

**Research Department**



**Bank of Israel**

**Nowcasting Israel GDP  
Using High Frequency Macroeconomic Disaggregates**

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Discussion Paper No. 2010.16  
December 2010

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The authors would like to acknowledge Ya'acov Ritov from the Statistics Department of the Hebrew University in Jerusalem and Amit Friedman, Tomer Kriaf and Tanya Suhooy from the Research Department in the Bank of Israel for their valuable advice throughout the research process

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## אומדן מוקדם לתמ"ג הרבעוני של ישראל באמצעות נתונים כלכליים בתדירות גבוהה

גיל דפנאי ויונתן סידי

### תקציר

במאמר זה נציג מודל דינמי לחיזוי בזמן הווה – Nowcasting – של התמ"ג הרבעוני בישראל. בישראל, בדומה לרוב מדינות העולם, אף על פי שמדיניות מוניטארית נבחנת ונקבעת מדי חודש נתוני התוצר הינם רבעוניים ומתפרסמים כשישה שבועות לאחר סיום הרבעון. כך נוצר מצב בו לא עומדים לרשות קובעי המדיניות בבנק ישראל נתונים עדכניים לגבי שינויים בתמ"ג, בבואם לקבוע את המדיניות המוניטארית כשלושה שבועות לאחר סיום הרבעון.

המטרה של פרויקט החיזוי המוצג במאמר זה היא לייצר אומדן בזק עבור נתוני התוצר, כשלושה שבועות לאחר סיום הרבעון, וכך לאפשר לקובעי המדיניות להסתמך על נתונים עדכניים יותר בתהליך קביעת הריבית. חיזוי התוצר מתבצע באמצעות ניצול האינפורמציה הגלומה בתוך קבוצה גדולה של נתונים חודשיים הזמינים בתאריך הרלוונטי.

בחירת האינדיקטורים מתוך הקבוצה הנ"ל מתבצעת באמצעות מספר טכניקות שונות להורדת מימד, הנבחנות לאורך המאמר בשני אופנים עיקריים: התנייתן במשתנה המוסבר וללא התניה.

נמצא, כי מבין השיטות שנבחנו, טכניקת ה-Elastic Net היא היעילה ביותר לבחירת המודל, בכך שהיא מייצרת את התחזיות היציבות והמדויקות ביותר עבור נתוני התוצר הרבעוני. כמו כן, נמצא כי המשתנים אשר נבחרים למודל החיזוי הסופי באופן העקבי ביותר על פני זמן הינם: מחיר הנפט, סקר מעסיקים, מדד מנהלי הרכש, מדד היצור התעשייתי, ומדד המועסקים בענף ייצור המנועים והרכיבים האלקטרוניים.

# **Nowcasting Israel GDP**

## **Using High Frequency Macroeconomic Disaggregates**

**Gil Dafnai and Jonathan Sidi**

### **Abstract**

This paper presents a dynamic nowcasting model for Quarterly GDP in Israel. Currently, monetary policy in Israel is evaluated and updated on a monthly basis. The recent GDP figure is, however, unavailable for monetary policy makers, at the Bank of Israel, at the month following the end of the quarter, due to a six-week lag of the GDP data publication.

The aim of this nowcasting project is to derive "flash" estimates of GDP at a four-week lag, in order to gain four weeks in terms of data availability when updating the interest rate. This is done by utilizing the information contained within a large group of monthly indicators that are available at the relevant date.

Indicator selection, from a pool of these high frequency series, is applied through a variety of dimension reduction techniques. The ability to apply these techniques while conditioning them on the predicted indicator will be examined and discussed in this article.

The Elastic Net is found to be the most comprehensive model selection technique, generating the lowest mean absolute forecast error of only 1.62%. In addition, the Elastic Net successfully captures the timing and magnitude of the 2008-2009 economic cycle. Notable variables that have model inclusion persistence are: The Price of Oil, Employers Survey, Purchasing Managers Index, Industrial Production Index, and Employed Persons Index in Manufacturing of Electronic Motors, Components, and Transport Equipment.

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## Part I

# Introduction

## 1 Background

Central Bank assessments of the current state of the economy play a vital role in the conduct of monetary policy. However, providing an accurate assessment of economic growth in real-time is a challenge that many central banks have to overcome. The challenge is found in the delay between the end of the quarter and the publication of the GDP data. In Israel, quarterly GDP is published at a six-week lag off the end of the relevant quarter.

Many attempts have been made to create real time projection models for the quarterly GDP figures based on monthly indicators that have a short publication lag, e.g. *Zheng and Rossiter, Central Bank of Canada, (2006)* [18]. The aim of this paper is to present variable selection methodology from various fields other than economics, as to test its application in generating real-time estimation of the current quarter GDP.

We use a large sample of different monthly indicators which are chosen according to their timing of publication, in order to nowcast quarterly GDP. An indicator will enter the initial set of explanatory variables only if it has at least a value for the first two months of the projected quarter.

Since the number of monthly indicators is larger than the number of observations there has been extensive application of dimension reduction and variable selection techniques via Bridge Equations. These techniques project quarterly data through monthly variables. Many of the previous papers in the field applied Factor Analysis, e.g. *Angelini et al. (2008)*[2] and *Banbura, Giannone and Reichlin*[3], largely following *Giannone, Reichlin, & Small, (2007)*[10]. Table 1 shows the different methods applied in the leading central banks and research centers. This paper will approach the problem from different directions, comparing five different approaches of dimension reduction and variable selection in order to select the optimal model for projection.

Table 1: Variable Selection Methods Applied in Leading Central Banks

Organization	Method	Number of Variables
Bank of Canada	Simple bridge equation with backward selection	30
ECB (ECARES)	Bridge via Factor Analysis	200
Central Bank of Ireland		41
Central Bank of Portugal		45
Bank de France	Bridge via PCA	Business surveys
<b>This Research</b>	<b>Model Selection via: PCA, SPCA, LASSO, Elastic Net</b>	<b>155</b>

Following *Klein and Sojo (1989)*[13] we use Principal Component Analysis (PCA) to reduce the dimensionality of the dataset. Since PCA does not set to zero any of the coefficients in the principal components we also apply Sparse PCA, *Zou et al. (2004)* [19]. This method was constructed to improve the inference ability compared to the PCA method. These methods have a serious drawback since they do not incorporate the target variable, i.e. the quarterly GDP, into the dimension reduction procedure. To accommodate this drawback we also examine the application of two new and promising variable selection techniques: Least Absolute Shrinkage Selection Operator (LASSO), *Tibshirani (1996)* [17] and the Elastic Net, *Zou and Hastie (2003)* [20]. Both techniques were developed to address variable selection problems in the field of Bio-Informatics, and both select variables correlated to the response variable by setting constraints on the coefficient of the Least Squares problem.

We find that Elastic Net and LASSO indeed improve the proficiency of now-casting when compared with both unconditional methods (PCA and SPCA) and univariate multiple regression. We also find that the price of oil, purchasing manager’s index, employer’s survey, industrial production index, and employed person’s index in manufacturing of electronic motors, components, and transport equipment are the variables with the highest probability to enter the final set of the projection model.



## 2 The Problem

Modern day econometrics attempts to construct models that represent economic processes by defining a set of variables and a set of logical and quantitative relationships between them. Economic models use structural parameters to create various desired properties of interest. Assumptions pertaining to the relationships between the structural parameters are based on previous research or on widely accepted economic theory.

In this paper we attempt to predict the percent change of the current quarter GDP through the application of nonparametric and semi-parametric dimension reduction methodology, in order to identify underlying structures in large data sets. The advantage in this approach is two-fold:

1. Minimizing the use of confining structural assumptions on the data.
2. The common properties are deduced from within the data.

We define the general set of variables as  $\Omega^d$ , which consists of the relevant monthly time series  $\omega_p^d$  up to date  $d$ :  $\Omega^d := \{\omega_p^d | \omega_1^d, \dots, \omega_P^d\}$ .

Due to different publication lags in the data we redefine  $\Omega^d$  as two different subsets:  $\Omega_1^m$  and  $\Omega_2^m$ , where  $\Omega^d = \Omega_1^m \cup \Omega_2^m$ .  $\Omega_1^m$  contains monthly series which have values up to and including the last month in quarter  $q$ , and  $\Omega_2^m$  contains series in which one month is missing. We forecast the missing month in  $\Omega_2^m$  series using Holt and Winters Exponential Smoothing. Once this preprocessing is complete a "bridge regression" can be run, which utilizes the higher frequency monthly variables to forecast a lower frequency of the dependent variable, the GDP.

While there are a number of candidate goodness of fit statistics to compare the out of sample efficiency of each method, we apply the Mean Absolute Forecast Error (MAFE). We define the MAFE as  $\frac{1}{n} \sum_{i=1}^n |\widehat{GDP^q} - GDP^q|$ , denoting the current quarter forecast, i.e. nowcast, of the GDP as  $\widehat{GDP^q}$ . The reason for choosing this statistic instead of the Mean Square Error or the Root Mean Square Error is due to the presence of a possible outlier observation in the published GDP. The robust nature of the MAFE enables us to produce a comparison of the methods without removing the problematic observation from the dataset, while preserving the original units.

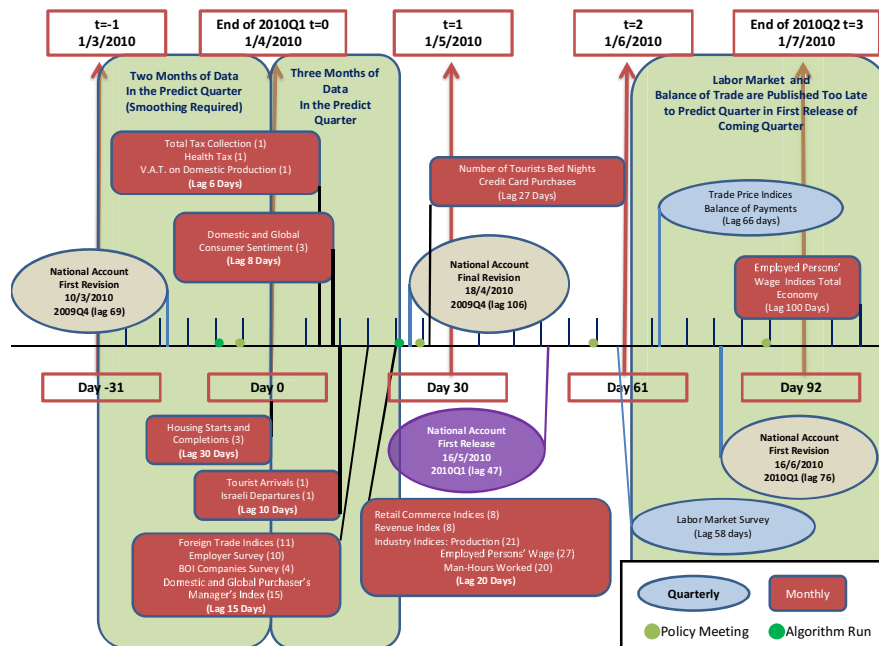
# Part II

## Data Selection & Preprocessing

### 3 Indicator Selection Methodology

The general set of indicators,  $\Omega^d$ , is compiled of indicators in the general monthly appendix that is used in monthly meetings in the bank in addition to past research [10] [2] [18]. In total there are 140 domestic indicators and 15 global indicators<sup>1</sup>. These indicators are characterized by their availability and stability. The availability of monthly data received from the Central Bureau of Statistics (CBS) in comparison to the target meetings, determines the number of indicators included in the initial set. Figure 1 depicts a stationary time line illustrating the indicator publication chronology from the Central Bureau of Statistics.

Figure 1: Central Bureau of Statistics Publication Timeline



<sup>1</sup>The full list of indicators and their description can be found in Appendix B.2 9

Indicator selection occurs twice:

- Structural Criteria

The initial selection occurs prior to data transformation fulfilling two conditions:

SC(a). Minimum history of series is the first month of 1998, 1998m1.  
This condition is set to allow for a large number series that have been recently generated through surveys.

SC(b). At most one month missing from the current quarter.  
This condition is set to minimize the variability of forecasts to the original series.

While the algorithm can be run up to twelve days prior to the end of the relevant quarter, the number of variables that have up to one month missing is too small and they all have to be extended to a third month to complete the quarter. Currently, running the algorithm prior to the end of the quarter has shown inferior results, and will be focused upon in further research.

Defining the resulting data set after the first selection procedure as:

$$\widehat{\Omega}^d := \{\widehat{\omega}^d \in \Omega^d : \widehat{\omega}^d \text{ fulfills SC(a) and SC(b)}\}$$

- General Information Criteria:

The indicator accounts for a high proportion of the total variability in  $\widehat{\Omega}^d$ .

Defining the resulting data set after the second selection procedure as

$$\Theta^d := \{\theta^d \in \widehat{\Omega}^d : \theta^d \text{ fulfills General Information Criteria}\}$$

## 4 Data Preprocessing

Data transformation  $\tilde{\omega}^d = f(\hat{\omega}^d)$  is applied in order to begin the secondary selection procedure given that all the indicators are seasonally adjusted and have the same end point. In addition we remove any trend from the data through log differencing and standardize each series.

1. Seasonal adjustment is carried out on all series using X12-ARIMA. The specification file uses the default SARIMA model selection procedure, automatically finds outliers from 1999 to the end of the sample, and the Jewish calendar and trading days are exogenous variables in the model. This is done in order to transform the data to be as similar as possible to seasonally adjusted data of the CBS.
2. Different publication lags for each indicator causes jagged edges in the data, i.e. different end dates from series to series. The Holt and Winters exponential smoother is applied to each indicator that is missing the final month in the current quarter. The estimates for the additive coefficients of each series are estimated<sup>2</sup> <sup>3</sup>.
3. The log-difference of each seasonally adjusted series is calculated.
4. The percent changes are standardized.

## Part III

# Methodology

The importance of dimension reduction techniques in modern statistics can be dated back to R.A. Fisher. Fisher is responsible for laying the foundations for modern theoretical and applied statistics. In an article published by Fisher in 1922 [8] he defined one of the main goals in statistics as:

”... the objective of statistical methods is the reduction of data. A quantity of data...is to be replaced by relatively few

---

<sup>2</sup>Estimation was done using the BFGS optimization routine.

<sup>3</sup>The option to forecast the missing month using X12-ARIMA was not applied in this paper due to technical issues found in the use of EVIEWS 6.0 for seasonal adjustment.

quantities which shall adequately represent...the relevant information contained in the original data.”

Further paraphrasing Fisher’s 1924 [9] text he stated that the variables that are employed as predictors must be chosen without reference to the variable of interest, e.g. current quarter GDP. The article concentrated on the subject of transforming an “ $n < p$ ” into an “ $n > p^*$ ” without the dependency of the transformation on the response variable.

In conjunction with Fisher’s articles other researchers formulated methods of dimension reduction, including: *Adcock (1878)* [1], *Pearson (1901)* [15], *Spearman (1904)* [16], *Hotelling (1933)* [11]; which are today known as Principal Component Analysis (PCA).

The study of principal components in regression is a case in which the vector of predictors is reduced prior to the regression on the response variable. This is predominantly done in order to mitigate the effects of collinearity and to facilitate model specification by allowing visualization of the regressors in low dimensions, *Cook (1998)* [4]. Additionally it provides a parsimonious set of predictors on which to base interpretation, *Cook (2007)* [5].

As Fisher stated in his 1924 article, these methods are solely transformations on the explanatory variables. This is the main drawback of dimension reduction when applied to regression. It may be possible to contain the same information in a subset of  $M$  leading principal components as in the population set, but their relationship to the response variable is not addressed. Moreover, an additional drawback is the absence of a conventional method to decide which principal components should be included in  $M$ , this was addressed by Cox [6]:

”A difficulty seems to be that there is no logical reason why the dependent variable should not be closely tied to the least important principal component.”

To overcome these inherent problems in the application of principal components in regression we use sparse regression methodology, [17] [19], to redefine the PCA model as a least squares model, i.e. the Elastic Net.

In the following subsections we define the methods described above beginning with those that are *not conditioned* on the response variable,[5.1] and [5.2], and then discuss the methods which condition the data reduction on the response variable, [6.1]-[6.3], a flowchart in Figure 2 describes how each method is applied in order to identify the final variable set  $\Theta^d$ .

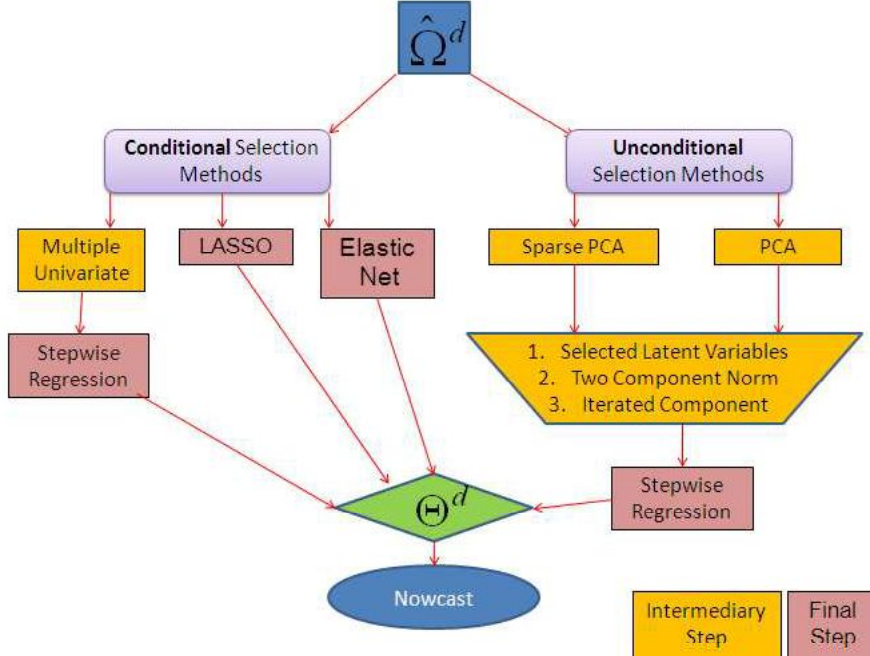


Figure 2: Flowchart of Methods Applied

## 5 Unconditional Methods

### 5.1 Principal Component Analysis

Principal Component Analysis (PCA) is a standard tool in modern data analysis. Its application can be found in a number of diverse fields from Computer Graphics to Neuroscience. This is because of the simple non-parametric method it uses to extract relevant information from condense data sets. The subsequent section will provide the basic intuition behind PCA, after which it's utilization concerning nowcasting will be explained.

#### 5.1.1 Background

The objective of PCA is to find a linear transformation that reduces the dimension of a multivariate sample  $X_{(n \times p)}$  defined as:  $X_1, X_2, \dots, X_p$  into  $\hat{X}_{(n \times q)}$  where  $q < p$ . The transformed set has the following desirable properties:

1. The elements of  $\hat{X}$  are uncorrelated.

2. Each element in  $\hat{X}$  should account for as much of the combined variance of the elements in  $X$  as possible.  $\hat{X}$  is selected so as to minimize redundant information in the latent variables by maximizing variance of the relevant variables in the data, thereby minimizing information loss.

The objective of PCA is to minimize the target function  $F(\cdot)$ , where  $F(\cdot) = c_i^t R c_i$  subject to constraints:

$$\max \quad c_i^t R c_i \quad (1a)$$

$$s.t.(1) \quad \|c_i\|_2 = 1 \quad (1b)$$

$$s.t.(2) \quad c_i^T C_{i-1} = (0, 0, \dots, 0) = 0_{i-1}^t \quad (1c)$$

In this problem we define  $R$  as the standardized covariance or correlation matrix where the vectors  $c_i$  are the solution to the maximization problem. The formulation in (1) gives an intuitive insight into the main purpose of PCA, which is to find the directions which maximize the variance in  $\mathbf{X}$ . Notice that (1b) is necessary in order to ensure the problem to have a finite solution and (1c) assures that each successive solution  $c_i$  is orthogonal to all the previous solutions,  $C_{i-1}$ .

A practical solution to (1) is through the use of Singular Value Decomposition (SVD). SVD allows us to take any matrix  $\mathbf{X}$  and decompose it into the eigenvalues and eigenvectors.

Defining the SVD of  $\mathbf{X}$  as

$$XV = U\Lambda \quad (2a)$$

$$X = U\Lambda V^t \quad (2b)$$

We define two orthogonal matrices  $\mathbf{U}$  and  $\mathbf{V}$ , and a diagonal matrix  $\Lambda$ . The construction of  $Diag(\Lambda)$  is of the form  $(\sigma_1 \dots \sigma_p, 0 \dots 0)$ , where  $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_p$ . The columns of  $\mathbf{U}$  are called principal components of unit length, and the columns of  $\mathbf{V}$  are the corresponding loadings of the principal components, i.e. each  $V_i$  are eigenvectors of matrix  $R$ , defined  $R = X^t X$ . By applying some linear algebra:

$$X = U\Lambda V^t \quad (3a)$$

$$X^t X = V\Lambda U^t U\Lambda V^t = V\Lambda^2 V^t \quad (3b)$$

$$tr(X^t X) = tr(VV^t \Lambda^2) \quad (3c)$$

$$tr(X^t X) = \sum_{i=1}^p \lambda_i^2 \quad (3d)$$

In (3d) we conclude that  $\lambda_i$  are eigenvalues of  $R$ , furthermore they represent the variance of  $R$  along a given  $V_i$ . This is due to the fact that  $\|Xv_i\|^2 = \lambda_i \equiv \sigma_i^2$ . After we solve for the eigenvectors we can use them to change the base of  $\mathbf{X}$  to the orthogonal base  $\hat{X}$ :

$$V^t X^t = \hat{X} \quad (4)$$

### 5.1.2 Application in Regression

In regression there are many cases in which there are more variables,  $p$ , than observations,  $n$ . In these cases, PCA is used in order to create new variables, the latent scores  $\hat{X}$  as described above, which are used as the observed variables in the regression. This is done to decrease the effects of collinearity and simplify the interpretation of the regressors in the model. We test three methods in the use of principal components in multivariate regression. The first uses the leading principal components as independent variables in regression, while the other two apply variable selection from within the components.

#### Selected Latent Variables

Defining the subset  $\Theta^d$  consisting of  $q$  principal components. The number of components may be set equal to the number of components that their corresponding  $\lambda$  is greater than some  $\lambda_0$ . The level of  $\lambda_0$  used in this method was set at 0.7 [12]. The use of the components in now-casting is less favorable because the components used are linear combinations of all the variables in  $\hat{\Omega}^d$ . Using such a large set for policy decisions is not practical.

#### Two Component Norm

In this procedure the norm of the first two principal components from



each variable is calculated and then sorted in ascending order. This is a naive variable selection procedure because it only utilizes the first two components. The maximum variation that can be explained is  $\frac{\sum_{i=1}^2 \lambda_i}{\sum_{i=1}^p \lambda_i}$ . Within this portion of the total variance we are choosing the subset of variables that have the largest weights.

Algorithm 1 Two Component Norm	
1:	Define $\tilde{V} = [v_1 v_2]$
2:	Calculate by rows $Q = \ \tilde{V}\ $
3:	Sort Ascending Q
4:	$\Theta^d = \bigcup_{j=1}^8 Q_j$

### Iterated Component

This variable selection method, introduced by Jolliffe (1972) [12], forms a subset of  $K$  independent variables for multivariate regression using principal components. A subset of the first  $K$  components is formed by setting a constraint  $\lambda_k \geq \lambda_0$ , for a given  $\lambda_0$ . The same level of  $\lambda_0$  was used as in the classic approach (0.7). The variable with the largest coefficient in  $K_1$ , the component with the largest eigenvalue, is placed in subset  $\Theta^d$ . Then iteratively one variable is chosen which is associated with the remaining  $K-1$  components under consideration and which has not already been placed in  $\Theta^d$ .

Algorithm 2 Iterated Component	
1:	Define $\lambda_0$
2:	if $\lambda_i \geq \lambda_0$ then $PC_i \in K, i = 1 \dots P$
3:	$v_j = \arg \max (K_1)$
4:	$v_j \in \Theta^d$
5:	for $k=2$ to $\text{length}(K)$
6:	$v_j = \arg \max (K_k)$
7:	if $v_j \notin \Theta^d$
8:	then add to $\Theta^d$
9:	else check next largest $v_j$
10:	end if
11:	next

## 5.2 Sparse PCA

### 5.2.1 Background

In the previous section we discussed different methods to identify variables after applying PCA to the data set. The main drawback of PCA is that the loadings are inherently nonzero. This makes it difficult to interpret PCs when applying it to large data sets. In the following section we will describe a technique that produces modified principal components, i.e. sparse loadings. This will be done by formulating the PCA optimization as a regression-type optimization problem, and imposing two exterior penalty functions on the regression coefficients.

### 5.2.2 Formulation

The application of this algorithm on principal components was first introduced by *Zou, Hastie, and Tibshirani, 2004* [19]. The exterior penalty functions are defined as:

1.  $L_1$ -norm constraint:  $P_1(\cdot) = \sum_{j=1}^p |\beta_j|$

This constraint effectively act as a scaling parameter on the solution to the minimization problem.

2.  $L_2$ -norm constraint:  $P_2(\cdot) = \sum_{j=1}^p |\beta_j|^2$

The Ridge penalty/Tikhonov regularization is a well known method for reducing variability of coefficients in a regression through the bias-variance trade off. This constraint allows for highly correlated coefficients to be grouped together.

The PCA problem (2) can be viewed as a simple regression problem with a ridge penalty, where  $X_i$  is the  $i$ -th row vector,  $\alpha_{(p \times k)}$ ,  $\beta_{(p \times k)}$ , and  $\forall \lambda > 0$  we derive:

$$(\hat{\alpha}, \hat{\beta}) = \arg \max_{\alpha, \beta} \sum_{i=1}^n |\mathbf{X}_i - \alpha \beta^t \mathbf{X}_i|^2 + \lambda |\beta|^2 \quad (5a)$$

$$s.t. \quad |\alpha|^2 = I_k \quad (5b)$$

Then we get  $\hat{\beta}_i \propto \mathbf{V}_i$  for  $i = 1 \dots k$ .

The intuition behind (5) is that if we set  $\alpha = \beta$  then  $\sum_{i=1}^n |\mathbf{X}_i - \alpha \alpha^t \mathbf{X}_i|^2$  then we get an alternative formulation of the standard PCA problem that gives the same solution, under orthonormal constraints. As in the case of the nowcasting problem setting, "  $p > n$  ", requiring  $\lambda > 0$  ensures (5) yields the exact PCA solution.

Finally defining the SPCA problem by adding the  $L_1$ -norm constraint to (5) in order to obtain sparse loadings.

$$(\hat{\alpha}, \hat{\beta}) = \arg \max_{\alpha, \beta} \overbrace{\sum_{i=1}^n |\mathbf{X}_i - \alpha \beta^t \mathbf{X}_i|^2}^{pca} + \lambda \sum_{j=1}^k |\beta_j|^2 + \overbrace{\sum_{j=1}^k \lambda_{1,j} |\beta_j|_1}^{coefficient\ calibration} \quad (6a)$$

$$s.t. \quad |\alpha|^2 = I_k \quad (6b)$$

The different  $\lambda_{1,j}$  are allowed for penalizing the loading of different PCs, and the ridge penalty is constant over all  $k$  components.

While Sparse PCA is a marked improvement in the ability to interpret the loadings in comparison to PCA the method lacks the qualities we are aiming for as a tool for model selection in nowcasting. The main drawbacks to this method are:

- To form sparse components where no other variable selection method is needed causes the proportion of  $\frac{\text{variance explain}_{spca}}{\text{variance explain}_{pca}}$  to decrease beyond accepted levels.
- The solution to the optimization problem is not conditioned on the variable of interest.

## 6 Conditional Methods

### 6.1 Univariate Regression

#### 6.1.1 Background

The production of real-time forecasts from univariate equations by regressing current-quarter GDP on all variables in a general set has been used extensively in central banks e.g the Central Bank of Canada.

Although this method is quite simplistic, it produces low forecasting errors in the absence of full quarter data, we have included this method as a benchmark of the methods that condition the subset selection on the GDP.

#### 6.1.2 Formulation

For each variable in the data set a univariate regressions is run on the current quarter GDP.

$$GDP^d = c + \beta_i \hat{\omega}_i^d, \quad \forall \hat{\omega}_i \in \hat{\Omega}$$

A Subset of the 25 variables with the highest  $R_{adj}^2$  is taken, after which a stepwise backward regression is applied with the subset as the independent variables and the GDP as the dependent variable. The stopping criterion, removal p-value, in the selection is 10 percent.

Algorithm 3 Univariate Regressions	
1:	Run $GDP^d = c + \beta_i \hat{\omega}_i^d, \quad \forall \hat{\omega}_i \in \hat{\Omega}$
2:	$Q = \bar{R}_i^2$ sorted in descending order
3:	$\hat{Q} = \bigcup_{j=1}^{25} Q_j$
4:	Run Stepwise Backward Regression on
	$GDP^d = c + \sum_{i=1}^{25} \beta_i \hat{Q}_i + \epsilon$
	The variables that are in the final regression make up the subset $\Theta$

While improving the simple regression by selecting a subset of the original data the stepwise method activated on a subset of variable has two drawbacks:

1. The accepted stepwise methods used do not calculate all the models possible if there are more than eight possible variables in the model. [12]
2. Prediction stability is a problem because small changes in the data can result in very different models being selected.

## 6.2 Regression with a Tuning Parameter

### 6.2.1 Background

Given the ordinary least squares (OLS) formulation where target variable  $Y^4$  and the estimation compromised of the training data  $\mathbf{X}$ . The object of OLS is to minimize the residual square error.

$$\hat{\beta} = \arg \min_{\beta} \sum_{i=1}^N (y_i - x_i^t \beta)^2 \quad (7)$$

There are two central reasons why an analyst would be unsatisfied with the OLS estimators.

#### Prediction accuracy

OLS estimates are defined to have zero bias (BLUE) but this causes the variance of the model to increase, therefore prediction accuracy can be improved by setting some of the coefficients to zero.

#### Interpretation

In the case of a large number of predictors, we often would like to determine a canonical subset that exhibits the strongest effects, increasing the ability to inference the results.

In an attempt to address the issues with variable selection procedures the LASSO (Least Absolute Selection and Shrinkage Operator) was introduced by *Tibshirani (1996)* [17].

### 6.2.2 Formulation

The LASSO solves the minimization problem (7):

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<sup>4</sup>The target variable is centered prior to optimization.

$$\hat{\beta} = \arg \min_{\beta} \sum_{i=1}^N (y_i - x_i^t \beta)^2 \quad (8a)$$

$$st \sum_j |\beta_j| \leq t \quad (8b)$$

The same penalty function was used in the case of Sparse PCA,  $L_1$ -norm [1]. The LASSO tends to shrink the OLS coefficients toward zero, and setting some exactly to zero leaving only the most important ones. This often improves prediction accuracy, while trading off decreased variance for increased bias.

### 6.3 Regression with a Tuning Parameter and a Grouping Penalty

As we have seen in the previous section penalizing a regression with the  $L_1$ -norm improves OLS in sense of variability of coefficients and sparsity. Although it has shown success in many situations, it has limitations:

1. Where " $p > n$ " the LASSO selects at most  $n$  variables.
2. Where " $n > p$ " , if there are high correlations between predictors, it has been empirically observed that the prediction performance of the LASSO is dominated by ridge regression [17].<sup>5</sup>
3. If there is a group of variables among which the pairwise correlations are very high, then the LASSO selects only one variable from the group and does not differentiate which one is selected.

Concerning nowcasting the third limitation theoretically makes the LASSO an inferior variable selection method. This problem was first addressed by data analysts who worked with micro arrays, where the number of variables is extremely high and grouping is a desirable property. *Zou, Hastie, and Tibshirani (2004)* [19] have proposed a new method called the Elastic Net, which integrates the  $L_1$ -norm and  $L_2$ -norm penalties together thus gaining the desirable property of grouping. The Elastic Net is a convex combination of the ridge penalty and the LASSO penalty.

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<sup>5</sup>It should be noted that this shortcoming is irrelevant to this paper.

The Elastic Net solves the following problem, Friedman, Hastie and Tibshirani (2008) [7]:

$$\min_{\beta \in \mathbb{R}^{p+1}} \left[ \frac{1}{2N} \sum_{i=1}^N (y_i - x_i^t \beta)^2 + \lambda P_\alpha(\beta) \right] \quad (9)$$

where

$$P_\alpha(\beta) = (1 - \alpha) \frac{1}{2} \|\beta\|_{l_2}^2 + \alpha \|\beta\|_{l_1} = \sum_{j=1}^p \left[ \frac{1}{2} (1 - \alpha) \beta_j^2 + \alpha |\beta_j| \right] \quad (10)$$

## Part IV

# Results

## 7 Israel GDP: Descriptive Analysis

Prior to presenting the results of each method described above we will briefly discuss the descriptive attributes of the quarterly seasonally adjusted GDP released by the CBS. Additionally, we will mark points of interest that will be expanded upon in a case study following the results.

Table 2: Descriptive Statistics: Israel GDP

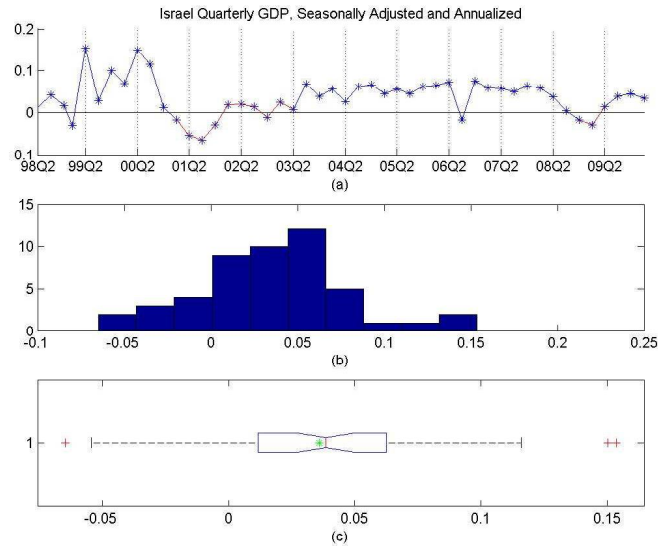
Israel GDP 1998q1-2010q1			
Seasonly Adj. and Annual Percent Change			
Mean	Median	Std	Kolmogorov-Smirnov (Pvalue)
3.60%	3.85%	4.50%	47.42%

During the sample period (1998q1-2010q1), the economic climate in Israel which was characterized by steady growth accompanied by two business cycles that contributed to the variation, as seen in Figure 3(a). Furthermore, we see that the published GDP is distributed normally, as we do not reject the hypothesis tested in the Kolmogorov-Smirnov test. Although, there is

evidence of positive skewness and elevated levels of kurtosis to the distribution, as seen in Figure 3(b). Using the boxplot in Figure 3(c) we locate possible outliers of the series at the peaks (1999q2 and 2000q2) and the gully (2001q3) of the high tech bubble.

A case study will concentrate on the recessions marked in red<sup>6</sup> in Figure 3(a), to test how well the model reacts to external shocks to the economy. In addition, as part of a more general test of robustness of the algorithm, we will focus on how the model reacts to the data with and without the Second Lebanon War (2006q3). This will test if the event can be treated as an outlier of the published series.

Figure 3: Analysis of Distribution Properties of the Published Quarterly GDP.



## 8 Constraint Levels

The selection of the constraint levels in the optimization problems applied in this paper is paramount in producing results with low error rates and correct

<sup>6</sup>The NBER method was used for recession identification.



levels of sparsity. In this section we will discuss the dynamic constraint level selection procedures applied in the general algorithm.

As part of the algorithm which solves optimization problem<sup>7</sup>, we select values of  $\alpha$  in the range [0.1, 0.2, ..., 1.0]. For each value of  $\alpha$  the algorithm generates solutions to the optimization problem between two extremes on the range of  $\lambda$ ,  $[\lambda_L(\alpha), \lambda_H(\alpha)]$ , where:

$$\|\beta(\lambda)\| = \begin{cases} \|\beta_{OLS}\| & \lambda < \lambda_L(\alpha) \\ 0 & \lambda > \lambda_H(\alpha) \end{cases}$$

It was found that the predictions of the GDP in the range of  $[\lambda_L(\alpha), \lambda_H(\alpha)]$  are locally robust with relation to close values of  $\lambda$ . On the other hand, the variability increases when comparing the predictions between the quantiles of  $[\lambda_L(\alpha), \lambda_H(\alpha)]$ . We use three points, designated as  $q25$ ,  $q50$ , and  $q75$ , from this range as representative solutions.

Table 3 shows the out of sample deviation statistic, Mean Absolute Forecast Error (MAFE), for each level of  $\alpha$  and  $\lambda(\alpha)$ . This procedure is applied to determine which constraint levels give the best out of sample results each time the general algorithm is run. In the data tested we found that for the Elastic Net  $\alpha = 40\%$  and  $\lambda(\alpha) = q50$ , and  $\lambda = q50$  for the LASSO produced the smallest forecast errors.

Table 3: Comparison of MAFE for Different Levels of Constraints  $\alpha$  and  $\lambda(\alpha)$  Applied in the Elastic Net Procedure, Sample 2004Q2-2010Q1

		Elastic Net									LASSO
	$\alpha$	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
$\lambda(\alpha)$	25%	1.91%	1.91%	1.89%	1.89%	1.92%	1.96%	2.00%	2.04%	2.09%	2.12%
	50%	1.66%	1.65%	1.63%	<b>1.62%</b>	1.63%	1.66%	1.70%	1.76%	1.77%	<b>1.80%</b>
	75%	1.73%	1.73%	1.76%	1.78%	1.80%	1.83%	1.86%	1.90%	1.93%	1.98%

Additionally, as discussed in the previous section, the SPCA problem can also be reformulated as an Elastic Net problem. The values of  $\alpha$  and  $\lambda$  are set automatically by the algorithm<sup>8</sup> in order to solve for a maximum 20 nonzero coefficients per component. This amount of nonzero coefficients is sufficient

<sup>7</sup>The glmnet package which solves the Elastic Net optimization problem is available for both Matlab and R can be found at <http://www-stat.stanford.edu/~tibs/lasso.html>

<sup>8</sup>The LARS Matlab package, found in the same link referred to the glmnet, was used to calculate the solution for the SPCA problem.

to run the subsequent variable selection methods, i.e. Classic Approach, Two Component Norm and Iterated Component, and stepwise regressions applied in the unconditional methods.

## 9 Main Results

In order to assess the goodness of fit of our models we have conducted a rolling regression of 24 periods, beginning at 2004Q2. We then calculated the out of sample projection for each period accompanied by an identical set of statistics for each of the methods. The statistics included are: Standard error of the projection (S.E), Adjusted R2, Akaike Information Criterion (AIC), Root Mean Square Error (RMSE), Durbin Watson statistic (DW) and the Kolmogorov-Smirnov (KS) test. Table 4 summarizes the results of the different methods by averaging each statistic over the 24 periods on which we conducted an out of sample projection. In addition we constructed a series called  $CBS_{first}$  which consists of the first vintage data published by the CBS in each quarter. This will serve as our control series to compare out of sample results. Figure [4] shows a comparison of the projected and published GDP series within the conditional and unconditional methods.

Figure 4: Comparison of Out of Sample Projection

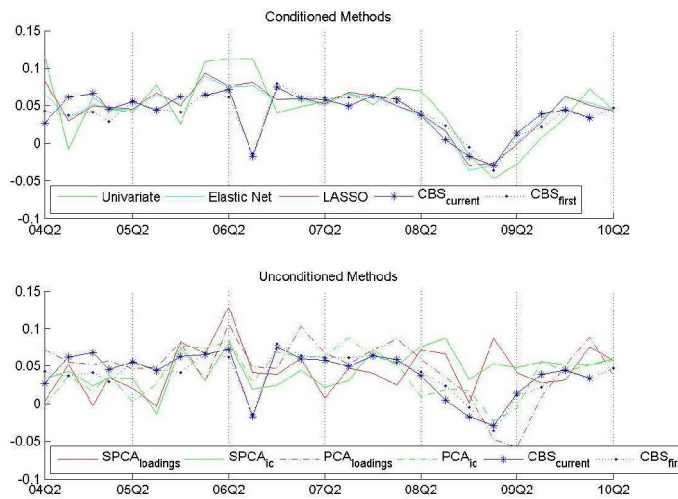


Table 4 may be quite misleading at first, since it displays data from the rolling regression which by construction takes into account the in-sample projections. Therefore, even though this table is quite informative regarding the goodness of each model it does not provide an answer to the most important question for this paper: Which selection method best projects the quarterly GDP?

Table 4: Statistics of Goodness of Fit of the Selection Methods

Method		S.E.	R2Adj	AIC	RMSE	DW	KS test
Conditional	Univariate	3.0%	58.9%	-125.6	2.9%	2.5	86.9%
	LASSO	3.5%	51.3%	-123.6	3.3%	2.6	72.6%
	Elastic Net	3.6%	50.8%	-123.2	3.3%	2.7	81.3%
Unconditional	IC	3.6%	42.9%	-112.9	3.4%	2.3	70.0%
	TCN	4.0%	29.5%	-104.6	3.8%	1.8	84.7%
	PCA: Loadings	3.5%	46.8%	-114.8	3.3%	2.5	73.5%
	SPCA: Loadings	3.4%	48.8%	-117.6	3.2%	2.4	74.0%

Table 5 presents the mean, median, minimum and maximum value of the absolute error for each model over the 24 periods of the out of sample projections. This table clearly shows that the Elastic Net, followed by the LASSO, provides the best projection. In Figure 5 we show the distribution of the absolute projected errors for each method.

Table 5: Distribution of the Absolute Errors Using Out of Sample Forecasts

Method		Mean	Median	Min	Max	
Conditional	Univariate	3.18%	3.05%	0.06%	12.99%	
	LASSO	<b>1.80%</b>	1.24%	0.10%	9.89%	
	Elastic Net	<b>1.62%</b>	1.20%	0.00%	9.47%	
Unconditional	PCA	Loadings	2.62%	2.51%	0.34%	7.13%
		IC	1.93%	1.65%	0.01%	7.13%
		TCN	2.73%	2.33%	0.29%	8.29%
	SPCA	Loadings	2.57%	2.03%	0.01%	8.94%
		IC	1.96%	1.41%	0.37%	5.92%
		TCN	2.97%	2.39%	0.42%	9.98%

Figure 5 is reinforced by Table 6 which shows Welch's t test. Welch's t test tests for equal forecast performance between methods. This is done by testing the hypothesis that a pair of series have equal means, while allowing for different variation within each one. From this table we conclude that

the Elastic Net and the LASSO are similar to first release of the GDP in the context of performance, while the other methods reject this hypothesis. Furthermore, this table gives a simple method to compare all the methods to each other.

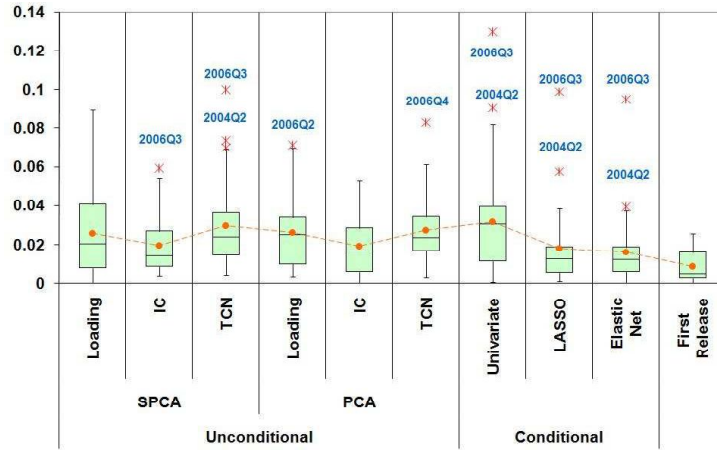


Figure 5: Graphical Comparison of Out of Sample Forecasts Performance

Table 6: Test for Equal Forecast Performance Between Methods

Welch's t Test (Pvalues)									
	First Release	PCA			SPCA				
		Elastic Net	LASSO	Univariate	Loading	IC	TCN	Loading	IC
Elastic Net	<b>0.13</b>	-	-	-	-	-	-	-	-
LASSO	<b>0.08</b>	<b>0.75</b>	-	-	-	-	-	-	-
Univariate	0.00*	0.05*	0.09	-	-	-	-	-	-
Loading	0.00*	0.12	0.25	0.38	-	-	-	-	-
IC	0.01*	0.54	0.82	0.10	0.29	-	-	-	-
TCN	0.00*	0.04*	0.10	0.63	0.57	0.09	-	-	-
SPCA	Loading	0.00*	0.12	0.22	0.52	0.82	0.25	0.79	-
	IC	0.01*	0.46	0.72	0.12	0.35	0.89	0.12	0.30
	TCN	0.00*	0.04*	0.08	0.84	0.43	0.09	0.75	0.61

\*Reject hypothesis that a pair of series have equal means at 0.95

Another question that is of interest to us is to what extent the density of the results from the different models is similar to the actual GDP, and even more importantly, whether the out of sample predictions are unbiased. Figure 6 displays the out of sample density of each projection method. The figure shows very clearly that the LASSO and the Elastic Net are as expected slightly biased, though comparatively less than the other methods.

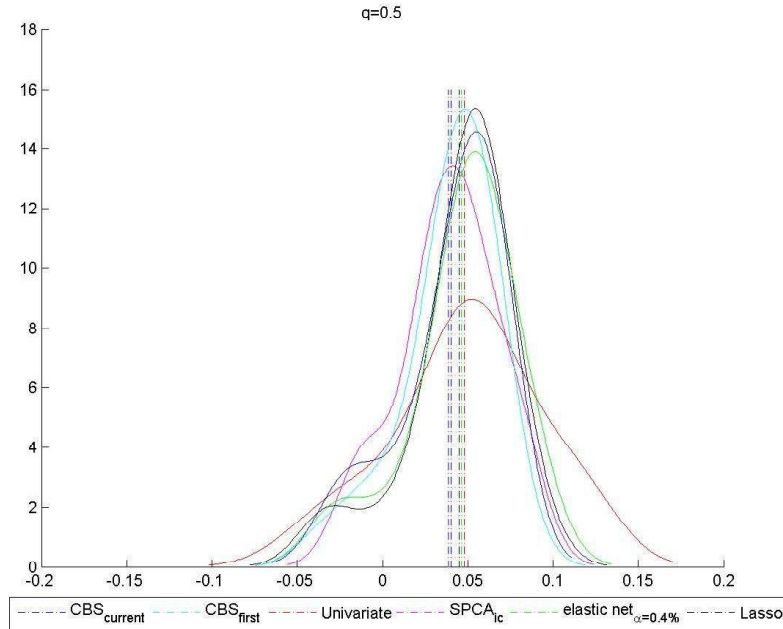


Figure 6: Comparison of Density and Bias Between Methods

## 9.1 Importance of Different Series

Table 8 in Appendix B show the probability of each variable to enter each model. We find that the conditional methods present very high consistency over time, in contrast to the unconditional methods which showed many substitutions of different variables over time. This implies that there are strong correlations between several variables and GDP, however these variables do not necessarily account for a large part of the data variability over time. In the conditional methods we find that the variables that enter the final set  $\Theta^d$  of the projection model can be categorized into three groups; a) Domestic Indicators b) Market Expectations Indices c) Global Variables.

The variables with the highest probability of entering  $\Theta^d$  include: Price of Oil, Purchasing Managers Index, Employers Survey, Industrial Production Index, and Employed Persons Index in Manufacturing of Electronic Motors, Components, and Transport Equipment. Figures 10a-10c in Appendix A show the size of the coefficients of all the variables chosen in each period of the rolling regressions.

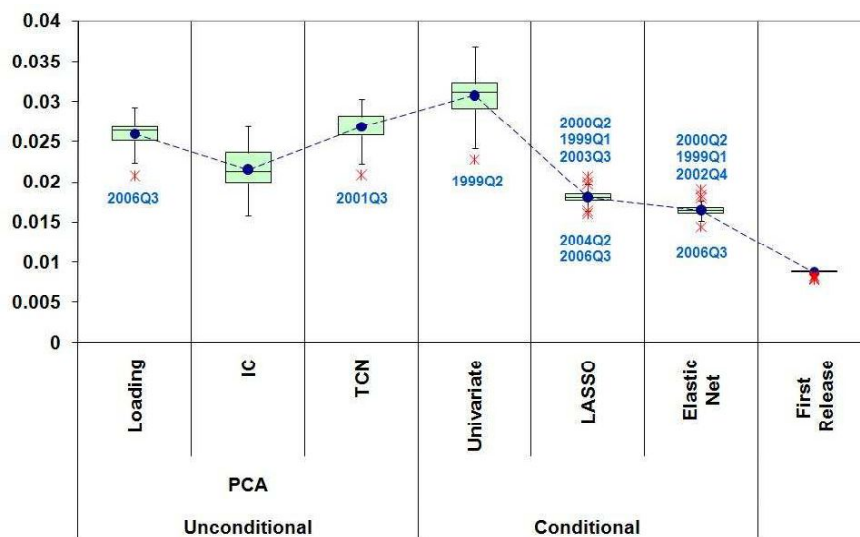
Table 7: Comparison of the Variables with the Highest Probability of Selection Across Methods

Rank	Description	LASSO	Elastic Net	Univariate	SPCA IC
1	TPR : Electronic Components and Different Industrial Equip.	100%	100%	24%	4%
2	TMB : Motors, Electronic Components	100%	100%	92%	-
3	Cushing Crude Oil Price	92%	96%	4%	28%
4	US Exports : Goods	92%	100%	24%	-
5	PMI : Employment : Level Component	68%	100%	-	-
6	PMI : Euro 16	68%	92%	4%	-
7	Industrial Firms Stock Index	60%	56%	-	-
8	Health Tax	60%	60%	-	4%
9	Tourist Bed-nights in Authorized and Unauthorized Hotels	60%	60%	-	8%
10	TPR : Electronics Communication Equipment	60%	68%	-	4%
11	TMB : Jewelry, Goldsmith's and Silversmith's Articles	60%	60%	-	8%
12	Employer Survey : Transportation	60%	64%	4%	-
13	General Stock Index	52%	80%	-	16%
14	TPR : Textiles	52%	48%	-	8%
15	PMI : Production : Output Component	40%	56%	32%	8%

## 9.2 Robustness of the Algorithm

Finally we test if the results from each method is robust in its prediction. This is done by applying the Jackknife technique to the sample. We use the Jackknife to estimate the bias and the variance of the absolute error of each method, by leaving out one observation at a time from the sample set. From Figure 7 we see the conditional methods are more stable than the unconditional ones. Within the conditional methods the oversensitivity to changes in the data in the univariate is highlighted, the Elastic Net has both lower mean absolute errors and variation compared to the LASSO. Detailed results of the each method within each period chosen in the Jackknife can be found Figures 12a-12h in Appendix A.

Figure 7: Mean Absolute Errors in Jackknife Procedure



## 10 Inference

While conventional econometrics is based on structural models, which by construction produce unbiased estimates, the methods utilized in this paper break those assumptions by adding a penalty to the minimization problem. It has been shown in the previous section that the out of sample predictions produced by the LASSO, and its general form the Elastic Net, out perform simple regression and classic dimension reduction techniques.

The question left unanswered is the ability to inference using the coefficients produced in sparse regression. The constraints used in the the Elastic Net were formulated for variable selection and do not solve the inherent multi-collinearity problem found in OLS. Thus, reaching conclusions as to the effect of each variable on the response variable may be tenuous. Conclusions that can be extracted from the Elastic Net are the characteristics of the variables chosen in the final subset and the proportion which each variable contributes to the forecast level.

To facilitate economic policy decisions a comparison can be conducted to understand changes in composition to the final subset and the different magnitude of the persistent variables throughout the economic business cycle.

## 11 Case Study: The Effect of External Shocks on Model Accuracy

There are two prominent episodes during our sample period, that exemplify abnormal economic activity. We examine these periods in order to assess and analyze algorithm performance in unusual times. The first is the Second Lebanon War, 2006Q3, in which the largest absolute error from the published GDP occurred and the previous global economic crisis 2008Q2-2009Q4. We discuss the subject of short comings of the algorithm and if they have economic explanations or if they are methodological errors.

### 11.1 Anomalies in the Data: The Second Lebanon War

The GDP growth rates which preceded and follow the Second Lebanon War, 2006Q3, were 7.5% and 7.8%, respectively. During this period there was steady global economic growth. Expectations of conflict in the northern border were minimal, and consequently the macroeconomic impact of the Second Lebanon War was unexpected and instantaneous. There was minimal impact to the growth levels of all the major indicators of economic activity: private consumption (3.4%), government consumption (13.6%), fixed capital formation (26.1%), unemployment (6.9%), business sector labor hours (-1.2%). It is evident that the abrupt deviation from steady growth in the GDP (-1.5%) was the consequence of an unexpected drop in inventories.

This conclusion is not surprising when recalling the immediate impact that the war had on the Israeli economy and in particular the labor market. During the 34 days of conflict the northern region was paralyzed due to the constant shelling, and a major portion of the workforce was enlisted to reserve duty. These factors led to a reduction in production capabilities of the Israeli economy, reflected in the steep decline of 12% in the utilization of machinery and equipment index during the quarter. A review of the variable contribution, Figure 8, in the LASSO model reveals that compared to the previous period the level of the variables have a similar behavior which we see in the National Accounts data, i.e. not reflecting the unexpected drop in the GDP. A variable that could have captured the shock is Number of Tourist Bed-Nights<sup>9</sup>. Internal research<sup>10</sup> in the Bank of Israel has found this variable

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<sup>9</sup>The Number of Tourist Bed-Nights exhibited an 8.4% drop in 2006Q3.

<sup>10</sup>Unpublished discussion paper by Menashe and Sharhabani



to be highly correlated with the level of security concerns in Israel. However, since this variable does not show high predictive ability during steady economic activity, it was not chosen by the LASSO when it could have actually been most indicative.

This special case provides an important insight to the importance of variable selection to the general set of indicators. The current general set is almost exclusively comprised of uses related variables. This in turn causes the algorithm to be insensitive to changes in the GDP which are induced by short term and unexpected factors, such as local conflicts or natural disasters.

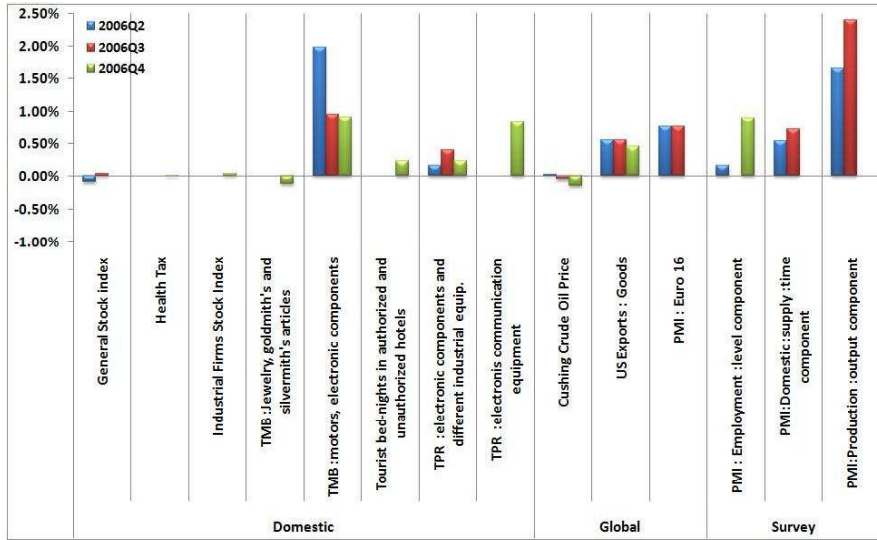


Figure 8: Variable Contribution for Nowcast 2006Q2-2006Q4 (LASSO)

## 11.2 Unmatched Market Expectation: Emerging out of Crisis

Compared to the global markets Israel has encountered more of an economic slowdown than a financial and real estate crisis in the past two years (2008Q2-2009Q4). Nevertheless, Israel did suffer from four consecutive quarters of comparatively low economic activity, two of which included contraction of the GDP. Analysis of the performance of the model in signaling the entrance and the emergence from an economic slowdown is prudent to understand how the algorithm reacts to economic instability. The algorithm captured

the timing, magnitude, and depth of the downturn. While capturing the timing of the beginning of the recovery it miss-timed the end of the recovery. We will briefly discuss the reasons we believe the persistent high growth rate continued through 2009Q4 (5.9%) while the published GDP tailed off (4.4%). Level shifts in time series are difficult to capture, one may argue that this contributed to the temporary inaccuracy of the model. However, the model performed fairly well during the level shifts in the aforementioned global crisis. In addition, a closer examination of the variables that were chosen, Figure 9, reveals that even though the variables with the largest coefficients actually decreased during this period, the Purchasing Managers and the Employers Indices had higher than expected levels of optimism in 2009Q4, causing the extended increase in projected growth<sup>11</sup>.

This case study reveals a source for potential inaccuracies in the projected data, due to the fact that it uses market expectations and not only market data. This drawback shall be examined in future research in order to improve model accuracy.

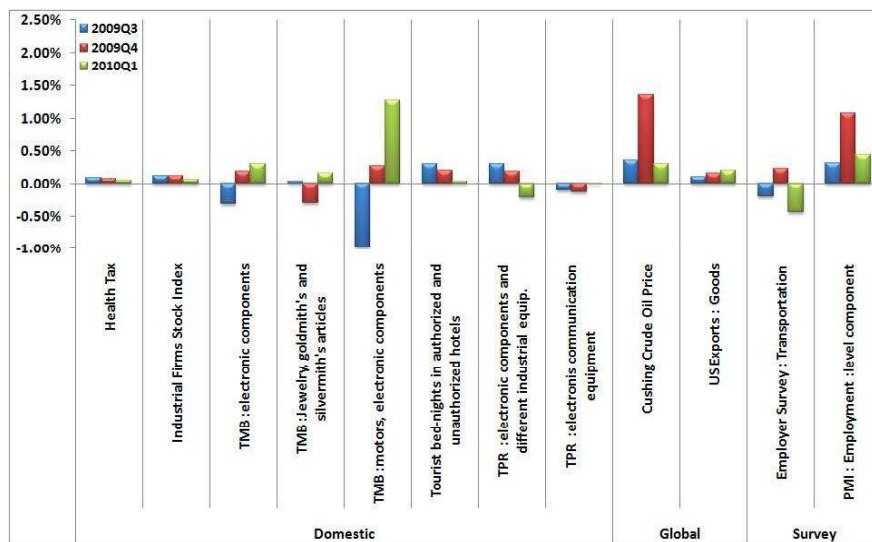


Figure 9: Variable Contribution for Nowcast 2009Q3-2010Q1 (LASSO)

<sup>11</sup>A full decomposition of coefficient levels is available in Figures 10-11, Appendix A.

## Part V

# Conclusions

Policy decision making in central banks is dependent on real-time data analysis as it is published. The ability to produce precise nowcasts through canonical models has evolved with the methodological progress of model selection techniques. Advances in different fields of research have improved model selection for large scale problems. These advances in nowcasting have yet to be fully utilized.

This paper compared model selection techniques applied in leading central banks today with a new method, the Elastic Net. The application of nowcasting with the Elastic Net to the Israel GDP yielded more precise and stable results. Moreover, the dynamic nature of the model allows it to adapt to shocks in the economy producing a more robust model.

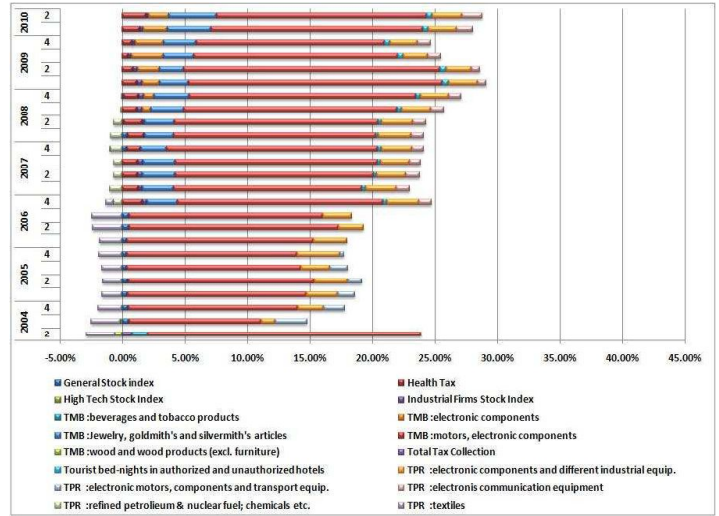
A distinguishing feature of the Elastic Net is the ability to isolate influential variables which contribute to the real-time assessment. This refinement of the results separates this method from current ones used in nowcasting and allows the model to be a more comprehensive tool in economic policy decisions. Finally, this research highlighted the contribution of advanced data mining techniques in a policy driven economic setting. Further development and adaptation of the inference ability of these techniques could broaden the insight into many structural econometric models applied today.

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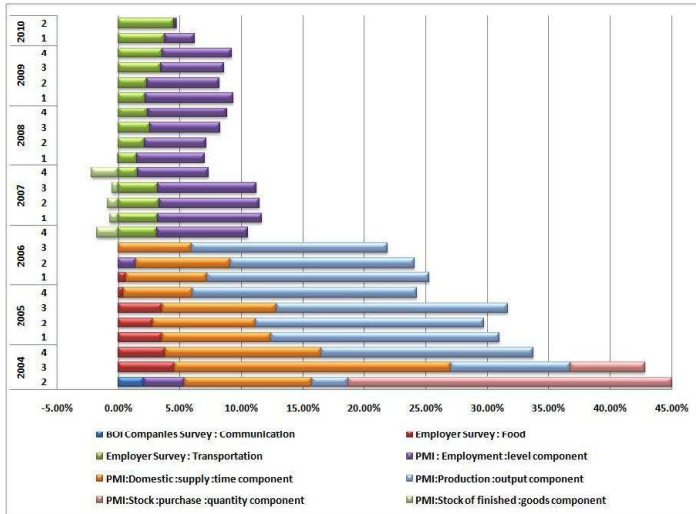
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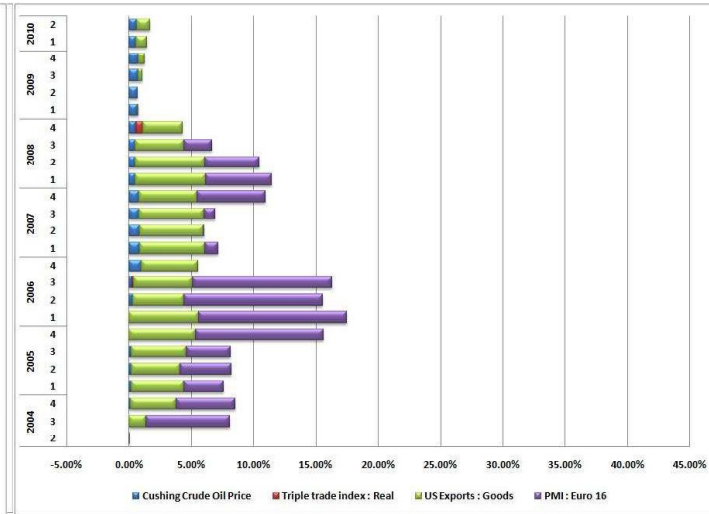
Part VI  
**Appendix**  
A Figures



(a) Domestic

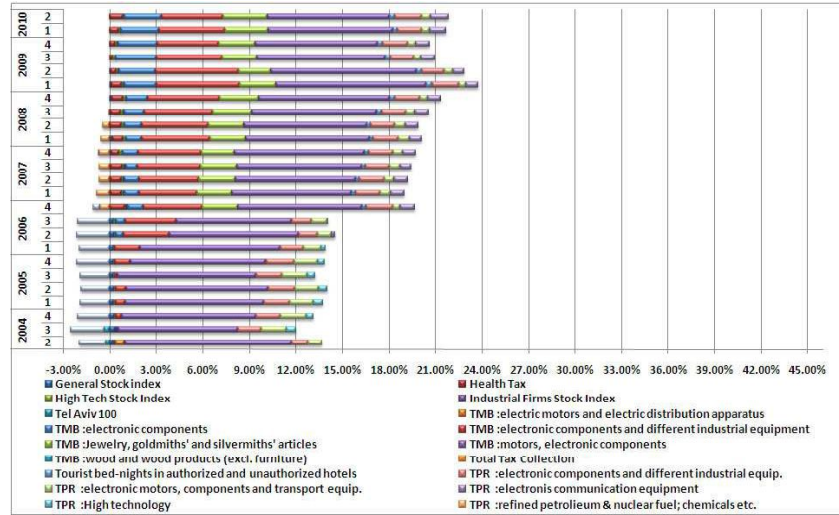


(b) Market Expectations

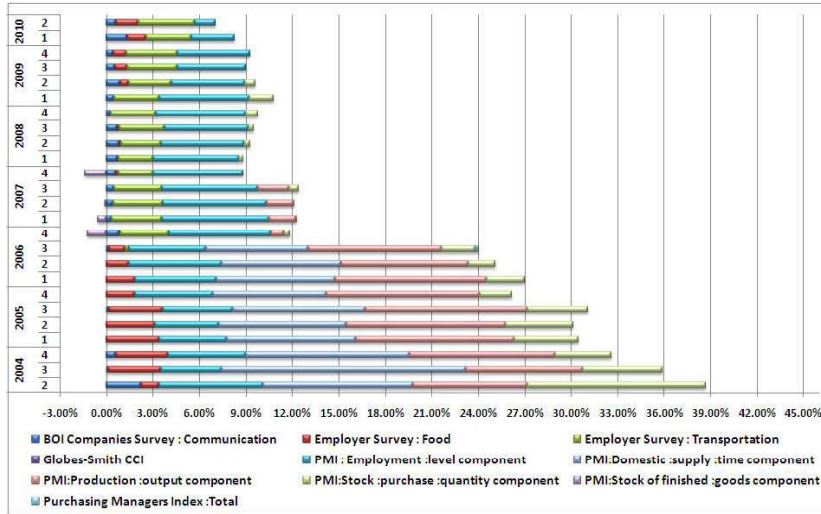


(c) Global

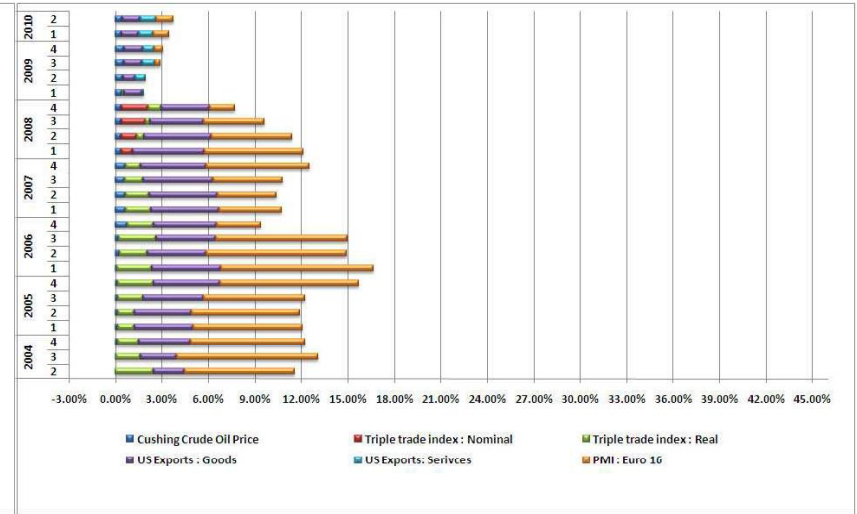
Figure 10: Size of Coefficients in each Period (LASSO)



(a) Domestic



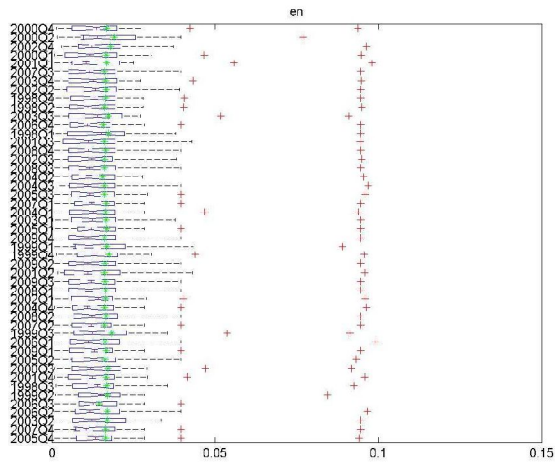
(b) Market Expectations



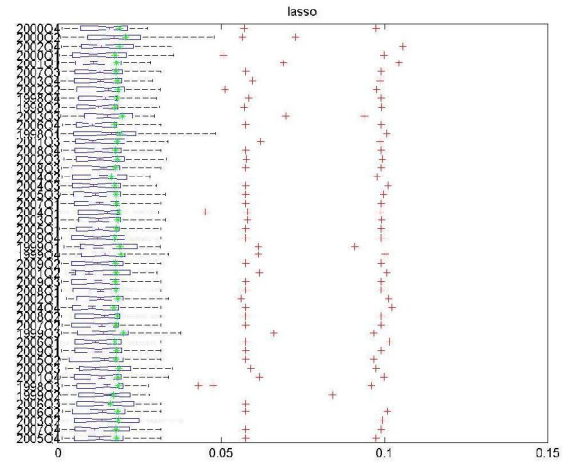
(c) Global

Figure 11: Size of Coefficients in each Period (Elastic Net  $\alpha = 0.4$ )

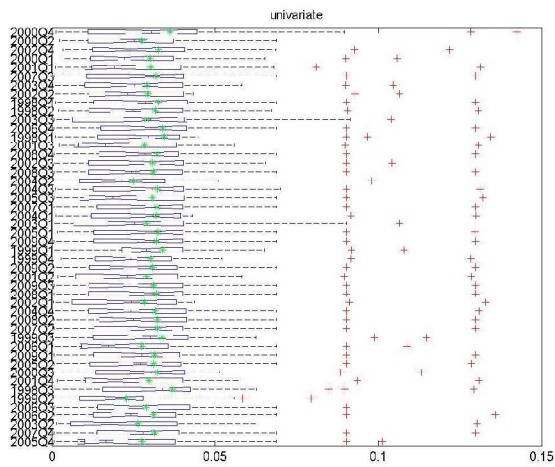




(a) Elastic Net



(b) LASSO



(c) Univariate

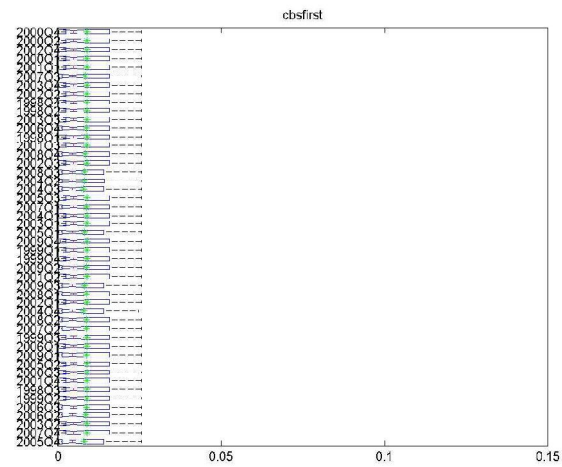
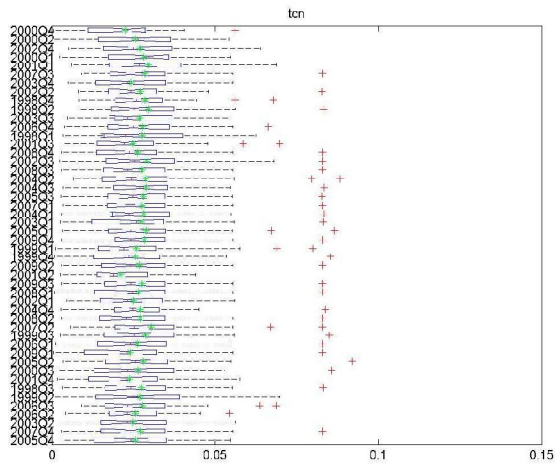
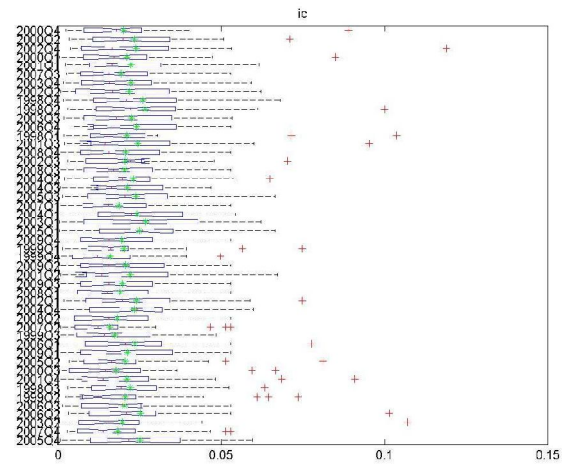
(d)  $CBS_{First}$ 

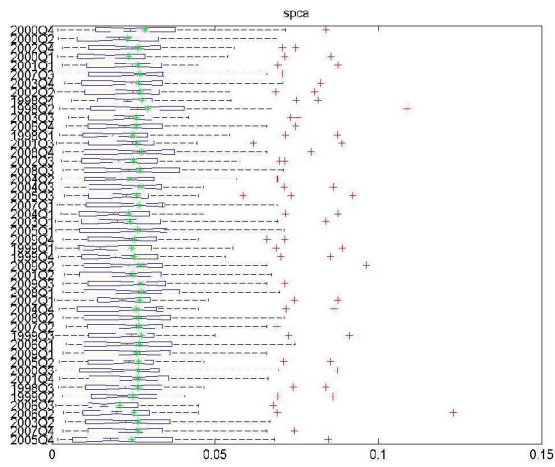
Figure 12: Jackknife Boxplots of Conditional Methods



(e) TCN



(f) IC



(g) PCA

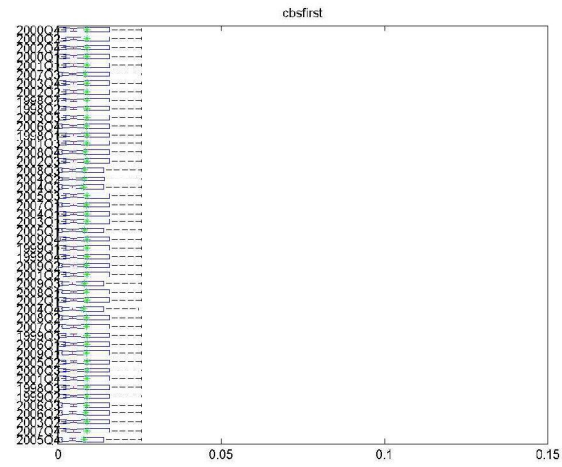
(h) CBS<sub>First</sub>

Figure 12: Jackknife Boxplots of Unconditional Methods

## B Tables

Table 8: Probability of Variables to Enter the Final Subset

<b>Table 8</b>	
<b>Conditional Methods</b>	
<b>Univariate Multiple Regression</b>	
Prob Chosen	Description
92.00%	TMB :motors, electronic components
32.00%	PMI:Production :output component
28.00%	BOI Companies Survey : Communication
24.00%	US Exports : Goods
24.00%	TPR :electronic components and different industrial equip.
16.00%	PMI:Domestic :supply :time component
4.00%	Cushing Crude Oil Price
4.00%	US Exports: Serivces
4.00%	PMI:Stock :purchase :quantity component
4.00%	Employer Survey : Transportation
<b>Elastic Net</b>	
<b>Alpha=0.4</b>	
<b>Quantile=0.75</b>	
Prob Chosen	Description
100.00%	High Tech Stock Index
100.00%	Cushing Crude Oil Price
100.00%	US Exports : Goods
100.00%	PMI : Employment :level component
100.00%	Tourist bed-nights in authorized and unauthorized hotels
100.00%	TPR :textiles
100.00%	TPR :electronic motors, components and transport equip.
100.00%	TPR :electronic components and different industrial equip.
100.00%	TMB :motors, electronic components
96.00%	Globes-Smith CCI
<b>Quantile=0.50</b>	
Prob Chosen	Description
100.00%	US Exports : Goods
100.00%	PMI : Employment :level component

Continued on next page

<b>Table 8 – continued from previous page</b>	
100.00%	TPR :electronic motors, components and transport equip.
100.00%	TPR :electronic components and different industrial equip.
100.00%	TMB :motors, electronic components
96.00%	Cushing Crude Oil Price
96.00%	TMB :electronic components and different industrial equipment
92.00%	High Tech Stock Index
92.00%	PMI : Euro 16
80.00%	General Stock index
<b>Quantile=0.25</b>	
Prob Chosen	Description
100.00%	US Exports : Goods
100.00%	PMI : Employment :level component
100.00%	PMI:Stock :purchase :quantity component
100.00%	TPR :electronic components and different industrial equip.
100.00%	TMB :motors, electronic components
100.00%	TMB :electronic components
100.00%	TMB :electronic components and different industrial equipment
96.00%	PMI : Euro 16
92.00%	TPR :electronic motors, components and transport equip.
84.00%	BOI Companies Survey : Communication
<b>LASSO</b>	
<b>Quantile=0.75</b>	
Prob Chosen	Description
100.00%	Tourist bed-nights in authorized and unauthorized hotels
100.00%	TPR :textiles
100.00%	TMB :Machinery and equipment
100.00%	TMB :motors, electronic components
96.00%	Cushing Crude Oil Price
92.00%	High Tech Stock Index
92.00%	US Exports : Goods
92.00%	TPR :electronic components and different industrial equip.
84.00%	RT : Consumption goods :Other
84.00%	Health Tax
<b>Quantile=0.50</b>	
Prob Chosen	Description
Continued on next page	

<b>Table 8 – continued from previous page</b>	
100.00%	TPR :electronic components and different industrial equip.
100.00%	TMB :motors, electronic components
92.00%	Cushing Crude Oil Price
92.00%	US Exports : Goods
68.00%	PMI : Employment :level component
68.00%	PMI : Euro 16
60.00%	Industrial Firms Stock Index
60.00%	Health Tax
60.00%	Tourist bed-nights in authorized and unauthorized hotels
60.00%	TPR :electronic communication equipment
<b>Quantile=0.25</b>	
Prob Chosen	Description
100.00%	TMB :motors, electronic components
84.00%	US Exports : Goods
84.00%	TPR :electronic components and different industrial equip.
80.00%	PMI:Stock :purchase :quantity component
72.00%	PMI : Euro 16
60.00%	BOI Companies Survey : Communication
36.00%	PMI : Employment :level component
36.00%	PMI:Production :output component
36.00%	TMB :electronic components
16.00%	PMI:Domestic :supply :time component
<b>Unconditional Methods</b>	
<b>PCA</b>	
<b>Two Component Norm Selection</b>	
Prob Chosen	Description
28.00%	Industrial Firms Stock Index
20.00%	PMI:Domestic :supply :time component
20.00%	TMB :Jewelry, goldmiths' and silvermiths' articles
16.00%	Employer Survey : Education
8.00%	General Stock index
8.00%	Israel Exports :Services (NIS)
4.00%	High Tech Stock Index
4.00%	TMB :food products
4.00%	TMB :food products, beverages and tobacco products
Continued on next page	

<b>Table 8 – continued from previous page</b>	
4.00%	TMB :textiles
<b>Iterated Componet Selection</b>	
<b>Prob Chosen</b>	<b>Description</b>
44.00%	PMI:Domestic :supply :time component
32.00%	TMB :Jewelry, goldmiths' and silvermiths' articles
32.00%	Employer Survey : Building
28.00%	Cushing Crude Oil Price
28.00%	Manufacturing exports (NIS)
24.00%	Industrial Firms Stock Index
20.00%	DOLLAR/NIS EXCHANGE RATE
16.00%	Health Tax
16.00%	Israel Exports :Services (NIS)
16.00%	PMI:Domestic :orders component
<b>SPCA</b>	
<b>Two Component Norm Selection</b>	
<b>Prob Chosen</b>	<b>Description</b>
16.00%	Triple trade index : Nominal
16.00%	PMI : Employment :level component
16.00%	Employer Survey : Food
12.00%	PMI:Domestic :supply :time component
12.00%	TPR :electronic motors, components and transport equip.
12.00%	TMB :motors, electronic components
12.00%	PMI : Euro 16
8.00%	US Exports: Serivces
8.00%	Revenue index :Community, social, personal and other services
8.00%	Revenue index :Commerce and services
<b>Iterated Componet Selection</b>	
<b>Prob Chosen</b>	<b>Description</b>
32.00%	Tel Aviv 100
28.00%	Cushing Crude Oil Price
28.00%	TMB :industrial equipment for control and supervision
24.00%	PMI:Domestic :supply :time component
20.00%	Hotels :no. of bed-nights in tourist hotels :Israeli
20.00%	TMB :wearing apparel
16.00%	General Stock index
Continued on next page	

<b>Table 8 – continued from previous page</b>	
12.00%	US Exports: Services
12.00%	Israel Exports :Services (NIS)
12.00%	PMI:Stock of finished :goods component

Table 9: List of Variables in the General Set

<b>Index</b>	<b>Description</b>
	BOI: Bank of Israel, RT: Retail Trade, TPR: Industrial Production Index, TMB: Employed Persons' Index, PMI: Purchasing Manager's Index, CCI: Consumer Confidence Index, RT: Retail Trade, THP: Man-Hours Worked Index
1	BOI Companies Survey : Building
2	BOI Companies Survey : Communication
3	BOI Companies Survey : Industry
4	BOI Companies Survey : Retail
5	Capital Utilization Index
6	Cushing Crude Oil Price
7	Dollar/NIS Exchange Rate
8	Employer Survey : Agriculture
9	Employer Survey : Building
10	Employer Survey : Education
11	Employer Survey : Financial
12	Employer Survey : Food
13	Employer Survey : Health
14	Employer Survey : Industry
15	Employer Survey : Real
16	Employer Survey : Trade
17	Employer Survey : Transportation
18	EURO/NIS EXCHANGE RATE
19	General Concert Bonds Stock index
20	General Stock index
21	Globes-Smith CCI
22	GOLD : Market Rate
23	Gross Capital Stock : Business Sector
24	Health Tax

Continued on next page

**Table 9 – continued from previous page**

Index	Description
	BOI: Bank of Israel, RT: Retail Trade, TPR: Industrial Production Index, TMB: Employed Persons' Index, PMI: Purchasing Manager's Index, CCI: Consumer Confidence Index, RT: Retail Trade, THP: Man-Hours Worked Index
25	High Tech Stock Index
26	Hotels : No. of bed-nights in tourist hotels :total
27	Hotels : No. of bed-nights in tourist hotels :Israeli
28	Housing completions :Total
29	Housing starts :public sector
30	Housing starts :Total
31	Imports :Consumer goods (NIS)
32	Imports :Investment goods (NIS)
33	Imports :Net (NIS)
34	Industrial Firms Stock Index
35	Israel Exports :Goods (NIS)
36	Israel Exports :Services (NIS)
37	Israel Imports :Goods (NIS)
38	Israel Imports :Services (NIS)
39	Manufacturing exports (NIS)
40	Michigan CCI
41	MSCI : Currency(NIS)
42	No. of tourist arrivals :total
43	No. of tourist arrivals, by air passengers
44	PMI : Employment :level component
45	PMI : Euro 16
46	PMI : USA
47	PMI : Domestic :orders component
48	PMI : Domestic :supply :time component
49	PMI : Global :orders component
50	PMI : Import :supply :time component
51	PMI : Production :output component
52	PMI : Raw :material :stock :levels component
53	PMI : Stock :purchase :prices component
54	PMI : Stock :purchase :quantity component
55	PMI : Stock of finished :goods component
Continued on next page	



**Table 9 – continued from previous page**

Index	Description
	BOI: Bank of Israel, RT: Retail Trade, TPR: Industrial Production Index, TMB: Employed Persons' Index, PMI: Purchasing Manager's Index, CCI: Consumer Confidence Index, RT: Retail Trade, THP: Man-Hours Worked Index
56	Price index of dwellings
57	Purchasing Managers Index :Total
58	Real Effective Exchange Rate
59	Residential building :Completions :private sector
60	Residential building :Completions :public sector
61	Residential building :Starts :private sector
62	Revenue index :Accommodation services and restaurants
63	Revenue index :Banking, insurance and other Financial institutions
64	Revenue index :Business activities
65	Revenue index :Commerce and services
66	Revenue index :Community, social, personal and other services
67	Revenue index :Education
68	Revenue index :Health, welfare & social work services
69	Revenue index :Wholesale and retail trade, and repairs
70	RT : Consumption goods :Other
71	RT : Durables
72	RT : Footwear
73	RT : Textile and clothing
74	RT : Total excl. gas, fertilizers and petroleum
75	RT :Food
76	RT :Kitchen and house accessories
77	RT :Petroleum
78	Tel Aviv 100
79	THP : basic metal
80	THP : beverages and tobacco products
81	THP : chemicals and their products
82	THP : components
83	THP : electronics communication equipment
84	THP : furniture
85	THP : High technology
86	THP : industrial equip. for control
Continued on next page	

**Table 9 – continued from previous page**

Index	Description
	BOI: Bank of Israel, RT: Retail Trade, TPR: Industrial Production Index, TMB: Employed Persons' Index, PMI: Purchasing Manager's Index, CCI: Consumer Confidence Index, RT: Retail Trade, THP: Man-Hours Worked Index
87	THP : industry : index
88	THP : Jewelry, goldsmiths' and silversmiths' articles
89	THP : Low technology
90	THP : Machinery and equipment
91	THP : Medium-high technology
92	THP : Medium-low technology
93	THP : metal products
94	THP : motors, electronic components and equip.
95	THP : other mining and quarrying
96	THP : textiles & wearing apparel
97	THP : textiles
98	THP : Transport equipment
99	THP : wood and its products & furniture
100	TMB : chemicals and their products
101	TMB :basic metal
102	TMB :beverages and tobacco products
103	TMB :electric motors and electric distribution apparatus
104	TMB :electronic components
105	TMB :electronic components and different industrial equipment
106	TMB :food products
107	TMB :food products, beverages and tobacco products
108	TMB :footwear, leather and its products
109	TMB :furniture
110	TMB :industrial equipment for control and supervision
111	TMB :Jewelry, goldsmiths and silversmiths articles
112	TMB :Machinery and equipment
113	TMB :Manufacture of plastic and rubber products
114	TMB :manufacturing n.e.c
115	TMB :metal products
116	TMB :motors, electronic components
117	TMB :non-metallic mineral products
Continued on next page	

**Table 9 – continued from previous page**

Index	Description
	BOI: Bank of Israel, RT: Retail Trade, TPR: Industrial Production Index, TMB: Employed Persons' Index, PMI: Purchasing Manager's Index, CCI: Consumer Confidence Index, RT: Retail Trade, THP: Man-Hours Worked Index
118	TMB :other mining and quarrying
119	TMB :paper and its products
120	TMB :publishing and printing
121	TMB :textiles
122	TMB :textiles & wearing apparel
123	TMB :Transport equipment
124	TMB :wearing apparel
125	TMB :wood and its products & furniture
126	TMB :wood and wood products (excl. furniture)
127	Total Tax Collection
128	Tourist bed-nights in authorized and unauthorized hotels
129	TPR :Other Branches
130	TPR :beverages and tobacco products
131	TPR :building products :paper and its products
132	TPR :electronic components and different industrial equip.
133	TPR :electronic motors, components and transport equip.
134	TPR :electronics communication equipment
135	TPR :food products, beverages and tobacco products
136	TPR :furniture
137	TPR :High technology
138	TPR :industrial equipment for control and supervision
139	TPR :Jewelry, goldsmiths and silversmiths articles
140	TPR :Low technology
141	TPR :Medium-high technology
142	TPR :Medium-low technology
143	TPR :other mining and quarrying
144	TPR :refined petroleum & nuclear fuel chemicals etc.
145	TPR :textiles
146	TPR :textiles & wearing apparel & footwear etc.
147	TPR :total (excl. diamonds)
148	TPR :Transport equipment
Continued on next page	

**Table 9 – continued from previous page**

<b>Index</b>	<b>Description</b>
	BOI: Bank of Israel, RT: Retail Trade, TPR: Industrial Production Index, TMB: Employed Persons' Index, PMI: Purchasing Manager's Index, CCI: Consumer Confidence Index, RT: Retail Trade, THP: Man-Hours Worked Index
149	TPR :wood and its products & furniture
150	TPR :Metal and machinery
151	Treasury bills: Fixed interest 1 month to redemption
152	Triple trade index : Nominal
153	Triple trade index : Real
154	US Exports : Goods
155	US Exports: Services
156	V.A.T. on Domestic Production