes the Joshi-Schultz effect to be zero when $TFR_{(r),t} = \overline{JS}$, and the parameter js determines the magnitude of the effect of siblings on the future number of years of schooling.

As for cohorts belonging to older age groups ($a \in \{35-29, ..., 80+\}$), we assume they gradually increase the number of years of schooling they have acquired until the age of 34 toward the benchmark rate:

$$YSC_{(a,r,s),t} = (1-\tilde{\rho}^{YSC}_{(a,r,s),t})YSC_{(a-1,r,s),t-1} + \tilde{\rho}^{YSC}_{(a,r,s),t}YSC^{LR}_{(a,r,s),t} \quad \text{for} \quad a \in \{35-39, ..., 80+\} \tag{30}$$

Note that here the auto regressive component, $YSC_{(a-1,r,s),t-1}$, is the cohorts' previous period schooling rather than the age groups' previous period schooling as in equations (27) and (28).

We now turn to the benchmark value for years of schooling ($YSC^{LR}_{(a,r,s),t}$). For Jews we generally assume that in the long run, cohorts characterized by $TFR = \overline{JS}$ at the time of birth will reach ysc^{LR} average years of schooling: 12 years before mandatory army services and an additional $ysc^{LR} - 12$ years directly following the service (in our baseline model we set $ysc^{LR} = 18$). Cohorts characterized by higher TFR, and therefore a negative Joshi-Schultz effect (see equation 29), will have a lower benchmark level for years of schooling:

$$YSC^{LR}_{(a,J,s),t} = \begin{cases} Min\left\{age(a) - 6;\ ysc^{LR} + JS_{(a,J),t}\right\} & \text{for} \quad a \in \{15-19\} \\ Min\left\{age(a) - army_{J,s} - 6;\ ysc^{LR} + JS_{(a,J),t}\right\} & \text{for} \quad a \in \{20-34\} \\ YSC^{LR}_{(a-1,J,s),t-1} & \text{for} \quad a \in \{35+\} \end{cases} \tag{31}$$

where $age(a)$ is the average age of group a (for example $age(15-19) = 17$), and $army_{r,s}$ is the overall length of army service for religion group r of gender s (Army service, conducted mainly by the non-ultra-Orthodox Jewish population starts at age 18 and lasts for 2.5 to 3.5 years when we include a customary post–service extended vacation). For the group aged 30-34, $age(a) - army_{J,s} - 6$ is at least 22.5. Therefore according to the second row in equation (31) the benchmark rate will normally be determined by $ysc^{LR} + JS_{(a,J),t}$ since it is typically lower than 22.5. According to the third row of equation (31) the long run rate stays fixed for each cohort of age group 35–39.

Historical data point to similar patterns between Orthodox and Jewish women, where it doesn't seem that the Orthodox women are following the educational level of Jewish women. Therefore, for Orthodox women, we adopt an identical system for the benchmark value of years of schooling of the ($YSC^{LR}_{(a,O,W),t}$) as that of the Jewish population as described in equation (31).

We model the Orthodox men and Arab population groups' ($r \in \{U, A\}$) benchmark average years of schooling so that it is equal to the actual average years of schooling of the parallel Jewish group, except for differences resulting from historical differences in the TFR (through the JS effect):[11]

$$YSC^{LR}_{(a,r,s),t} = \begin{cases} YSC_{(a,J,s),t-1} & \text{for} \quad a \in \{15-19\}\,; r \in \{O, A\} \\ (1-w) \cdot YSC_{(a,J,s),t-1} + w \cdot YSC_{(a+1,J,s),t-1} & \text{for} \quad a \in \{20-24\}\,; r \in \{O, A\} \\ YSC_{(a,J,s),t-1} + JS_{(a,r),t} - JS_{(a,J),t} & \text{for} \quad a \in \{25+\}\,; r \in \{O, A\} \end{cases} \tag{32}$$

The benchmark rate for the 20–24 age group is a weighted average between its parallel non-ultra-Orthodox Jewish group and the actual years of schooling of the 5-year older agegroup. This reflects the lack of army service in the Arab and Orthodox groups which may allow them to begin tertiary education earlier. The relative weight on the older group is a function of the army service of the parallel's group: $w = (army_{J,s} - army_{r,s})/5$.

3.2.6 Calculating labor market experience

Since there is no observation on labor market experience, it is customary to use a simple age- and schooling-based calculation for experience:

$$\text{Experience} = (\text{age}) - (\text{actual years of schooling}) - (\text{years of army service}) - 6$$

However, there are two complicating issues that need to be addressed: (1) for the actual years of schooling we need to count all years, including those which are not effective in terms of labor market human capital; (2) we need to account for cohort-specific increases in labor force participation (i.e., entrance of new, non-experienced, workers to the labor force) which introduces an upward bias in the simple age- and schooling-based calculation of experience.

[11] For the sake of brevity we mark the group as $r \in \{U, A\}$ although it includes only the men of the Orthodox group.

Noneffective years of schooling The ultra-Orthodox men are characterized by non-labor-market-effective years of schooling. Their primary and secondary education systems include massive religious study at the expense of core curriculum (mathematics, English and science). Moreover, they attend additional years of schooling in Yeshiva schools which offer primarily religious studies.[12] These years of study are formally recorded in the Labor ForceSurvey as regular years of study. When constructing our data for effective years of schooling, YSC, which affects the human capital from schooling ($h^S_{(a,r,s),t}$), we correct for this bias by limiting the Orthodox mens' years of schooling to 10 years.[13] However, it is the actual, rather than the effective, years of schooling that determines the worker's labor market experience (given that students do not work during effective and noneffective studies).

For the purpose of calculating labor market experience, we define the actual years of schooling ($YSC^{Act}_{(a,r,s),t}$), reported in the original data, by:

$$YSC^{Act}_{(a,r,s),t} = YSC_{(a,r,s),t} + YSC^{ext}_{(a,r,s),t} \tag{33}$$

where $YSC_{(a,r,s),t}$ is the effective years of schooling and $YSC^{ext}_{(a,r,s),t}$ is the extra (noneffective) years of schooling.[14] The dynamic forecast model for extra years of schooling simply assumes that the reduction in non-effective schooling, to the extent that it exists, will follow the convergence process in effective years of schooling:

$$YSC^{ext}_{(a,r,s),t} = \begin{cases} (1-\tilde{\rho}^{YSC}_{(a,r,s),t})YSC^{ext}_{(a,r,s),t-1} + \tilde{\rho}^{YSC}_{(a,r,s),t}\overline{YSC}^{ext}_{(r,s)} & \text{for} \quad a \in \{15-34\} \\ (1-\tilde{\rho}^{YSC}_{(a,r,s),t})YSC^{ext}_{(a-1,r,s),t-1} + \tilde{\rho}^{YSC}_{(a,r,s),t}\overline{YSC}^{ext}_{(r,s)} & \text{for} \quad a \in \{35+\} \end{cases} \tag{34}$$

where $\overline{YSC}^{ext}_{(r,s)}$ is a religion- and gender-specific long-run value of extra (non effective) years of schooling. In the baseline forecast we set $\overline{YSC}^{ext}_{(r,s)} = 0$ for all groups. The group-

[12] We identify individuals in the Labor Force Survey as ultra-Orthodox if their, or their family member's, last educational institute was a Yeshiva school.

[13] For details on this calibration see Argov (2018)

[14] The equations are written in general form and not only for the ultra-Orthodox men.

and cohort-specific speed of convergence, $\tilde{\rho}^{YSC}_{(a,r,s),t}$, is the same as in the dynamic process for years of schooling (section 3.2.5). Notice that for age groups younger than 35, the AR process is on the previous period group of the same age, $YSC^{ext}_{(a,r,s),t-1}$ whereas for older age groups, the process is on the same cohort's previous period extra years of schooling - $YSC^{ext}_{(a-1,r,s),t-1}$; just as in the processes for effective years of schooling.

Given this correction, the calculation for average experience, $EXP_{(a,r,s),t}$, of the young working-age groups ($a \in \{15-29\}$) is given by:

$$EXP_{(a,r,s),t} = \begin{cases} Max\left(age(a) - YSC^{Act}_{(a,r,s),t} - 6; 0\right) & \text{for} \quad a \in \{15-19\} \\ Max\left(age(a) - YSC^{Act}_{(a,r,s),t} - army_{r,s} - 6; 0\right) & \text{for} \quad a \in \{20-29\} \end{cases} \tag{35}$$

where $age(a)$ is the average age of group a, and $army_{r,s}$ is the overall length of army services for religion group r of gender s (details appear after equation 31). Since experience may not be negative, we bound the calculation by zero.

Increases in labor market participation The simple age- and schooling-based calculation of labor market experience does not take into account that increases in prime age (+30) participation rates means that new workers enter the labor force with low experience and therefore reduce the age-group's average years of experience. This issue is particularly relevant if we assume that convergence in labor force participation rates (between religion groups) occurs at all working ages, and not only through the young-age new entrants. There is no exact way of calculating and simulating this effect, however we do take it into account in the forecasting process by following each cohort's average labor force participation rate as of age 25. Increases in a cohort's average over time means there are new entrants with low experience.

Formally, the average experience, $EXP_{(a,r,s),t}$, of the older prime age groups ($a \in \{+30\}$) is a weighted average of the regular age- and schooling-based calculation (as in equation 35) and 2.5 years (which reflects the average experience of new entrants during the 5-year cycle):

$$EXP_{(a,r,s),t} = \left(1 - w^{new}_{(a,r,s),t}\right)\left(age(a) - YSC^{Act}_{(a,r,s),t} - army_{r,s} - 6\right) + w^{new}_{(a,r,s),t} \cdot 2.5 \quad \text{for} \quad a \in \{+30\} \tag{36}$$

The weight for new entrants is defined by:

$$w^{new}_{(a,r,s),t} = 1 - \text{Min}\left(\frac{LFPR^{(25+)hist}_{(a-1,r,s),t-1}}{LFPR_{(a,r,s),t}}; 1\right) \tag{37}$$

where $LFPR^{(25+)hist}_{(a,r,s),t}$ is the (a, r, s) cohort's average participation rate as of age 25:

$$LFPR^{(25+)hist}_{(a,r,s),t} = \text{Mean}\left(LFPR_{(a,r,s),t}, LFPR_{(a-1,r,s),t-1}, ..., LFPR_{(a-i,r,s),t-i}\right) \quad \text{for} \quad (a-i) \geq (25-29) \tag{38}$$

The idea behind $w^{new}_{(a,r,s),t}$ is that out of a group's current participation rate ($LFPR_{(a,r,s),t}$), the cohort's average historical rate ($LFPR^{(25+)hist}_{(a-1,r,s),t-1}$) has participated since the end of their schooling years, and the rest are new participants. We use this logic only when the cohort's current participation rate is higher than the average ($LFPR_{(a,r,s),t} - LFPR^{(25+)hist}_{(a-1,r,s),t-1} > 0$). When it is as below, i.e., the cohorts' participation rate is on a downward path, we assume it is the less experienced workers that left the workforce and therefore the regular age- and schooling-base calculation applies.

3.2.7 Initial data for the baseline simulation

The initial disaggregated data for the simulation of the human capital model is mainly based on the 2015 Labor Force Survey (LFS). From the individual level data we construct initial average data matrices for the labor force participation rate ($LFPR_{(a,r,s),0}$), the unemployment rate ($UE_{(a,r,s),0}$), actual weekly hours worked per employed person ($HO_{(a,r,s),0}$), effective years of schooling ($YSC_{(a,r,s),0}$) and extra non-effective years of schooling ($YSC^{ext}_{(a,r,s),0}$). The matrices consist of 84 population cells divided by 5-year age groups, $a \in (0-4, 5-9, \cdots, 75-79, 80+)$; gender, $s \in (M, W)$; and religion groups – non-ultra-Orthodox Jews (hereinafter Jews or Jewish), ultra-Orthodox Jews (hereinafter Orthodox) and Arabs – $r \in (J, O, A)$. Historically, the LFS didn't include a question on the level of religion which could directly identify ultra-Orthodox Jews. We therefore indirectly identify them by individual Jews for which a Yeshiva was the last school for them or for one of their family members. An alternative data set which allows direct identification of ultra-Orthodox Jews is the Social Survey, last available in 2014. This survey was also the basis for the long-term demographic forecasts made by the Central Bureau of Statistics. However the Social Survey does not include labor force statistics which are the bread and butter of our growth model. For comparison, in the CBS demographic forecast's point of departure, 8.2% of the 2015 working age (15+) population were ultra-Orthodox Jews. Our identification from the LFS points to a somewhat lower share of 7.1%.

The initial population matrix ($N_{(a,r,s),\mathbf{0}}$) is also derived from the 2015 LFS which generally records individuals in the working age population (15+). We construct initial data on the younger groups through the question on the surveyed individuals' children.

For the Joshi-Shultz effect we need historical estimates of the total fertility rate $TFR_{(a,r),\mathbf{0}}$. For Arab women we use data from the CBS's Annual Data (from the years 2012–16) and for Jewish ultra-Orthodox and non-ultra-Orthodox women we use estimates from an updated database of Hleihel (2011). For the simulation of average experience we need historical estimates of cohort average labor force participation rates ($LFPR^{(25+)hist}_{(a,r,s),\mathbf{0}}$). These are constructed from the LFS between 1987–2015.

3.2.8 Baseline parameter calibration

The parameter sets for the human capital model include three main groups - (1) general group-independent parameters (Table 2), (2) age independent parameters (Table 3), and (3) group dependent parameters (Tables 4 and 5).

For the population forecast we adopted the assumptions from the CBS's 2016 demographic forecast (medium scenario).[15] Specifically, we used the assumptions on the rate of

[15] The 2016 forecast is an updated version of Paltiel et al. (2012).

girl birth (w_g), the time-varying group specific survival-rate ($Sur_{(a,r,s),t}$), the fertility-rate ($F_{(a,r),t}$) and net-migration ($Mig_{(a,r,s),t}$). The assumptions on the fertility rate translate into the Total Fertility Rate of non-ultra-Orthodox Jewish women remaining stable at 2.5 births per woman until 2040, after which it gradually decreases to 2.3. A gradual downward TFR path is assumed for ultra-Orthodox women (from 6.7 to 5.2 in 2065) and Arab women (from 3.1 to 2.3 in 2065). Based on historical averages, the annual number of net migrators (Mig) is assumed to remain 21,800 until 2035, after which it will gradually decrease to zero. Most net migrators belong to the Jewish religion group.

For the forecast of human capital from schooling we assume the general level of long run number of years of schooling (ysc^{LR}) is 18. Though this number seems extremely high, our calibration of convergence rate ensures it is not nearly reached in the forecast horizon (in 2065, the average years of schooling of young Jews is forecasted to be 16 years—approximately 0.8 years more than the 2015 figure). As detailed in Section 3.2.5, the long- and short-run number of years of schooling negatively depends on the groups' TFR at the time of birth (quality-quantity trade-off). The calibration of the parameter $\overline{TFR}$ is a normalization indicating that the long-run years of schooling will be lower than 18 if $TFR > 2.1$. The magnitude of the effect of TFR on the expected years of schooling (both in the short- and long-run) is governed by js. For its calibration we use the results from Joshi and Schultz (2007), who analyzed a randomized intervention in Matlab, Bangladesh. They found that a TFR reduction of 15%, resulting from the intervention, led to an increase of 0.52 years of schooling (i.e., $js = 0.52/0.15$). Following Argov (2018) we calibrate the return to schooling of all groups (θ) to 8% – a value in line with both macro and micro estimates.

The calibration of the experience effect on human capital (ϕ and ψ) is taken from Friedman and Zussman (2009) which estimated micro level wage regressions for Israel.

The annual expansion rate of foreign workers (g_{EM}^{FRN}) is assumed be 1%. Based on historical estimates, each foreign worker is assumed to work 45 hours per week. Foreign workers are assumed to have the human capital equivalent of 8 years of schooling and 18 years of experience.[16]

Table 2: General Parameter Values for Human Capital Model

Parameter		Value
Rate of girl birth	w_g	0.49
Net migration (avg.)	$\overline{Mig}$	17,036
Long-run YSC	ysc^{LR}	18
TFR benchmark for JS	$\overline{TFR}$	2.10
TFR effect on YSC	js	3.47
Return to schooling	θ	0.080
Return to experience	ϕ	0.052
Return to experience (sqr.)	ψ	-0.001
FRN growth rate	g_{EM}^{FRN}	0.010
FRN hours per worker	HO^{FRN}	45
FRN equivalent years of schooling	YSC^{FRN}	8
FRN equivalent experience	EXP^{FRN}	18

We now turn to the group specific but age independent parameters in Table 3. We assume a negative effect of childbearing on labor market participation and hours per-worker governed through the religion-specific parameters $cc_{(r)}^{LF}$ and $cc_{(r)}^{HO}$. For Jewish women, we calibrate a cost in line with the finding in Bloom et al. (2009) for developed economies that a marginal child reduces 4.4 years of labor supply. We assume half of this effect works through participation and half through hours per-worker.[17] For ultra-Orthodox women, characterized with much higher fertility rates, the cost of a marginal child should be lower due to economies of scale in child-rearing. We use the calibration from Ashraf, Weil, and Wilde (2013) of half a year cost which was targeted for a simulation for Nigeria. For the Arab population, which has an intermediate level of TFR, we use a cost of 2.5 years. Once

[16] Although non-Israeli workers are generally younger, and therefore less experienced than the average Israeli worker, we set the foreign workers' experience equal to the average Israeli worker. In doing so we folded all of the foreign workers lower productivity into the calibration of their years of schooling.

[17] Accordingly, $cc_{(J)}^{LF} = 0.5 * (-4.4) = -2.2$ and $cc_{(J)}^{HO} = 0.5 * (-4.4) * (32/0.85) = -82.8$ when using the actual average of 32 weekly working hours and 0.85 employment rate for Jewish women as a measure of normal employment in terms of hours.

again, for both groups we split the cost equally between participation and hours per worker (while using the actual average weekly hours adjusted for the employment rate of women as a measure of normal work hours).

The length of army service ($army_{r,s}$) for Jewish men and women is set to 3.5 and 2.5 years, respectively, which includes both the actual length of service and a 1/2 year customary trip following the service. In general, the Orthodox and Arab populations do not conduct proper army service. Extra non-effective years of schooling ($\overline{YSC}^{ext}_{(r,s)}$) are assumed to be zero in the long-run for all population groups (including ultra-Orthodox men for which their initial extra years of schooling reaches up to 6.7 years).

Table 3: Age Independent Parameter Values

Parameter		Jews		Ultra-Orthodox		Arabs	
		Men	Women	Men	Women	Men	Women
Child time cost - participation	$cc^{LF}_{(r)}$	-	-2.20	-	-0.25	-	-1.25
Child time cost - hours	$cc^{HO}_{(r)}$	-	-82.8	0.0	-8.0	0.0	-86.1
Army service length	$army_{(r,s)}$	3.5	2.5	0.0	0.0	0.0	0.0
LR Non-effective YSC	$\overline{YSC}^{ext}_{(r,s)}$	0.0	0.0	0.0	0.0	0.0	0.0

The group specific convergence in labor market variables ($LFPR_{(a,r,s)}$, $UE_{(a,r,s)}$, $HO_{(a,r,s)}$ and $YSC_{(a,r,s)}$) are mainly determined by the speed of convergence parameters ($\rho_{(a,r,s)}$) and the cohort indicators ($ind_{(c,r,s)}$) for which a value of 1 indicates that the group is in a convergence process (see for example equations 17 and 18 for the participation rate). We followed two general guiding rules in calibrating the speed of convergence parameters. First, we tried to maintain consistency in the calibration between labor market variables or between different groups within the same variables. Second, we closely inspected historical and resulting simulated group specific labor market variable figures in order to set convergence rates that are in line with historical developments in the groups' convergence. Such graphs are available in Appendix A. We are aware that various changes made in the LFS as of 2012 resulted in a data break. When inspecting how detailed labor force data developed over time in the past, one should be aware of this data break which makes it difficult to assess whether historical convergence continued in the most recent past. For this reason, the historical data in the figures of Appendix A are broken between 2011 and 2012.

We assume that the Jewish womens' participation rate will continue to converge to that of Jewish men so that 35% of the gap will be closed each 5-year period (see Table 4). In addition, we assume between sector convergence will also occur, but at a moderate pace: the Ultra-orthodox men and Arab groups' convergence rate to their Jewish counterparts is set to 0.1. Moreover, for Orthodox men and Arab women convergence will occur only for cohorts that in the year of departure were aged younger than 30 (Orthodox men) and 45 (Arab women). Older cohorts will keep the initial year's age-specific participation rate. As for Orthodox women, who showed significant increases of participation rates in the past, we calibrate a faster speed of convergence of 0.25.

For the unemployment rate and hours per worker we assume the convergence rate for all groups will be 1/2 of the convergence rate for the participation rate ($\rho^{UE}_{(a,r,s)} = \rho^{HO}_{(a,r,s)} = 0.5\rho^{LFPR}_{(a,r,s)}$), with a similar structure of cohort convergence indicators.

For young Jews and Orthodox women we assume the average years of schooling will close in each 5-year period 5% of the gap between the actual and the assumed long-run years of schooling (see Table 5). Young (25-34) ultra-Orthodox men and Arabs will close in each period 40% and 20%, respectively, of the gap between them and their counterpart non-ultra-Orthodox Jews. Age groups between 35 and 64 will gradually close the gap between the number of schooling years the cohort has studied until the age of 34 and their group specific long run or benchmark rate.

3.3 A model for TFP

This part of the model generates the forecast of the growth rate in Total Factor Productivity (TFP, A_t). To do so, we employ an econometric cross-country "conditional convergence" model. The details of the model, including the conceptual background, the structure of the

Table 4: Parameter Values for Labor Force Participation Rate Convergence

	Jews		Ultra-Orthodox		Arabs	
	Men	Women	Men	Women	Men	Women
Age group			Variable: $\rho^{LFPR}_{(a,r,s)}$			
15-19	0.02	0.02	0.10	0.00	0.10	0.10
20-24	0.02	0.02	0.10	0.00	0.10	0.10
25-29	0.02	0.02	0.10	0.25	0.00	0.10
30-34	0.35	0.35	0.10	0.25	0.10	0.10
35-39	0.35	0.35	0.10	0.25	0.10	0.10
40-44	0.35	0.35	0.10	0.25	0.10	0.10
45-49	0.35	0.35	0.10	0.25	0.10	0.10
50-54	0.35	0.35	0.10	0.25	0.10	0.10
55-59	0.35	0.35	0.10	0.25	0.10	0.10
60-64	0.60	0.60	0.10	0.25	0.10	0.10
65-69	0.60	0.60	0.10	0.25	0.10	0.10
70-74	0.60	0.60	0.10	0.25	0.10	0.10
75-79	0.60	0.60	0.10	0.25	0.10	0.10
80+	0.02	0.02	0.10	0.25	0.10	0.10
Cohort			Variable: $ind^{LFPR}_{(c,r,s)}$			
15-19	1	1	1	1	1	1
20-24	1	1	1	1	1	1
25-29	1	1	1	1	1	1
30-34	1	1	0	1	1	1
35-39	1	1	0	1	1	1
40-44	1	1	0	1	1	1
45-49	1	1	0	1	1	0
50-54	1	1	0	1	1	0
55-59	1	1	0	1	1	0
60-64	1	1	0	1	1	0
65-69	1	1	0	1	1	0
70-74	1	1	0	1	1	0
75-79	1	1	0	1	1	0
80+	1	1	0	1	1	0

Table 5: Parameter Values for Years of Schooling Convergence

	Jews		Ultra-Orthodox		Arabs	
	Men	Women	Men	Women	Men	Women
Age group			Variable: $\rho^{YSC}_{(a,r,s)}$			
15-19	0.00	0.00	0.40	0.00	0.15	0.15
20-24	0.00	0.00	0.40	0.00	0.20	0.20
25-29	0.05	0.05	0.40	0.05	0.20	0.20
30-34	0.05	0.05	0.40	0.05	0.20	0.20
35-39	0.03	0.03	0.03	0.03	0.03	0.03
40-44	0.03	0.03	0.03	0.03	0.03	0.03
45-49	0.03	0.03	0.03	0.03	0.03	0.03
50-54	0.03	0.03	0.03	0.03	0.03	0.03
55-59	0.03	0.03	0.03	0.03	0.03	0.03
60-64	0.03	0.03	0.03	0.03	0.03	0.03
65-69	0.00	0.00	0.00	0.00	0.00	0.00
70-74	0.00	0.00	0.00	0.00	0.00	0.00
75-79	0.00	0.00	0.00	0.00	0.00	0.00
80+	0.00	0.00	0.00	0.00	0.00	0.00
Cohort			Variable: $ind^{YSC}_{(c,r,s)}$			
15-19	1	1	1	1	1	1
20-24	1	1	1	1	1	1
25-29	1	1	1	1	1	1
30-34	1	1	0	1	1	1
35-39	1	1	0	1	1	1
40-44	1	1	0	1	1	1
45-49	1	1	0	1	1	1
50-54	1	1	0	1	1	1
55-59	1	1	0	1	1	1
60-64	1	1	0	1	1	1
65-69	1	1	0	1	1	1
70-74	1	1	0	1	1	1
75-79	1	1	0	1	1	1
80+	1	1	0	1	1	1

model and the econometric estimations, are described in a companion paper - Tsur and Argov (forthcoming). Here, for the sake of brevity, we will lay out only the parts relevant to our forecast model and its parameterization.

The TFP model is built from three equations. The first equation determines the 5-year growth rate of TFP:

$$\Delta a_t = a_g + \rho Gap^y_{t-1} + \lambda_t \tag{39}$$

where $a_t = \log(A_t)$ and Δ is the 5-year difference operator. The growth rate of TFP is determined by the global frontier growth rate parameter, a_g, and the initial labor productivity gap ($Gap^y_{t-1} = \hat{y}_{t-1} - y_{t-1}$)—that is, the gap between the country specific potential labor productivity ($\hat{y}_{t-1}$), to be detailed below, and the actual one y_{t-1}. The inclusion of this gap generates the conditional convergence process in which countries that are initially below their own potential will grow faster than the global frontier growth rate through the TFP component. The speed of convergence is set by the parameter ρ. In addition, we include in the equation an exogenous temporary shock to TFP growth λ_t. The dynamic process for the labor productivity gap is:

$$Gap^y_t = Gap^y_{t-1} + \Delta\hat{y}_t - \left(\frac{1}{1-\alpha}\right)\Delta a_t$$

where $\Delta\hat{y}_t$ - is the log difference in country specific potential labor productivity (in terms of GDP per worker); The last term in the equation reflects the actual increase in labour productivity that is driven by increases in TFP.[18]

The third equation describes the dynamic evolvement of the country specific potential level of labor productivity ($\hat{y}$) as a function of changes in fundamental- and policy-effected variables:

[18] Given the endogeneity of the capital stock, a 1% increase in TFP leads in the long run to a $\left(\frac{1}{1-\alpha}\right)$% increase in labor productivity.

$$\Delta \hat{y}_t = \left(\frac{1}{1-\alpha}\right) a_g + b_1 \Delta \overline{Fundementals}_t + b_2 \Delta \overline{Policy}_t + e_t \qquad (40)$$

where $\overline{Fundementals_t}$ is a set of country level fixed fundamental attributes such as geography, culture and luck;[19] $\overline{Policy_t}$ is a set of changeable, possibly by means of policy, variables; e_t is a permanent shock to the level of potential labor productivity. According to equation (40), given that policy or fundamental variables remain unchanged, the potential level of productivity grows over time according to the global frontier growth in TFP, adjusted through the fraction $\left(\frac{1}{1-\alpha}\right)$ to its dynamic long-run effect on labor productivity.

To complete the description of the TFP model we need to explain how we derive the initial labor productivity gap (Gap_0^y) and the parameter estimates of equations (39) and (40). These are derived from cross country econometric estimations.

The initial level of country specific productivity gap as well as the b parameters are derived from cross country regressions of the (log) level of labor productivity (GDP per worker) on a set of fundamental and policy affected variables for the years t=1965, 1970,...,2010:

$$y_{i,t} = \alpha + \beta_{t,1} \overline{Fundementals}_i + \beta_{t,2} \overline{Policy}_{it} + \epsilon_{i,t} \qquad (41)$$

The full data set includes approximately 70 developing and advanced economies (indexed by i), 42 of them with GDP per capita above 5000$ (among them Israel).

The fundamental variables included are taken from a variety of studies that explored the deep roots of growth, as organized in Ashraf and Galor (2013): (1) **Neolithic transition** is the number of years (in thousands) that elapsed since agriculture became the primary mode of subsistence; (2) **Arable land** is the fraction of total land area that is arable, as reported by the World Bank's World Development Indicators; (3) **Population in tropical** is the percentage of a country's 1995 population that lives in tropical areas; (4) **Distance to waterway** is the average across the grid cells of a country, in thousands of km, from

[19] Although these variables are fixed, we mark them by under-script t in order to allow us to conduct a purely hypothetical simulation for GDP had one of the fixed attributes would be different.

Table 6: Regression Estimation Results for TFP Model

(1) Dependent variable: Log of GDP per worker in 2010 (PPP adjusted)	
Variable	Avg. Coefficient[1]
Neolithic transition	0.113
Arable land	-0.156
Population in tropical	-0.428
Distance to waterway	0.065
OPEC dummy	0.4505
Genetic diversity	-1.484
Genetic diversity sq'	2.501
Ethnic fractionalization	0.073
Doing business	0.010
Economic freedom	0.002
Roads quality	0.197
School grades	0.009
Phones per capita	0.252
Educational inequality	-0.142
Constant	6.553
Religon controls	Yes

(2) Dependent variable: 5-year TFP growth	
Variable	Coefficient
Lagged prod. Gap	0.0438***
	(0.0146)
Dummy for years:	
1976-1980	0.0783***
	(0.0215)
1981-1985	
1986-1990	0.0781***
	(0.0207)
1991-1995	0.0712***
	(0.0207)
1996-2000	0.0704***
	(0.0207)
2001-2005	0.0627***
	(0.0207)
2006-2010	0.0384*
	(0.0207)
Constant	-0.0441***
	(0.0149)
Observations	279
R-squared	0.084

Standard errors in parentheses

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

[1] The average estimates are based on 8 regressions which are different combinations of the listed variables.

an ice-free coastline or sea-navigable river; (5) **OPEC dummy** equals 1 for countries that are members in the Organization of the Petroleum Exporting Countries; (6) **Genetic diversity** is the expected heterozygosity (genetic diversity) as predicted by migratory distance from East Africa (Ashraf and Galor (2013));(7) **Ethnic fractionalization** is the probability that two randomly selected individuals, will belong to different ethnic groups; (8) **Religion controls** include variables that represent the share of Muslims, the share of Catholics and the share of Protestants in the country.

As for the policy variables, we include two from each of the following three groups - (a) institutions, (b) infrastructure and (c) education. (a-1) **Doing business** is the country's "Distance to Frontier" in the World Bank's indicator which measures the easiness of doing business in several areas; (a-2) **Economic Freedom** is an index that covers 12 areas, such as property rights and financial freedom, in 186 countries since 1970 (b-3) Data on **the quality of roads** - a principal component of indicators for the quality of roads, based on indices taken from the "International Road Federation". (b-4) data on **communication infrastructures** - main telephone lines and mobile phones per 1000 workers, as published by the World Bank, based on the International Telecommunications Union; (c-5) Data on the quality of education: **Test scores** for the years 1995 2010, standardized over time, across subjects (Math, Reading and Science), across schooling levels, and across various international and regional assessments. These data will be obtained from the World Bank, based on a study by Angrist, Patrinos, and Schlotter (2013). (c-6) **Inequality in education** is represented by Gini coefficients of education provided by Ziesemer (2016) for 146 countries for the years 1950–2010, based on data from Barro and Lee (2013). These Gini coefficients were calculated based on a methodology that was first developed by Thomas, Wang, and Fan (2001) and Castelló and Doménech (2002).

We described 6 policy variables along with 8 fundamental categories so we have a total of 14 controls. Including all of them in a single regression naturally yields some non-significant variables. Given that we do not use all the variables as regressors, there is a huge number of combinations of regressors that may be entered into the regression, and choosing between them might be arbitrary and simplistic. Therefore, we decided to focus on specifications that include the full set of fundamentals, one institutions variable, one infrastructure variable, and one education variable.[20] This rule creates 8 combinations of level regressions which could be estimated using the full sample of countries or a sub-sample that includes only advanced economies with GDP per capita above 5000$. Israel is included in the advanced economies, but at the same time some of its policy variables - mainly in education and institution - are in the proximity of poorer countries. Therefore we decide to take the approach of basing our estimates of the initial gap and coefficient values on two sets of regressions (2*8=16 level regressions). There are several options to weight the results from the 16 regressions. We decided (at least at this point) to simply average between the predicted gaps and parameter estimates from the specifications. Column 1 in Table 6 presents the average estimates for each variable across the 16 specifications.

The labor productivity gap in 2010 is the negative value of the country-specific error term in the regression; put otherwise, the potential level of labor productivity in 2010 is the fitted value of labor productivity given the country specific fundamental and policy variables. Since our simulations start in 2015, we extrapolated the estimated gap from the regression using the actual development of labor productivity in Israel between 2010 and 2015.

Israel's average productivity gap in 2015 was near 8% while the gap in OECD countries was -4%. This means that in the long term the Israeli productivity is expected to grow faster than OECD countries' average and get 12productivity.

In order to test for the robustness of the policy variable coefficient estimates to basic differences between the economies in our sample that were not captured by the fundamen-

[20]This strategy is somewhat similar to the one adopted by Sala-i Martin (1997), who ran around 2 million regressions in order to test which variables are the most correlated with prosperity. Sala-i Martin (1997) decided to include three fixed variables and three variables that changed from one specification to the other.

tals variables, we also estimated panel regressions with policy variables and country fixed effects (in this setting we do not receive country specific productivity gaps which are crucial for our model setting). Generally, the sizes of the coefficient estimates of the three policy variables are very similar to their sizes in the cross section regressions that control for fundamental variables. Importantly, the relative effect of each policy area—infrastructure, institutions and educational quality — is kept. This similarity suggests that controlling for the fundamentals improved the validity of the estimates for the policy variables, including those policy variables that are available for use only in the cross section regressions (more details on this set of regressions appear in Tsur and Argov (forthcoming)). Nevertheless, we are aware that our coefficients do not represent clean causal effects, and they may also include reverse effects (from growth to policy effected variables). Our methodology balances between estimating the "clean" causal effect of policy variables on the level of productivity and achieving estimates with external validity. Our estimates will have higher external validity compared to research that exploits a specific exogenous event in order to find a causal relationship between policy and growth, and higher internal validity compared to cross section regressions with policy variables alone. Estimating the effect of policy variables on the level of productivity in cross section regressions, after controlling deep roots, gets us closer to the causal effect of policy measures on long-run standards of living.

The parameters ρ and a_g are derived from a cross country 5-year panel regression running from 1960 to 2010. The econometric specification is:

$$\Delta a_{i,t} = \rho \times Gap^{y}_{i,t-1} + YearFixedEffects + Cons \tag{42}$$

The dependent variable, $\Delta a_{i,t}$, is the 5-year TFP growth rate of country i ending in period t. The main explanatory variable is the labor productivity gap at the end of the previous five year period ($Gap^{y}_{i,t-1}$). The productivity gap for each period is estimated through labor productivity level regressions just as discussed above, however, including only 3 policy variables for which we have historical data available. In addition, the explanatory variables include a constant and a year fixed effects (where the base year is 1961-1965). The estimates of the constant and the year fixed effect are used to parameterize the average level of frontier growth (a_g).

The estimation results are given in the second column of Table 6. The estimated speed of convergence, ρ, is 0.0438, meaning that half of an existing gap is expected to close within 40 years.[21] For the simulations we set the frontier 5-year growth rate to 2% based on the average year fixed effects as of 1990–2010 plus the constant of the equation. This implies an annual TFP frontier growth rate of approximately 0.4%.

3.4 The demographic investment rate model

The investment rate of the economy affects the long-run growth forecast through its effect on the accumulation of capital (see equation 3). Demographic developments, mostly effect the investment rate and the saving rate through two main channels. First, the longevity channel reflects the increase in life expectancy which, given the stickiness in actual retirement ages, operates to increase the saving rate during the working years. Second, the dependency channel reflects the effect that a greater weight of elderly or children in the population reduces the average saving rate because there is no income in those ages. In addition, a high weight of dependent groups may reduce the saving rate of the working ages that need to economically provide for the dependent ages. Theoretical, overlapping generation models to support these channels are given in Li, Zhang, and Zhang (2007) and Tobing (2012).

In order to incorporate demographic effects in the long-run growth model, we allowed, in equation (5), the long run of the investment rate (f_t^{LR}) to depend on accumulated demographic development ($\widehat{fi}_t^{DEM}$) through three basic demographic variables:

$$\widehat{fi}_t^{DEM} = -0.007 \cdot \widehat{TFR}_{t-1} + 0.002 \cdot \widehat{Ex}_t - 0.671 \cdot \widehat{OldDep}_t \tag{43}$$

[21] Half life is equal to $\frac{\log(1/2)}{\log\left(\frac{1-\alpha-\rho}{1-\alpha}\right)}$.

where TFR_t is the economy wide Total Fertility Rate, Ex_t is life expectancy at birth and $OldDep_t$ is the Old (aged 65+) to middle-age (15-64) population ratio. The forecast for all of these variables is generated consistently with the demographic forecast described in Section (3.2.1). The hat ($\widehat{\ }$) sign stands for the difference of the variable from some benchmark rate (e.g. $\widehat{TFR}_t = TFR_t - \overline{TFR}$) that will be calibrated consistently with the constant term in the long-run investment rate equation ($\overline{fi^{LR}}$). Specifically, since we will set $\overline{fi^{LR}} = 0.19$ which is the average of the investment rate during 2000–10, we will also set the benchmark rates of the demographic variables to their 2000–10 average:

$$\overline{TFR} = 2.98; \overline{Ex} = 80.3; \overline{OldDep} = 0.159$$

The choice of the variables in 43, as well as the calibration of the coefficients, is based on the empirical cross-country panel estimation Li, Zhang, and Zhang (2007). In the basic regression, from which we employed the coefficients, the dependent variable is the investment to GDP ratio, and the independent variables are the three variables above (controlling for time and country fixed effects). The negative coefficients on $\widehat{TFR}_{t-1}$ and $\widehat{OldDep}_t$ capture the dependency channel, and the positive coefficient on $\widehat{Ex}_t$ captures the longevity channel. The estimation covers 149 countries over the period 1963 to 2003, where in order to reduce short-term cyclical influences, annual data were converted to 5-years averages (in total, due to lags and some missing data, the estimations included 775 observations).

3.5 The static general equilibrium model

In order to allow long-run effects of macroeconomic structural changes, we combine into our long-term growth model a steady-state version of the Bank of Israel's general equilibrium model (MOISE).[22] For the time being, we derive from the GE model its prediction for the real investment rate which will be used to quantify effects on the long-run value of the fixed

[22] See Argov et al. (2012)

investment rate (fi_t^{LR}, see equation 5). For the purpose of shortness, in this section we will not Lay out the entire MOISE model, but rather sketch the key macro relations and functions underling the model, while removing as much as possible parts of the equations that do not effect the steady state (like temporary shocks). The economic agents in the GE model include households, firms of several types in the production sector, a government and an inflation-targeting central bank (in these models monetary policy does not affect the long run of the real side of the economy, and therefore we will not elaborate on it here). Households consume, supply labor and make investment decisions (since they are the owners of the capital stock). The production sector includes monopolistic producers of intermediate goods (who employ labor and physical capital as production inputs), competitive producers of final goods (one of which is investment goods) which combine domestic and imported intermediate goods to produce final goods (one of which is investment goods), importers of intermediate goods, and exporters.

The section is structured according to the economic agents of the model and ends with the equilibrium condition and the basic calibration of the model. We should note that the MOISE is a quarterly model, so that the notation t in the description of MOISE stands for a quarter (rather than a five-year interval as in the long-run growth model).

3.5.1 Households

The model consists of a continuum of households, indexed by $h \in [0, 1]$. Households derive their lifetime utility from the discounted flow of private consumption (with external habit formation) and leisure:

$$E_t \sum_{k=0}^{\infty} \left[\beta^k \left(\ln \left(C_{h,t+k} - \kappa C_{t+k-1} \right) - \frac{1}{1+\zeta} \left(N_{h,t+k} \right)^{1+\zeta} \right) \right], \qquad (44)$$

where E_t is the mathematical expectations operator, $C_{h,t}$ denotes the consumption composite consumed by household h in period t and $N_{h,t}$ denotes working hours. The parameter β is the discount factor and ζ is the inverse of the Frisch elasticity of labor supply. Households are subject to external habit persistence, where the parameter κ measures its degree and C_t denotes the composite of *aggregate* consumption in period t. The period-by-period budget constraint faced by household h is given by:

$$
\begin{aligned}
& \left(1+\tau^{C}\right) P_{C,t} C_{h,t} + P_{I,t} I_{h,t} + P_{I,t} \Delta inv Y_t \\
& + \left(R_t\right)^{-1} B_{h,t+1} + \left(R_t^{*}\right)^{-1} S_t B_{h,t+1}^{*} \\
= \; & \left(1-\tau^{N}-\tau^{W_h}\right) W_{h,t} N_{h,t} + \left(1-\tau^{K}\right) R_{K,t} K_{h,t} \\
& + \left(1-\tau^{D}\right) D_{h,t} - T_t + B_{h,t} + S_t B_{h,t}^{*} \; .
\end{aligned}
\tag{45}
$$

The first term, $\left(1+w_{\tau^C}\tau^{C}\right) P_{C,t} C_{h,t}$, denotes nominal expenditure on consumption, τ^C is the rate of value added tax (VAT), w_{τ^C} is the share of goods subject to VAT and $P_{C,t}$ is the pre-tax price of the consumption good. The term $P_{I,t} I_{h,t}$ is the expenditure on fixed capital investment (which is conducted by households) and $P_{I,t}\Delta inv Y_t$ is the exogenously determined expenditure associated with the change in inventories (Y_t is GDP and Δinv is the calibrated steady-state value for the change in inventories as a share of GDP). In the second row of the budget constraint (45), $B_{h,t}$ and $B_{h,t}^{*}$ denote bond holdings at the beginning of period t, denominated in domestic and foreign currencies, respectively. The market price of the local currency bond, $\left(R_t\right)^{-1}$, is driven by the short-term gross nominal interest rate, R_t. With respect to the foreign-currency bonds, S_t denotes the nominal exchange rate, while R_t^{*} is the foreign risk-free nominal interest rate.

Households provide labor services at an hourly wage rate of $W_{h,t}$. A household's labor income is subject to two taxes: a direct income tax τ^N and a social security tax τ^{W_h}. The household also has capital income, where $R_{K,t}$ denotes the nominal price of capital services and $K_{h,t}$ denotes the capital stock owned by household h. The tax rate on capital income is τ^K.[23] Household h earns a flow of dividends, $D_{h,t}$, deriving from its ownership of monopolistic firms. τ^D is the tax rate on dividend income. Finally, the variable T_t denotes a lump sum tax.

[23] The budget constraint in the original model included a tax exemption on depreciation ($\tau_t^K \delta P_{I,t} K_{h,t}$). We dropped this term in order to allow the calibration of the investment rate given updated values of other parameters (mainly α in the production function)

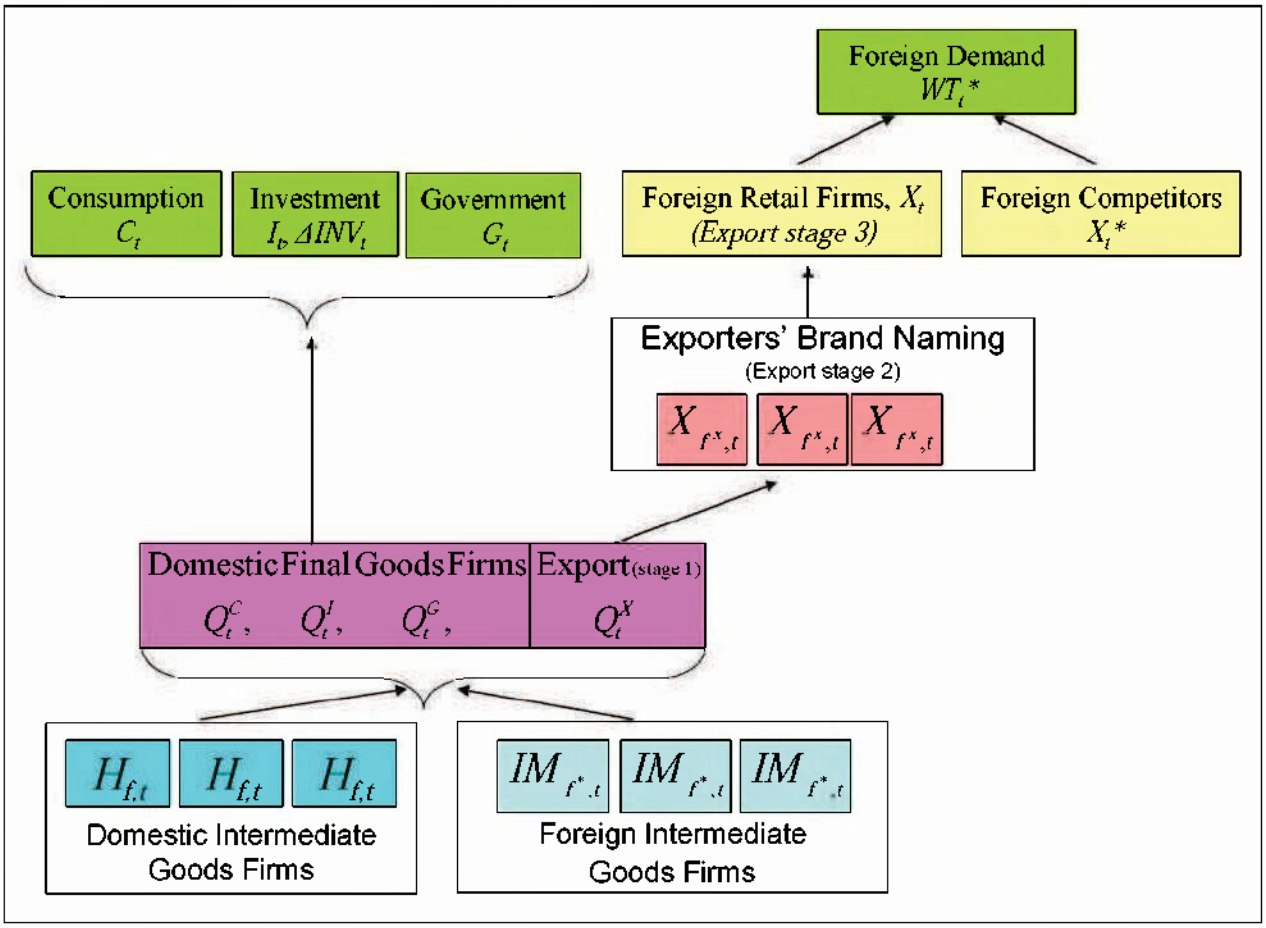
The stock of physical capital evolves as follows:

$$K_{h,t+1} = (1-\delta) K_{h,t} + I_{h,t}, \tag{46}$$

where δ is the quarterly depreciation rate.

3.5.2 Firms

Figure 4: The Structure of the Production Sector



Turning to the firms, Figure 4 illustrates the structure of the production sector, which is comprised of five types of firms:

- Monopolistically competitive domestic firms which produce differentiated intermediate goods, $H_{f,t}$, where $f \in [0,1]$.

- Monopolistically competitive foreign firms which produce differentiated intermediate goods, $IM_{f^*,t}$, where $f^* \in [0,1]$. These goods are imported to the domestic economy.

- Perfectly competitive firms which produce final goods for consumption, investment, government consumption and export (Q_t^C, Q_t^I, Q_t^G, and Q_t^X, respectively). The production inputs of these firms are the differentiated intermediate goods, both domestically produced ($H_{f,t}$) and imported ($IM_{f^*,t}$).

- Monopolistically competitive exporters who buy the final homogenous domestic export good (Q_t^X) and differentiate (i.e., brand name) it. The differentiated good, $X_{f^X,t}$ where $f^x \in [0,1]$, is then sold to foreign retail firms.

- Foreign retail firms which combine the differentiated export goods ($X_{f^X,t}$) into a homogenous exported good (X_t).

We will now turn to a detailed description of each firm type.

Domestic intermediate goods firms A continuum of domestic firms, indexed by $f \in [0,1]$, produce differentiated intermediate goods, $Y_{f,t}^s$. The production technology combines capital, $K_{f,t}^s$, and differentiated labor services hired from households, $N_{f,t}$:

$$Y_{f,t}^s = \max\left[\left(K_{f,t}^s\right)^\alpha \left(z_t N_{f,t}\right)^{1-\alpha} - \psi z_t,\ 0\right]. \tag{47}$$

z_t is a difference-stationary labor-augmenting productivity shock that determines the balanced growth path of all real variables (all of which are symmetric across firms). Its steady-state gross growth rate is denoted by g_z.

The variable $K_{f,t}^s$ is (homogenous) capital services rented under perfect competition. Labor services employed by the $f'th$ firm, $N_{f,t}$, is given by a Dixit and Stiglitz (1977)

Constant Elasticity of Substitution (CES) composite of household-specific labor inputs, $N^h_{f,t}$:

$$N_{f,t} = \left(\int_0^1 \left(N^h_{f,t}\right)^{\frac{1}{\varphi^W}} dh \right)^{\varphi^W}. \tag{48}$$

The exogenous CES between differentiated labor services is defined to be $\varphi^W / \left(\varphi^W - 1\right) > 1$, where $\varphi^W > 1$ may be interpreted as an exogenous wage markup.

Finally, the production technology (47) includes a fixed cost term ψz_t, where the parameter ψ is calibrated to ensure zero profits in the steady state. This is consistent with the assumption of no entry or exit of firms in the steady state.[24]

Total variable production cost is given by :

$$TVC_t = R_{K,t} K^S_{f,t} + R^F_t \left(1 + \tau^{W_f}\right) W_t N_{f,t}, \tag{49}$$

where $\tau^{W_f}_t$ is the rate of the social security tax levied on firms. The model includes a working capital channel, $R^F_t = 1 + \nu^F \left(R_t - 1\right)$, where each firm borrows a fraction ν^F of its wage bill ahead of production at an interest rate of R_t.

Foreign intermediate goods firms A continuum of foreign firms, indexed by $f^* \in [0,1]$, produce differentiated intermediate goods, $IM_{f^*,t}$, which are imported to the domestic economy. We assume consumer-currency pricing subject to the following nominal marginal cost:

$$MC^*_t = S_t \left(\overline{\Pi}^*_Y P^*_{OIL,t-1}\right)^{\omega^*} \left(P^*_{Y,t}\right)^{1-\omega^*}. \tag{50}$$

Except for the nominal effective exchange rate, S_t, all variables in (50) are expressed in terms of producer currency: $\overline{\Pi}^*_Y$ is the gross steady-state inflation rate in the foreign economy, $P^*_{OIL,t}$ is the global price of oil and $P^*_{Y,t}$ is the global price of foreign intermediate

[24] When analyzing the steady state effect of parameter changes, only for the purpose of our long-term growth model, we may also assume the fixed cost ψ does not change so that the "new" steady state includes profits.

goods. The parameter ω^* is the share of oil in the import basket. Once differentiated, the imported intermediate goods are supplied as inputs to the final goods firms in monopolistically competitive markets.

Domestic final goods firms Domestic firms producing final goods are divided into four categories: producers of consumption goods Q_t^C, producers of investment goods Q_t^I, producers of government-consumption goods Q_t^G and producers of exported goods Q_t^X. This section describes the first category, i.e., the producers of final consumption goods. A similar description can be applied to the other categories as well.[25,26]

The final consumption good is a CES composite of domestically produced and imported aggregates of intermediate goods (Y_t^C and IM_t^C, respectively):

$$Q_t^C = \left(\nu_C^{\frac{1}{\mu_C}} \left[Y_t^C\right]^{1-\frac{1}{\mu_C}} + (1-\nu_C)^{\frac{1}{\mu_C}} \left[IM_t^C\right]^{1-\frac{1}{\mu_C}}\right)^{\frac{\mu_C}{\mu_C-1}}. \tag{51}$$

The parameter μ_C is the CES between domestic and imported goods while the parameter ν_C measures the degree of home bias ($1-\nu_C$ is the steady-state import intensity in the Q_t^C sector).

The aggregates of the domestically produced and imported intermediate goods are, respectively:

$$Y_t^C = \left(\int_0^1 \left(Y_{f,t}^C\right)^{\frac{1}{\varphi^H}} df\right)^{\varphi^H}, \tag{52}$$

and

$$IM_t^C = \left(\int_0^1 \left(IM_{f^*,t}^C\right)^{\frac{1}{\varphi^*}} df^*\right)^{\varphi^*}. \tag{53}$$

Thus, φ^H and φ^* are the optimal markups of the intermediate goods producers.

[25] With the appropriate changes in parameterization.

[26] Section 3.5.2 below elaborates on some additional steps in the production and marketing of export goods.

Deriving the analogous equations for the other sectors (Q_t^I, Q_t^G and Q_t^X) is straightforward and is accomplished by replacing the index C in equations (51) to (53) with I, G or X.

Exporters This subsection focuses on the so-called exporters (see figure 4), who are indexed by $f^x \in [0,1]$. They buy the homogenous export good, Q_t^X, and brand-name it so as to provide a differentiated good, $X_{f^x,t}$.[27] Thus, exporter f^x buys the amount $Q_{f^X,t}^X$ of the homogenous export good and brand-name it to become $X_{f^X,t}$ units of differentiated good using a simple production function:

$$X_{f^X,t} = Q_{f^X,t}^X - \psi^X z_t. \tag{54}$$

As in the case of monopolistic producers of domestic intermediate goods, brand naming involves a fixed cost, $\psi^X z_t$.

Foreign retail firms Foreign retail firms purchase the differentiated export goods $X_{f^X,t}$, where $f^X \in [0,1]$, and combine them into a homogenous export good, X_t (see Figure 4). The homogenous export good, in turn, is a CES aggregate of the differentiated export goods:

$$X_t = \left(\int_0^1 (X_{f^X,t})^{\frac{1}{\varphi^X}} df^X \right)^{\varphi^X}. \tag{55}$$

Since there are infinitely many foreign retailers who sell a homogenous good, the price of the good is equal to their marginal cost of production, namely $P_{X,t}$ (in foreign currency). The homogenous export good is combined with other countries' export goods to form a CES aggregate of world trade, WT_t^*. Thus, the demand for Israeli exports is:

$$X_t = \nu^* \left(\frac{P_{X,t}}{P_{X,t}^{c,*}} \right)^{-\mu^*} WT_t^* \quad , \tag{56}$$

[27] With an additional sector of differentiated goods in place, monopolistic competition can be imposed on the exporting sector, thus allowing for price rigidity in terms of the foreign currency.

where the parameter μ^* is the price elasticity of exports, the parameter ν^* is a country-specific share in world trade and $P^{c,*}_{X,t}$ is the price aggregate of world trade ($p^{c,*}_{X,t} = \frac{P^{c,*}_{X,t}}{P^*_{Y,t}}$ is its relative price which may be thought of as an exogenous part of the terms of trade).

3.5.3 The government and the current account

The government purchases homogenous final goods (G_t), issues bonds (B_t) and imposes taxes both distortionary and lump sum. The period-by-period budget constraint faced by the government is given by:

$$\begin{aligned} P_{G,t}G_t + B_t &= \tau^C P_{C,t}C_t + \left(\tau^N + \tau^{W_h}\right)\int_0^1 W_{h,t}N_{h,t}dh + \tau^{W_f}W_tN_t \\ &\quad + \tau^K\left[R_{K,t} - \delta P_{I,t}\right]K_t + \tau^D D_t + T_t + R_t^{-1}B_{t+1}. \end{aligned} \quad (57)$$

We assume that the steady state of government expenditure as share of GDP ($s_g = \frac{P_G G}{P_Y Y}$) is exogenous. The allocation between lump-sum taxes (T_t) and the issue of debt (B_{t+1}) to finance government spending (in order for the budget constraint (57) to be satisfied), is determined by the following rule:

$$s_{T,t} = \phi_B\left(s_{B,t+1} - s_B\right). \quad (58)$$

The variables $s_{T,t} \equiv \frac{T_t}{P_{Y,t}Y_t}$ and $s_{B,t+1} \equiv \frac{B_{t+1}}{P_{Y,t}Y_t}$ are, respectively, lump-sum taxes and the outstanding government debt, both in terms of their share in GDP. Note that since distortionary taxes are exogenous, "Ricardian equivalence" holds and the (somewhat arbitrary) specification of the financing rule (58) does not affect the rest of the model. Also note that (58) ensures the convergence of government debt to its steady-state value in the long run ($E_t\left[s_{B,t+\infty}\right] \rightarrow s_B$).

The current account is defined by:

$$CA_t = P_{X,t}S_tX_t - P_{IM,t}IM_t + s_{FTR}P_{Y,t}Y_t \quad (59)$$

where s_{FTR} is the exogenous share of foreign transfers in GDP to the domestic economy. In order to insure that the domestic economy's share in global assets does not grow indefinitely, we assume that in the steady state the current account is balanced ($CA = 0$).

3.5.4 Solving and calibrating the baseline steady state

There are a few steps needed before solving the steady state: setting down market clearing conditions, stationarizing the model and calibrating parameters that affect the steady state of the model.

Market clearing conditions define the equilibrium between demand and supply. For example, clearing conditions in the competitive domestic final goods market are as follows:

$$Q_t^C = C_t \ , \tag{60}$$

$$Q_t^I = I_t + \Delta INV_t \tag{61}$$

and

$$Q_t^G = G_t \ . \tag{62}$$

Stationarizing the model involves manipulating the variables so that the model will be expressed in terms of variables that have a defined state (that is, they do not grow over time). This involves normalizing real variables by the permanent technology factor z_t (for example normalized GDP is $y_t = \frac{Y_t}{z_t}$) and prices are expressed as a ratio to the consumption price ($p_{y,t} = \frac{P_{Y,t}}{P_{C,t}}$). Then we may solve for the model's steady state.

Table 7 presents the basic calibration of the structural parameters that affect the steady state. The guiding principle for the calibration is to set the model's basic steady-state values and ratios to those observed in the data over long horizons or those that were viewed to be the convergence values for the economy. Most values are taken from Argov et al. (2012) but some are updated due to changes in ratios observed in the past ten years.

Table 7: Calibrated Parameters

Parameter		Value	Parameter		Value
Discount factor	β	0.997	Wage markup	φ^W	1.3
Inverse of labor EoS	ζ	2.0	Dom. price markup	φ^H	1.5
Capital share in prod.*	α	0.45	Imp. price markup	φ^*	1.3
LR productivity growth*	g_z	1+0.008/4	Exp. price markup	φ^X	1.2
Depreciation rate*	δ	0.07/4	Home bias - C*	ν^C	0.75
EoS in consumption	μ^C	1.1	Home bias - I*	ν^I	0.665
EoS in investment	μ^I	1.1	Home bias - G	ν^G	0.95
EoS in government	μ^G	0.2	Home bias - X*	ν^X	0.67
EoS in exports	μ^X	1.1	Gov. to GDP*	s_G	0.24
Foreign EoS	μ^*	0.5	Consumption tax	τ^C	0.17
X's competitors' price	p_X^C	1.0	Capital tax*	τ^K	0.45
Relative technology	$\widetilde{z}$	1.0	Labor income tax	τ^N	0.33
X's weight in IM*	ν^*	0.012	Payroll tax - h	τ^{W_h}	0.07
Working capital weight	ν^F	0.5	Payroll tax - f	τ^{W_f}	0.05
Foreign transfers to GDP	s_{FTR}	0.0	Gov. transfers to GDP	s_{TR}	0.153
LR inflation rate	$\overline{\Pi}$	1.005	Δ Inventories in GDP	Δinv	0.01
Share of taxed goods	w_{τ^C}	0.78			

* Updated from original MOISE claibration.

The steady-state inflation objective ($\overline{\Pi}$) was calibrated at an annual rate of 2%, which is located in the middle of the inflation target. The labor productivity growth parameter (g_z) was set so as to reflect a growth rate of 0.8% in annual terms, which is approximately the historical average. We set the following exogenous shares in nominal GDP: inventory investment (Δinv) of 1%, Government consumption (s_G) of 24% and foreign transfers (s_{FTR}) of 0%. These are approximately the average ratios observed in the historical data.

The weight of capital in the production function (α) was calibrated to obtain a wage bill share in GDP (at factor cost) of 55%. For the basic calibration of the steady state we set the depreciation rate (δ) at 7.0% per annum, approximately the implied levels for the last years (see Figure 2). The discount factor β was calibrated so that the steady-state real interest rate equals 2.0%.

For the calibration of certain parameter values, Argov et al. (2012) followed what is common practice in the literature of DSGE models. Thus, they calibrated the inverse of the labor supply elasticity (ζ) to 2.0. The steady-state markups (φ^W, φ^*,φ^H, φ^X) were set at 30% in the wage and import sectors; in the domestic sector a higher markup of 50% was set, and in the export sector a smaller markup was chosen (20%) since the monopolistic exporters' price of inputs (P_t^{DX}) is already marked up over marginal cost due to the domestic and import price markups. The steady-state elasticities of substitution between domestic and imported intermediate goods in the private consumption, investment and export sectors (μ^C, μ^I, μ^X) were calibrated to 1.1, which is lower than the values commonly used in the literature but higher than the estimate of 0.4 found for the Israeli economy by Friedman and Lavi (2007). A very low elasticity (0.2) of substitution in government consumption (μ^G) was assumed, given that the government's main expenditure is public sector wages, which cannot be substituted for. The foreign elasticity of substitution between imports from different countries (μ^*) was set to 0.5, a value that generates export elasticities with respect to the real exchange rate that are in line with common findings for Israel. The home bias parameters (ν^C, ν^I, ν^G, ν^X) were calibrated according to the following import intensities in the steady state: 25% in private consumption, 33.5% in investment, 5% in government consumption and 33% in exports.

The following tax rates were imposed in order to finance government consumption in the steady state: 17% for the consumption tax (τ^C), which is levied on 78% of the consumption basket (w_{τ^C}),[28] 33% for the labor income tax (τ^N), 7% for the payroll tax paid by households (τ^{W_h}) and 5% for the payroll tax paid by firms (τ^{W_f}). In addition, the capital income tax (τ^K) was calibrated to 45%, partly in order to fine tune the steady-state investment-to-GDP ratio. The share of government transfers in GDP (s_{TR}) was calibrated to ensure that the government's budget is balanced in the steady state.

The parameters in the export demand function were set as follows: the export competitors' relative price (p_X^C) and the steady-state relative level of foreign technology ($\tilde{z} = \frac{z*}{z}$) were normalized to 1.0 and the steady-state weight of Israel's exports in world trade was calibrated to 0.5%. In order to allow for the working capital channel to have an effect, the weight of wage-bill loans (ν^F) was calibrated to 0.5.

Additional details on the equations in steady state and their basic calibration are outlined in Appendix C and Section 3.4.1 of Argov et al. (2012). However, we do apply a different strategy to solve the steady state. While Argov et al. (2012) arbitrarily normalized the real exchange rate to 1, and then analytically solved the steady state (including the normalized size of world trade wt^*), we instead fix the steady state of world trade and numerically solve for steady state, including the real exchange rate. The reason for the shift of approach is that we would also like to allow analysis based on changes in the steady state (due to parameter shifts) and these should affect the long-run real exchange rate and not world trade.

The basic steady state solution results in the following endogenous relations:

[28] In Israel, housing services and fresh fruits and vegetables, comprising 22% of the consumption basket, are not subject to VAT.

		Steady state solution	Data (2005-15)
Real fixed investment rate (fi_t^{LR})	$\frac{I_t}{Y_t}$	21.6	19.2
Nominal investment to GDP	$\frac{P_{I,t}I_t}{P_{Y,t}Y_t}$	20.2	19.6
Nominal consumption to GDP	$\frac{P_{C,t}C_t}{P_{Y,t}Y_t}$	56.5	56.1
Nominal exports to GDP	$\frac{P_{X,t}X_t}{P_{Y,t}Y_t}$	30.8	32.0
Nominal imports to GDP	$\frac{P_{IM,t}IM_t}{P_{Y,t}Y_t}$	30.8	31.5

Indeed, for the basic calibration of the steady state we set the depreciation rate (δ) to 7.5% per annum. However, in order to be consistent with the upward trend in the calibration of the depreciation in the Unifying Model, (see Figure 2), in the baseline simulation below we include the marginal effect of a shift in the depreciation rate from 7.0% to 8.2% on the long-run investment rate. For that, after solving the steady state with basic calibration (part of which $\delta = 7.0\%$) and retrieving a steady-state investment rate, $\left(\frac{I}{Y}\right)_0$, we resolve the steady state with $\delta = 8.2\%$ and retrieve a higher investment rate - $\left(\frac{I}{Y}\right)_1$. For the baseline growth simulation below we will set the GE model effect on the long-run investment rate to $\widehat{fi}^{GE} = \left(\frac{I}{Y}\right)_1 - \left(\frac{I}{Y}\right)_0$.

4 The baseline forecast

4.1 Forecast of aggregate macro variables

The baseline forecast scenario is detailed in Table 8 and in Figures 5 and 6. The average annual growth rate of GDP over the entire forecast horizon is expected to be 2.4%, substantially lower than the 4.1% historical average since 1980. GDP per capita growth is expected to slow as well to 0.7% compared to an historical rate of 1.8%. The expected growth rate is driven down mainly due to expected demographic developments: average overall population growth is expected to grow at a rate of 1.7% – 0.5 percentage points (p.p.) lower than the historical average.[29] Moreover, the expected growth rate of the prime

[29] The historical comparison period (1980–2016) includes the great immigration wave from the former Soviet Union in the first half of the 1990s. However, even when excluding these years, the growth rate of population was somewhat higher than the forecasted growth, as can be seen by the 2000–15 column in

working age population is expected to slow by more - 0.9 p.p. - and to a slower pace than the growth rate of total population.

Table 8: Average Annual Growth Rates - Actual (1980Ű2016) and Baseline Forecast (2015Ű65), percent

	Actual		Forecast			Forecast-Actual
	1980 to 2016	2000 to 2016	2015 to 2035	2035 to 2065	2015 to 2065	2015-2065 *less* 1980-2016
GDP	4.1	3.3	2.7	2.2	2.4	-1.7
GDP per capita	1.8	1.4	0.9	0.6	0.7	-1.1
Total population	2.2	1.9	1.8	1.6	1.7	-0.5
Prime age population	2.5	2.0	1.5	1.5	1.5	-0.9
Total human capital input	3.5	2.8	1.8	1.8	1.8	-1.8
Total employment (incl. foreign)	2.9	2.5	1.6	1.6	1.6	-1.3
Hours per employed	-0.0	-0.2	-0.0	-0.0	-0.0	-0.0
Capital	3.7	3.0	2.8	1.8	2.2	-1.5
TFP	0.5	0.5	0.5	0.4	0.4	-0.0

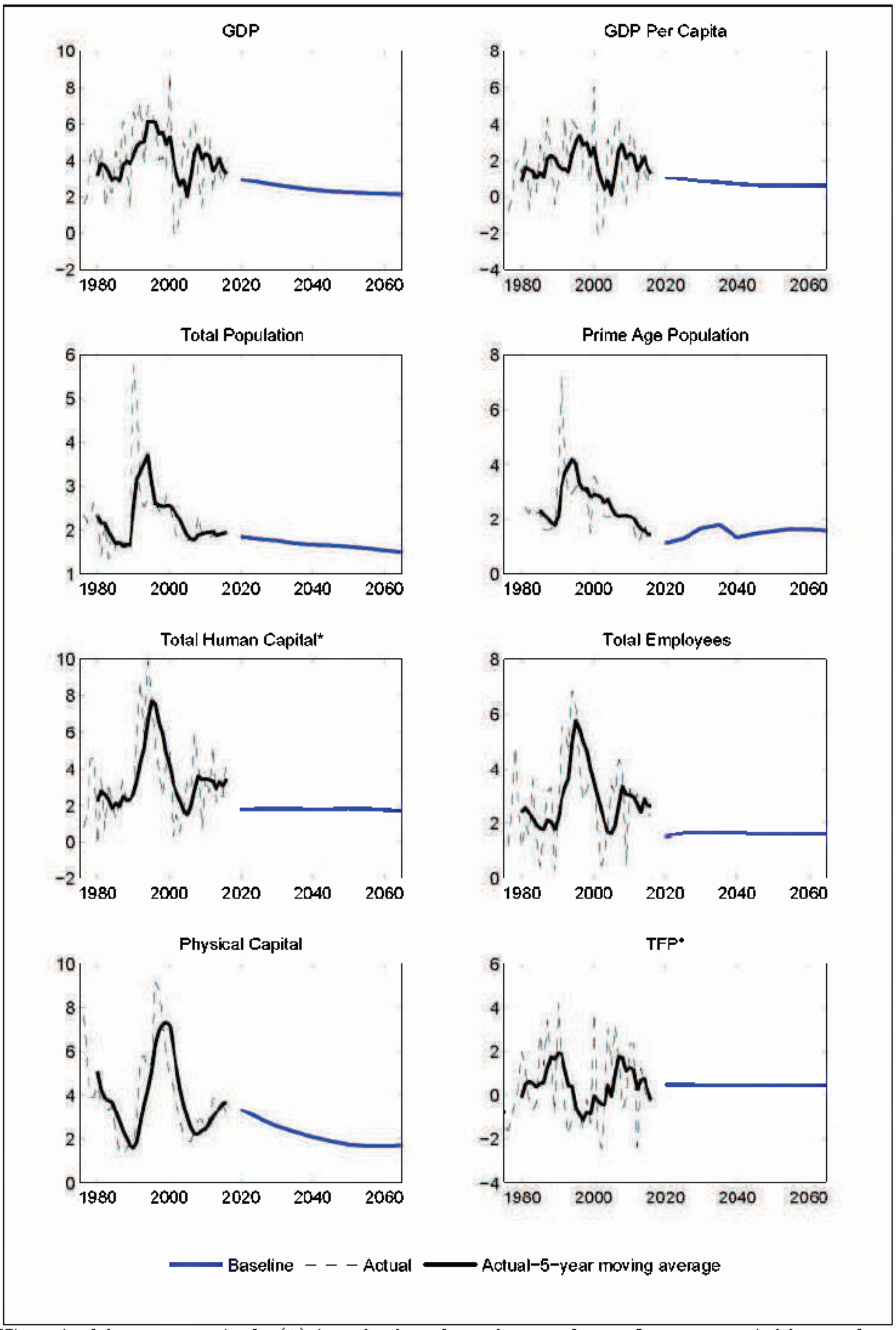
The expected decrease in the overall employment growth rate is more substantial than in the prime working age population (1.3 p.p compared to 0.9 p.p.). The excess decrease results from the assumption that the future entrance rate of foreign workers (1% per year) will be lower than the historical pace (4% per year[30]) and from a forecasted reduction in the pace of increase in the labor force participation rate: from an historical annual average increase of 0.36 p.p. (1980–16) to 0.09 p.p. in the forecast. There are two main contradicting trends operating on the participation rate. Our ambitious (in terms of policy) assumption that populations characterized by low participation rates, mainly Arab women and ultra-Orthodox men, will gradually converge towards the participation rate in the Jewish population serves as a backwind to the overall participation rate.[31] Yet, in contrast, the

Table 8.

[30] This high rate of foreign worker growth mainly occurred in the second half of the 1990s as the number of foreign workers doubled. Since then, the government pressured to reduce the number so that, in practice, the absolute number of foreign workers in 2015 is similar to 2000.

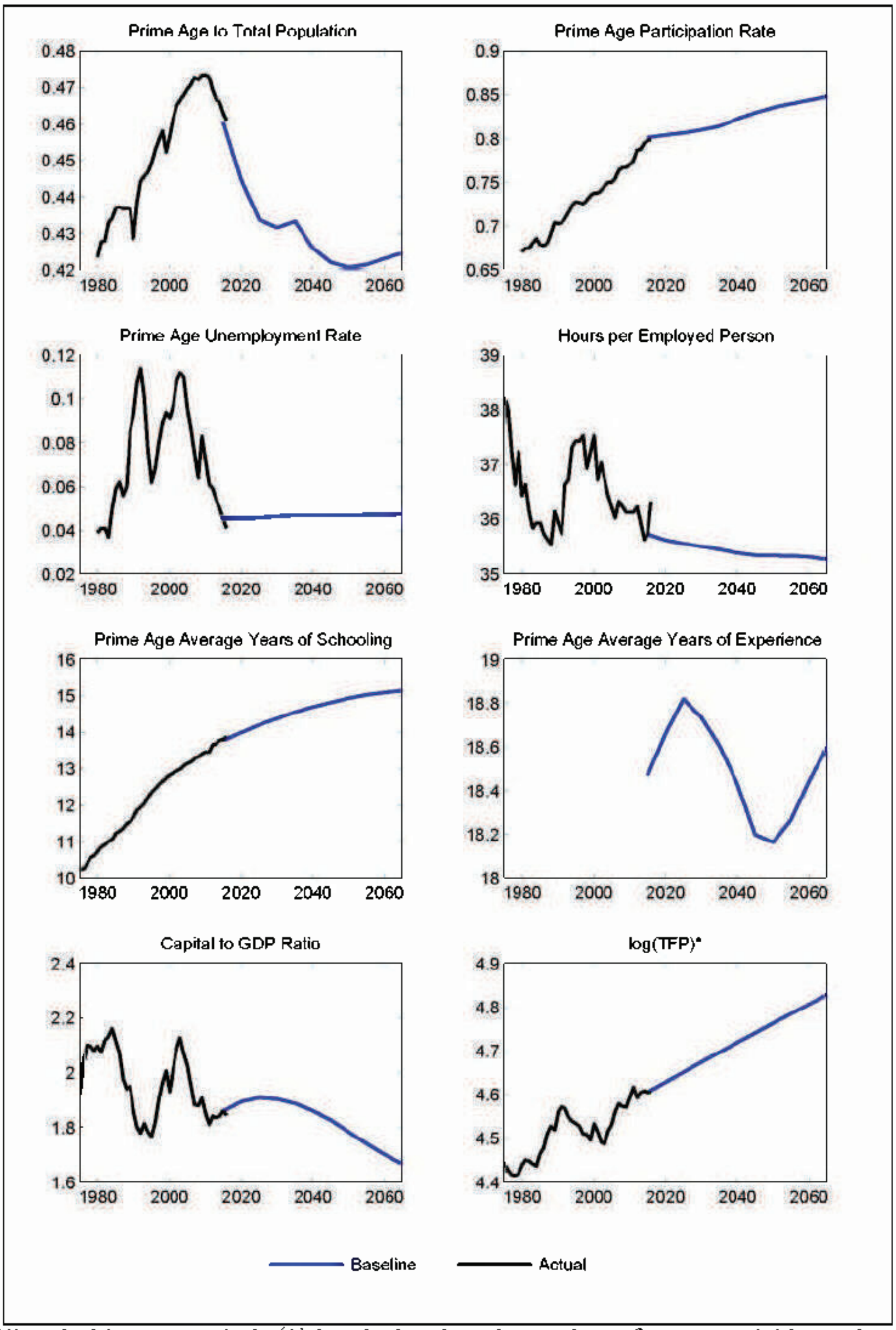
[31] We also assume that Jewish womens' participation rate will increase toward that of mens'.

Figure 5: Growth of Main Macro Aggregates: Actual 1975-2015 and Baseline Forecast 2015-2065 (percent, annualized terms)



* Historical human capital: (1) is calculated as the product of macro variables and not as a sum of micro-level products. (2) human capital from experience is assumed to be constant.

Figure 6: Main Macro Aggregates: Actual 1975-2016 and Baseline Forecast 2015-2065



* Historical human capital: (1) is calculated as the product of macro variables and not as a sum of micro-level products. (2) human capital from experience is assumed to be constant.

expected increase in the population share of the low participating groups gives a headwind to the overall increase in the participation rate (Subsection 4.2 details the labor market developments in each group).

The forecasted growth of total human capital input, which includes hours worked adjusted for human capital from education and experience, is expected to grow on average by 1.8%. The decrease compared to historical growth, 1.8 p.p., is larger than the decrease in total employment growth. This is because the contribution of the increase in average years of schooling to growth of human capital will decrease compared to the past as the growth of average years of schooling is exhausting.

Annual TFP growth is expected to be 0.4% on average, similar to the historical average. Since our TFP model estimates that Israel's productivity gap in 2015 is moderate - approximately 8% below potential - and since we assumed that the fundamental and policy variables will remain unchanged, this growth rate is only slightly higher than the global trend growth of TFP. The numerical value is based on the average TFP growth of developed countries during the period 1990–10 (after controlling for initial differences in productivity).

Given the projected growth rate in human capital and TFP, the reduced rate of labor share (0.55-0.52 compared to an average historical rate of 0.61) that translates into higher elasticty of GDP to physical capital increases the growth rate of GDP by 0.1 p.p. The mechanism at work is that any change in TFP translates into higher growth in physical capital. As the mentioned elasticity increases, the growth rate in GDP increases.

When splitting the forecast period between 2015–35 and 2035–65 we can see that the reduction of growth is gradual and this is due to the dynamic process of the physical capital stock. There are several endogenous forces operating on the growth rate of the capital stock. First, our demographic-investment rate model forecasts that the ongoing increase in old age dependency ratio will induce a gradual decrease in the investment rate from 19% to 16.5% in 2065. In addition, the capital stock plays a role in smoothing the transition process induced by factors like demographic changes. Given a fixed level of investment rate, as the growth rate of total human capital input decreases, the steady state of physical capital to GDP increases.[32] This means that during the transition, the growth rate of the physical capital stock is expected, all else equal, to be higher than GDP and therefore contribute to its growth. As the transition stabilizes, the growth rate of physical capital moderates. The bottom left plot in Figure 6 shows that in the first few periods of the forecast the latter force is dominant inducing an increase in the capital to GDP ratio, while afterwards the reduction in the investment rate is dominant and the ratio gradually drops.

Interestingly, our baseline forecast for productivity growth during the first half of the sample (1.1%) is close to that projected by Gordon (2016) for the United States (1.2%). He emphasizes the slowing rate of advance of educational attainment in explaining the reduction of productivity growth compared to the previous decade (2004–15). Fernald et al. (2017) also support the view that US cyclically adjusted growth won't pick up in the coming years. This is based on their observation that the softening of US growth experienced since the financial crisis was due to noncyclical and noncrisis related factors that resulted in weak TFP growth and declining participation rates (according to their estimates, 2/3 of the decline in participation is due to demographic changes). Gordon (2016) goes further and argues that the US median income per person will grow even slower (0.4%) if the rise of inequality will continue at roughly the same rate experienced from 1975 to 2014. While this point is very interesting and may be relevant to the Israeli experience, it is beyond the scope of the current model and left for future research.

4.2 Forecast of group specific human capital

In this subsection we discuss in greater detail the expected labor market developments by subgroups of the population (Jewish, Arab, and ultra-Orthodox). The developments

[32] According to the capital dynamics equation (3), in steady state the capital to GDP ratio is$\left(\frac{K}{Y}\right) = \frac{s\cdot(1+g_y)}{g_y+\delta}$, where g_y is the average growth rate of GDP.

are governed mainly by the between group convergence process discussed in the model for human capital (Section 3.2). The group specific historical data and baseline forecasts are detailed in figures appearing in Appendix A. We should stress that due to changes in the Labor Force Survey the historical data contains a break between 2011 and 2012 which makes it difficult to estimate the most current trends in group specific labor force attributes (as participation).

4.2.1 Non-ultra-Orthodox Jews

The participation rate and hours per worker of most Jewish men age-groups are assumed to remain at their 2015 level (figures 13 and 18). As a result, the overall participation rate and hours per worker of prime age Jewish men are expected to remain stable, at 91% and 42 hours per week, respectively.

Jewish women are assumed to continue their labor market assimilation process toward Jewish men. Most of the gap in the age specific participation rates among the prime working age groups have already been closed during the past 30 years (Figure 13). According to the forecast, remaining gaps (up to the age of 59) will be almost fully closed by 2040. In contrast to the participation rate, we do not assume that gender gaps in age specific hours per worker will narrow during the forecast period (Figure 18), so that the average hours per prime age woman worker will remain stable at 32 hours compared to 41 for men. There are several theoretical forces that should operate on hours per worker. On the one hand as the labor market assimilation process of women proceeds, one would expect them to increase their hours worked. However, as more women join the labor force—meaning that they devote less time to household duties, their spouses can devote less time for work and should reduce hours. Empirically, since 1980 there was only a mild gender convergence in hours. Extending to a more complete forecast of hours (of women and men) is left for future research.

Average years of schooling of Jewish men and women are expected to continue increasing throughout the forecast (Figure 23). The average years of schooling of the prime-age men (women) is expected to increase from 14.3 (14.6) years in 2015 to 15.6 (16.0) at the end of the forecast. When looking at the age group of 30-34 (the youngest age group that completed most of its schooling), we can see that the average years of schooling increases until 2025, and thereafter stabilizes at approximately 16 years. The increase until 2025 is supported both by the convergence process toward the long-run and by the historical decrease in the total fertility rate that occurred until the mid 1990s (from 3.1 in the 1970s to 2.25 in the mid 1990s). The latter trend increases future acquiring of schooling as governed by the quantity-quality trade off (the Joshi-Sultz effect - see equations 29 and 31). As the forecasted average years of schooling (for the 30-34 age group) nears its long-run of 17.5 years, and as the historical TFR stabilized and even slightly increased after the mid 1990s, the increase in the average years of schooling comes to an halt after 2025.

4.2.2 Ultra-Orthodox Jews

The general assumption regarding the Orthodox men is that cohorts below the age of 30 in 2015 will begin a very gradual assimilation process toward the labor market attendance and education of Jewish men. Due to these cohorts, which by the end of the forecast horizon cover the entire working age population, the overall participation rate of prime-age Orthodox men is forecasted to increase from 54% in 2015 to 75% in 2065 (16 p.p lower than the Jewish men). The overall average of weekly hours per worker is also expected to increase from 32 to 35 in 2065 (6.5 hours less than the Jewish men). Although we do assume some process of convergence in effective schooling, substantial gaps will remain by the end of the forecast: the average years of schooling of Orthodox men aged 30-34 is forecasted to be 12.5 years compared to 16 for parallel Jewish group.

Orthodox women are less integrated in the labor market, in terms of labor input, than the Jewish women with ambiguous signs of convergence over the past decade. In 2015, their participation rate was below that of Jewish women, mainly in the prime working ages, while it did show a path of convergence toward the Jewish women. As for hours per worker, that of the Orthodox women was substantially lower than the Jewish women (24 compared to 32 hours per week), without any clear increase in the past 30 years. In the forecast, Orthodox women do increase their integration in the labor market in terms of labor input. The participation rate in the prime-age group is expected to increase from 77.5% in 2015 to 88% in 2065, only 1 p.p. lower than the Jewish women. We expect that the assimilation in the participation will eventually be accompanied by an increase in the Orthodox women's hours per worker, but major gaps are expected to remain even in 2065; the average hours of the prime-age group is expected to increase from 24 in 2015 to 30 in 2065, 3 hours below that of Jewish women. The level of schooling in the Orthodox women population is rather similar to the Jewish women (approx. 15 for the 30-34 age group). However we forecast that contrary to Jewish women, the average years of schooling of Orthodox women will not increase. The reason is that the high historical and expected fertility rate predicts a long-run level of years of schooling of only 14 years (compared to 17.5 for Jewish women). In other words - Orthodox women are already near their long run level of schooling.

4.2.3 Arabs

The overall participation rate of Arab men is lower than that of Jewish men (78% compared to 91% in 2015), their hours per worker, for most age groups, are somewhat smaller than that of Jewish men (1 hour less on average in the prime age), and their average years of schooling are substantially lower (11.7 compared to 14.3 years in 2015). The historical data hardly shows any trend of participation rate convergence between the groups that has been taking place. In the forecast, labor force characteristics of the Arab men are expected to very gradually converge to that of Jewish men. However, nontrivial gaps in the labor force participation rate are expected to remain even in the end of the forecast horizon.

Arab women are much less integrated in the labor market. Their participation rate is very low - 34% in 2015 (compared to 84.5% for Jewish prime age women) and their average years of schooling is well below that of Jewish women (approximately 11.5 years compared to 14.6 in 2015). The hours per worker (of those who work) are only slightly lower than Jewish women. For some prime age groups we can see in the past 30 years a very slow process of convergence. We expect this convergence process to continue throughout the forecast horizon. In terms of the participation rate, the convergence will take place only for cohorts younger than 45 years in 2015. However, due to the large gaps in the beginning of the forecast, significant gaps will remain even in 2065: the average participation rate of the prime age group is expected to be 20 p.p. below Jewish women. In terms of years of schooling, the convergence trends that prevailed in the past, which are assumed to continue in the next 50 years, generate a forecast by which the average years of schooling of Arab women aged 30-34 will be almost similar to Jewish women by 2050.

5 Evaluation of uncertainty

In Section 4 we described a baseline scenario. Of course there is uncertainty regarding the point estimate. Since most of the model is not econometric, we can not construct classical statistical confidence intervals. Therefore, in order to provide an idea of the degree of uncertainty we take a different approach. We will run many alternative simulations made up of combinations of alternative assumptions reflecting four main sources of uncertainty: the demographic assumptions, the group-specific developments in human capital, the initial labor productivity gap and the future path of the investment rate Let us explain how we calibrated the degree of uncertainty with respect to each component.

Demography: The CBS's demographic forecast included three scenarios ("High", "Medium", "Low") for each of the three population groups (see Table 11 for details on the scenarios). The "High" and "Low" scenarios reflect a 95% confidence interval. This allows us to construct $3^3 = 27$ demographic forecasts.[33]

[33] More details on the alternative demographic assumption and their effect on growth are provided in

Human capital: The dominant components of human capital are the group-specific participation rate, hours per worker and years of schooling. We used expert consultation to construct the degree of uncertainty regarding the future development of these variables for each of the 84 population cells. The consultations were done by means of questionnaires that we sent to economists at the Bank of Israel and other economic institutions. In the questionnaires we asked the respondents to assess what will be the participation rate, the average hours per worker and the average schooling years of the 35-40 age group in 2035 and 2065 by gender and sector. To ease the response burden, we only asked about one age group, and operated constant formulas to reflect the respondents' answers on other age groups. In addition, we asked the respondents for which of the 27 demographic assumptions their assessment is relevant (may be relevant for more than one or even all demographic scenarios). When running the various combinations of simulations (by demographic assumptions and expert assessments) we will consider for each expert-simulations only the demographic assumptions the respondent pointed out as relevant. In order to allow the exploration of boundary scenarios, respondents that provided a small amount of relevant demographic scenarios, were asked again about their assessments, assuming that an alternative demographic scenario will materialize (for these cases 1 of 3 scenarios was chosen randomly - full-low, full-medium or full-high).[34]

Productivity gap: The initial level of labor productivity gap, a key factor in the TFP model, is derived from the average of Israel's residuals from several cross-country regressions of the (log) level of actual labor productivity in 2010 on sets of fundamental and policy variables (see Section 3.3). The various regressions differ in the exact set of policy variables and whether they include all countries or only advanced economies. For

Section 7.2.

[34]Out of the 28 respondents, one respondent described a world where "work is only an option" and therefore the participation rate is extremely small. We discarded this answer since it cannot be evaluated within our model. Simply taken, it would generate extremely low growth forecasts while the expert actually described a world where technological advances will allow robots to work instead of humans—a scenario which should go with high growth rates.

the current analysis we will use the specific initial levels. In total we used results from 16 regressions in which the initial gap ranges from 19% below potential to 5% above (in the baseline case the initial gap was 8% below potential).

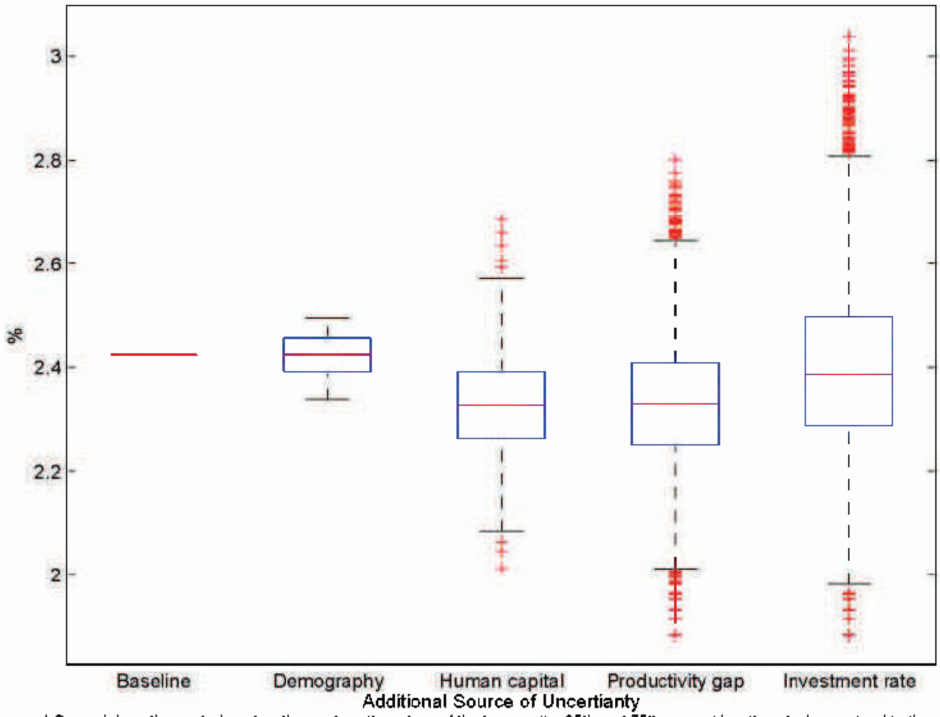
The investment rate: Israel's initial investment rate, 19%, is low by international standards (Figure 12). Nevertheless, in the baseline scenario the investment rate gradually decreases to 16.5% in 2065 due to the ongoing increase in old age dependency ratio. For the uncertainty analysis we will consider two alternative options: the investment rate remaining unchanged at 19% and the investment rate gradually increasing to the current OECD average of 21%. Hence this part of the uncertainty tilts the distribution of future growth rates upward since both alternative scenarios are characterized by a higher investment rate than baseline.

The boxplot in Figure 7 summarizes the results; it shows the distribution of the 2015 to 2065 GDP growth rate, by source of uncertainty, on a cumulative basis. To explain, the far left line labeled "baseline" is the baseline scenario of 2.4% average growth. The next box, labeled "demography", shows the distribution of GDP growth rate forecast under the 27 demographic scenarios (keeping all other assumptions as in the baseline). In the next box labeled "Human capital" we show the distribution given both the variety of demographic scenarios and expert assessments. In the next box ("Productivity gap") we add to the previous two sources of uncertainty that from the initial labor productivity gap, and in the far left box we add as well the uncertainty from the investment rate. The last box is composed of approximately 12,000 simulations.

A few conclusions may be drawn from the boxplot:

1. With all of the considered uncertainties (far right hand box labeled "Investment rate"), the growth rate forecast distribution (including outliers) spans from 1.8% to 3.0%. Thus the entire distribution of forecasts is below the historical averages of growth (3.3% between 2000 to 2016).

Figure 7: Boxplots* of the Cumulative Uncertainty Bands Regarding Average 2015-65 GDP Growth Rate



* On each box, the central mark is the median, the edges of the box are the 25th and 75th percentiles, the whiskers extend to the most extreme data points not considered statistical outliers, and outliers are plotted individually. Points are drawn as outliers if they are larger than q3 + 1.5*(q3 – q1) or smaller than q1 – 1.5*(q3 – q1), where q1 and q3 are the 25th and 75th percentiles, respectively. This range corresponds to approximately 99.3 coverage if the data are normally distributed.

2. We do not detect an upward or downward bias in our baseline simulation, as the median growth forecast from the full uncertainty simulations is very close to our baseline scenario.

3. Most of the uncertainty derives from the variety in expert assessments on human capital. We assert this from the observation that the span of the boxplot extend most when adding this source of uncertainty.

4. The experts tended to be less optimistic than the baseline scenario regarding the contribution of human capital to growth. Their median growth forecast is 2.3%. This mainly reflects their softer predictions on the group-specific participation rates and average years of schooling among the Arabs and the female Orthodox. In addition, they tended to

embrace the "Low" demographic forecast at a greater weight than the "High" one.

6 Ex post in-sample forecast

In this section we try to evaluate to what degree our model can forecast the actual historical growth rate. Specifically we will conduct an in-sample forecast for the years 2000–15, and compare the results with the actual outcomes.

As described in the previous section, since our model is not a statistical model, we cannot conduct a pure out-of-sample exercise. Therefore, let us first briefly describes the main assumptions taken to conduct the in-sample forecast. Since we don't have demographic forecasts from 2000, we take the population data, fertility rates and mortality rates as given. This is not a very strong assumption since in the short run the main demographic figures that affect the growth forecast are known (the prime age population for the first 25 years are already born). We also use actual 2000–15 data on the labor share, the depreciation rate and number of foreign workers.

For most parameters we use the values as set in the baseline simulation—namely, the speeds of convergence, the demographic effects (as the quantity-quality trade-off and the cost of childbearing), the long-run values for the number of years of schooling (18 in general), the return to schooling and the parameters of the Demographic Investment Rate and the General Equilibrium models.

For the TFP model, we set the initial 2000 productivity gap as estimated from a cross country level regression (as in 41) for 2000. Since we do not have time series for all policy variables, we estimate only one regression based on available variables for 2000. For the speed of convergence and global frontier growth rate of TFP we use the same panel regression (as in 42) that was based on the full sample. However, for the global frontier TFP growth parameter we use an average for the years up to 2000.

One set of crucial calibrated variables are the long-run benchmark values for the labor-market human capital attributes of the non-Ultra-orthodox Jews (mainly the participation rate). In the baseline simulation we set those of men to their actual level in 2015, and for women we set them to the values of men of the same age. This seems reasonable for the baseline simulation as these reached historically and internationaly high levels. However, had we conducted this exercise in 2000, we probably would have set higher values as the participation rates were much lower at the time. Therefore we held two ex post scenarios. In the first, labeled Ex post A, we naively set the benchmark values according to 2000 levels. In the second, labeled Ex post B, we set the same benchmark values as in the baseline scenario (i.e., based on 2015 values).

Table 9: Average Anuual Growth Rates - Actual and Ex post Forecast (2000Ű15), percent

	1980-2000	2000-2015		
	Actual	Actual	Expost A	Expost B
GDP	4.7	3.3	2.9	3.2
GDP per capita	2.2	1.4	1.1	1.4
Total population	2.5	1.9	1.9	1.9
Prime age population	2.8	2.1	2.1	2.1
Total human capital input	4.2	2.6	2.2	2.6
Total employment (incl. foreign)	3.3	2.5	2.0	2.5
Hours per employed	0.1	-0.3	-0.1	-0.2
Capital	4.2	3.0	3.0	3.1
TFP	0.5	0.5	0.4	0.4

The results are described in Table 9. The first two columns show the actual growth rates in 1980–2000 and 2000–15; they show the reduction in the average growth rate of the economy from 4.7% to 3.3%. The next two columns are the ex post scenario results. We can clearly see that the ex post forecasts point to the reduction in the growth rate, and that the reduction is due to a decrease in the growth rate of human capital. In Ex post A we can see that the reduction is even overestimated as the model does not internalize the major increase in the non-Ultra-orthodox Jews' participation rate that occurred between 2000 and 2015. Once we allow the model to internalize this process we can see that the model forecasts the actual growth rate of GDP very well.

These are very encouraging results as they indicate that at the macro level our model is able to reproduce past developments - both in GDP growth and in the main components. However, this does not mean that the model perfectly replicates all ingredients that appear in the model. It may be that the human capital of some population groups are biased upward and some downward. Moreover, in the TFP model we get an initial large negative TFP gap (that is, above potential) for 2000 that should have pointed to a relatively low TFP growth rate. However, this is offset by a relatively high global TFP growth rate when using the year fixed-effects up to 2000.[35] This is part of the nature of a model with many components—in the forecast, some upward errors are set off by other downward errors, making the overall forecast more stable.

7 Alternative scenarios

7.1 Labor market convergence

In the baseline scenario we assumed the next 50 years will be characterized by gradual convergence between group and gender in labor market attributes: participation rates and hours per worker. Namely, the Orthodox and Arab groups will converge toward the non-ultra-Orthodox Jews, and women will converge toward men. In this section we will inspect the sensitivity of the forecast to smaller degrees of convergence.

In the first alternative (alt 1) we will assume that the most labor market absent groups—Orthodox men and Arab women—will remain throughout the forecast with their initial level of participation rate and hours per-worker. Formally, we set:

$$\rho^{LFPR}_{(a,O,M)} = \rho^{LFPR}_{(a,A,W)} = \rho^{HO}_{(a,O,M)} = \rho^{HO}_{(a,A,W)} = 0$$

In the second alternative (alt 2) we will assume that all Orthodox and Arab groups will

[35] According to the actual data, the negative gap was to a large extent closed and turned over by PPP price developments and high growth in other advanced economies, as opposed to real TFP developments in Israel that we measure.

remain throughout the forecast at their initial level. Formally, we set:

$$\rho^{LFPR}_{(a,O,s)} = \rho^{LFPR}_{(a,A,s)} = \rho^{HO}_{(a,O,s)} = \rho^{HO}_{(a,A,s)} = 0$$

In the third alternative (alt 3) we will assume all groups will remain at their initial labor market levels, that is, the "static scenario". Mainly this alternative removes the convergence of Jewish women toward the attributes of men, on top of the assumptions in alternatives 2 and 3. Formally, we set:

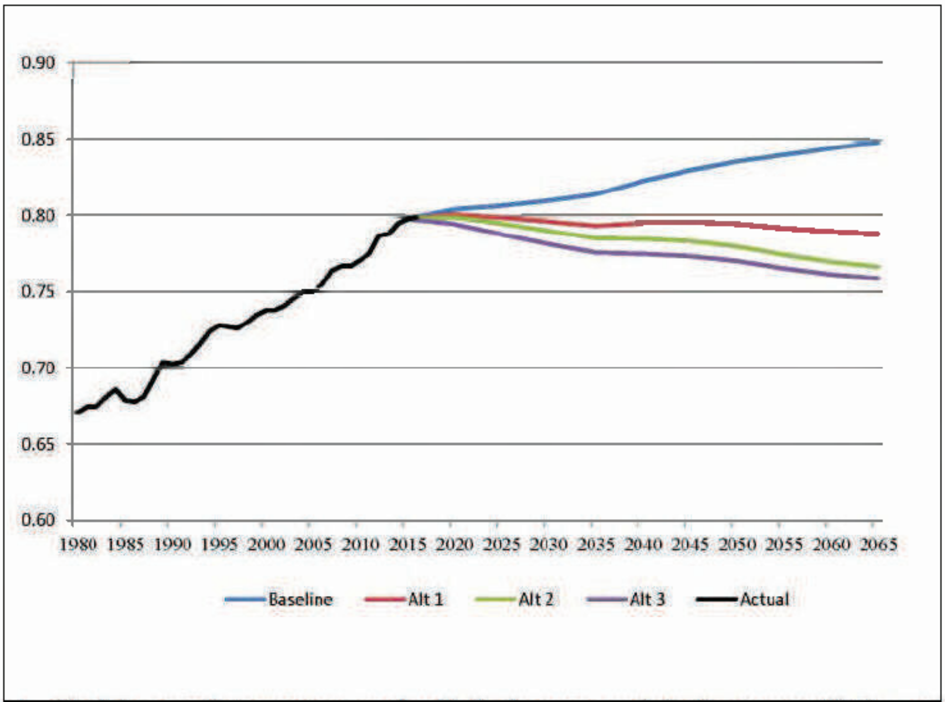
$$\rho^{LFPR}_{(a,r,s)} = \rho^{HO}_{(a,r,s)} = 0$$

Table 10: Average Growth Rate Forecasts for 2015-2065: Baseline and Alternative Convergence Scenarios, percent

	Baseline	Alternatives		
		1	2	3
	Average annual growth rate			
GDP	2.4	2.3	2.2	2.2
GDP per capita	0.7	0.6	0.6	0.5
Total population	1.7	1.7	1.7	1.7
Prime age population	1.5	1.5	1.5	1.5
Total human capital input	1.8	1.6	1.5	1.5
Total employment (incl. foreign)	1.6	1.5	1.4	1.4
Hours per employed	-0.0	-0.0	-0.1	-0.1
Capital	2.2	2.1	2.1	2.0
TFP	0.4	0.4	0.4	0.4
GDP (2065; pct. deviation from baseline)		-5.6	-8.8	-9.5

The results of the alternative scenarios are summarized in Table 10. All three scenarios reduce the average growth rate of GDP. Although the effects in terms of growth rate seem small 0.1-0.2 pct. points, after 50 years this sums up to a lower level of GDP (and GDP per capita) of 5% to 10%. The largest marginal effect occurs when we remove the convergence of the most absent groups (alt. 1). The loss of GDP is mainly due to the removal of group convergence in the participation rate that operates to reduce the growth rate of aggregate employment. For most, in the static simulation (alt. 3) the overall prime age participation rate decreases until 2065 by 4 percentage points toward its mid-2000s level (Figure 8).

Figure 8: Prime Age Participation Rate - Actual, Baseline Forecast and Convergence Alternatives



Alt 1 - No labor market convergence for Orthodox men and Arab women; Alt 2 - no labor market convergence for Orthodox and Arabs; Alt 3 - No labor market convergence.

We should mention that we also tested an alternative where we set the long-run benchmark value of the participation rate among the non-ultra-Orthodox Jewish population aged 25 54 ($LFPR^{LR}_{(25-54,J,s),t}$) to its maximum level among OECD economics (see Figure 3). The forecast macro results do not change significantly, since we already set in the baseline forecast the long-run benchmark value of Jewish womens' LFPR to that of Jewish men—which is higher than the maximum value among OECD women.

7.2 Alternative demographic scenarios

For the baseline scenario we used for all population groups the demographic assumptions that were used in the "medium" scenario from the CBS's long term demographic forecast. The CBS's forecast also included two alternative "Low" and "High" scenarios for each group. The "Low" ("High") scenario assumes lower (higher) fertility and higher (lower) mortality rates. Table 11 summarizes the main demographic parameters that derive for the baseline ("medium") and alternative scenarios. We can see that while the baseline forecast assumed a slight reduction in the TFR of all religion groups, the "Low" scenario assumes a deeper reduction and the "High" scenario assumes increases in the TFR. The higher mortality rates in the "Low" scenario cut off almost all the expected increase in the life expectancy while the "High" scenario induces a major increase. While overall population growth is expected to differ between the scenarios, prime age population growth will be similar because by 2040 newborns during the forecast horizon will not have reached the prime age groups.

Table 11: Main Demographic Parameters in Baseline and Alternative demographic Scenarios

	2015	2040		
		Baseline	Low	High
Total Fertility Rate (TFR):				
Jewish (non-ultra-Orthodox)	2.7	2.5	2.1	2.9
Ultra-Orthodox	6.9	6.2	4.7	7.8
Arabs	3.3	2.6	2.1	3.0
Life expectancy at birth:				
Jewish (including ultra-Orthodox)	82.6	86.9	84.3	89.3
Arabs	79.0	83.4	80.8	85.6
Population growth (5 year avg.), %	1.9	1.7	1.3	2.0
Prime age population growth (5 year avg.), %	1.5	1.3	1.3	1.3
Old to middle age dependency ratio, %	17.7	25.0	24.0	25.8

The full results for the "High" and "Low" scenarios are provided in Table 12 (we will

Table 12: Average Growth Rate Forecasts for 2015Ű65: Baseline and Alternative Demographic Scenarios, percent

	Baseline	Demographic Alternatives	
		Low	High
	Average annual growth rate		
GDP	2.4	2.3	2.5
GDP per capita	0.7	1.1	0.4
Total population	1.7	1.2	2.1
Prime age population	1.5	1.3	1.7
Total human capital input	1.8	1.6	1.9
Total employment (incl. foreign)	1.6	1.4	1.8
Hours per employed	-0.0	0.0	-0.1
Capital	2.2	2.2	2.2
TFP	0.4	0.4	0.4
GDP (2065, pct. deviation from baseline)		-4.1	3.6
GDP per capital (2065, pct. deviation from baseline)		21.4	-16.4

describe the results for the "High" scenario, the opposite follows for the "Low" scenario). We can see that GDP growth is expected to be somewhat higher in the "High" alternative. The main channel at work is the higher increase in prime age population that increases the growth rate of employment. However there are a few more channels at work with smaller magnitudes of effect: first, the increase in fertility reduces the hours per worker of women in fertility ages due to the cost of childbearing (see Section 3.2.4 for details). Second, the increase in the TFR results in a future decrease in years of schooling as dictated by the "quantity-quality" trade-off (see Section 3.2.5 for details). The combination of these two channels explains the smaller increase above baseline in total human capital (compared to employment). The increase in the capital stock (compared to baseline) is similar to that in GDP, as the investment rate paths in all scenarios are similar. The increase in life expectancy offsets the dependency effects (note from Table 11 that the old to middle age dependency ratio hardly varies until 2040).

The mild increase in 2065 GDP (3.6%) is in parallel to the substantial increase in total population resulting in GDP per capita that is 16.4% lower than baseline (remember that the increase in young ages and the extension of old age longevity is immediate, the increase in prime age population is gradual).

7.3 Economic environment improvement

In this set of alternative scenarios we will consider an improvement in the business environment that may be achieved though policy steps. For this we will use the TFP model (described in Section 3.3). In the baseline forecast we fixed the "policy variables" to their current level. Here we will execute hypothetical scenarios where Israel leaps to the top 5% among countries with GDP per capita above 5000$ in each policy group : "doing business" and "economic freedom" in the institutions scenario (Figure 9); "transportation infrastructure" and "communications infrastructures" in the infrastructure scenario (Figure 10); and "national tests" and "educational inequality" in the education scenario (11). We also run a scenario where all improvements are achieved together.

Table 13: Average Growth Rate Forecasts for 2015-2065: Baseline and Alternative TFP Scenarios, percent

	Baseline	TFP Alternatives from better:			
		Inst.	Infr.	Educ.	All
	Average annual growth rate				
GDP	2.4	2.5	2.5	2.6	2.8
GDP per capita	0.7	0.9	0.9	0.9	1.1
Total population	1.7	1.7	1.7	1.7	1.7
Prime age population	1.5	1.5	1.5	1.5	1.5
Total human capital input	1.8	1.8	1.8	1.8	1.8
Total employment (incl. foreign)	1.6	1.6	1.6	1.6	1.6
Hours per employed	-0.0	-0.0	-0.0	-0.0	-0.0
Capital	2.2	2.3	2.3	2.3	2.5
TFP	0.4	0.5	0.5	0.5	0.7
GDP (2065, pct. deviation from baseline)		5.9	6.0	6.5	19.4

Table 13 summarizes the main results from these forecast scenarios. Each of the first three scenarios ("Inst.", "Infr.", "Educ.") results in an increase in average growth rate

Figure 9: Cross-Country Comparison of "Policy Variables" in TFP Model: Quality of Institutions

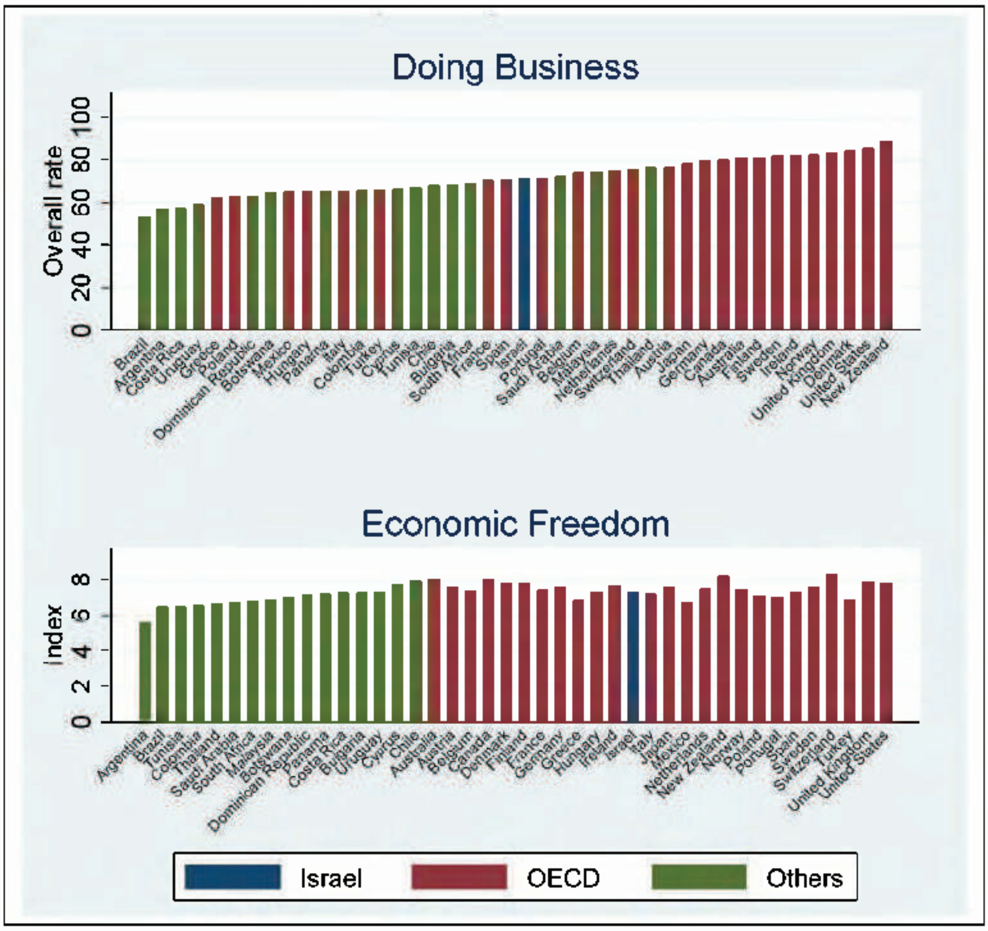


Figure 10: Cross-Country Comparison of "Policy Variables" in TFP Model: Infrastructure

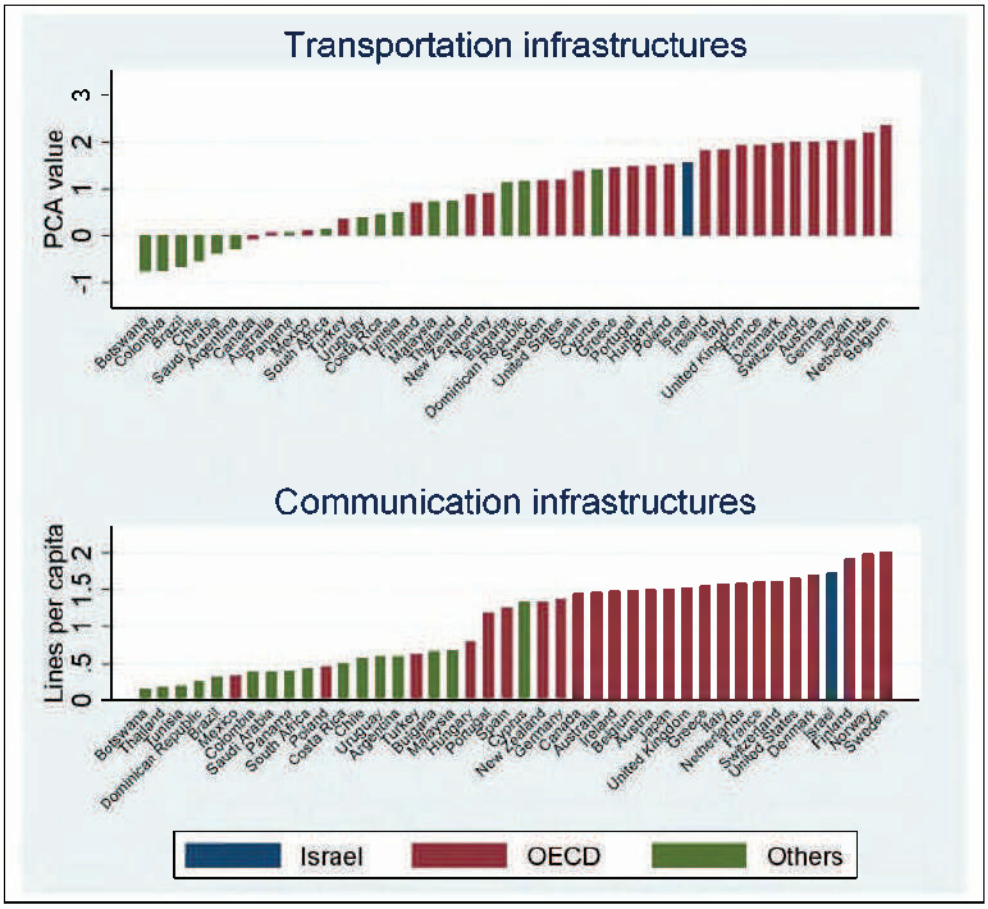
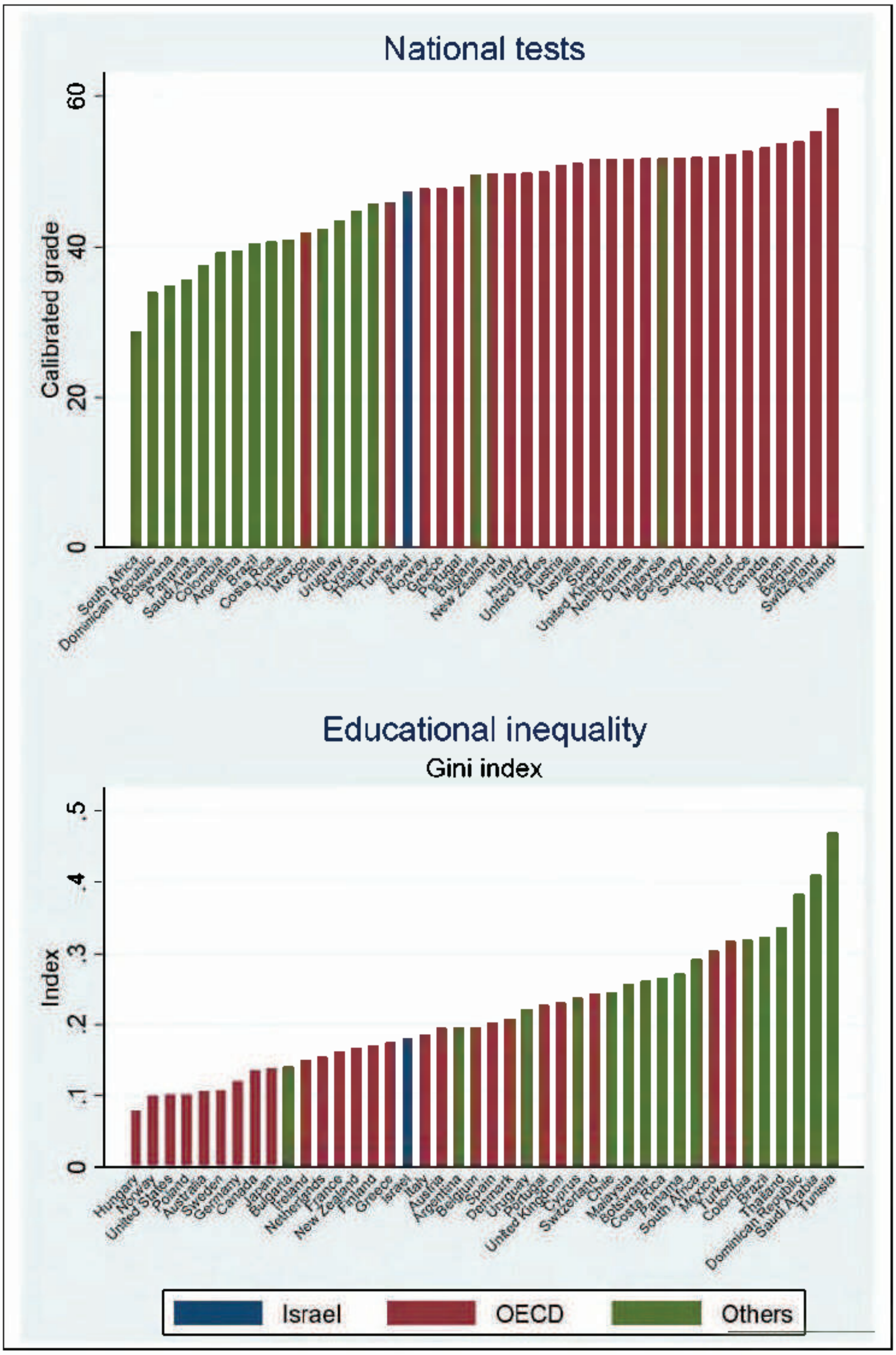


Figure 11: Cross-Country Comparison of "Policy Variables" in TFP Model: Education



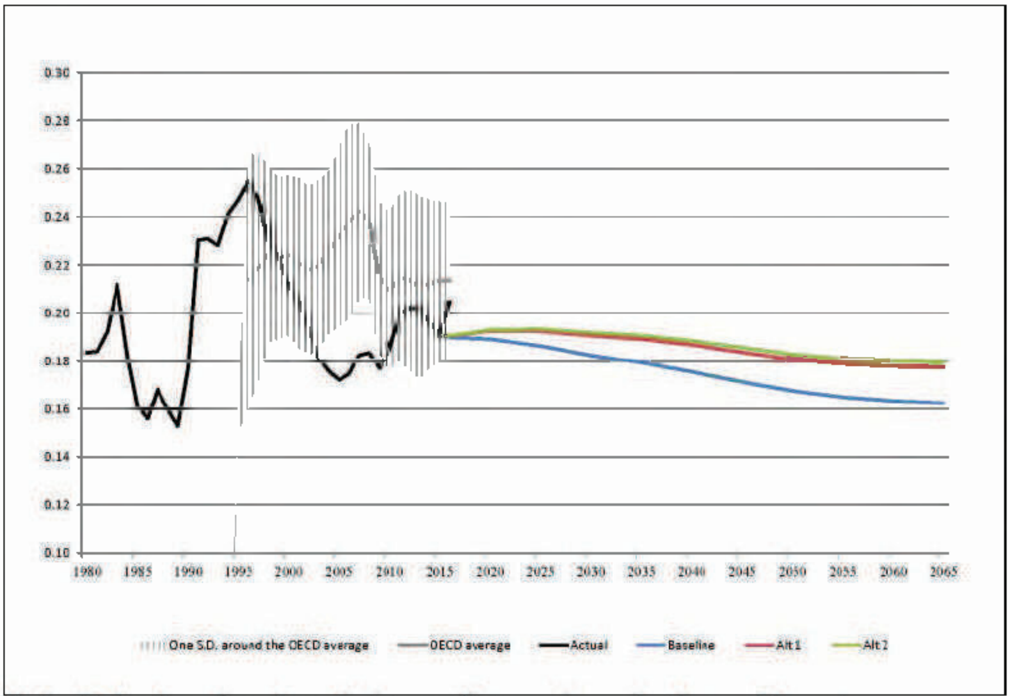
(compared to baseline) of 0.1 p.p. that originates from an increase in TFP growth. These accumulate to an increase of approximately 6% in 2065 GDP (compared to baseline). For the case of the institutions and education scenarios, the GDP improvement is due to Israel's substantial initial distance from best practice in the institutions and education indicators. For the cost of infrastructure, the magnitude of the marginal effect is dominant as Israel is not very far for the top in these indicators. Finally, combining all three groups of improvements generates by 2065 a GDP increase of 19% above baseline.

7.4 Structural changes on the investment rate

In this section we will use the static general equilibrium model (outlined in Section 3.5) to produce different paths for the investment rate due to changes in the fundamental parameters of the economy. In the first alternative (alt 1) we will consider an increase in market competition (i.e., decrease in monopolistic power) that will result in a reduction of markups in the intermediate goods sectors. We will reduce the markups in the domestic and imported intermediate goods sector (φ^H and φ^* in equations 52 and 53) to only 10% above marginal costs (compared to the baseline parametrization of 50% in the domestic sector and 30% in the import sector). The reduction of markups generates a permanent increase in real wages and capital rental rates which supports an equilibrium increase in demand for domestic products and increase in supply of production factors, one of which is capital. Therefore the steady state of the investment rate increases by 1.7 p.p, out of which 2/3 is due to the reduction in domestic markups.

In the second alternative (alt 2) we will increase the exogenous component of Israel's share in world trade from 0.5% to 1.0% (ν^* in equation 56). We can think of this shift as a change in global tastes tilted toward Israeli goods or as policy steps to support the penetration of Israel's exporters in foreign markets. The increase in exports finances, through the current account, an increase of intermediate goods imports. The economic mechanism supporting this transition is an endogenous steady state real appreciation of the exchange rate (of approximately 25%) and an improvement in the terms of trade. The increase in imports and improvement in terms of trade support the real increase of all uses, including the steady state of the investment rate by approximately 2 p.p.

Figure 12: Investment to GDP Ratio - Actual Israel and OECD Average, Baseline Forecast and General Equilibrium Model Alternatives



Alt 1 - Reduction of markups in intermediate goods production and imports;
Alt 2 - Increase in share in world trade (from 0.5 pct to 1.0 pct).

Since the quantitative results of both scenarios in terms of the investment rate and GDP growth are similar we will discuss them together. Figure 12 plots the historical and simulated paths of the investment rate. The figure shows that in the last decade Israel's investment rate of 19% on average was low compared to OECD countries: in 2015 the investment rate was in the lower part of the one-S.D. band around the OECD average. Remember that since 2009 investment rates in most OECD countries dropped due to cyclically weak economic growth. Before that, a 19% investment rate is around the bottom of the one-S.D. band. In the baseline forecast the investment rate gradually decreases to

Table 14: Average Growth Rate Forecasts for 2015-2065: Baseline and Alternative Steady State Model Scenarios, percent

	Baseline	GE Model Alternatives	
		1	2
	Average annual growth rate		
GDP	2.4	2.5	2.6
GDP per capita	0.7	0.9	0.9
Total population	1.7	1.7	1.7
Prime age population	1.5	1.5	1.5
Total human capital input	1.8	1.8	1.8
Total employment (incl. foreign)	1.6	1.6	1.6
Hours per employed	-0.0	-0.0	-0.0
Capital	2.2	2.5	2.5
TFP	0.4	0.4	0.4
GDP (2065, pct. deviation from baseline)		6.1	7.0

16.5% due to demographic changes (increase in dependency rate). In both the alternative scenarios, the investment rate hardly goes down and remains stable at around 18% to 19%. As is evident in Table 14 the increase in the investment rate boosts the growth rate of capital as dictated by equation 3. GDP growth increases by 0.1 p.p. which in 2065 totals to a GDP increase of 6.1% (alt 1) and 7.0% (alt 2).

We should mention that within the general equilibrium model the reduction of markups (alt 1) also triggers a steady state increase in labor input (approximately 10%). However, at this stage we only connect between the general equilibrium and the unifying models through the investment rate, so the effect may be considered as partial. In future versions we may consider allowing structural changes in the economy to have a macroeconomic effect on labor input as quantified by the general equilibrium model.

8 Summary and conclusions

In this paper we documented the development of a model built to create simulations regarding the long-term growth of GDP in Israel. The complete model is constructed from five blocks of models, each with a different target variable and methodology that fits best for its purpose. We preferred this strategy, rather than building one complete unified model, because it allows us to extract various strengths of policy modeling, be it use of international data, use of theoretical general-equilibrium conditions or detailed empirical data. For example, any long-run view on Israel's human capital must consider the developments in the fast growing ultra-Orthodox sector which has different labor market habits. These factors cannot be dealt with within an international panel model or a general equilibrium model. In contrast, research has shown that an important determinant of productivity is the extent to which it can converge to international levels (conditioned on different characteristics of each economy), particularly through technology spillover. This sort of economic consideration is built best from international data on productivity and models of cross-country conditional convergence. The weakness of our eclectic modeling strategy is that the different blocks of the model are not necessarily coherent about all aspects.

At the heart of the paper we presented a baseline simulation of Israel's average GDP growth rate between 2015–65. This simulation, and the evaluation of the uncertainty surrounding it, clearly indicates that Israel's future growth rate is expected to be lower than historical averages. The decrease in the growth rate is mainly due to demographic factors: the slowing growth rate of the prime working age population, the growing share in population of sectors with weak attachment to the labor market and shortages in human capital, and the aging of the population is increasing the old-age dependency ratio and reducing saving and investment rates. In addition, a few important drivers of growth which were very important in the past are coming to exhaustion: the increase in years of schooling, and the labor market participation rate of women.

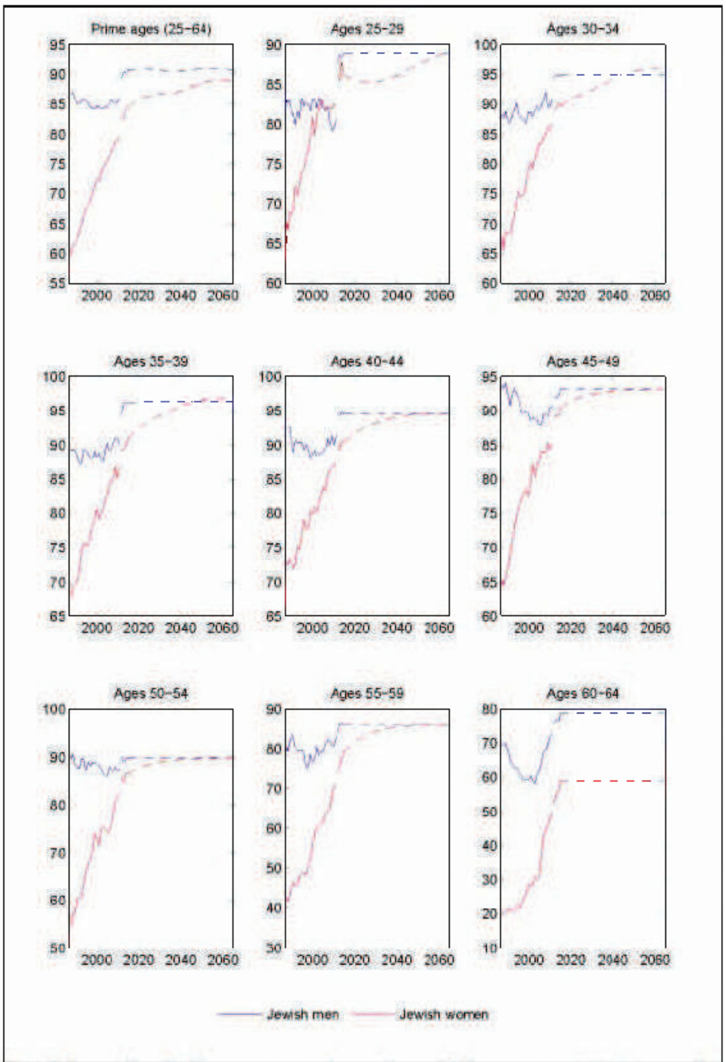
Beyond this somewhat gloomy baseline simulation, we have shown there are policy areas that can raise future growth and help decrease the current GDP per capita gap with respect to other developed economies. Our TFP model points to potential benefits from better educational quality of wider schooling opportunities as well as improvement in the business environment and infrastructure.

This project is far from being completed. The model was built on simplicity and modularity which allows the addition of other model blocks or the estension of existing blocks rather simply. Some areas we have intendedly left for future work are: The extension of human capital to include abilities as measured by PIAAC surveys (as an alternative to the centrality of yeas of schooling in the determination of human capital beyond labor input), to extend the amount of cross-block effects (for example, to introduce an effect from TFP on years of schooling or on the return to schooling); to better identify the causal relationship between policy and TFP growth; to include concrete policy variables in the human capital model block; to extend the model beyond GDP growth to national income which is closer to the concept of utility, and more. These extensions can be added along the way as the model becomes operational for the use of policy making in Israel.

Appendices

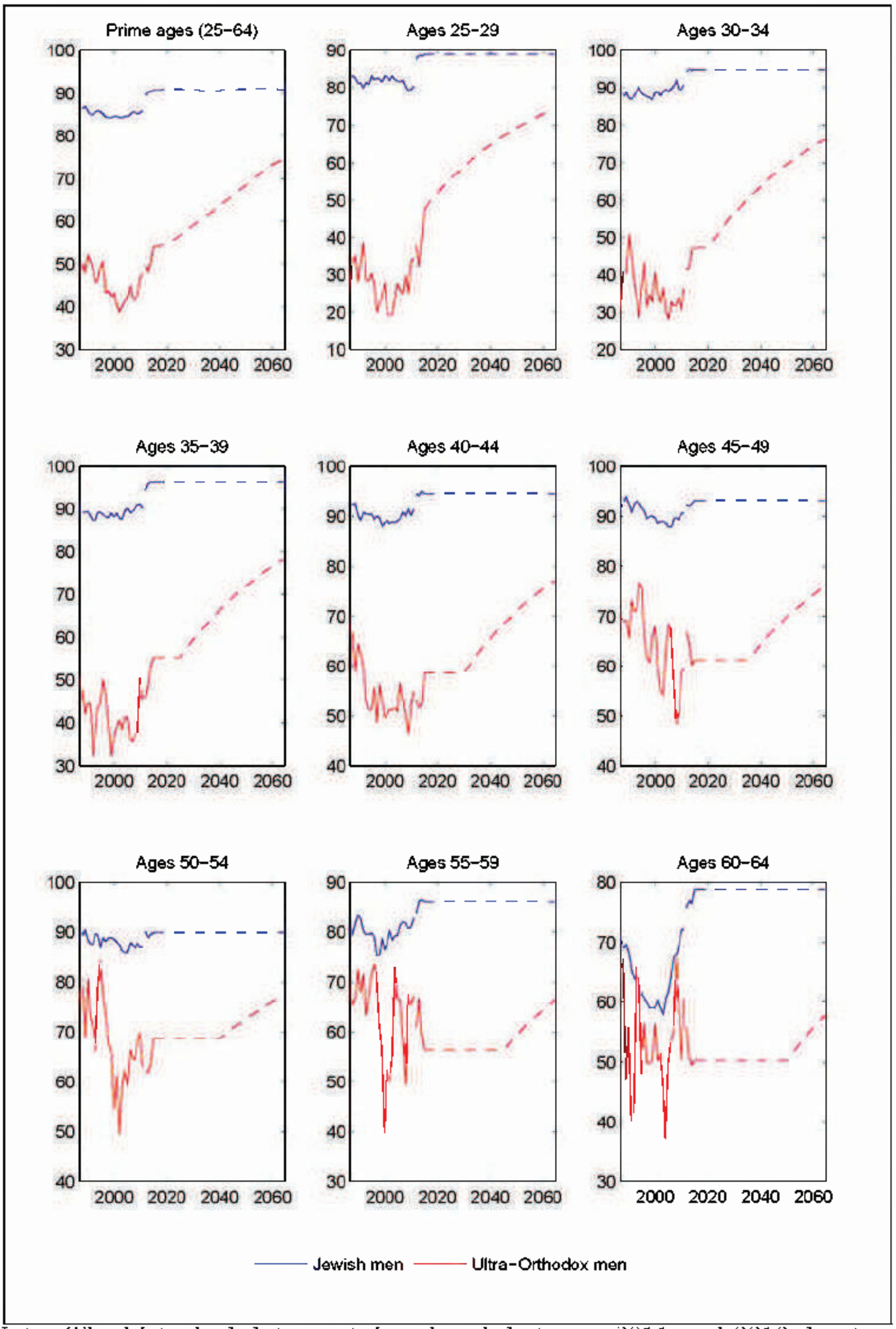
Appendix A Group specific convergence graphs

Figure 13: Convergence in Group Specific Labor Force Participation Rate - Jewish



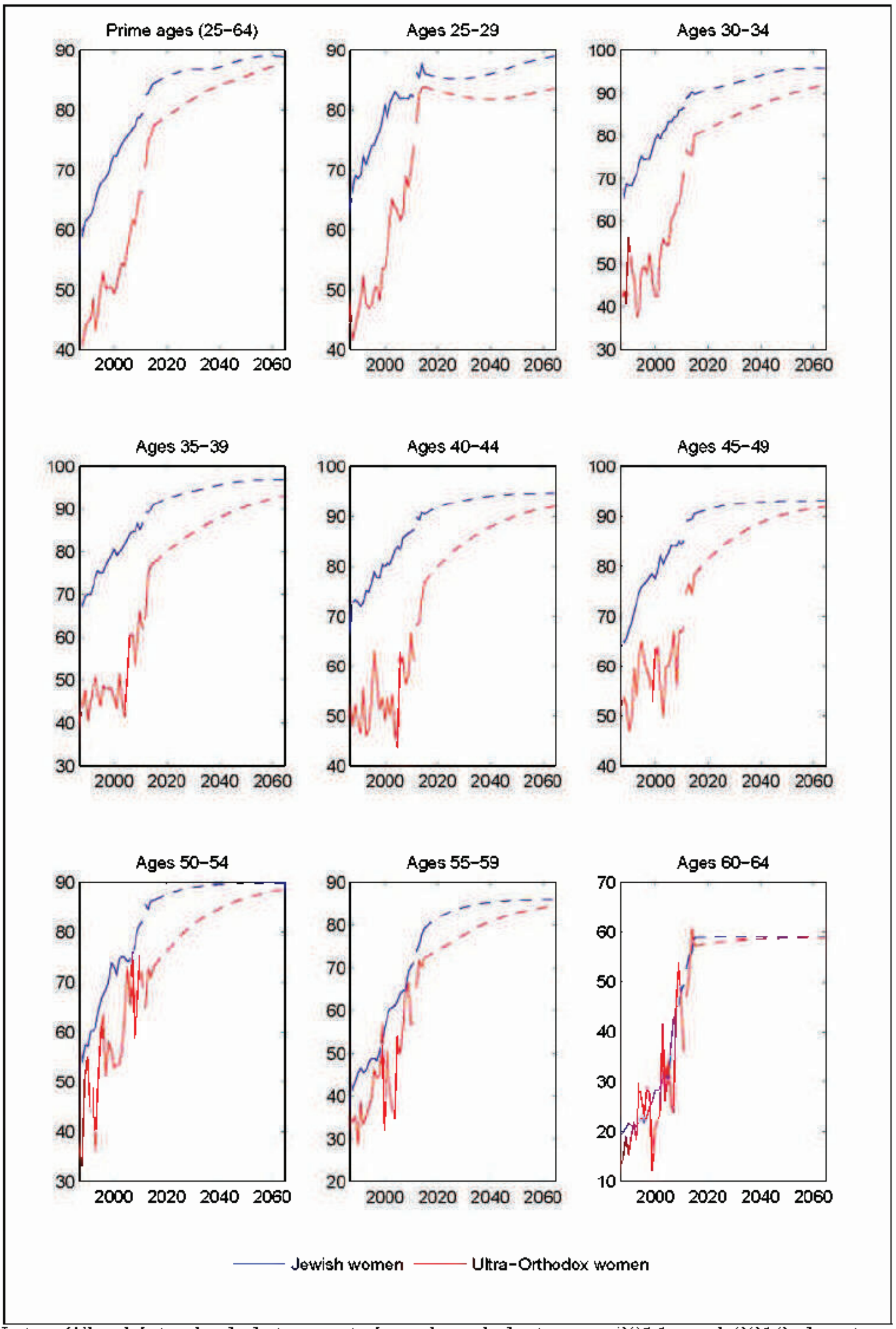
Note: The historical data contain a break between 2011 and 2012 due to changes in the Labor Force Survey.

Figure 14: Convergence in Group Specific Labor Force Participation Rate - Ultra-Orthodox Men



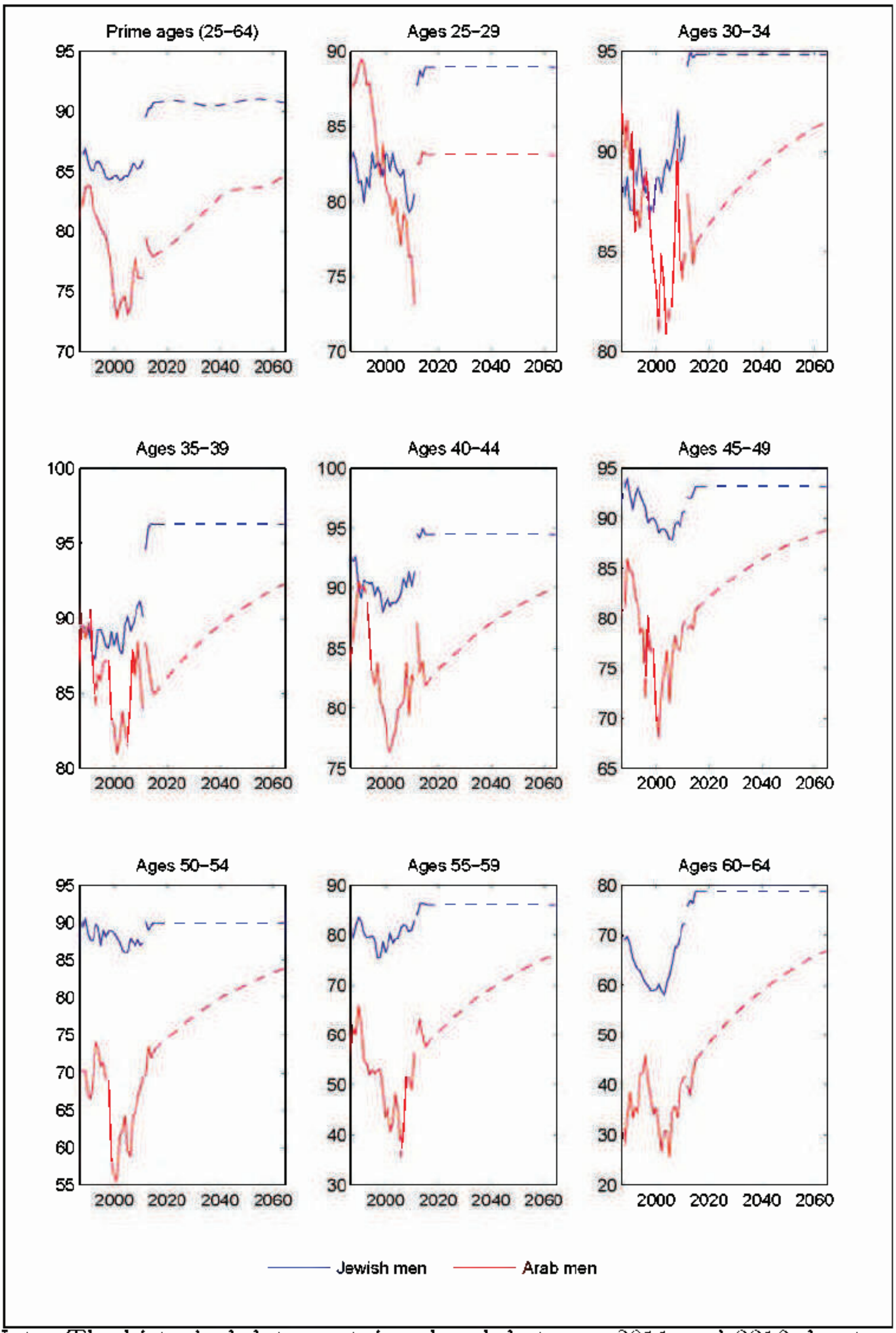
Note: The historical data contain a break between 2011 and 2012 due to changes in the Labor Force Survey.

Figure 15: Convergence in Group Specific Labor Force Participation Rate - Ultra-Orthodox Women



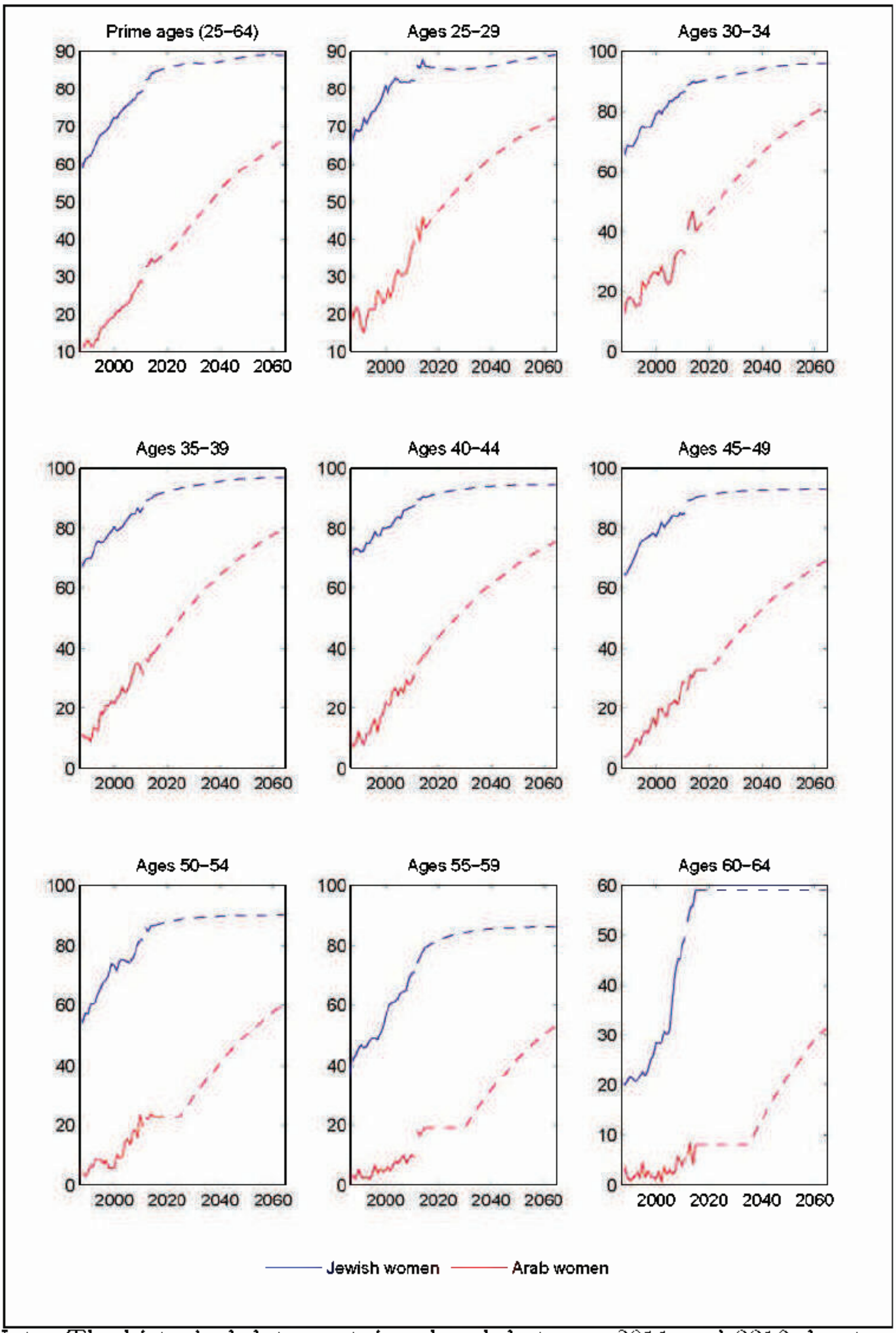
Note: The historical data contain a break between 2011 and 2012 due to changes in the Labor Force Survey.

Figure 16: Convergence in Group Specific Labor Force Participation Rate - Arab Men



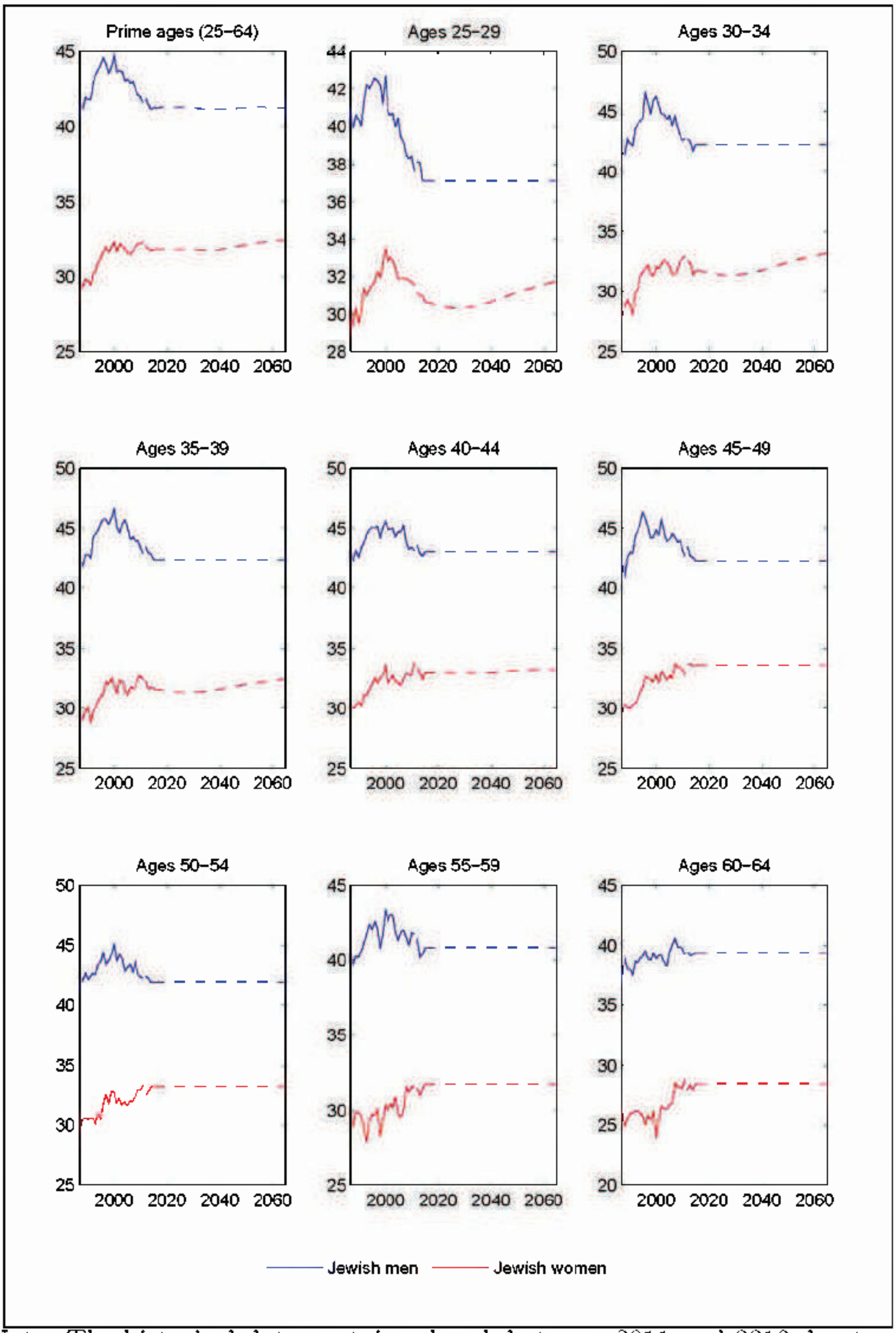
Note: The historical data contain a break between 2011 and 2012 due to changes in the Labor Force Survey.

Figure 17: Convergence in Group Specific Labor Force Participation Rate - Arab Women



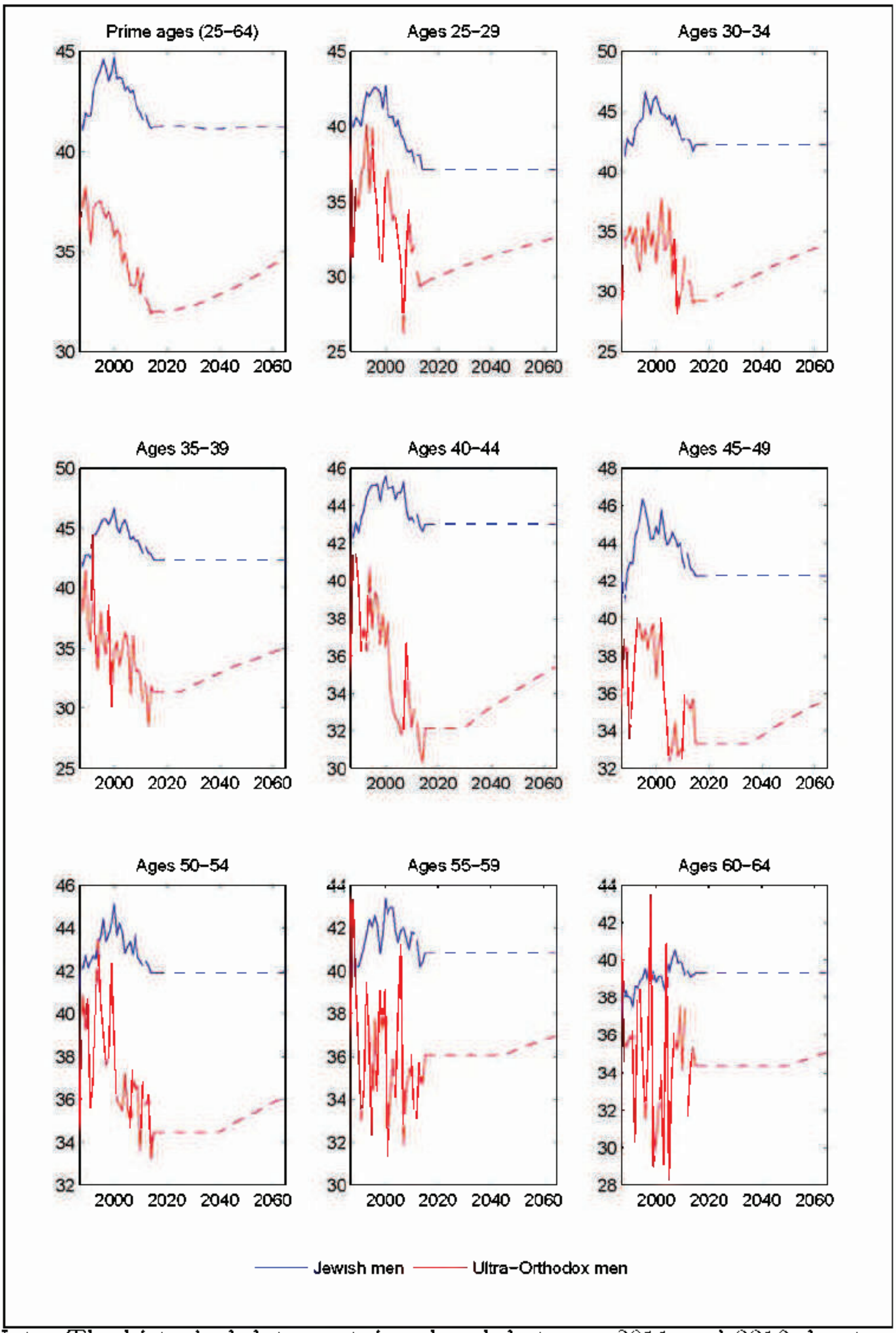
Note: The historical data contain a break between 2011 and 2012 due to changes in the Labor Force Survey.

Figure 18: Convergence in Group Specific Hours per Worker - Jewish



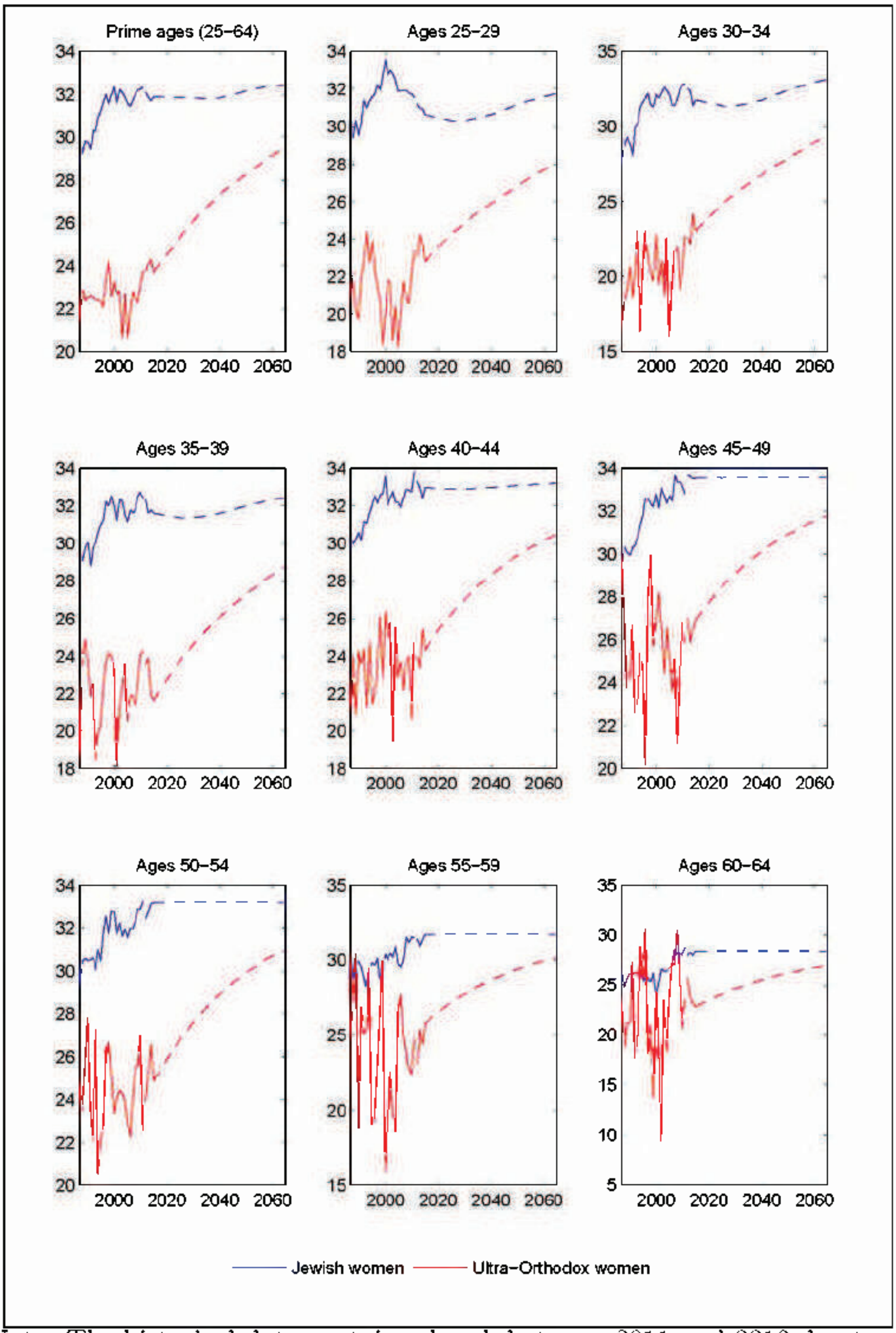
Note: The historical data contain a break between 2011 and 2012 due to changes in the Labor Force Survey.

Figure 19: Convergence in Group Specific Hours per Worker - Ultra-Orthodox Men



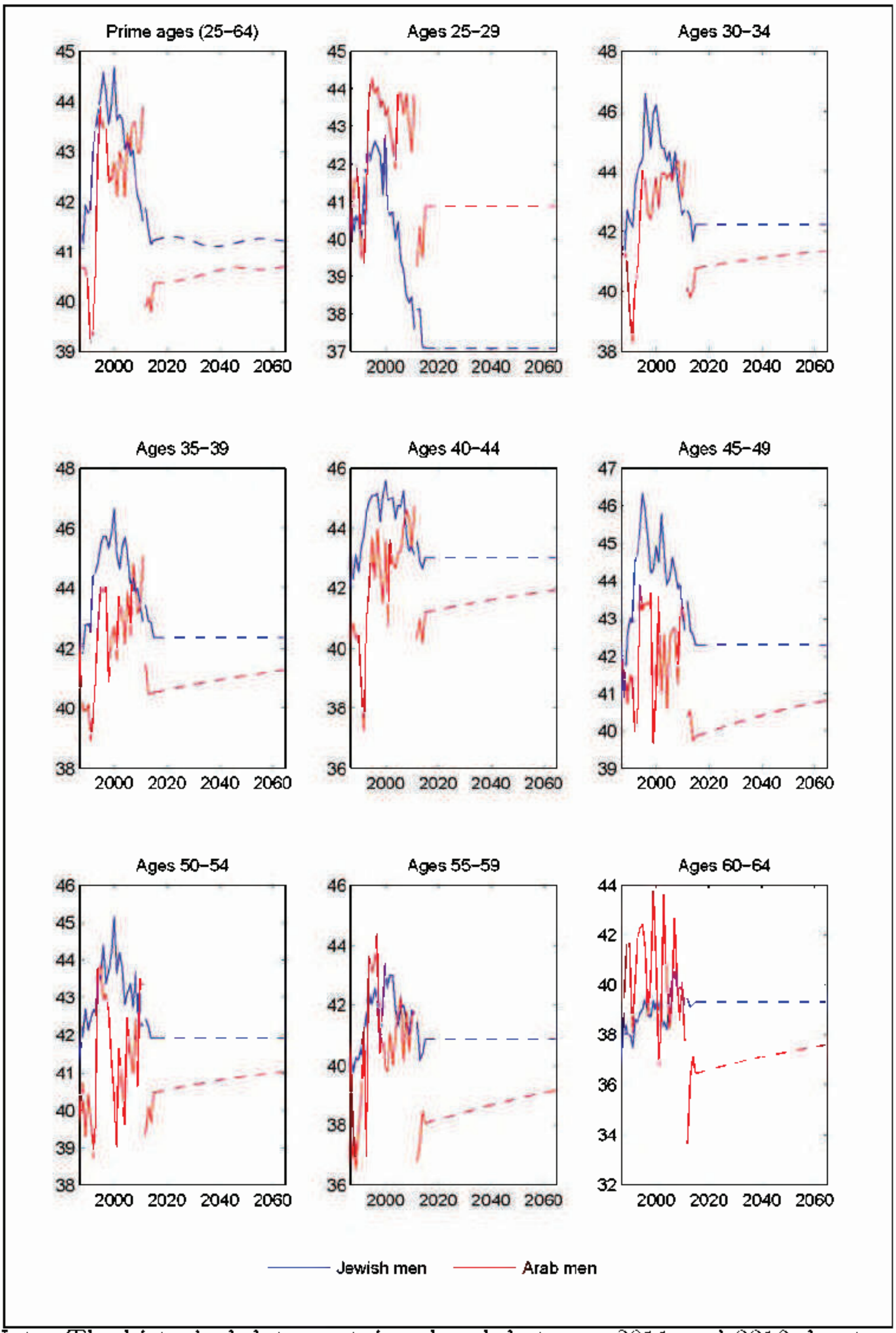
Note: The historical data contain a break between 2011 and 2012 due to changes in the Labor Force Survey.

Figure 20: Convergence in Group Specific Hours per Worker - Ultra-Orthodox Women



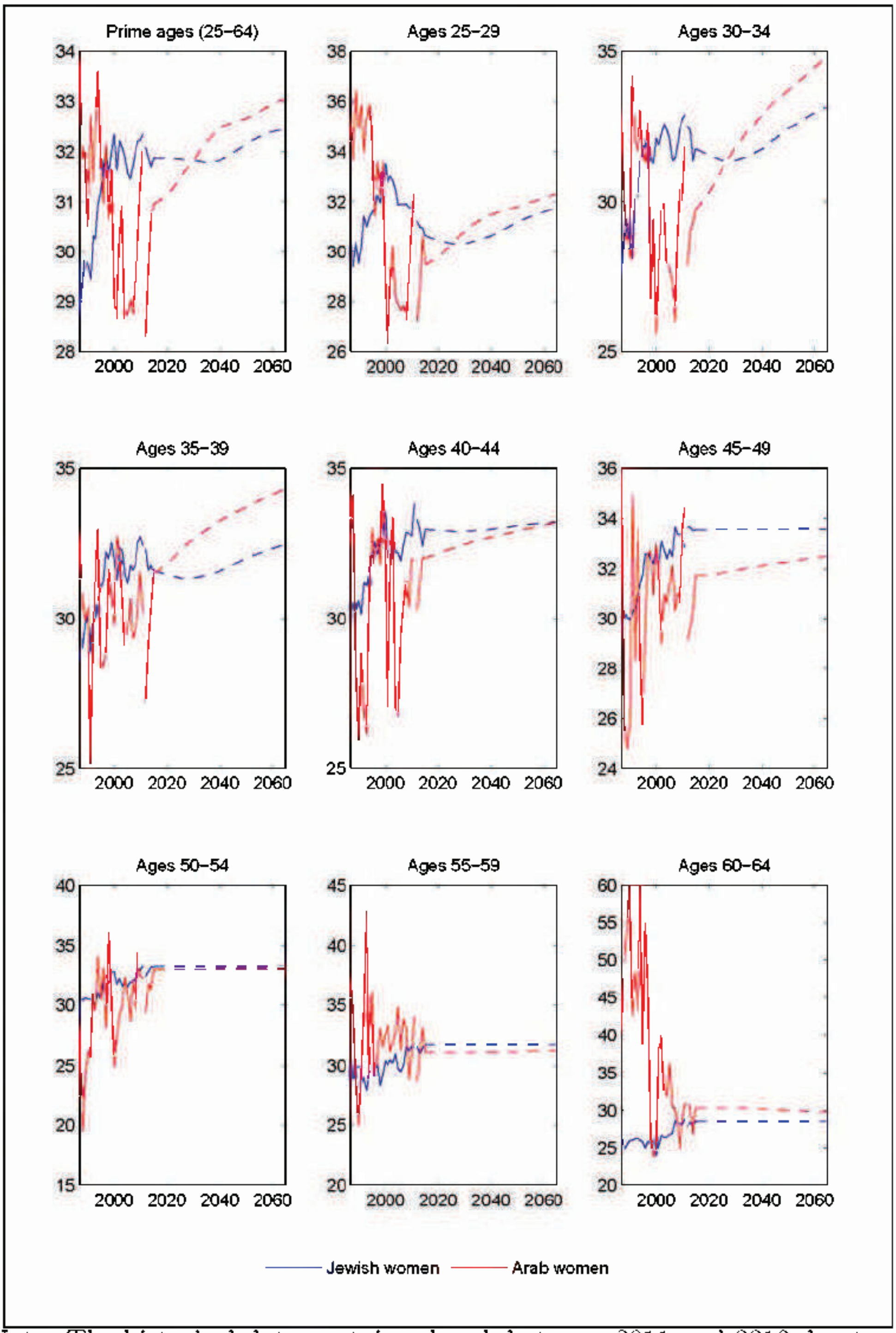
Note: The historical data contain a break between 2011 and 2012 due to changes in the Labor Force Survey.

Figure 21: Convergence in Group Specific Hours per Worker - Arab Men



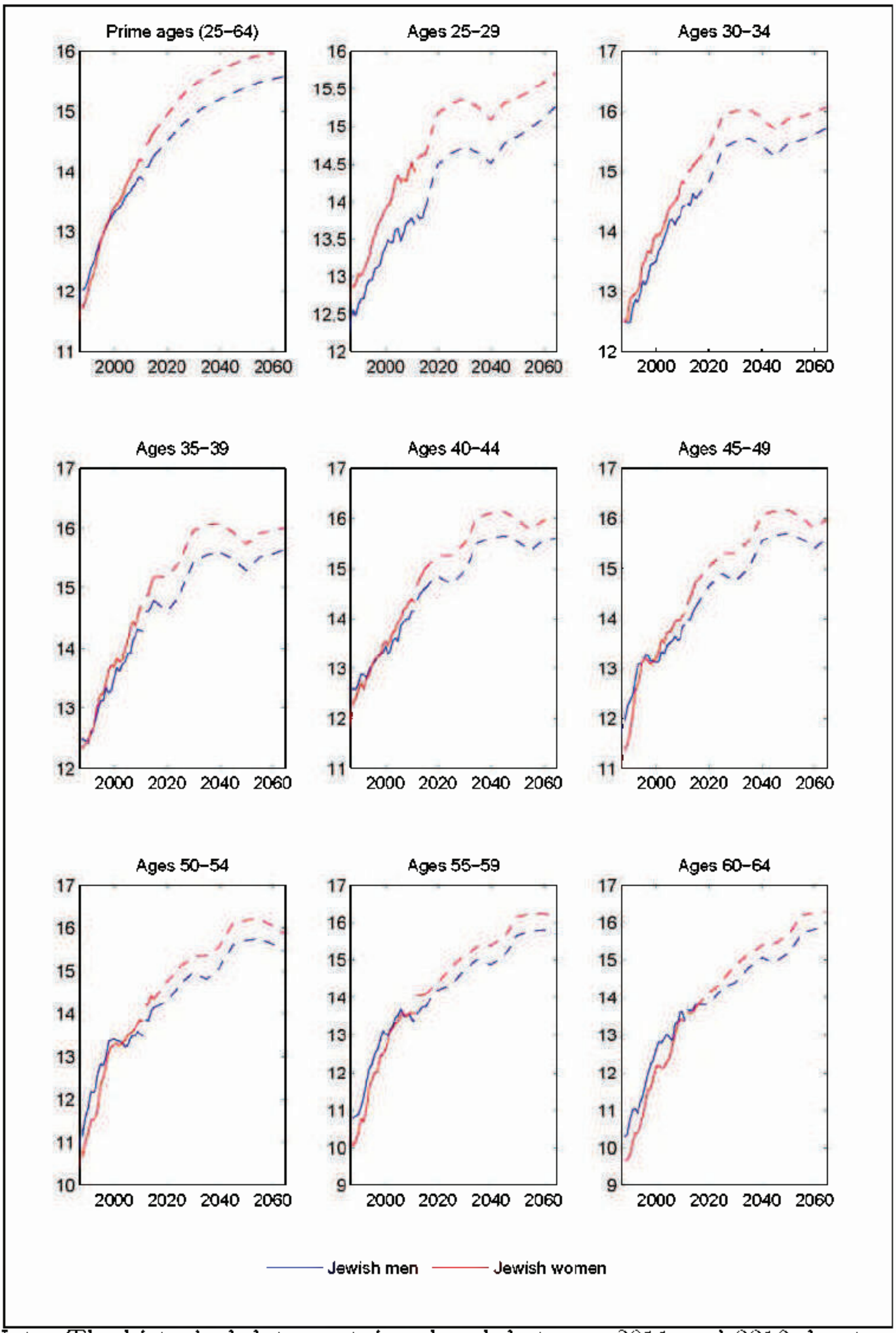
Note: The historical data contain a break between 2011 and 2012 due to changes in the Labor Force Survey.

Figure 22: Convergence in Group Specific Hours per Worker - Arab Women



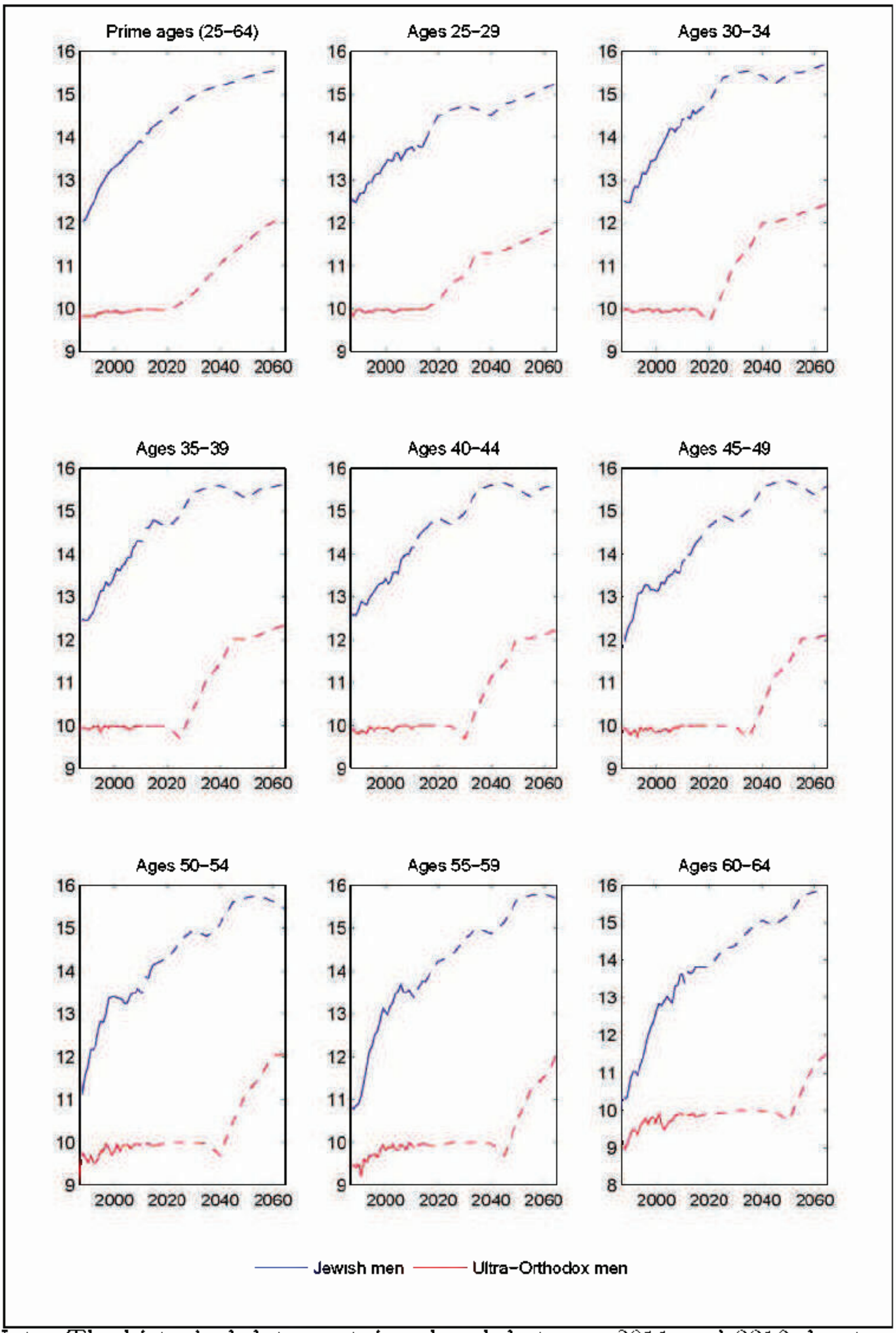
Note: The historical data contain a break between 2011 and 2012 due to changes in the Labor Force Survey.

Figure 23: Convergence in Group Specific Years of Schooling - Jewish



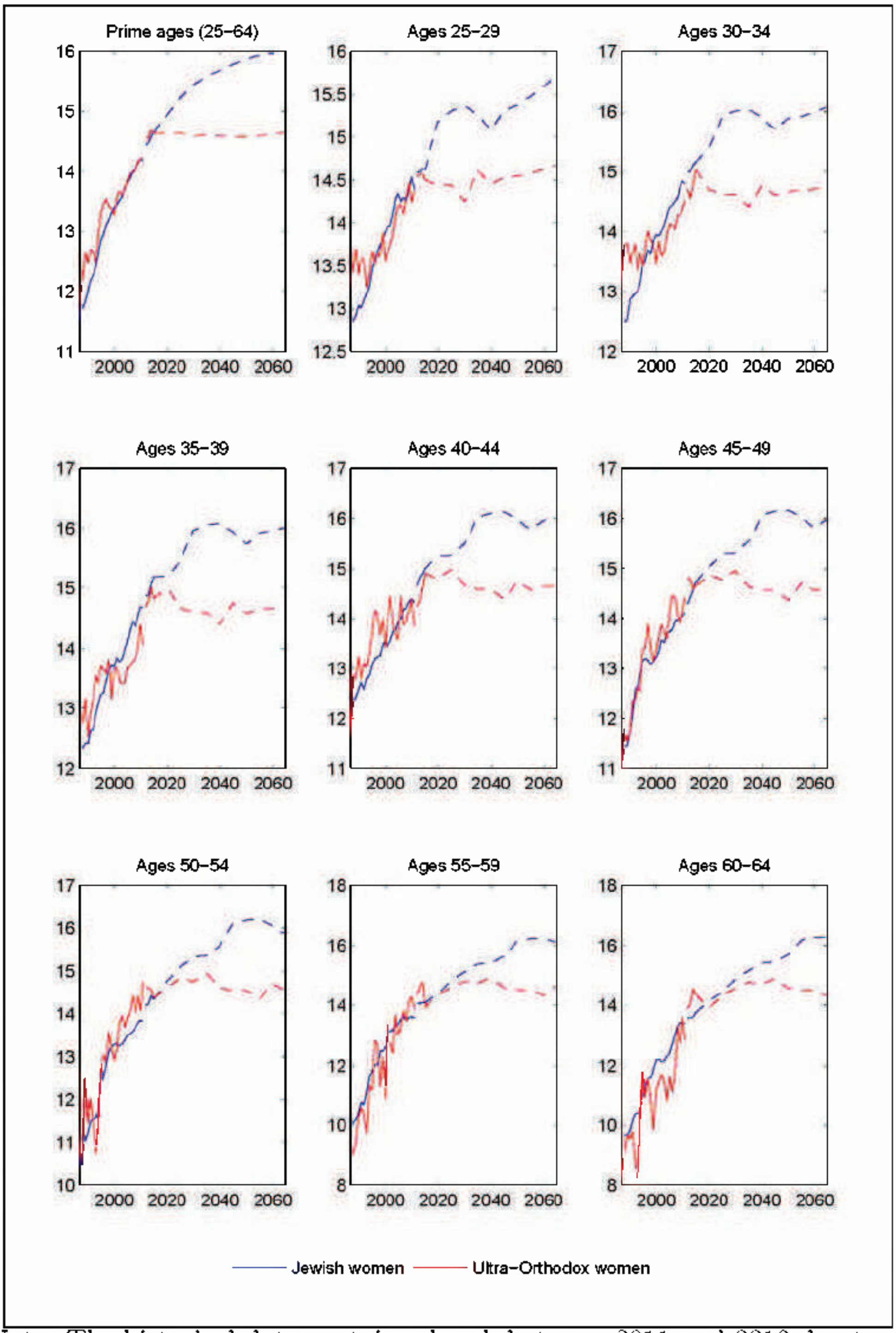
Note: The historical data contain a break between 2011 and 2012 due to changes in the Labor Force Survey.

Figure 24: Convergence in Group Specific Years of Schooling - Ultra-Orthodox Men



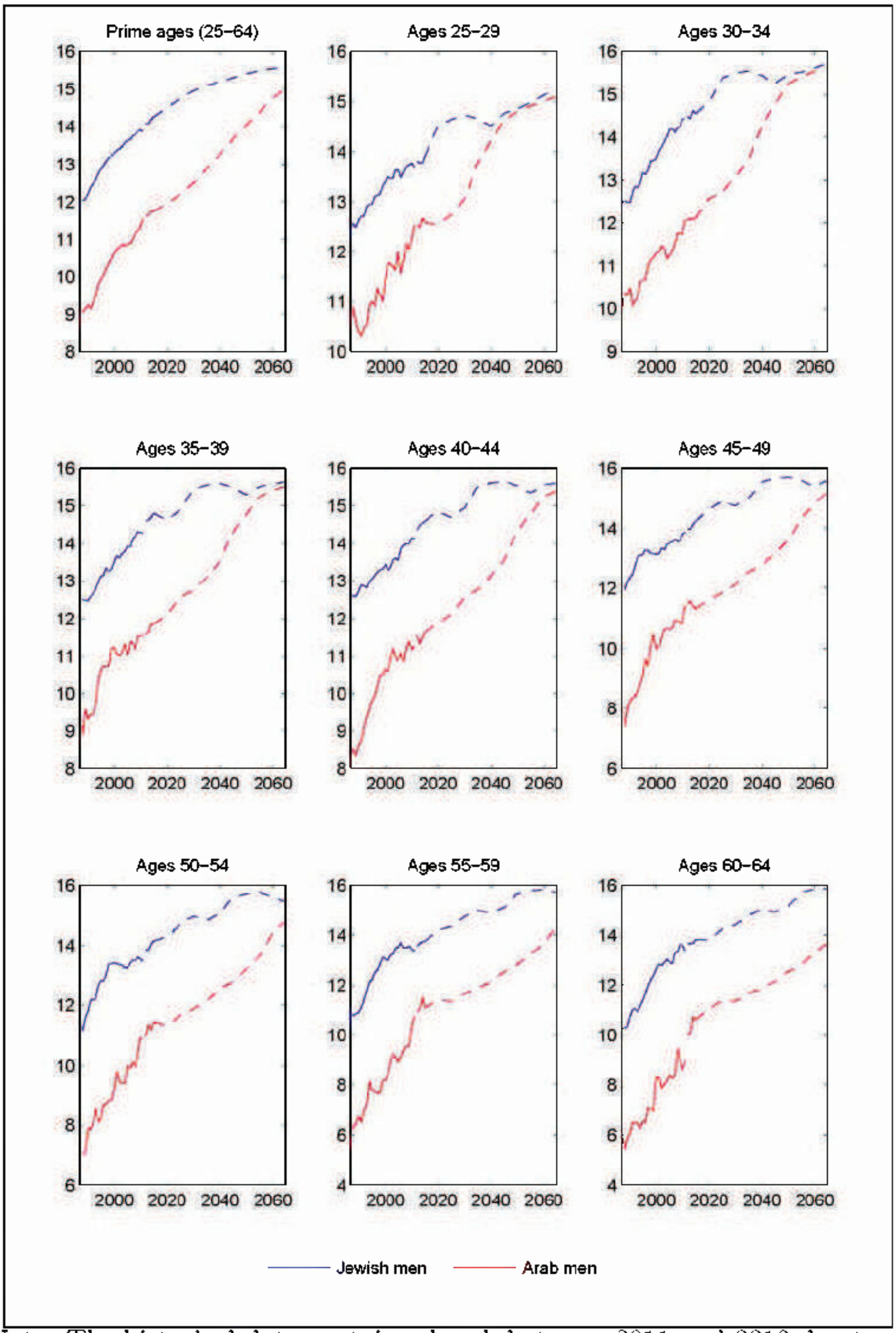
Note: The historical data contain a break between 2011 and 2012 due to changes in the Labor Force Survey.

Figure 25: Convergence in Group Specific Year of Schooling - Ultra-Orthodox Women



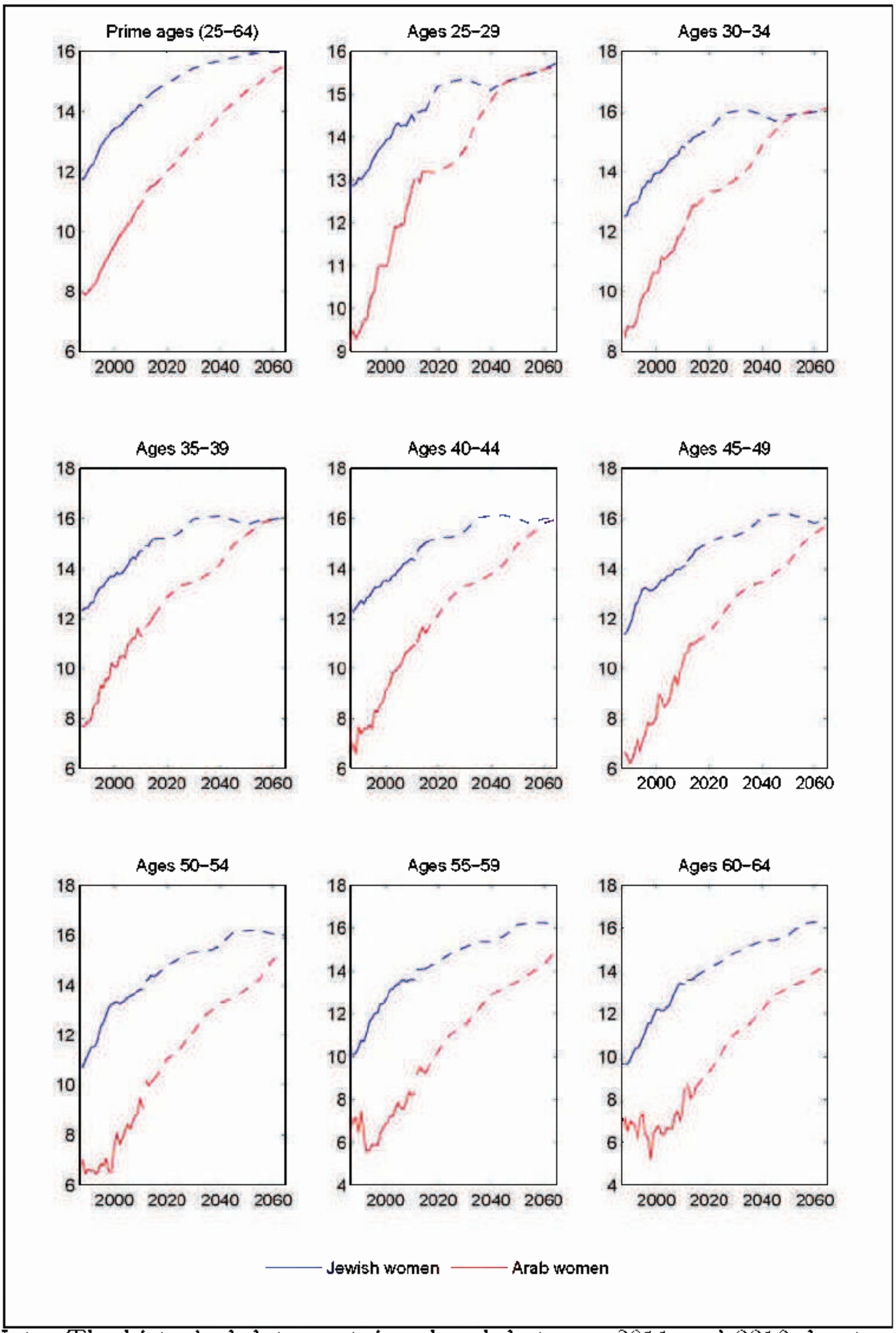
Note: The historical data contain a break between 2011 and 2012 due to changes in the Labor Force Survey.

Figure 26: Convergence in Group Specific Years of Schooling - Arab Men



Note: The historical data contain a break between 2011 and 2012 due to changes in the Labor Force Survey.

Figure 27: Convergence in Group Specific Years of Schooling - Arab Women



Note: The historical data contain a break between 2011 and 2012 due to changes in the Labor Force Survey.

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