



**Optimal monetary policy under heterogeneous  
beliefs of the central bank and the public**

**Alex Ilek\* and Guy Segal\*\***

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\* Alex Ilek, Research Department – Email: [alexei.ilek@boi.org.il](mailto:alexei.ilek@boi.org.il), Tel. 972-2-6552636,  
Fax: 972-2-6669521.

\*\* Guy Segal, Research Department – Email: [guy.segal@boi.org.il](mailto:guy.segal@boi.org.il), Tel. 972-2-6552529,  
Fax: 972-2-6669529.

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# Optimal monetary policy under heterogeneous beliefs of the central bank and the public

Alex Ilek and Guy Segal

## Abstract

We propose a novel approach to modeling and conducting optimal monetary policy under heterogeneous beliefs between the central bank and the public concerning the structure of the economy.

Given this heterogeneity in beliefs, we assert that the optimal monetary policy requires a central bank to model the expectations formation of the public and to combine it with its own structural model. This is because the public's expectations affect the economy.

This approach minimizes the welfare loss, as the central bank exploits the interrelations (contemporaneous and dynamic) among the economic variables which affect the public's expectations. If the central bank does not account for this discrepancy in beliefs, but instead ignores the expectation formation mechanism of the public, it generates noticeable welfare losses.

## מדיניות מוניטרית אופטימלית כאשר לבנק המרכזי ולציבור ציפיות שונות

אלכס אילק וגיא סגל

### תמצית

אנו מציעים גישה חדשה למידול ולביצוע של מדיניות מוניטרית אופטימלית כאשר לבנק המרכזי ולציבור יש הנחות שונות לגבי האופן שבו הכלכלה מתנהגת. בהינתן הבדל זה בין הבנק המרכזי לציבור, אנו מראים כי כדי לנהל מדיניות מוניטרית אופטימלית על הבנק המרכזי למדל את מנגנון יצירת הציפיות של הציבור ולשלב במודל המבני של הבנק. זאת מאחר שהציפיות של הציבור משפיעות על הכלכלה. גישה זו ממקסמת את הרווחה של הציבור משום שהבנק המרכזי מנצל בצורה יעילה את הקשרים בין המשתנים הכלכליים המשפיעים על הציפיות של הציבור. אם הבנק המרכזי מתעלם מההבדל שבינו לבין הציבור ומתעלם ממנגנון יצירת הציפיות של הציבור, אזי הדבר מוביל לפגיעה משמעותית ברווחה.

# 1. Introduction

The mainstream literature of optimal monetary policy (henceforth OMP) assumes that the expectations of the public are rational. According to the assumption of rational expectations (hereinafter, RE), both the central bank (hereinafter, CB) and the public possess full information concerning the same model of the economy, and therefore their expectations are consistent with each other (e.g., Clarida, et al., 1999, Woodford, 2003a, and Walsh, 2010).

In recent years, a growing body of literature has focused on OMP when the public is boundedly rational (learning public). In this setting, although the model specification of the public is fully equivalent to the model of the CB, the public does not know its parameters and learns them over time. Based on the perceived model and its estimated parameters, the public forms its expectations, which jointly with other factors determine the development of the economy.

Evans and Honkapohja (2003, 2006) proposed an expectations-based optimal policy rule which is derived under RE. They showed that when agents are boundedly rational, policy should respond to the expectations of the public in order to guarantee determinacy and learnability of the RE equilibrium. If instead the policy rule is expressed in terms of shocks only<sup>1</sup>, the economy has a high likelihood to diverge and cause noticeable welfare losses. Vestin, et al. (2006a, 2006b) and Molnar and Santoro (2010) proposed a novel monetary policy framework where the CB exploits the learning algorithm of the agents by putting it as an additional dynamic constraint in the optimization problem of the CB. Thus the CB takes into account its effect on the public's estimated parameters and therefore on the public's future expectations when it chooses the interest rate.

Honkapohja and Mitra (2005) examined the E-stability properties of the RE equilibrium when the CB's interest rate rule is specified in terms of its own internal forecasts. In this setting, they assumed identical models in terms of specification for the CB and the public. The main difference between the models appeared in the learning process and the initial beliefs about the state of the economy.

Muto (2011) considered the case where the public learns from the CB forecasts, while the CB reacts to its own expectations concerning inflation and the output gap rather

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<sup>1</sup> Under RE, there is no difference at all in which terms the optimal policy rule is represented. Different representations of the policy rule result in the same dynamics of the economy.

than directly to the public's expectations. Muto (2011) found that to insure learnability of the RE equilibrium the CB should respond more strongly to the expected inflation than the extent to which the Taylor principle suggests.

Adam and Woodford (2012) proposed a general approach for OMP when agents' beliefs are close to satisfying RE but the exact formation of their beliefs is not known to the CB. Thus the CB chooses the policy that deals with the worst-case beliefs. They applied their approach to a micro-founded New Keynesian model and showed that the long term inflation rate underreacts to the cost push shock.<sup>2</sup> Their overall conclusion, however, is that the policy prescriptions arising under RE are robust to the environment of near-rational expectations of the public.

The literature also examined OMP when agents misspecify the model of the CB. Orphanides and Williams (2008) showed that when the public is uncertain of the true model and forms its expectations using an estimated model (VAR) the policy derived under RE assumption "*can perform poorly*". Berardi (2009) considered the case where a subgroup of agents has a correct model whereas another subgroup has an underspecified model. Using a forward-looking policy rule derived under timeless perspective regime and RE assumption<sup>3</sup>, Berardi (2009) tested whether the CB should react to the average expectations of the public or only to the expectations of a specific group. He concluded that the CB should only consider the correct expectations and disregard the misspecified expectations of the public.

Berardi and Duffy (2007) and Cone (2005) also assumed that agents misspecify the model of the CB. They examined the value of the transparency of the CB. In the transparent regime the CB announces its model and objectives to the public and thus enables the public to adopt the model of the CB (under a nontransparent regime, agents keep underspecifying the model of the CB). Berardi and Duffy (2007) concluded that in the discretion case the advantage of the transparency is not unequivocal and depends on policy target values.

In this paper, we propose a novel framework for modeling OMP under heterogeneous beliefs of the CB and the public regarding the structure of the economy. We extend the existing approach in one very fundamental aspect—we assume that there is a

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<sup>2</sup> The authors applied commitment regime of the CB. Under the robust policy rule, the initial response of inflation to cost-push shock is smaller than implied under RE, but in the subsequent periods the dynamics are the same.

<sup>3</sup> Following the approach of Evans and Honkapohja (2003).

discrepancy in beliefs between the CB and the public<sup>4</sup>, which cannot be eliminated even by full transparency and communication of the CB. Such a discrepancy may stem from several causes: 1) a potential limited credibility of the CB's short-medium horizon forecasts; 2) a practical inability of the public to adopt the model and the perceptions of the CB.

**1) Limited Credibility of the CB's short-medium horizon forecasts**—The mentioned literature assumes full credibility of the CB. This implies that when the CB decides to be fully transparent, just by announcing its model and its forecasts the CB can easily shift the public from its own beliefs to be in line with the CB's beliefs. In practice, there is no convincing evidence for such a behavior; although the CB regularly announces its projections of the economy and even posts its models to the public, capital market participants, professional forecasters (henceforth PF) and households (based on surveys) seem to perceive the economy differently from the CB by reporting different projections. One explanation might be incentives of PF not to report their true projections to gain publicity (see Frenkel, et al., 2013 and Laster, et al., 1999). However there is no such incentive for capital markets, firms and households.

Agents may think that although the CB may have a rich and complicated theoretical model, this does not necessarily provide better forecasts of the economy than their own forecasts. Thus there is a subjective probability of the public that its own model is better. We can go even further and claim that agents also realize the existence of a principle-agent problem as regards the CB: even if the CB has an incorrect model of the economy, it always has the incentive to convince the public to adopt its own model and its projections of the economy. Therefore, the CB's announcement "please take my model because I know how the economy works" could be not credible.

**2) Operational difficulty**—There is another reason for potential reluctance of the public to adopt the model of the CB (or its forecasts), which has nothing to do with credibility. Usually, the models of the CB are complicated and hardly operational in practice for the majority of the public. Therefore the suggestion of

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<sup>4</sup> By discrepancy in beliefs we mean that the model of the CB and the public are different and can step beyond the near-rational expectations neighborhood considered by Adam and Woodford (2012).

Berardi and Duffy (2007), Cone (2005) and others that the CB should educate the public on the correct model seems a very desirable purpose, but hardly implementable in practice. Another difficulty arises from the fact that the projections of the CB are published on a low frequency basis and hence are not useful for high frequency usage.

A survey among PF in Israel (see Appendix C for full description of the survey and its results) supports our assumption of discrepancy in perceptions of the economy between the CB and the public.

Subject to discrepancy in beliefs mentioned above, the best thing the CB can do to achieve its objectives in the most efficient way is to learn the expectation formation mechanism (perceived model) of the public instead of vainly trying to convince the public to adopt the expectations of the CB. Then the CB should utilize the expectations formation of the public along with its own structural model, and to then derive the optimal policy rule. Thus, the policy rule is derived from the combined model, where the structural model of the CB comprises the specific expectations formation of the public. Under this approach the welfare loss is minimized because the CB exploits the relationship (contemporaneous and dynamic) among the economic variables appearing in the model of the public. This is in contrast to the methodology of deriving the CB's policy rule when it ignores the expectations formation of the public. Thus the CB reacts directly via this rule to the expectations of the public, irrespective of how well these expectations were formed.

To our knowledge this is the first paper proposing this framework of OMP under heterogeneous beliefs of the CB and the public. There is an additional channel absent in our paper and we leave it for future research, which enables the CB to minimize its loss function even more. The CB should take into account not only the expectation formation of the public with respect to the variables contained in its model but also with respect to the dynamics of the parameters via the learning algorithm of the public.<sup>5</sup>

The rest of the paper is organized as follows: Section 2 presents the canonical New Keynesian model and the expectation-based policy rule which ignores the different

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<sup>5</sup> Vestin, *et al.* (2006a, 2006b), and Molnar and Santoro (2010) examined this channel of the monetary policy in a very parsimonious framework and the main challenge is to apply it to richer models

process of the expectations formation of the public. Section 3 describes the OMP and the solution of the economy where the CB takes into account the discrepancy in beliefs. Section 4 describes the solution of the economy under the expectation-based policy rule. Section 5 presents welfare analysis comparing the novel and the standard approaches of the OMP. Section 6 concludes.

## 2. The structural model of the economy

### 2.1 Description of the New Keynesian model

In this section we assume that the economy follows the canonical New Keynesian model as presented by Clarida, et al. (1999). All variables are expressed as deviations from the steady state values. The model consists of two behavioral equations:

**The Philips curve (inflation equation):**

$$\pi_t = \beta E_t \pi_{t+1} + kx_t + \varepsilon_t^\pi, \quad (1)$$

where  $\pi_t$  is the inflation rate,  $x_t$  is the output gap which is defined as the gap between actual output,  $y_t$ , and potential output. For simplicity we assume that the potential output is constant at the level of the output in the steady state,  $y$ .  $\varepsilon_t^\pi$  is a white-noise shock to inflation.  $E_t \pi_{t+1}$  denotes inflation expectations of the public.

**The demand equation (IS equation):**

$$x_t = E_t x_{t+1} - \alpha(i_t - E_t \pi_{t+1}) + \varepsilon_t^x, \quad (2)$$

where  $i_t$  is the interest rate of the CB (in terms of deviation from steady state) and  $E_t x_{t+1}$  are the public's expectations for the output gap in the next period.  $\varepsilon_t^x$  is a white-noise demand shock. In matrix form (1) and (2) are given by:

$$z_t = F_1 E_t z_{t+1} + F_2 i_t + F_3 \varepsilon_t, \quad (3)$$

where  $z_t = [\pi_t, x_t]'$ ,  $\varepsilon_t = [\varepsilon_t^\pi, \varepsilon_t^x]'$ ,  $F_1 = \begin{bmatrix} \alpha k + \beta & k \\ \alpha & 1 \end{bmatrix}$ ,  $F_2 = \begin{bmatrix} -\kappa \alpha \\ -\alpha \end{bmatrix}$ ,  $F_3 = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$ .

This is the state representation of the economy expressed in terms of the public expectations (which are not necessarily rational), the policy rule and the shocks.

## 2.2 The Central Bank's objective function

We assume that the CB acts under discretion. It minimizes the following loss function:

$$E_t L_t = E_t \sum_{j=0}^{\infty} \beta^j [\pi_{t+j}^2 + \lambda x_{t+j}^2], \quad (4)$$

where  $\lambda$  is the relative weight of the output gap with respect to the inflation in the loss function and  $\beta$  is the discount factor. From now on we assume that the CB holds the correct structural model of the economy and its loss function represents the true welfare loss of the economy.

## 2.3 Expectations-based interest rate rule ignoring the expectations formation of the public

The starting point for our analysis is that there is a difference between the CB's model and the model of the public, which is reflected in the different expectations of the CB and the public. Given this difference the CB can: 1) ignore this difference and act according to its model-consistent expectations optimal response 2) respond to the expectations of the public without modeling their formation mechanism 3) model the public expectations formation and then to derive the optimal response.

Clarida, et al. (1999, p. 1672) showed that under case (1) (which is also true in case (2)) the optimal discretionary first-order condition is given by:

$$k\pi_t + \lambda x_t = 0 \quad (5)$$

We follow Evans and Honkapohja (2003) and represent the optimal discretionary interest rate rule in terms of both expected inflation and expected output gap. Using (1), (2) and (5), the expectations-based policy rule is given by:

$$i_t = \left(1 + \frac{\beta k}{\alpha(\kappa^2 + \lambda)}\right) E_t \pi_{t+1} + \frac{1}{\alpha} E_t x_{t+1} + \frac{k}{\alpha(\kappa^2 + \lambda)} \varepsilon_t^\pi + \frac{1}{\alpha} \varepsilon_t^x. \quad (6)$$

Note, that the expectations-based policy rule (6) applies to any kind of expectations and assumes nothing about their formation, that is, whether they are rational or not.

### 2.3.1 The CB ignores the different expectations formation mechanism of the public

In the case when the CB ignores the discrepancy in beliefs between it and the public and acts according to its model-consistent expectations optimal response, then the



optimal discretionary policy (6) becomes (in the case that there is no serial correlation in the shocks the model-consistent expectations are zero):

$$i_t = \frac{k}{\alpha(k^2 + \lambda)} \varepsilon_t^\pi + \frac{1}{\alpha} \varepsilon_t^x. \quad (7)$$

Evans and Honkapohja (2003) showed that if the CB responds according to (7) when the public expectations are not rational but boundedly rational, then this rule leads to divergence of the economy implying enormous welfare losses. Thus, ignoring the public expectations in the policy rule is not an option.

### 2.3.2 The CB responds to the public's expectations without modeling them

Evans and Honkapohja (2003) showed that if the public is learning, then incorporating the public's expectations into (6) instead of the CB's model-consistent expectations rule in (7) leads to convergence of the economy towards RE equilibrium. Hence, the CB can act according to:

$$i_t = \left(1 + \frac{\beta k}{\alpha(k^2 + \lambda)}\right) E_t \pi_{t+1}^P + \frac{1}{\alpha} E_t x_{t+1}^P + \frac{k}{\alpha(k^2 + \lambda)} \varepsilon_t^\pi + \frac{1}{\alpha} \varepsilon_t^x. \quad (8)$$

where  $E_t \pi_{t+1}^P$  and  $E_t x_{t+1}^P$  are the expectations of the public.

However, the question is not only whether the economy will eventually converge to the RE equilibrium but what is the most efficient way, in terms of the loss function, during the path of convergence to RE equilibrium.

Our proposal is to model the public's expectations formation and to take it into account when deriving the optimal policy rule as is described in Section 3 below. We note that while the public as a whole doesn't have a model, we assume that its expectations can be modeled by a VAR model (with one lag) of three variables: inflation, interest rate and an indicator of real activity.

Specifically, we assume that as the indicator of real activity the public uses the growth rate of output (instead of the output gap in the model of the CB).

The structure of the subsequent analysis is summarized in Table 1: Case 1 considers the standard approach in the literature where the CB uses the expectations-based policy rule shown in (8). Case 2 considers the proposed approach where the CB derives its policy rule exploiting the expectations formation mechanism of the public. We analyze two scenarios regarding how the public treats the parameters in

the VAR: updating them each period (learning) or keeping the parameters fixed (not learning). We examine the convergence point in the economy in each case and finally, we make a welfare comparison between the two approaches of the monetary policy.

**Table 1: Optimal monetary policy (OMP) when the CB and the public have different models of the economy**

Case 1- OMP when the CB uses the expectations-based policy rule	Case 2- OMP when the CB derives the policy rule exploiting the expectations formation mechanism of the public
(1) public is learning (2) public is not learning	(1) public is learning (2) public is not learning
Convergence of the economy under different policy rules	
Welfare analysis: Case 1 versus Case 2	

### 3. Optimal monetary policy when the CB takes into account the public's expectations formation

The policy rule under Case 1 is given by (8) and we postpone the discussion about the properties of the economy under such a rule to Section 4.

At this stage we start deriving an optimal policy rule based on Case 2. We assume that the public has a VAR model which includes the growth rate of output instead of the output gap (which appears in the model of the CB). This assumption seems highly plausible: most, if not all, PF and surveys worldwide report their projections for the output growth rate rather than for the output gap, and there is a big doubt that these projections are consistent with the output gaps derived from the CB's model. A support for this assumption appears in the literature: Branch and Evans (2006), for example, modeled the expectations of the PF in US using, among other variables, the growth rate of output. The same picture is sketched by a survey we conducted among Israeli PF (see Appendix C).

Next, we depict our methodology. Consider the VAR model of the public:

$$\begin{pmatrix} \tilde{z}_t \\ \tilde{i}_t \end{pmatrix} = A \begin{pmatrix} \tilde{z}_{t-1} \\ \tilde{i}_{t-1} \end{pmatrix} + u_t, \quad (9)$$

where  $\tilde{z}_t = [\pi_t \ \Delta y_t]'$  is a vector consisting the inflation rate and the growth rate of the output, respectively.<sup>6</sup>  $u_t$  is  $3 \times 1$  matrix of the residuals. We will concentrate on submatrix  $\tilde{z}_t$  and rewrite the VAR model as:<sup>7</sup>

$$\begin{pmatrix} \pi_t \\ \Delta y_t \end{pmatrix} = \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{bmatrix} \begin{pmatrix} \pi_{t-1} \\ \Delta y_{t-1} \\ i_{t-1} \end{pmatrix} + u_t^z. \quad (10)$$

We assume that the CB observes the model of the public (in practice the CB should model the expectations formation of the public). The purpose of the CB is to derive the optimal policy rule by exploiting the expectations formation of the public in (10). Observing (10), first, we should express the variables in the VAR model in terms of the CB's model. That is, the growth rate of the output should be represented in terms of the output gap. As  $\Delta y_t = y_t - y - (y_{t-1} - y) = \Delta x_t$ , the VAR model (10) in terms of the output gap is given by<sup>8</sup>:

$$\pi_t = A_1 \pi_{t-1} + A_2 x_{t-1} - A_2 x_{t-2} + A_3 i_{t-1}. \quad (11)$$

$$x_t = B_1 \pi_{t-1} + (1 + B_2) x_{t-1} - B_2 x_{t-2} + B_3 i_{t-1} \quad (12)$$

Hence, the expectations for period  $t+1$  based on the information set at period  $t$ <sup>9</sup> are:

$$E_t \pi_{t+1} = A_1 \pi_t + A_2 x_t - A_2 x_{t-1} + A_3 i_t. \quad (13)$$

$$E_t x_{t+1} = B_1 \pi_t + (1 + B_2) x_t - B_2 x_{t-1} + B_3 i_t. \quad (14)$$

Note that (11) and (12) are equivalent to those in (10). However, the parameters in (11) and (12) are restricted.

Next we rewrite (13) and (14) in a matrix form in terms of  $z_t = [\pi_t \ x_t]'$  as:

$$E_t z_{t+1} = S_1 z_t + S_2 x_{t-1} + S_3 i_t, \quad (15)$$

$$\text{where } S_1 = \begin{bmatrix} A_1 & A_2 \\ B_1 & 1 + B_2 \end{bmatrix}, S_2 = \begin{bmatrix} -A_2 \\ -B_2 \end{bmatrix}, S_3 = \begin{bmatrix} A_3 \\ B_3 \end{bmatrix}.$$

The variables in (15) are expressed in the same terms as in the structural model of the CB, namely  $z_t = [\pi_t \ x_t]'$ . From (15) we observe that the public expectations are a

<sup>6</sup> We made a complete analysis also when the VAR of the public is expressed in terms of output gap and it can be provided upon request.

<sup>7</sup> We ignore the last equation for the interest rate because in this framework it does not play any role in the subsequent analysis.

<sup>8</sup> We omit residuals since they do not play any role in the subsequent analysis.

<sup>9</sup> The information set at period  $t$  consists of the realizations of variables up to period  $t$

function of the realization of the current inflation rate, the past and current output gap (reflecting the output growth rate) and the current interest rate.

Inserting the public's expectations in (15) into the state space representation in (3) yields:

$$z_t = F_1 S_1 z_t + F_1 S_2 x_{t-1} + (F_1 S_3 + F_2) i_t + F_3 \varepsilon_t. \quad (16)$$

Equation (16) reveals an existence of a simultaneity problem: the realization of the inflation rate, the output gap and the interest rate depend on the public's expectations, which in turn, depend on the realization of the economy at the current period. The solution of the system will determine the fixed point of this simultaneity problem.

After moving  $F_1 S_1 z_t$  to the LHS of (16) we derive the state space representation of the economy implied by the public's expectations:

$$z_t = \Lambda_1 x_{t-1} + \Lambda_2 i_t + \Lambda_3 \varepsilon_t, \quad (17)$$

where  $\Lambda_1 = (I - F_1 S_1)^{-1} F_1 S_2$ ,  $\Lambda_2 = (I - F_1 S_1)^{-1} (F_1 S_3 + F_2)$ ,  $\Lambda_3 = (I - F_1 S_1)^{-1} F_3$ .

Now the CB has to choose the optimal policy rule which minimizes the expected loss function in (4) subject to the constraint in (17).

Due to the existence of the lagged output gap in (17) the analytical solution for finding the optimal policy rule is very cumbersome, because the expected losses in the future depend on the reduced form solution of the economy, which is unknown unless the policy rule is determined. At the same time, the determination of the policy rule depends on the expected losses, which depend on the reduced form solution of the economy. We follow Dennis (2007), who proposed a numerical method to resolve this kind of problem. From (17), based on the MSV solution we observe that the specification of the optimal policy rule is based on the state variables such that the interest rate will react to the lagged output gap and to the current shocks. For simplicity, from now on we will denote this optimal policy rule POR (proposed optimal rule).

The POR is given by:

$$i_t = H_1 x_{t-1} + H_2 \varepsilon_t, \quad (18)$$

where  $H_1, H_2$  are obtained numerically using Dennis's (2007) method.<sup>10</sup> Plugging the POR in (18) back into (17) we get the actual law of motion (henceforth ALM) of the economy implied by the VAR model of the public and the POR:

$$z_t = \Psi_1 x_{t-1} + \Psi_2 \varepsilon_t, \quad (19)$$

where  $\Psi_1 = \Lambda_1 + \Lambda_2 H_1$ ,  $\Psi_2 = \Lambda_3 + \Lambda_2 H_2$  and

$\Lambda_1 = (I - F_1 S_1)^{-1} F_1 S_2$ ,  $\Lambda_2 = (I - F_1 S_1)^{-1} (F_1 S_3 + F_2)$ ,  $\Lambda_3 = (I - F_1 S_1)^{-1} F_3$  and  $H_1, H_2$  are obtained from the numerical simulations.

Note that the parameters  $\Psi_1, \Psi_2$  are time variant as long as the parameters in the VAR model of the public are time variant (if the public is learning). Eq. (19) reveals that both the inflation and the output gap are affected not only by the current shocks (with time variant elasticities if the public updates its parameters) but also by the lagged output gap. While a dependence on the historical output gap also exists under the canonical commitment regime, the source of this dependence here is completely different and it stems from the specification of the VAR model of the public. From (19) it is also clear that the impulse response function of the inflation and the output gap to shocks depends on the elasticities, which are directly affected by the public's beliefs concerning the parameters in the VAR model. These beliefs can affect not only the magnitude of these responses but even change their sign. For example, the output gap can increase after a positive cost push shock—a result which is still in line with the policy of achieving the minimum expected loss function.

Figure 1 illustrates the difference between the canonical New Keynesian model under RE (model consistent expectations) and the model where the public uses a VAR model to form its expectations. Figure 1 indicates that under the VAR model there is inertia both in the inflation and the output gap (pink) which does not exist under RE (blue). Moreover, the elasticities of the variables to shocks can be different from the RE case.<sup>11</sup>

One can erroneously interpret these impulses as the non-optimal monetary policy. Thus the CB can just raise the interest rate to push down both inflation and the

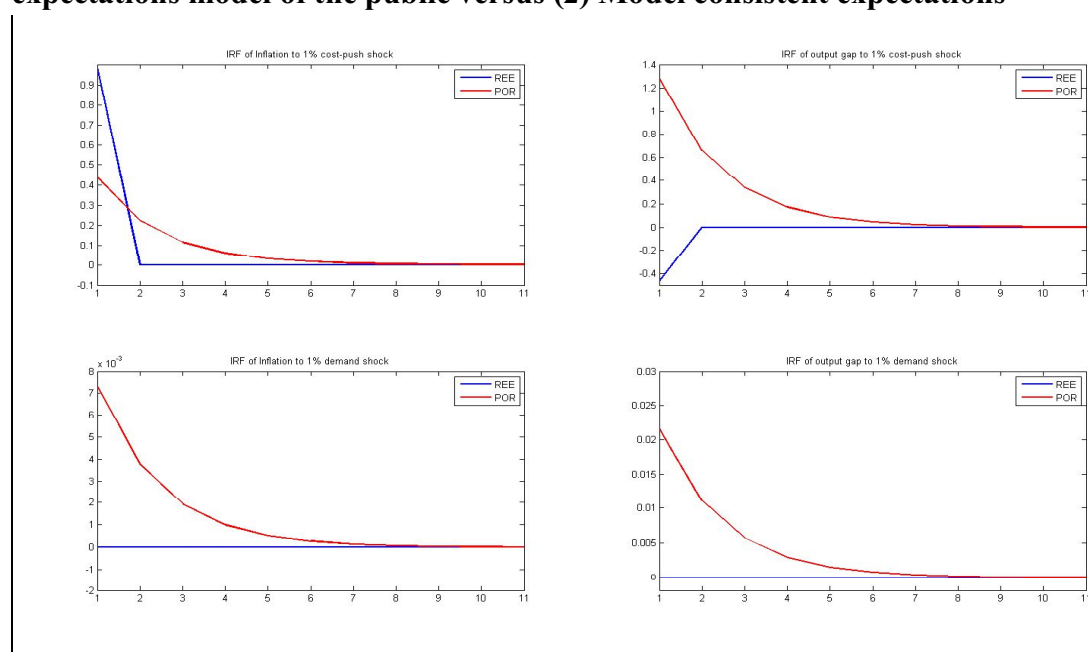
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<sup>10</sup> The Dynare and Matlab codes for derivation of the POR are available upon request.

<sup>11</sup> Specifically, we made 1,000 replications of Monte-Carlo simulations. In a third of them, the sign of the elasticity of the output gap to the cost-push shock was positive, as opposed to the RE case. Figure 1 depicts the average IRFs derived from the positive sign replications of the inflation and the output gap to cost-push and demand shocks (pink) versus the standard IRFs under RE (blue).

output gap. This is true under RE but not under the VAR expectation. In the latter case, raising the interest rate will increase the expectations for inflation and/or for the output gap, destabilizing the economy. That is, the decision to raise or to cut the interest rate eventually depends on the reaction of the public's expectations to the interest rate (and to other variables) and also on the relative weights of inflation and the output gap in the loss function of the CB. Returning to Figure 1, we assert that the CB cannot do better to reduce the expected loss function, given its model, its loss function and the public VAR model with respect to the specification and the parameters' values.

**Figure 1 – Impulse response function of inflation and output gap to cost-push shock and demand shock, in a combined New Keynesian model with (1) VAR expectations model of the public versus (2) Model consistent expectations**



### 3.1 Updating parameters in the VAR model of the public

We saw that the VAR model perceived by the public (henceforth PLM) is:<sup>8</sup>

$\tilde{z}_t = A\tilde{W}_{t-1} + u_t$ , where  $\tilde{z}_t = [\pi_t \ \Delta y_t]'$  and  $\tilde{W}_{t-1} = [\tilde{z}_{t-1} \ i_{t-1}]'$ , where  $A$  is a matrix of the parameters. We make two assumptions concerning how the public update the parameters:

- 1) **Learning public**—the public imposes priors on the parameters in the VAR model and then updates them each period using decreasing gain learning

algorithm (see Evans and Honkapohja, 2001)<sup>12</sup>. This learning process is given by:

$$A_t = A_{t-1} + t^{-1} R_t^{-1} W_{t-1} (\tilde{z}_t - A_{t-1} W_{t-1})$$

$$R_t = R_{t-1} + t^{-1} (W_{t-1} W_{t-1}' - R_{t-1})$$

where  $R_t$  is a matrix of second moments and  $t^{-1}$  is a decreasing gain.

### **Prior formation:**

In practice we don't know how the public forms its priors. Their priors can be derived from prejudices, estimations or both. In this paper we assume that agents form their priors concerning the parameters in the VAR model by exploiting the historical data in a pre-sample period. Specifically, they estimate the parameters by OLS. For simplicity we assume that the pre-sample period was generated by RE equilibrium. To make the whole analysis beneficial we want to create an error in the public's expectations relative to the RE. To do so we assume that the pre-sample period is small ( $T=50$ ), which leads to small-sample bias when agents estimate the parameters in the VAR model.

**2) Non-Learning public**—the public imposes priors on the parameters in the VAR model but does not update them over time. Under this presumption the parameters in each period are always equal to the priors, namely

$$A_t = A^{prior} \quad \forall t.$$

One can consider the case which is between (1) and (2) reflecting some friction in the learning process of the public. This friction could be justified, for example, by information costs or calculation time (see Branch, 2004), such that models are reestimated on low frequency basis (e.g., once a year) and not on a monthly or quarterly basis.

## ***3.2 Convergence of the economy under POR when public is learning***

Here we examine the convergence point of the economy. Specifically, the question is whether the economy will converge to the MSV solution under RE.

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<sup>12</sup> In practice, agents can use alternative learning algorithms: for example if agents believe that the economy experiences structural breaks they will use the constant gain (see Evans and Honkapohja, 2001). If agents are uncertain about the stochastic structure of the economy they can use the endogenous gain (see Ilek, 2013).

$$\pi_t = P_1 \varepsilon_t^\pi, \quad x_t = P_2 \varepsilon_t^\pi, \quad \text{where } P_1 = \frac{\lambda}{k^2 + \lambda}, \quad P_2 = -\frac{k}{k^2 + \lambda}.$$

For future needs, it is useful to rewrite the solution for the output gap in terms of the same variables as in the VAR model:

$$\pi_t = P_1 \varepsilon_t^\pi, \quad \Delta x_t = P_2 \varepsilon_t^\pi - \frac{P_2}{P_1} \pi_{t-1} \quad (\text{by exploiting } \varepsilon_{t-1}^\pi = \frac{1}{P_1} \pi_{t-1}). \quad (20)$$

To find the convergence point of the economy, we should examine the ALM of the economy in (18) with the PLM of the public in (10). First we should represent the ALM of the economy by the same variables as appear in the PLM of the public: (see Appendix A for details). This representation of the ALM is given by:

$$\begin{pmatrix} \pi_t \\ \Delta x_t \end{pmatrix} = \begin{bmatrix} -R_\pi^1 \\ -R_x^1 \end{bmatrix} \pi_{t-1} + \begin{bmatrix} \Psi_1^\pi - R_\pi \Psi_1 \\ \Psi_1^x - 1 - R_x \Psi_1 \end{bmatrix} \Delta x_{t-1} + \begin{bmatrix} \frac{\Psi_1^\pi}{H_1} \\ \frac{\Psi_1^x - 1}{H_1} \end{bmatrix} i_{t-1} + \begin{bmatrix} R_\pi \Psi_1 - R_\pi^2 \\ R_x \Psi_1 - R_x^2 \end{bmatrix} x_{t-1} + \Psi \varepsilon_t, \quad (21)$$

$$\text{where } R_\pi = \frac{\Psi_1^\pi H_2}{H_1} \Psi_2^{-1} = [R_\pi^1 \quad R_\pi^2] \text{ and } R_x = \frac{(\Psi_1^x - 1) H_2}{H_1} \Psi_2^{-1} = [R_x^1 \quad R_x^2].$$

Note that all the parameters in (21) are determined both by the structural parameters of the CB and by the parameters of the VAR model of the public.<sup>13</sup>

It is convenient to rewrite here the PLM of the public:

$$\begin{pmatrix} \pi_t \\ \Delta x_t \end{pmatrix} = \begin{bmatrix} A_1 \\ B_1 \end{bmatrix} \pi_{t-1} + \begin{bmatrix} A_2 \\ B_2 \end{bmatrix} \Delta x_{t-1} + \begin{bmatrix} A_3 \\ B_3 \end{bmatrix} i_{t-1} + u_t^z \quad (22)$$

The PLM of the public in (22) is underparameterized relative to (21) because it excludes  $x_{t-1}$ . However, the parameters in (21) are mutually dependent, that is, exclusion of one variable will automatically lead to exclusion of other variables, like a domino effect. For example, if  $\Psi_1 = 0$  then all parameters in the inflation dynamics in (21) will be equal to zero.

The fixed point of the differential equations in (23) below (for inflation and output gap, respectively) determines the convergence point of the economy:

$$\frac{d(A_1 \ A_2 \ A_3)}{d\tau} = \begin{bmatrix} -R_\pi^1 - A_1 \\ \Psi_1^\pi - R_\pi \Psi_1 - A_2 \\ \frac{\Psi_1^\pi}{H_1} - A_3 \end{bmatrix} \quad \text{and} \quad \frac{d(B_1 \ B_2 \ B_3)}{d\tau} = \begin{bmatrix} -R_x^1 - B_1 \\ \Psi_1^x - 1 - R_x \Psi_1 - B_2 \\ \frac{\Psi_1^x - 1}{H_1} - B_3 \end{bmatrix}, \quad (23)$$

<sup>13</sup> The definitions of all parameters are provided by Appendix A.



where  $R_\pi, R_x, \Psi_1, \Psi_2, H_1, H_2$  are implicit functions of the parameters  $A_j, B_j$  ( $i=1,2,3$ ) in the VAR. Although it is impossible here to derive the explicit analytical solution for (23), we find the convergence point numerically. The numerical results show that the economy converges to the MSV solution under RE shown in (20), such that the convergence point of the estimated parameters in the VAR is:

$A_j = 0, (j=1,2,3) B_1 = -\frac{P_2}{P_1}, B_2 = B_3 = 0$ . This implies that the parameters of the ALM

$$\text{in (18) converge to } \Psi_1 \rightarrow 0, \Psi_2 = \begin{bmatrix} \Psi_{21}^\pi & \Psi_{21}^x \\ \Psi_{22}^\pi & \Psi_{22}^x \end{bmatrix} \rightarrow \begin{bmatrix} P_1 & 0 \\ P_2 & 0 \end{bmatrix}.$$

### ***3.3 Convergence of the economy under POR when public is not learning***

Here we consider the case where the public uses its VAR model keeping the parameters fixed. This implies that the parameters  $\bar{\Psi}_1, \bar{\Psi}_2$  in the ALM of the economy in (18) will be fixed as well. Moreover, as long as the public believes that the lagged output growth should be included in the VAR model, the ALM of the economy will always depend on the lagged output gap, namely  $\bar{\Psi}_1 \neq 0$ . Since the public always uses its biased priors without updating them over time, the economy will systematically deviate from the RE equilibrium.

## **4. The OMP based on expectations-based policy rule**

In Section 3 we analyzed Case 2 where the CB considers the expectations formation of the public and then utilizes it in the derivation of the optimal policy rule. In this section we analyze Case 1 (the standard approach), where the CB utilizes the expectations-based policy rule shown in (8)<sup>14</sup>.

The key difference between this approach and our approach is that in the latter the policy rule is derived from the combined model, where the structural model of the CB comprises the specific expectations formation of the public. In the existing approach, in contrast, the policy rule is derived from the structural model of the

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<sup>14</sup> Evans and Honkapohja (2003) also examined the case where the CB policy rule is expressed in terms of shocks only (this specification is fully equivalent to the expectations-based rule under RE). They showed that under such a rule the economy will diverge with probability one, carrying enormous welfare losses to the economy. For that reason we also ignore this case in our analysis.

economy when the CB ignores the expectations formation of the public (or it assumes that they are model-consistent): and only at the second stage the interest rate reacts to the expectations, irrespective of their formation mechanism. This case was examined by Evans and Honkapohja (2006a, 2006b), Berardi and Duffy (2007) under discretion regime and by Berardi (2010) under commitment regime. Here we apply the discretionary monetary policy as in Evans and Honkapohja. We denote this policy rule as EHR (Evans and Honkapohja rule).

To derive the solution of the economy under the HER we first plug (8) into the state space of the economy in (3), (see Appendix B for proof). We get the following:

$$\begin{aligned}\pi_t &= (\beta - \kappa\alpha\mu)E_t\pi_{t+1} + \frac{\lambda}{k^2 + \lambda}\varepsilon_t^\pi, \\ x_t &= -\alpha\mu E_t\pi_{t+1} - \frac{k}{k^2 + \lambda}\varepsilon_t^\pi\end{aligned}\quad (24)$$

Note that at this stage we haven't yet specified the expectation formation of the public, therefore (24) holds for any type of expectations. Now we shall explicitly express the inflation expectations of the public by using VAR model.

#### ***4.1 The solution of the economy under EHR and VAR***

To derive the solution of the model in terms of past variables and current shocks we need solve for  $E_t\pi_{t+1}$  in (24). Appendix B shows that after solving for  $E_t\pi_{t+1}$  the solution of inflation and the output gap is given by:

$$z_t = \Phi_1 x_{t-1} + \Phi_2 \varepsilon_t, \quad (25)$$

where the specification of the parameters  $\Phi_1, \Phi_2$  is given in Appendix B.

Eq. (25) reveals that the specification of the solution under the EHR is equivalent to that under the POR (see eq. 19): both inflation and the output gap react to the lagged output gap and to current shocks. The main difference is reflected in the elasticities. The bottom line is that the solution in (19) minimizes the welfare losses of the economy, whereas the solution in (25) does not. We will see the implications of two solutions on the welfare loss in Section 5.

## ***4.2 Convergence of the economy with EHR when public is learning***

Once the CB utilizes the EHR and the public is learning, the economy will converge to the MSV solution under RE shown in (20). The analysis is the same as with the POR: we obtain the fixed point of the ALM and the PLM numerically and find that asymptotically the inflation dynamic will be implied by the RE equilibrium, such that  $A_j = 0$ , ( $j=1,2,3$ )  $B_1 = -\frac{P_2}{P_1}$ ,  $B_2 = B_3 = 0$ . This implies that the parameters of the

ALM in (25) converge to  $\Phi_1 \rightarrow 0, \Phi_2 = \begin{bmatrix} \Phi_{21}^\pi & \Phi_{21}^x \\ \Phi_{22}^\pi & \Phi_{22}^x \end{bmatrix} \rightarrow \begin{bmatrix} P_1 & 0 \\ P_2 & 0 \end{bmatrix}$ .

## ***4.3 Convergence of the economy with EHR when public is not learning***

Since the agents keep the parameters in the VAR constant over time, the equilibrium of the economy is given by:

$$z_t = \bar{\Phi}_1 x_{t-1} + \bar{\Phi}_2 \varepsilon_t,$$

where  $\bar{\Phi}_1, \bar{\Phi}_2$  are constant parameters implied by the public's VAR and the structural parameters of the CB's model. The economy will never be at the RE equilibrium.

## **5. Welfare analysis under POR and EHR**

In this section we analyze the consequences on the welfare of the economy of using POR versus EHR by the CB. In Sections 3 and 4, we showed that when agents are learning with decreasing gain learning algorithm, the economy will eventually converge to RE equilibrium both under the POR and EHR, therefore asymptotically there is no difference between these two rules with respect to the welfare of the economy. However, the dynamics toward the convergence point could be very different implying serious welfare consequences. These consequences will be even more severe when agents are not learning, because the difference in the solution under two rules will not disappear even asymptotically.

### **Explanation of the simulations:**

The analysis is made under Monte-Carlo simulation with 1,000 replications. In each replication there are two steps:

- 1) In each simulation, agents form their priors of the parameters in the VAR model based on the pre-sample data, as explained previously in Section 3.
- 2) The main sample consists of 1,000 periods. In each period new shocks hit the economy and the CB utilizes either the POR (by exploiting the expectations formation of the public) or the EHR (expectations-based policy rule). At the same time, agents either learn (update the VAR parameters as the new outcome of the economy is realized) or keep forming their expectations using the parameters fixed. This process continues from period  $T=1$  up to  $T=1,000$ .

### **Calibration**

We calibrated the structural parameters of the canonical CB's model following Woodford (2003b) and McCallum and Nelson (2004). Following Woodford (2003b), the discount factor  $\beta = 0.99$ ; the weight of the output gap in the loss function  $\lambda = 0.05$ ;<sup>15</sup> the elasticity of the output gap in the Philips curve  $k = 0.0238$ ; and the elasticity of the interest rate in the demand equation  $\alpha = 1/0.1571$ . Following McCallum and Nelson (2004) we calibrated the variances of the shocks to be  $\text{var}(\varepsilon_t^r) = 0.005^2$  in the Philips curve; and  $\text{var}(\varepsilon_t^x) = 0.020031^2$  in the demand equation.

## ***5.1 Results of Monte-Carlo simulations***

Table 2 presents the expected discounted loss under the POR and the EHR when agents use VAR.<sup>16</sup> The second and the third row in Table 2 consider the cases of learning and non-learning agents, respectively. The most outstanding result is presented in the last column: this is the ratio between the expected loss under the EHR and under the POR. The higher the ratio is (relative to 1), the more efficient the POR is relative to the EHR.

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<sup>15</sup> Woodford calibrated  $\lambda = 0.048$ .

<sup>16</sup> The welfare analysis with the VAR expressed in terms of output gap is even more in favor of the POR (results could be provided upon request)

**Table 2: The comparison of the expected loss function under POR versus the EHR (the VAR model of the public in terms of growth rate of the output)**

	Expected loss under POR, $EL_t(POR)$	Expected loss under EHR, $EL_t(EHR)$	Loss Ratio $\frac{EL_t(EHR)}{EL_t(POR)}$
Learning public	23.26	27.84	1.197
Non-learning public	22.76	35.65	1.578

The simulation results show that when agents are learning the loss ratio is 1.197, which means that the expected welfare loss under the EHR is higher by 19.7% than under the POR. When agents are not learning, the expected loss ratio is noticeably higher and stands on 1.578, which means that the expected welfare loss under the EHR is higher by 57.8% than under the POR.

The reason for the higher loss ratio in the second case is that the expectations of the public systematically deviate from the RE. Nevertheless, under the POR the CB can stabilize the economy more efficiently by anchoring the public's expectations. This is because the CB uses the expectations formation of the public as additional information in its optimization problem. Under the EHR, in contrast, the CB makes a systematic mistake in its policy rule because its specification is based on the RE assumption which never holds. This systematic mistake is reflected, as we saw in (24), in highly volatile inflation and output gap which boost the expected welfare loss.

In the case when the agents are learning, the dominance of the POR over the EHR is weaker. This is because the public updates the parameters in the VAR model over time and therefore its expectations become closer and closer to the RE over time. This implies that eventually the POR converges to the EHR.

## 6. Conclusions

In this paper we propose a novel approach for modeling and conducting optimal monetary policy when the CB and the public have different models of the economy. The proposed framework requires that the CB engage in a two stage process: first, the CB should learn the expectations formation of the public. Second, the CB should utilize the public's expectations formation into its own structural model to derive the optimal policy rule. In our paper we illustrated the benefits of implementing such a

policy in a parsimonious framework, where the CB has a simple structural New-Keynesian model and it fully observes the VAR model of the public. We show that when the CB exploits the expectations formation of the public instead of using the expectations-based rule, the expected loss is noticeably reduced. The magnitude of this reduction depends on whether the public is learning or not.

We are aware that modeling the expectations formation of the public is always subject to misspecification and errors, which could potentially reduce the advantage of the proposed framework of the monetary policy. Nevertheless, we believe that once a reasonable model of the expectations formation of the public is obtained, the CB can achieve its goals more efficiently.

There are several main challenges for future research: (1) To implement the proposed framework in richer models, and (2) to derive the optimal policy rule which exploits not only the expectations formation of the public with respect to the variables but also with respect to the parameters which vary over time as a result of the learning process of the public.

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## Appendix A

### The representation of the dynamics of the economy in the same terms as the VAR model of the public

The purpose of Appendix A is to express the ALM of the economy in the same terms as the PLM of the agents. We start with adding and subtracting  $x_{t-2}$  from the ALM in (19):

$$\begin{bmatrix} \pi_t \\ \Delta x_t \end{bmatrix} = \begin{bmatrix} \Psi_1^\pi \\ \Psi_1^x - 1 \end{bmatrix} \Delta x_{t-1} + \begin{bmatrix} \Psi_1^\pi \\ \Psi_1^x - 1 \end{bmatrix} x_{t-2} + \Psi_2 \varepsilon_t, \quad \Psi_2 = [\Psi_2^\pi \quad \Psi_2^x]' \quad (\text{A.1})$$

We start analyzing the second equation in (A.1):

$$\Delta x_t = (\Psi_1^x - 1) \Delta x_{t-1} + (\Psi_1^x - 1) x_{t-2} + \Psi_2^x \varepsilon_t \quad (\text{A.2})$$

From the POR  $i_t = H_1 x_{t-1} + H_2 \varepsilon_t$  we get  $x_{t-2} = \frac{1}{H_1} i_{t-1} - \frac{H_2}{H_1} \varepsilon_{t-1}$ , where  $H_1$  is a scalar and  $H_1$  is a vector.

$$\text{Substituting into (A.2): } \Delta x_t = (\Psi_1^x - 1) \Delta x_{t-1} + \frac{\Psi_1^x - 1}{H_1} i_{t-1} - \frac{(\Psi_1^x - 1) H_2}{H_1} \varepsilon_{t-1} + \Psi_2^x \varepsilon_t$$

Now we exploit the ALM of the economy  $z_t = \Psi_1 x_{t-1} + \Psi_2 \varepsilon_t$  to derive the vector of past shocks  $\varepsilon_{t-1}$ , that is  $\varepsilon_{t-1} = \Psi_2^{-1} z_{t-1} - \Psi_2^{-1} \Psi_1 x_{t-2}$ .

Now we add and subtract  $\Psi_2^{-1} \Psi_1 x_{t-1}$  to the previous equation:

$$\varepsilon_{t-1} = \Psi_2^{-1} z_{t-1} - \Psi_2^{-1} \Psi_1 x_{t-2} + \Psi_2^{-1} \Psi_1 x_{t-1} - \Psi_2^{-1} \Psi_1 x_{t-1} = \Psi_2^{-1} z_{t-1} + \Psi_2^{-1} \Psi_1 \Delta x_{t-1} - \Psi_2^{-1} \Psi_1 x_{t-1} \quad (\text{A.3})$$

Now we have:

$$\Delta x_t = (\Psi_1^x - 1) \Delta x_{t-1} + \frac{\Psi_1^x - 1}{H_1} i_{t-1} - \frac{(\Psi_1^x - 1) H_2}{H_1} [\Psi_2^{-1} z_{t-1} + \Psi_2^{-1} \Psi_1 \Delta x_{t-1} - \Psi_2^{-1} \Psi_1 x_{t-1}] + \Psi_2^x \varepsilon_t$$

After some algebra on the previous equation:

$$\Delta x_t = (\Psi_1^x - 1) \Delta x_{t-1} + \frac{\Psi_1^x - 1}{H_1} i_{t-1} - \frac{(\Psi_1^x - 1) H_2}{H_1} \Psi_2^{-1} z_{t-1} - \frac{(\Psi_1^x - 1) H_2}{H_1} \Psi_2^{-1} \Psi_1 \Delta x_{t-1} + \frac{(\Psi_1^x - 1) H_2}{H_1} \Psi_2^{-1} \Psi_1 x_{t-1} + \Psi_2^x \varepsilon_t$$

Continue with simplifications:

$$\Delta x_t = [\Psi_1^x - 1 - \frac{(\Psi_1^x - 1) H_2}{H_1} \Psi_2^{-1} \Psi_1] \Delta x_{t-1} + \frac{\Psi_1^x - 1}{H_1} i_{t-1} - \frac{(\Psi_1^x - 1) H_2}{H_1} \Psi_2^{-1} \begin{bmatrix} \pi_{t-1} \\ x_{t-1} \end{bmatrix} + \frac{(\Psi_1^x - 1) H_2}{H_1} \Psi_2^{-1} \Psi_1 x_{t-1} + \Psi_2^x \varepsilon_t$$

Define  $R_x = \frac{(\Psi_1^x - 1) H_2}{H_1} \Psi_2^{-1} = [R_x^1 \quad R_x^2]$ , so we have:

$$\Delta x_t = [\Psi_1^x - 1 - R_x \Psi_1] \Delta x_{t-1} + \frac{\Psi_1^x - 1}{H_1} i_{t-1} - [R_x^1 \quad R_x^2] \begin{bmatrix} \pi_{t-1} \\ x_{t-1} \end{bmatrix} + R_x \Psi_1 x_{t-1} + \Psi_2^x \varepsilon_t$$

Finally:

$$\Delta x_t = [\Psi_1^x - 1 - R_x \Psi_1] \Delta x_{t-1} + \frac{\Psi_1^x - 1}{H_1} i_{t-1} - R_x^1 \pi_{t-1} + (R_x \Psi_1 - R_x^2) x_{t-1} + \Psi_2^x \varepsilon_t \quad (\text{A.4})$$

Now we consider the inflation equation from (A.1) and in the same manner:

$$\pi_t = [\Psi_1^\pi - R_\pi \Psi_1] \Delta x_{t-1} + \frac{\Psi_1^\pi}{H_1} i_{t-1} - R_\pi^1 \pi_{t-1} + (R_\pi \Psi_1 - R_\pi^2) x_{t-1} + \Psi_2^\pi \varepsilon_t \quad (\text{A.5})$$

where  $R_\pi = \frac{\Psi_1^\pi H_2}{H_1} \Psi_2^{-1} = [R_\pi^1 \quad R_\pi^2]$ .

In compact form, (A.4–A.5) could be rewritten as:

$$\begin{pmatrix} \pi_t \\ \Delta x_t \end{pmatrix} = \begin{bmatrix} -R_\pi^1 \\ -R_x^1 \end{bmatrix} \pi_{t-1} + \begin{bmatrix} \Psi_1^\pi - R_\pi \Psi_1 \\ \Psi_1^x - 1 - R_x \Psi_1 \end{bmatrix} \Delta x_{t-1} + \begin{bmatrix} \frac{\Psi_1^\pi}{H_1} \\ \frac{\Psi_1^x - 1}{H_1} \end{bmatrix} i_{t-1} + \begin{bmatrix} R_\pi \Psi_1 - R_\pi^2 \\ R_x \Psi_1 - R_x^2 \end{bmatrix} x_{t-1} + \Psi \varepsilon_t.$$

## Appendix B

### The solution of the economy under the EHR and the VAR model of the public

The purpose of Appendix B is to derive the solution of the economy when the public forms its expectations using the VAR model and the CB utilizes the EHR.

After plugging the EHR in (8) into the state space of the economy in (3), we get:

$$\begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} \alpha k + \beta & k \\ \alpha & 1 \end{bmatrix} E_t \begin{bmatrix} \pi_{t+1} \\ x_{t+1} \end{bmatrix} + \begin{bmatrix} -\kappa \alpha \\ -\alpha \end{bmatrix} \left[ (1 + \mu) E_t \pi_{t+1} + \frac{1}{\alpha} E_t x_{t+1} + \frac{k}{\alpha(k^2 + \lambda)} \varepsilon_t^\pi + \frac{1}{\alpha} \varepsilon_t^x \right] + \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} \begin{pmatrix} \varepsilon_t^\pi \\ \varepsilon_t^x \end{pmatrix}$$

where  $\mu = \frac{\beta k}{\alpha(k^2 + \lambda)}$ .

After some algebra we get:

$$\begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} \alpha k + \beta & k \\ \alpha & 1 \end{bmatrix} E_t \begin{bmatrix} \pi_{t+1} \\ x_{t+1} \end{bmatrix} + \begin{bmatrix} -\kappa \alpha (1 + \mu) E_t \pi_{t+1} - k E_t x_{t+1} - \frac{k^2}{k^2 + \lambda} \varepsilon_t^\pi - k \varepsilon_t^x \\ -\alpha (1 + \mu) E_t \pi_{t+1} - E_t x_{t+1} - \frac{k}{k^2 + \lambda} \varepsilon_t^\pi - \varepsilon_t^x \end{bmatrix} + \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} \begin{pmatrix} \varepsilon_t^\pi \\ \varepsilon_t^x \end{pmatrix}$$

Continuing with simplifications:

$$\begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} (\alpha k + \beta) E_t \pi_{t+1} + k E_t x_{t+1} - \kappa \alpha (1 + \mu) E_t \pi_{t+1} - k E_t x_{t+1} - \frac{k^2}{k^2 + \lambda} \varepsilon_t^\pi - k \varepsilon_t^x + \varepsilon_t^\pi + k \varepsilon_t^x \\ \alpha E_t \pi_{t+1} + E_t x_{t+1} - \alpha (1 + \mu) E_t \pi_{t+1} - E_t x_{t+1} - \frac{k}{k^2 + \lambda} \varepsilon_t^\pi - \varepsilon_t^x + \varepsilon_t^x \end{bmatrix}$$

After rearrangement we get the dynamics of the economy implied by the EHR.

$$\begin{aligned}\pi_t &= (\beta - \kappa\alpha\mu)E_t\pi_{t+1} + \frac{\lambda}{k^2 + \lambda}\varepsilon_t^\pi, \\ x_t &= -\alpha\mu E_t\pi_{t+1} - \frac{k}{k^2 + \lambda}\varepsilon_t^\pi\end{aligned}\tag{B.1}$$

Notice that (B.1) holds for any type of expectations. In order to represent (B.1) in reduced form solution, we need first to express  $E_t\pi_{t+1}$  in terms of current shocks and past variables. To do so we substitute (B.1) into the VAR model of the public (taking the VAR one period ahead and applying conditional expectations for  $t+1$ ):

$$\begin{aligned}E_t\pi_{t+1} &= A_1[(\beta - \kappa\alpha\mu)E_t\pi_{t+1} + \frac{\lambda}{k^2 + \lambda}\varepsilon_t^\pi] + A_2[-\alpha\mu E_t\pi_{t+1} - \frac{k}{k^2 + \lambda}\varepsilon_t^\pi] \\ &\quad - A_2x_{t-1} + A_3[(1 + \mu)E_t\pi_{t+1} + \frac{1}{\alpha}E_tx_{t+1} + \frac{k}{\alpha(k^2 + \lambda)}\varepsilon_t^\pi + \frac{1}{\alpha}\varepsilon_t^x] \\ E_tx_{t+1} &= B_1[(\beta - \kappa\alpha\mu)E_t\pi_{t+1} + \frac{\lambda}{k^2 + \lambda}\varepsilon_t^\pi] + (1 + B_2)[-\alpha\mu E_t\pi_{t+1} - \frac{k}{k^2 + \lambda}\varepsilon_t^\pi] \\ &\quad - B_2x_{t-1} + B_3[(1 + \mu)E_t\pi_{t+1} + \frac{1}{\alpha}E_tx_{t+1} + \frac{k}{\alpha(k^2 + \lambda)}\varepsilon_t^\pi + \frac{1}{\alpha}\varepsilon_t^x]\end{aligned}$$

After some simplifications:

$$\begin{aligned}E_t\pi_{t+1} &= A_1(\beta - \kappa\alpha\mu)E_t\pi_{t+1} + A_1\frac{\lambda}{k^2 + \lambda}\varepsilon_t^\pi - A_2\alpha\mu E_t\pi_{t+1} - A_2\frac{k}{k^2 + \lambda}\varepsilon_t^\pi \\ &\quad - A_2x_{t-1} + A_3(1 + \mu)E_t\pi_{t+1} + A_3\frac{1}{\alpha}E_tx_{t+1} + A_3\frac{k}{\alpha(k^2 + \lambda)}\varepsilon_t^\pi + A_3\frac{1}{\alpha}\varepsilon_t^x \\ E_tx_{t+1} &= B_1(\beta - \kappa\alpha\mu)E_t\pi_{t+1} + B_1\frac{\lambda}{k^2 + \lambda}\varepsilon_t^\pi - (1 + B_2)\alpha\mu E_t\pi_{t+1} - (1 + B_2)\frac{k}{k^2 + \lambda}\varepsilon_t^\pi \\ &\quad - B_2x_{t-1} + B_3(1 + \mu)E_t\pi_{t+1} + B_3\frac{1}{\alpha}E_tx_{t+1} + B_3\frac{k}{\alpha(k^2 + \lambda)}\varepsilon_t^\pi + B_3\frac{1}{\alpha}\varepsilon_t^x\end{aligned}$$

Continue with the simplifications:

$$\begin{aligned}(1 - A_1(\beta - \kappa\alpha\mu) + A_2\alpha\mu - A_3(1 + \mu))E_t\pi_{t+1} - A_3\frac{1}{\alpha}E_tx_{t+1} &= \\ [A_1\frac{\lambda}{k^2 + \lambda} - A_2\frac{k}{k^2 + \lambda} + A_3\frac{k}{\alpha(k^2 + \lambda)}]\varepsilon_t^\pi + A_3\frac{1}{\alpha}\varepsilon_t^x - A_2x_{t-1} & \\ [-B_1(\beta - \kappa\alpha\mu) + (1 + B_2)\alpha\mu - B_3(1 + \mu)]E_t\pi_{t+1} + (1 - B_3\frac{1}{\alpha})E_tx_{t+1} &= \\ [B_1\frac{\lambda}{k^2 + \lambda} - (1 + B_2)\frac{k}{k^2 + \lambda} + B_3\frac{k}{\alpha(k^2 + \lambda)}]\varepsilon_t^\pi + B_3\frac{1}{\alpha}\varepsilon_t^x - B_2x_{t-1} &\end{aligned}$$

Now the expectations could be expressed in the compact matrix form:

$$G_1E_t\begin{bmatrix}\pi_{t+1} \\ x_{t+1}\end{bmatrix} = G_2\begin{bmatrix}\varepsilon_t^\pi \\ \varepsilon_t^x\end{bmatrix} + G_3x_{t-1},$$

where:

$$G_1 = \begin{bmatrix} 1 - A_1(\beta - \kappa\alpha\mu) + A_2\alpha\mu - A_3(1 + \mu) & -A_3\frac{1}{\alpha} \\ -B_1(\beta - \kappa\alpha\mu) + (1 + B_2)\alpha\mu - B_3(1 + \mu) & 1 - B_3\frac{1}{\alpha} \end{bmatrix}$$

$$G_2 = \begin{bmatrix} A_1\frac{\lambda}{k^2 + \lambda} - A_2\frac{k}{k^2 + \lambda} + A_3\frac{k}{\alpha(k^2 + \lambda)} & A_3\frac{1}{\alpha} \\ B_1\frac{\lambda}{k^2 + \lambda} - (1 + B_2)\frac{k}{k^2 + \lambda} + B_3\frac{k}{\alpha(k^2 + \lambda)} & B_3\frac{1}{\alpha} \end{bmatrix}$$

$$G_3 = \begin{bmatrix} -A_2 \\ -B_2 \end{bmatrix}$$

Solving for  $E_t\pi_{t+1}$  and  $E_t x_{t+1}$ :

$$E_t \begin{bmatrix} \pi_{t+1} \\ x_{t+1} \end{bmatrix} = L \begin{bmatrix} \varepsilon_t^\pi \\ \varepsilon_t^x \end{bmatrix} + Kx_{t-1}, \text{ where } L = G_1^{-1}G_2, K = G_1^{-1}G_3. \quad (\text{B.2})$$

$$\text{For convenience we partition the matrices as } L = \begin{bmatrix} L_\pi^{\varepsilon(\pi)} & L_\pi^{\varepsilon(x)} \\ L_x^{\varepsilon(\pi)} & L_x^{\varepsilon(x)} \end{bmatrix} \text{ and } K = \begin{bmatrix} K_\pi \\ K_x \end{bmatrix}. \quad (\text{B.3})$$

Since the expectations for the output gap do not play any role in the dynamics of the economy (see B.1), we focus only on the inflation expectations. From (B.2) applying (B.3) we get:

$$E_t\pi_{t+1} = L_\pi^{\varepsilon(\pi)}\varepsilon_t^\pi + L_\pi^{\varepsilon(x)}\varepsilon_t^x + K_\pi x_{t-1}. \quad (\text{B.4})$$

Substituting (B.4) into (B.1) we derive the solution of the economy:

$$\pi_t = (\beta - \kappa\alpha\mu)[L_\pi^{\varepsilon(\pi)}\varepsilon_t^\pi + L_\pi^{\varepsilon(x)}\varepsilon_t^x + K_\pi x_{t-1}] + \frac{\lambda}{k^2 + \lambda}\varepsilon_t^\pi,$$

$$x_t = -\alpha\mu[L_\pi^{\varepsilon(\pi)}\varepsilon_t^\pi + L_\pi^{\varepsilon(x)}\varepsilon_t^x + K_\pi x_{t-1}] - \frac{k}{k^2 + \lambda}\varepsilon_t^\pi$$

or

$$\pi_t = [(\beta - \kappa\alpha\mu)L_\pi^{\varepsilon(\pi)} + \frac{\lambda}{k^2 + \lambda}]\varepsilon_t^\pi + (\beta - \kappa\alpha\mu)L_\pi^{\varepsilon(x)}\varepsilon_t^x + (\beta - \kappa\alpha\mu)K_\pi x_{t-1},$$

$$x_t = -[\alpha\mu L_\pi^{\varepsilon(\pi)} + \frac{k}{k^2 + \lambda}]\varepsilon_t^\pi - \alpha\mu L_\pi^{\varepsilon(x)}\varepsilon_t^x - \alpha\mu K_\pi x_{t-1}$$

In matrix form:

$$z_t = \Phi_1 x_{t-1} + \Phi_2 \varepsilon_t, \quad (\text{B.5})$$

$$\text{where } \Phi_1 = \begin{bmatrix} (\beta - \kappa\alpha\mu)K_\pi \\ -\alpha\mu K_\pi \end{bmatrix}, \Phi_2 = \begin{bmatrix} (\beta - \kappa\alpha\mu)L_\pi^{\varepsilon(\pi)} + \frac{\lambda}{k^2 + \lambda} & (\beta - \kappa\alpha\mu)L_\pi^{\varepsilon(x)} \\ -[\alpha\mu L_\pi^{\varepsilon(\pi)} + \frac{k}{k^2 + \lambda}] & -\alpha\mu L_\pi^{\varepsilon(x)} \end{bmatrix},$$

$$\varepsilon_t = \begin{bmatrix} \varepsilon_t^\pi \\ \varepsilon_t^x \end{bmatrix}.$$

## **Appendix C**

### **Survey of professional forecasters in Israel**

In order to support our assumption of discrepancy in models between the CB and the public, we conducted a survey among ten PF in Israel. These PF provide to the Bank of Israel their projections for main economic variables on a monthly basis. They also attend regular meetings (once a quarter) with the Governor and the Bank's Monetary Committee to share their opinions concerning the state of the Israeli and the foreign economies.

We present the survey questionnaire for the PF and briefly explain the main results.

### **Questionnaire for professional forecasters in Israel**

**Explanation:** This short questionnaire is designated for the research needs of Alex Ilek and Guy Segal from the Research Department, Bank of Israel.

The questionnaire is completely anonymous.

For the questionnaire to be effective, please answer rightfully all listed questions.

Thank you for your cooperation.

**Question 1:** Which main macroeconomic variables do you forecast?

**Question 2:** Are your forecasts derived from a model? If yes, is it structural or statistical?

**Question 3:** If you have a model, does it contain the variable "output gap"? If yes, how it is defined and measured?

**Question 4:** How much do you rely on projections of the Israeli Central Bank in your own projections? (1-not at all, 5- very much). If you don't rely **very much** on the CB projections, please explain why?

**Question 5:** How much are your forecasts typically different from the CB's forecasts? (1 – not different at all, 5 – very much different)

**Question 6:** Have you ever used one of the models posted on the website of the Israeli Central Bank? If yes, which model have you used ? If not, please explain why.

#### **The main results:**

The most striking result is that none of the PF use currently, or had ever used in the past, the models of the BOI. Some PF pointed out that they would have liked to use

the BOI's models but these models are very complicated and hardly operational. One PF criticized these models. The PF rely on the projections of the BOI to a moderate extent. The PF forecasts differ from BOI forecasts to a moderate extent. Only one PF uses the "output gap" in his model, whereas the majority use the growth rate of output.