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Estimating the NAIRU using both the Phillips and the Beveridge curves[1]

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אמידת שיעור האבטלה הטבעי (NAIRU) לישראל תוך שימוש בעקומת פיליפס ובעקומת בווירז'

אלכס אילק ודוד אלקיים

תקציר

בעבודה זו אנו אומדים סדרה עיתית של שיעור האבטלה המבני (NAIRU) לישראל לתקופה 1998:Q1 עד 2012:Q4. זאת תוך שימוש הן בעקומת פיליפס (המתארת את הקשר השלילי שבין שיעור האבטלה וקצב האינפלציה) והן בעקומת בווירז' (המתארת את הקשר השלילי שבין שיעור האבטלה ושיעור המשרות הפנויות). אנו מראים שלצורך זיהוי "נכון" של התרומה של כל אחד מהקשרים לאמידת שיעור האבטלה המבני יש להשית מגבלות על מידת התנודתיות של משתנה זה, הנגזרות משיקולים של היגיון כלכלי. בהינתן מגבלות אלה נמצא כי לכל אחד מהקשרים יש תרומה משמעותית לאמידת שיעור האבטלה הטבעי. עוד נמצא כי החל משנת 2003 ועד 2012 חלה ירידה מתמדת של שיעור האבטלה הטבעי בישראל ואנו סבורים שניתן ליחס לפחות את חלקה למדיניות עידוד התעסוקה שננקטה על ידי הממשלה החל משנת 2002.

Estimating the NAIRU using both the Phillips and the Beveridge curves

David Elkayam and Alex Ilek

Abstract

We use both the Phillips curve (the relationship between inflation and unemployment) and the Beveridge curve (the relationship between unemployment and job vacancies) to estimate a time-varying NAIRU for Israel in the period from 1998:Q1 to 2012:Q4. After applying prior restrictions on NAIRU volatility, based on economic reasoning, we found that both relationships make a noticeable contribution to the identification of the NAIRU. The estimation results show a prolonged decline of the NAIRU from 2003 until 2012, which could be attributed to government policy that was operated since 2002, which aimed to increase the efficiency of the labor market.

I. Introduction

The Non-Accelerating Inflation Rate of Unemployment (the NAIRU, also frequently referred to as the natural rate of unemployment) is an important variable in the conduct of monetary and fiscal policies, the main idea being that changes in the NAIRU represent changes in the efficiency of the matching process in the labor market. The NAIRU is an unobserved variable that should be evaluated—preferably by the use of structural economic relationships, or at least by other statistical methods.

In a majority of studies (for example, Gordon, 1997; Laubach, 2001; Turner, *et al.*, 2001; Staiger, *et al*., 1997 and others) the NAIRU is obtained by exploiting the information available in an expectations-augmented Phillips curve (hereinafter, "PC"). That method involves estimating the relationship between the inflation rate, its lags, supply shock variables and the gap between the actual and the natural rate of unemployment (referred to below as the unemployment gap). In this framework the NAIRU, being a latent variable, is estimated by using a Kalman filter.

A main issue in the estimation of the NAIRU using the PC is how much high-frequency volatility of the estimated NAIRU should be assumed. Gordon (1997) discusses this issue at length, claiming that since the NAIRU is determined by the microeconomic structure and behavior of the economy, it should change slowly.[1] The NAIRU is the unemployment rate that would prevail in a situation of stable inflation and the absence of supply shocks. In such a situation we should not expect it to fluctuate too much from quarter to quarter. Based on this logic he proposed to use smoothness criteria: "the NAIRU could move around as much as

[1] It is reasonable to assume that Gordon is referring to normal times. Structural changes (such as large immigration or policy changes) can cause a big change in the NAIRU.

it likes, subject to the qualification that sharp quarter-to-quarter zig-zags are ruled out".

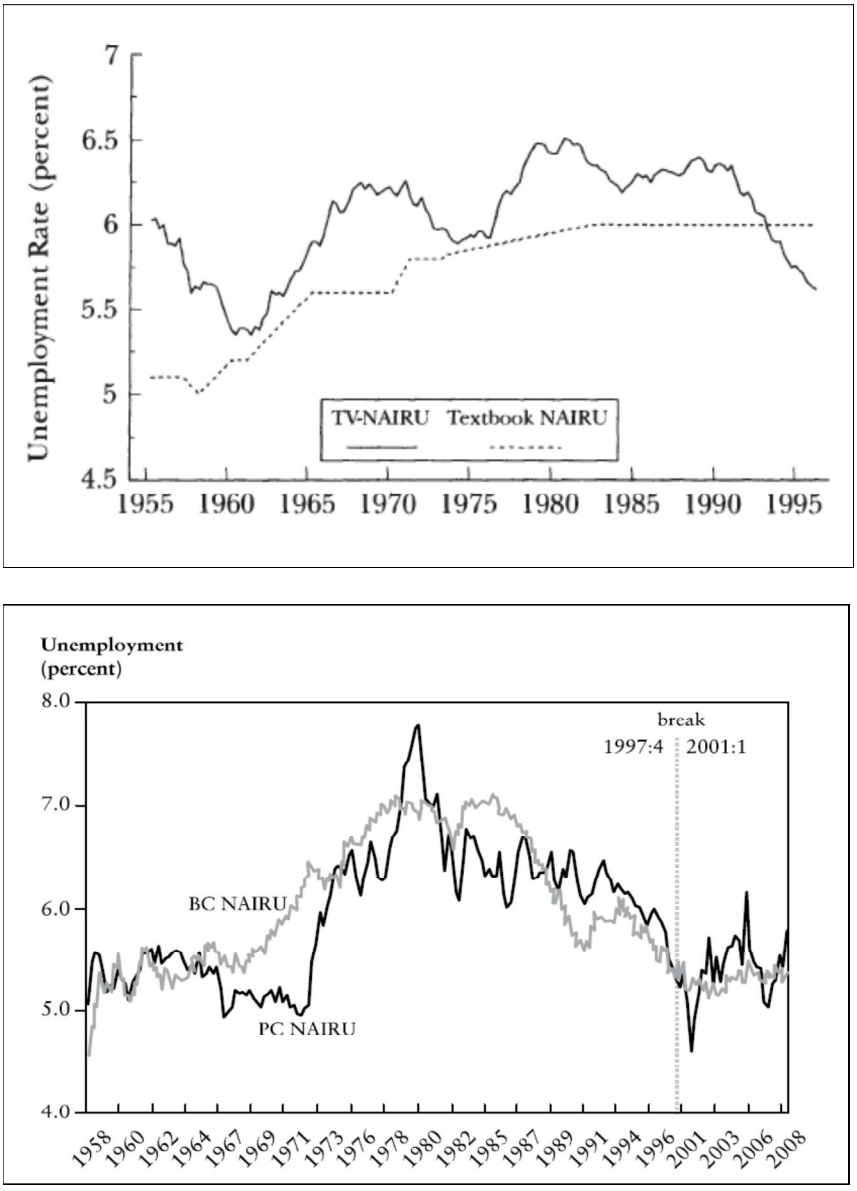
According to Dickens (2009) an additional source of information concerning the variation of the NAIRU is the Beveridge curve (hereinafter, "BC"), which postulates a negative relationship between the unemployment rate and the job vacancy rate. Dickens (2009) developed a framework in which changes in the NAIRU cause shifts of the Beveridge curve. That is, he concludes that such changes "are reflected not just in the relationship between inflation and unemployment, but also in the relationship between unemployment and job vacancies". Based on this idea, Dickens estimated the PC jointly with the BC and examined the relative contribution of each equation to the identification of NAIRU. He found that the BC is the main contributor to NAIRU identification, whereas the relative contribution of the PC was close to nil.

As we shall show later, the reason for this result is that Dickens, unlike Gordon (1997), did not use prior restrictions to limit the volatility of the estimated NAIRU. In addition, the BC, as specified by Dickens, imposes no restrictions (in contrast to the PC) on the dynamics of the NAIRU. The result, as we shall detail later on, is that the NAIRU estimated by Dickens is characterized by a very large high-frequency volatility that absorbs almost all the residual variation in the BC equation.

To shed light on the plausibility of Dickens's results, let us first compare the NAIRU estimated by Dickens (2009) (see BC-NAIRU on Figure 1, sample periods 1958–2008) with that estimated by Gordon (1997) (see TV NAIRU in Fig. 1, sample periods 1955–96). A large common sample of 1958–96 enables us to take a bird's eye view on the differences between these two series. The difference in the evolutions of the NAIRU is very noticeable: in Dickens (2009), the NAIRU is much more volatile than in Gordon (1997), and it is characterized

by frequent reversals ("zig-zags" in Gordon's words). In other words, the NAIRU from Dickens does not fulfill the smoothness criteria proposed by Gordon. As we shall show later, if Gordon's criteria is accepted, the contribution of the BC in Dickens (2009) is biased upward and that of the PC is biased downward.

Figure 1: The estimated NAIRU from Gordon (1997) (the upper plot) and from Dickens (2009) (the lower plot), the common sample is 1958-96



Source: Gordon (1997) and Dickens (2009).

To examine the evolution of the NAIRU and the relative contribution of the PC and the BC to its identification, under different assumptions concerning its volatility, we estimated the model for the Israeli economy under three versions. In each version we estimate the NAIRU and identify the contributions of the PC and the BC to its identification.

In the first version we exploit only the PC, by imposing the NAIRU volatility to be consistent with Gordon's smoothness criteria (as we shall explain later). In the second version we add the BC as in Dickens (2009), where no restrictions on the NAIRU volatility are imposed. We only imposed the restriction that the NAIRU coincide with the unemployment rate in the long run (namely, we assume a stationary unemployment gap with zero mean). We obtained generally reasonable results for the estimates of the parameters of both the PC and the BC; however, in our data, as in Dickens, the estimated NAIRU captured almost all the variations in the BC innovations, significantly increasing the contribution of the BC at the expense of the PC. Thus, without imposing prior smoothness restrictions on the NAIRU volatility we obtain results in our data which are similar those of Dickens (2009): a) the NAIRU is very volatile, is characterized by "zig-zags" and is close to the actual unemployment rate. b) The relative contribution of the BC to the NAIRU identification is appreciably greater than that of the PC.

In the third version, like in the second version, we exploit both the PC and the BC, but now we impose restrictions on the NAIRU volatility. Under this version we found that both the PC and the BC contributed meaningfully to the NAIRU identification.

The paper is organized as follows: Section II presents the model, Section III presents the estimation results of the model, Section IV presents the contributions of the signal equations and Section V concludes.[2]

II. Model

The Phillips Curve

Following the relevant literature[3], we start with a Phillips curve of the form:

$$\pi_t - \pi_t^e = \sum_{i=1}^{k\pi} \alpha_i (\pi_{t-i} - \pi_{t-i}^e) - \sum_{i=1}^{ku} \beta_i (u_{t-i} - un_{t-i}) + \sum_{i=0}^{kz} \gamma_i \Delta z_{t-i} + e_t^{pc} \quad (1)$$

where π_t is actual inflation in quarter t, π_t^e denotes expected inflation from quarter $t-1$ to quarter t, u_t is the unemployment rate, un_t is the NAIRU, Δz_t represents supply shocks and e_t^{pc} is an i.i.d. shock with zero mean and standard deviation σ_π. According to this equation, unexpected inflation is related to its lags and to lags of the unemployment gap and supply shocks. The equation contains two unobserved variables, π_t^e and un_t. With regards to π_t^e we shall follow the common practice of assuming $\pi_t^e = \pi_{t-1}$ which amounts to using $\Delta\pi_t$ as a proxy for unexpected inflation. With regard to un_t we shall apply the Kalman filter in a manner detailed below.

As a supply shock variable we used the change in the relative price of imported goods. More specifically, define $\Delta z_t = \Delta rer_t = \Delta e_t + \Delta pim_t - \pi_{t-1}$, where Δe_t represents the rate of change in the shekel/dollar exchange rate, Δpim_t stands for the rate of change in the world's price (in dollar terms) of Israeli imports and π_t is the inflation rate in Israel.

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[2] Some technical aspects, to which we shall refer later, are left to a technical appendix that can be provided upon request.

[3] See, for example, Laubach (2001) and the reference therein.

Essentially, the above equation is a reduced form equation for unexpected inflation. In the estimation of the system (that will be presented in the following subsections) we specified a Phillips curve in the form of Equation 1[4] where on the right hand side we allowed up to eight lags of the changes in inflation and four lags of each of the other right hand side variables. In addition, we allowed the current value of Δrer_t. After some experiments we obtained the following empirical specification of the PC that characterizes the Israeli data:

$$\Delta\pi_t = \sum_{i=1}^{6}\alpha_i\Delta\pi_{t-i} - \beta_1\Delta ugap_{t-1} - \beta_2 ugap_{t-3} + \gamma_1\Delta rer_t + \gamma_2\Delta rer^*_{t-1} + e^{pc}_t \qquad (2)$$

where $\Delta rer^*_{t-1} = 0.5\sum_{i=1}^{2}[\Delta e_{t-i} + \Delta pim_{t-i} - \pi_{t-i-1}]$ is a moving average of the changes in the real exchange rate during the two previous quarters. Note that Δrer_t contains contemporaneous changes in the nominal exchange rate and in the world price of imported goods. By including Δrer_t, we actually assume that shocks to inflation at period t do not affect the exchange rate at time t.[5] The world price of imported goods is clearly exogenous to the small domestic economy. Note also that Equation 2 contains not only the unemployment gap (at lag three) but also its changes from period $t-1$ to $t-2$.The change in the unemployment gap represents a possible nonlinearity in the effect of unemployment on inflation. A

[4] We use inflation of the CPI (π) excluding housing services, fruit and vegetables (f&v). This is because the f&v component is known for high volatility and irregularity. As for housing services, in 1999 the CBS changed the method of deriving this data (until 1999 they used house prices as a proxy, but from 1999 they started to use data on rents). Furthermore, until 2007 the housing component was almost fully linked to the shekel-dollar exchange rate. From 2007 onward, we observe a noticeable disruption in this linkage. The fact that the proxy for inflation expectations that we use (from the capital market) includes housing, f&v, but the inflation data does not, implies that in fact the inflation expectations included in the model have measurement errors. This increases the variance of the residuals in the inflation equation but does not bias the parameter estimates (note that we impose unit elasticity on the inflation expectations).

[5] To confirm this assumption we conducted a Durbin-Wu-Hausman test for the exogeneity of Δe_t . As additional instruments we used the current and four lags of the Libor rate and four lags of the Bank of Israel interest rate. The null hypothesis (the exchange rate is exogenous to inflation) was not rejected for significance levels up to 19%.

rapid decline in the unemployment rate may put upward short run pressure on inflation even at high levels of unemployment. This effect is sometimes referred to a "speed limit" effect.[6]

The Beveridge Curve

Dickens (2009) suggests a framework in which the NAIRU can be derived from the Beveridge curve (BC) as well. According to his analysis, the NAIRU is the main driver of the relationship between the unemployment rate and the vacancy rate.

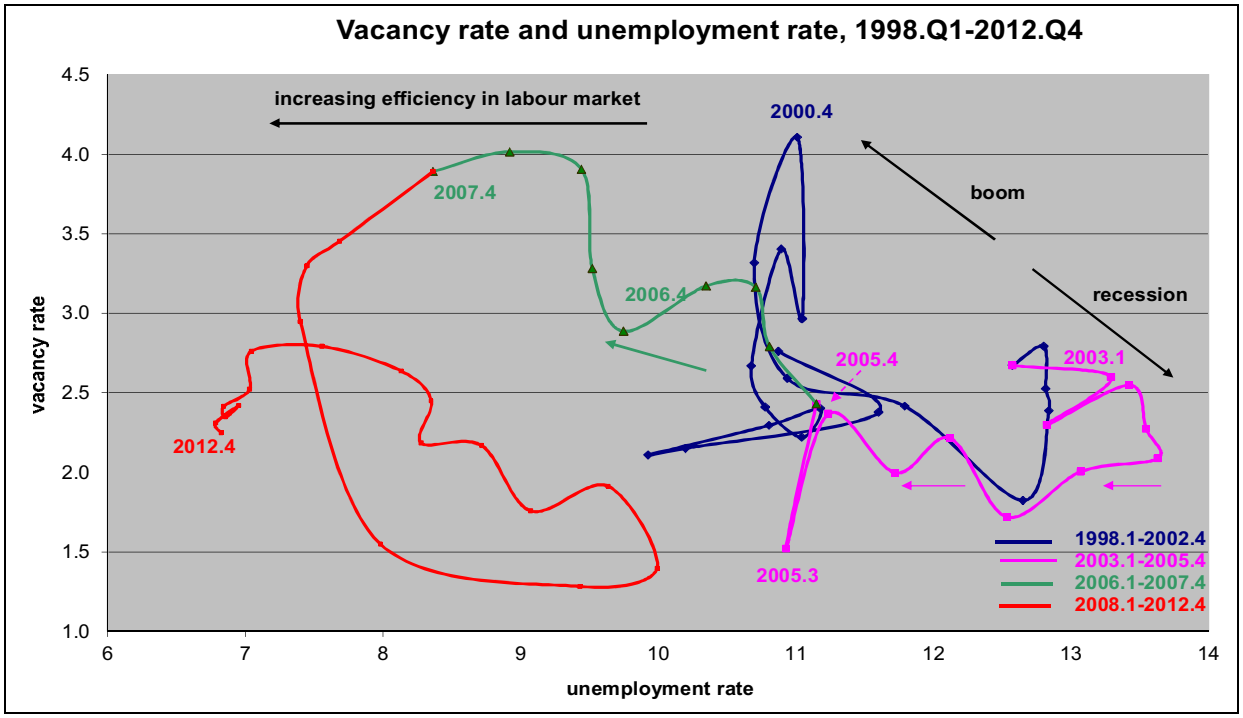
To get a sense of the kind of relationship between the unemployment rate and the vacancy rate in our data we present in Figure 2 a scatter plot of quarterly data on those two variables in Israel from 1998:Q1 to 2012:Q4. During the period 1998:Q1 to 2005:Q4 we can clearly observe a negative relationship between unemployment and vacancies (a "Beveridge curve")—the combined blue and pink lines in Fig. 2.[7] During 2006:Q1 to 2007:Q4 we can observe a shift of that "curve", the green line, and from 2008:Q1 to 2012:Q4 again we can see a "curve", the red line. The large shift of the curve that took place in 2006–07 can be attributed to an increase in efficiency in the labor market.[8]

[6] See for example Turner, *et al*. (2001).

[7] The period 1998:Q1 to 2005:Q4 can be divided into two distinct periods, until 2002:Q4 and from 2003:Q1 (the blue and pink lines). The pink line can be interpreted as a shift of the curve downwards. We can attribute that shift to a gradual reduction of the NAIRU due to government policy since 2002 with the aim of reducing structural unemployment (those steps included a cut in social security payments and unemployment benefits, and measures to moderate the increase in foreign workers).

[8] This possibility has been noted and discussed in the 2011 Bank of Israel Annual Report.

Figure 2: The "Beveridge Curve" 1998:Q1-2012:Q4



Now we turn to a more formal analysis of the NAIRU derivation from the BC. Following Blanchard (2009) the BC is given by:

$$s(1-u) = mF(u,v) \qquad (3)$$

where u stands for the unemployment rate and v is the vacancy rate. The left hand side of (3) represents the flow of separations from employment (s is separation rate) and the right hand side represents the flow of new hires, which is assumed to be captured by the matching function $F(u,v)$. m is a scale variable that represents the efficiency of the matching process. In equilibrium these two flows are equal. Along the BC (holding s and m constant) there is a negative relationship between unemployment and vacancies: in recession, unemployment is high and vacancies are low, and in booms, unemployment is low and vacancies are high. The factors that shift the BC are the changes in the separation rate and changes in the efficiency of the matching process.

Following Dickens (2009), we assume that the matching function has a Cobb-Douglas form:

$$F(u,v) = ku^{1-b}v^{b} \tag{4}$$

Substituting (4) into (3) and rearranging terms we get the following form of the BC:

$$(1-u)/u = (km/s)(v/u)^{b} \tag{5}$$

Following Blanchard (2009) and Dickens (2009) we assume that the NAIRU is a function of the separation rate (s) and the efficiency of the matching process (m),[9] and that it is the main driver of the relation between $(1-u)/u$ and v/u. Thus the NAIRU is determined as:

$$NAIRU = k'_0 + k'_1 \log(km/s) \tag{6}$$

Taking logs on the two sides of (5) and substituting (4) into (5) we get the following relation between unemployment, the NAIRU and vacancies:[10]

$$\ln[(1-u)/u] = k_0 + k_1 NAIRU + b\ln(v/u) \tag{7}$$

The rest of the model

The unemployment rate (u_t) is a sum of two unobserved components, the NAIRU (un_t) and the unemployment gap ($ugap_t$):

$$u_t = un_t + ugap_t \tag{8}$$

To distinguish empirically between un_t and $ugap_t$ we have to specify a data generating process for each. We model the data generating process of the NAIRU as a random walk with a stationary drift with unconditional zero mean (g_t) as follows:

[9] Blanchard claims that the NAIRU is also a function of the bargaining power of workers.

[10] Dickens was aware to the possibility that $\ln(v/u)$ is endogenous. He applied several kinds of tests and concluded that the resulting bias in the parameter b is small. We assume that this feature applies also to our data.

$$un_t = un_{t-1} + g_{t-1} \tag{9}$$

$$g_t = \varphi g_{t-1} + e_t^g \tag{10}$$

Here $0 < \varphi < 1$ measures the degree of the persistence of the change in the NAIRU, and e_t^g is an i.i.d. shock with mean zero and standard deviation σ_g. The inclusion of time variant and stationary drift g_t captures long lasting structural factors driving the NAIRU. This specification is consistent with the approach of Turner, *et al.* (2001) and implies that changes in the NAIRU are stationary and that in the long run the NAIRU is constant. Laubach (2001)[11] and others also included a stochastic drift in the NAIRU, but assumed that the drift follows a random walk - implying that only the second difference of the NAIRU is stationary (an assumption that does not seem reasonable for the Israeli unemployment rate).

As for the unemployment gap, following Laubach (2001) we assume that it follows an autoregressive process of the form:

$$ugap_t = \delta_1 ugap_{t-1} + \delta_2 ugap_{t-2} + e_t^{ugap} \tag{11}$$

where e_t^{ugap} is an i.i.d. shock with mean zero and standard deviation σ_{ugap}. We assume that all shocks are mutually uncorrelated and normally distributed. [12]

The model contains the following six equations:

$$\Delta\pi_t = \sum_{i=1}^{6} \alpha_i \Delta\pi_{t-i} - \beta_1 \Delta ugap_{t-1} - \beta_2 ugap_{t-3} + \gamma_1 \Delta rer_t + \gamma_2 \Delta rer_{t-1}^* + e_t^{pc} \tag{a}$$

$$\ln[(1-u_t)/u_t] = k_0 + k_1 un_t + k_2 \ln(v_t/u_t) + e_t^{bc} \tag{b}$$

$$u_t = un_t + ugap_t \tag{c}$$

$$un_t = un_{t-1} + g_{t-1} \tag{d}$$

[11] This specification was largely motivated by sample of European countries in which unemployment was trending up over the sample period used by Laubach (2001).

[12] To ensure convergence of the system, the parameters of equation (11) should fulfill: $\delta_1 + \delta_2 < 1,\ \delta_2 - \delta_1 < 1,\ -1 < \delta_2 < 1$.

$$g_t = \varphi g_{t-1} + e_t^g \quad \text{(e)}$$

$$ugap_t = \delta_1 ugap_{t-1} + \delta_2 ugap_{t-2} + e_t^{ugap} \quad \text{(f)}$$

The model contains three signal equations: the Phillips curve (PC), the Beveridge curve (BC) and the unemployment rate (UR).[13] In the next section we shall estimate the system of equations; (a)-(f) by means of the Kalman filter in order to obtain maximum likelihood estimates of the relevant parameters and the time series estimates of the unobserved NAIRU.

III. Estimation

Estimation of the system (a)–(f) without imposing prior restrictions on the variances of the shocks leads to results similar to those in Dickens (2009): the NAIRU is mainly determined by the BC while the contribution of the PC is close to zero.[14] In order to highlight the reasons for this result and to offer a way to progress we shall estimate the above system in three versions. First, we estimate the model excluding the BC; that is, estimating the system: (a), (c)–(f). (Call it Model 1). As we shall see later, the estimated NAIRU under this specification is quite smooth and is fully consistent with Gordon's "smoothness" criteria. Then we add the BC and estimate the model under two versions. In the first version (Model 2.1) we impose no restrictions on the variances, like in Dickens (2009). In the second version (Model 2.2), we impose two restrictions: $\frac{\sigma_g^2}{\sigma_\pi^2}$ and σ_π^2 are the same

[13] We abstain from including output as an additional observable signal equation, like in Basistha and Startz (2008) by exploiting information in the output gap concerning the unemployment gap (Okun's law). This is because the output gap is itself an unobservable variable and subject to misevaluation which might bias in the estimated NAIRU.

[14] The contribution is measured by the Kalman gain.

as in Model 1.[15] These two restrictions are consistent with Gordon's "smoothness" criteria and at the same time enable the BC to contribute to the NAIRU identification. Before we refer to detailed examination of the estimation results let us first take a bird's eye view of the estimated NAIRU under the various models.

A bird's eye view of the estimated NAIRU under the various models

In Figure 3 below we compare the estimated NAIRU from Models 1 and 2. Subplot (a) contains the actual unemployment rate[16] (U, blue line), the NAIRU estimated from Model 1(UN_1, red line) and the NAIRU estimated from Model 2.1 (UN_21, green line). In subplot (b) we retain U and UN_1 and replace UN_21 with the NAIRU estimated from Model 2.2 (UN_22, also in green line).

We can observe that UN_1 is quite smooth. It declines monotonically since 2003 and lies near U at the end of the sample period. The decline in the estimated NAIRU since 2003 is in line with the noticeable decline in the actual unemployment rate, while the inflation rate was roughly stable since 2003. In contrast to UN_1, UN_21 is very volatile and for most of the sample period it is tightly linked to the actual unemployment rate. Moreover, as in Dickens (2009), it is characterized by zig-zags, mainly in the first half of the sample (below we explain why this happens). Observing UN_22 in subplot (b), we see that it is smoother than UN_21 and close to UN_1 until the end of 2006. During 2007 and 2008, UN_22 is below UN_1 and is closer to U, which means that part of the decline in the NAIRU is not captured by the Phillips curve. This result is

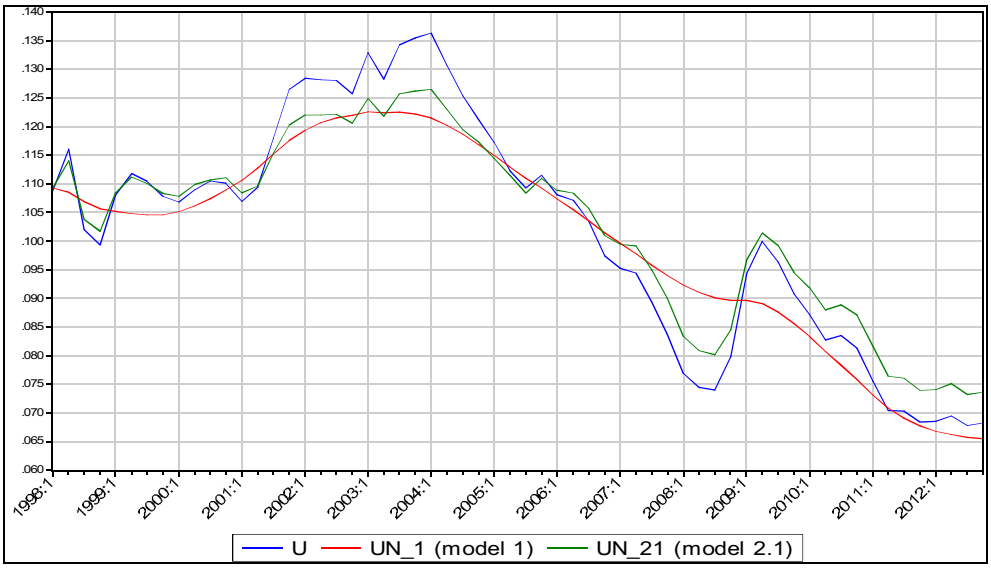
[15] Restricting $\frac{\sigma_g^2}{\sigma_\pi^2}$ only is not sufficient to fulfill Gordon's criteria. This is because given the imposed ratio, the estimated σ_g^2 (and σ_π^2) is high leading to highly volatile NAIRU.

[16] All the data refer to the new Labor Force Survey.

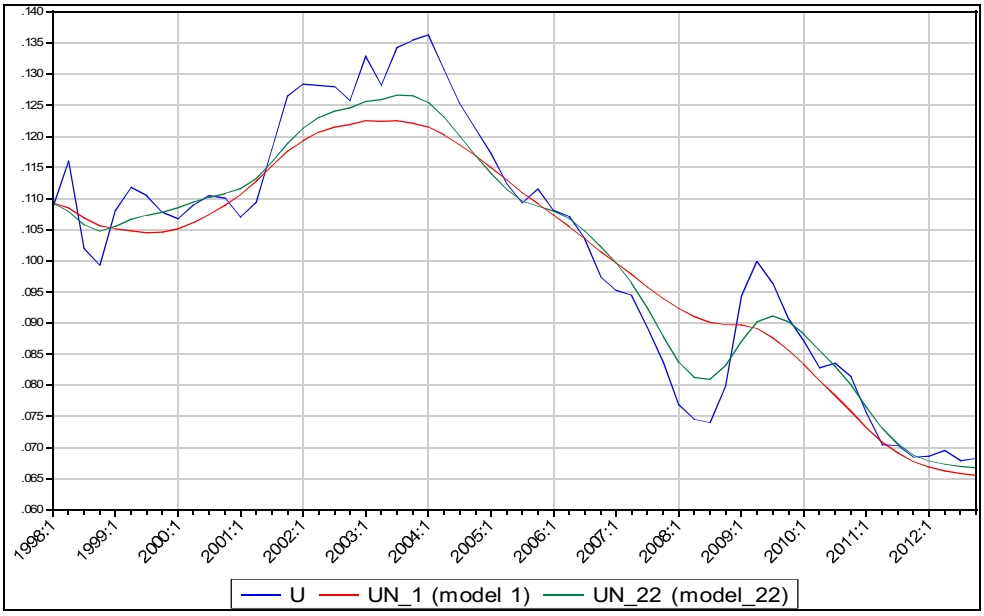
consistent with the shift of the Beveridge curve that occurred at that time (see Figure 2). From 2009, UN_22 is a bit above UN_1 and is closer to U.

Figure 3: The unemployment rate (blue) and the estimated NAIRU from Model 1 (red) and Models 2.1 and 2.2 (green), 1998:Q1-2012:Q4

(a):Model 1 versus Model 2.1



(b):Model 1 versus Model 2.2



Note: U is an actual unemployment rate, UN_1, UN_21, U_22 are estimated NAIRU from Model 1, Model 2.1 and Model 2.2, respectively.

The estimation results

In Table 1 we present the estimation results under the three versions of the model. First we describe the estimation of Model 1. The sample period is 1998:Q1-2012:Q4.[17] A maximum likelihood estimation of the system: (a), and (c)–(f), yields significant[18] estimates for most of the parameters[19] with the correct sign for all the parameters, but the estimate of the variance ratio σ_g^2/σ_π^2 was far from being significant. Difficulties in the estimation of the variance ratio of state variables are quite a common phenomenon in the application of the Kalman filter to such a system.[20] To identify the variance ratio σ_g^2/σ_π^2 the following procedure was implemented: we estimated the system for various values of the above ratio in the range {0.001 to 0.5} (in the unconstrained estimation the estimated value was 0.3 but insignificant). In the range {0.016 to 0.5} we found that the likelihood function is rather flat.[21] So we added another statistical criterion: the maximum t value of the parameter β_2 (the coefficient of the unemployment gap in the Phillips curve). The "preferred" estimate of σ_g^2/σ_π^2 under the above criterion is 0.05.[22] Note that this selected ratio satisfies not only the statistical criteria explained previously but also Gordon's economic criteria.

The resulting parameter estimates are presented in the second column of Table 1. Note that the parameter of the unemployment gap (with third lag) and the parameter of the change in the unemployment gap have reasonable values (-0.34

[17] The data on vacancy rates are available only from 1998:Q1.

[18] Under significance level of 5%.

[19] α_6 and γ_2 were not significant but with t values of 1.72 and 1.31, respectively.

[20] This is known as the "pile-up" problem, see Gordon (1997) and Laubach (2001) and the references there for discussions on that issue.

[21] Out of that range the value of the likelihood was lower.

[22] The simulation results can be provided by request.

and -0.42, respectively).[23] Both the unemployment gap and the changes in the NAIRU are stationary and highly persistent.

Now we turn to the extended model which combines the BC. The third column presents the estimation results of Model 2.1(unrestricted): here the parameter of the change in the unemployment gap is larger (in absolute value) than that of Model 1 (-0.55) and the parameter of the unemployment gap is smaller (-0.25). All the parameters in the BC equation are highly significant, although the parameter of the vacancy to unemployment ratio is quite small (0.03). Note also that the estimated variance of the changes in the NAIRU (the derived estimate of σ_g^2) is relatively high, implying a very low variance of the residuals in BC equation (σ_{BC}^2). This in turn is reflected in a drastic fall of the variance of the smoothed NAIRU estimator, $P_{t/T}$.[24]As we shall see below, this is reflected in a much smaller contribution of the PC than that of the BC to the identification of the NAIRU. A similar result in Dickens (2009) led him to conclude (erroneously, in our mind) that the PC contains no contribution to the identification of the NAIRU and that all the contribution comes from the BC.

Despite the fantastic fit of the BC (which is reflected in the very low value of the estimate of σ_{BC}^2) and the very low variance of the smoothed NAIRU estimator ($P_{t/T}$), the dynamics of the NAIRU are not consistent with Gordon's criteria (see UN_21, Figure 3), indicating that what is measured seems to have little in common with the "true" NAIRU.

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[23] For example Gordon's (1997) estimate of the unemployment gap parameter was in range of -0.58 to -0.68.

[24] Let un_t and $un_{t/T}$ present the true and the smoothed estimator of the NAIRU respectively. The variance of the smoothed estimator is: $P_{t/T} = E(un_{t/T} - un_t)^2$. For the definition of the term "smoothed estimator" see Hamilton (1994) or the technical appendix to this paper which can be provided upon request.

To see why the unrestricted estimation of the model leads to unreasonable volatility of the changes of the estimated NAIRU, note that the BC (Equation b in Section II) includes two unobservable variables: the NAIRU (un_t) in the form of a time varying intercept and the shocks (e_t^u). In this specification, we can substantially improve the fit of the BC by pushing up the volatility of the NAIRU and at the same time pushing down the estimated volatility of the shocks to the BC. If such a volatile NAIRU does not seriously harm the fit of the PC, the likelihood function would be vastly improved. Therefore, the contribution of the BC to the identification of the NAIRU increases on expense of the contribution of the PC. This phenomenon is somehow less serious if we estimate the NAIRU by using only the PC because in the PC the coefficient on the NAIRU is restricted to equal to the coefficient on the unemployment rate.

The last column presents the estimation results of Model 2.2 (restricted). The estimated effect of the unemployment gap in the PC equation is similar to that of Model 1 but the parameter of the vacancy to unemployment ratio in the BC is now much more prominent and reasonable (0.10). Clearly, since the model is now estimated under restrictions which impose lower variance of the NAIRU, the variance of the shocks in the BC is higher than in Model 2.1. We can observe that $P_{t/T}$ is much lower than in Model 1 but still much higher than in Model 2.1.

Table 1. The estimation results of Model 1 and Model 2

Parameters	Model 1	Model 2.1	Model 2.2
	The Phillips curve		
β_1	-0.42 (0.21)	-0.55 (1.06)	-0.48 (0.25)
β_2	-0.34 (0.13)	-0.25 (0.37)	-0.51 (0.21)
γ_1	0.18 (0.03)	0.18 (0.04)	0.18 (0.02)
γ_2	0.08 (0.05)	0.04 (0.07)	0.07 (0.05)
α_i	$\alpha_1 = -0.27$ (0.1) $\alpha_2 = -0.52$ (0.09) $\alpha_3 = -0.40$ (0.1) $\alpha_4 = -0.19$ (0.1) $\alpha_5 = -0.18$ (0.07) $\alpha_6 = -0.10$ (0.08)	$\alpha_1 = -0.21$ (0.16) $\alpha_2 = -0.43$ (0.15) $\alpha_3 = -0.36$ (0.16) $\alpha_4 = -0.11$ (0.18) $\alpha_5 = -0.13$ (0.12) $\alpha_6 = -0.07$ (0.13)	$\alpha_1 = -0.22$ (0.10) $\alpha_2 = -0.49$ (0.10) $\alpha_3 = -0.37$ (0.10) $\alpha_4 = -0.16$ (0.11) $\alpha_5 = -0.16$ (0.07) $\alpha_6 = -0.09$ (0.08)
	The Beveridge curve		
k_0		3.68 (0.00)	3.49 (0.03)
k_1		-14.09 (0.01)	-11.36 (0.32)
k_2		0.03 (0.00)	0.10 (0.01)
	The rest of the model		
φ	0.80 (0.09)	0.34 (0.13)	0.81 (0.08)
δ_1	1.09 (0.13)	1.07 (0.29)	0.80 (0.16)
δ_2	-0.30 (0.12)	-0.17 (0.26)	-0.18 (0.17)
	The variances		
σ_π^2	0.0052^2 (0.00)	0.0087^2 (0.00)	0.0052^2
$\frac{\sigma_g^2}{\sigma_\pi^2}$	0.05	0.10 (0.03)	0.05
σ_{ugap}^2	0.0039^2 (0.00)	0.0040^2 (0.00)	0.0039^2 (0.00)
σ_{BC}^2		0.0001^2 (0.00)	0.034^2 (0.01)
$P_{t/T}$	0.0039^2	$7.09E-06^2$	0.0014^2

Notes: The sample period is1998:Q1–2012:Q4. Numbers in parentheses are standard deviations of the parameters

IV. Assessing the relative contribution of the signal equations to the identification of the NAIRU by using the Kalman gain

In the previous sections we examined the estimation results of the model under different specifications and restrictions. In this section we shall apply the Kalman filter methodology to examine the contribution of each of the three signal equations ((a), (b) and (c)) to the NAIRU, especially focusing on the PC and BC. We do this by looking at the Kalman gain of each equation.[25]

Let $un_{t/t}$ stand for the filtered estimator of the NAIRU for the current period and let $un_{t/t-1}$ stand for the predicted estimator from the previous period. The difference between those two variables is the update of the NAIRU estimator due to new information that is provided by each of the signal equations as of time t. This can be written as:

$$un_{t/t} - un_{t/t-1} = g^{u}\varepsilon_t^{u} + g^{pc}\varepsilon_t^{pc} + g^{bc}\varepsilon_t^{bc} \qquad (12)$$

where g^{u}, g^{pc} and g^{bc} stand for the Kalman gain contributed by the unemployment equation (c), the PC equation (a) and the BC equation (b), respectively. The terms ε_t^{u}, ε_t^{pc} and ε_t^{bc} represent the forecast errors of the above signal equations. These gains reflect the direct contributions of the signal equations to the identification of the NAIRU.

Table 2 corresponds to Models 1, 2.1 and 2.2, showing the gains of the signal equations included in the models.[26] Starting with the second row we can see that the contribution of the PC in Model 1 is notable, meaning that applying statistical filters (HP filter and others) which exploit only data on the actual unemployment rate is not enough for the identification of the NAIRU.

[25] For a description of the Kalman filter methodology and the Kalman gain see Hamilton (1994). In a technical appendix to this paper, which can be provided upon request, we describe the concrete implementation of that methodology in this paper.

[26] The gains (in absolute terms) represent the values reached after convergence to a fixed point.

When we add the BC and estimate the model under no restrictions on the variances (Model 2.1, in Table 2), the relative contribution of the PC dramatically falls almost to zero, while at the same time, the relative contribution of the BC is quite big. This is because the NAIRU derived under this specification explains the BC almost perfectly, without seriously deteriorating the fit of the PC. Therefore more weight is given to the BC and less to the PC.

Continuing to Model 2.2 (the last row of Table 2) we see that the contribution of the PC increases at the expense of declining contribution of the BC, and that the PC and BC have similar contributions. That is, once we apply Gordon's criteria we see that both the PC and the BC make important contributions to the identification of the NAIRU.

Table 2. The Kalman gains under the three versions

	g^{u}	g^{pc}	g^{bc}
Model 1 (only PC)	0.561	0.306	_
Model 2.1 (both PC and BC - unrestricted)	3.3E−06	5.6E−12	0.07
Model 2.2 (both PC and BC - restricted)	0.148	0.031	0.041

Note: Numbers in Table 2 are Kalman Filter gains for three signal equations.

V. Conclusions

In this paper we estimate the NAIRU in Israel for the period 1998:Q1-2012:Q4 by exploiting both the Phillips curve and the Beveridge curve relationships. The estimation results show a prolonged decline of the NAIRU from 12% in 2003 to 6.5% at the end of 2012. This declining trend of the NAIRU is consistent both

with the inflation dynamics, described by the PC, and the vacancy-unemployment relationship, described by the BC.

When we conducted, as in Dickens (2009), unrestricted estimation of the NAIRU based on both the PC and the BC, we obtained similar results to Dickens: the contribution of the PC to the identification of the NAIRU was close to zero and the volatility of the estimated NAIRU was unreasonably high. When we added judgment concerning the NAIRU volatility, following the smoothness conception of Gordon (1997), we found that both the PC and the BC contain useful information concerning the NAIRU dynamics.

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