עיונים מוניטריים

Monetary Studies

Aggregation versus Disaggregation -

What can we learn from it?

Alex Ilek

March 2007

2007.02

Discussion Papers מאמרים לדיון





בנק ישראל המחלקה המוניטרית

Aggregation versus Disaggregation -

What can we learn from it?

Alex Ilek

March 2007

2007.02

The views expressed in this paper are those of the authors only, and do not necessarily represent those of the Bank of Israel.

Email: <u>alexilek@boi.gov.il</u>

© Bank of Israel Passages may be cited provided source is specified Catalogue no. 3111507002/6

Monetary Department, Bank of Israel, POB 780, Jerusalem 91007 <u>http://www.bankisrael.gov.il</u>

אגרגציה כנגד דיסאגרגציה-מה אנו יכולים ללמוד מזה ?

אלכס אילק

תקציר

עבודה זו מצביעה על אפשרות שזיהוי הרלוונטיות של המשתנים המסבירים את התפתחות האינפלציה (המדד הכללי) בשיטה המקובלת - האגרגטיבית - לוקה בהטיה, אשר עלולה להביא להוצאת משתנים מסבירים חשובים ממשוואת האינפלציה. לפיכך אני מציע שיטה חלופית לזיהוי הרלוונטיות של המשתנים הכלכליים באמידת תהליך האינפלציה - שיטה דיסאגרגטיבית.

אני משווה את הביצועים של שני מודלים מבניים חודשיים לתהליך האינפלציה בישראל - אגרגטיבי ודיסאגרגטיבי - שבמסגרתם ריבית בנק ישראל והמשתנים האחרים נקבעים סימולטנית עם האינפלציה. נמצא שהדיסאגרגציה מעלה את הדיוק בחיזוי האינפלציה, ובעקבותיו - את היעילות בניהול המדיניות המוניטרית.

העבודה בוחנת באמצעות שני המודלים גם את ההשלכות של אינפורמציה לא מלאה לגבי המשתנים הבלתי נצפים וכן את ההשלכות של זיהוי מוטעה של תהליך אינפלציוני במשק על פונקציית ההפסד של המדיניות המוניטרית. נמצא שלפי המודל הדיסאגרגטיבי ההפסד קטן יותר מאשר לפי המודל האגרגטיבי.

Aggregation versus Disaggregation -What can we learn from it?

Alex Ilek¹

Abstract:

This paper identifies a possible bias in the relevance of certain variables explaining CPI inflation when estimated on an aggregate basis. Since the bias can lead to an unjustified exclusion of variables from the CPI inflation regression, I propose the alternative technique in order to better identify the relevance of economic variables in explaining CPI inflation.

This paper also shows that the approach of modeling inflation through the disaggregation of its components is superior to the aggregated approach, because disaggregation increases both the accuracy of inflation forecasts and the efficiency of monetary policy management. I compare the performance of the aggregated and disaggregated monthly structural models for inflation processes in Israel, whereby the central bank interest rate and other economic variables are determined simultaneously with the inflation rate.

Within these two different structural models, I also examine the implications on the central bank loss function of incomplete information regarding certain unobserved economic variables, and misidentification of the true inflation process in the economy. It was found that the disaggregated model generates smaller loss to the economy then does the aggregated model.

¹ Monetary Department, Bank of Israel. I would like to thank A.Offenbacher, D. Elkayam and my colleges in the Monetary Department for their helpful comments on this work. Special thanks to E. Azoulay for his help and important advice.

1) Introduction

Most central banks in the world use price stability as their principal monetary policy objective². In order to achieve price stability, the central bank changes its policy instrument (generally the interest rate), according to its expectation of price developments in the future, thereby positioning the enhanced accuracy of inflation projections as a key monetary-policy task.

The quality of forecasts of any economic variable including inflation, depends on: the identification of the data generating process of the relevant economic variables (specification), and the identification of their parameters (through the use of estimation or calibration methods).

The New-Keynesian theory (Calvo (1983), (C.G.G [1997]) supports the theory that the process of price adjustment by firms is not continuous, mainly because these adjustments are costly. When firms set prices for their goods, they take account of the expected price level in the future and of the level of real activity. Price setting may also be influenced by past price developments (inflation inertia); (see Fuhrer and Moore [1995]) and, in the small open economy, prices are affected directly by exchange rate changes and world prices (see Svensson (2000)).

In reality however, changes in separate CPI components may not be consistent with these theoretical principles. Indeed, some components are affected by prices that are controlled/supervised by the government rather than determined by market forces (energy prices for example). In other components, prices are determined primarily by old habits of the different agents in the economy. Most noticeable in this respect are housing rentals, which are indexed to the shekel price of dollar, despite the fact that housing services are not considered a tradable commodity.

The fact that prices of certain CPI components react in Israel in a different manner than expected on the basis of their "theoretical" considerations calls for the development of a unique approach to modeling Israel's inflation³. Disaggregating the CPI to its main components and explaining each one through its unique explanatory variables could improve the estimation and reduce distortions identified in the aggregated CPI estimation. At the same time, disaggregation entails risks of mis-specifying some of the CPI components, undermining the forecast quality of price

² Israel's inflation target range has been between 1 to 3 percent since 2003, after a long disinflation process experienced during the 1990s.

³ Based on the New-Keynesian Theory.

changes in the different components and leading to a possible weak forecast of total CPI⁴. A decision on the optimal degree of disaggregation should therefore include considerations on the important tradeoff between the advantages and disadvantages of disaggregation. In this context, one important factor concerns our level of certainty about the data generating process of these components.

The question of aggregation versus disaggregation has been debated by many researchers in theoretical and empirical economic literature⁵ since the 1950s. Theil (1954), Grunfeld and Griliches (1960), and others mentioned the following advantages inherent in the disaggregation of economic variables: (1) Disaggregation enhances flexibility in the specification and in the dynamic structure of each component and thereby brings efficiency to the use of the relevant information that eventually improves the forecast of an aggregate variable such as the CPI. (Barker and Pesaran [1990]); (2) Disaggregation provides complete/partial offset of residuals derived from the separately estimated components. This offset reduces the variance of the unexplained part of the goal variable (CPI) and thus enables more efficient forecast of it; (3) Disaggregation enables examination of the difference between responses of the CPI components to shocks in the economy (see Bils, Klenow and Krystov (2003), Demers and Champlain (2005). This is crucial for monetary policymakers as they can more easily identify the source of inflation pressures in the economy; (4) Disaggregation could point to the nature of the inflation pressures whether they are transitory (seasonal factors, energy etc.) or whether they are permanent and reflected in core inflation. This issue has a crucial impact on the conduct of monetary policy. Along with the advantages of disaggregation, the following are its main drawbacks: (1) The wrong specification of the components may bias the inflation forecast for the goal variable (CPI); (2) Even when specification is handled correctly, the inclusion of a higher number of explanatory variables in the disaggregated equation tends to increase the effect of measurement errors on the accurate forecasting of the CPI; (3) The residuals derived from each component may not offset and more efficient forecasting of the aggregate variable will therefore not be achieved.

⁴ The forecast of total CPI is calculated as a weighted average of its components forecasts using their weight in the CP index.

⁵ In the economic literature the question of disaggregation is relevant not only to the CPI, but also in other fields. For example Marcellino, Stock and Watson (2001) examined if the forecast of inflation in the in the Euro area can be improved by making separate forecasts for each country included in eurozone and then aggregate this forecast - versus direct forecast of inflation in the eurozone.

The lack of unequivocallity about disaggregation has been reflected also in the empirical work of many researchers. Lutkepohl (1987) and Hubrish (2004) conclude, based on simulations in small samples, that it is not necessarily better to disaggregate CPI by its components (and model each component separately) in order to improve the forecast for the inflation of CPI. The superiority of the disaggregation versus direct forecasting of inflation, depends mainly on the correct identification of specification and on the estimated parameters in each component's equation. Reijer and Vlaar (2003) conducted empirical tests using the Netherlands data and did not find persuasive evidence in favor of disaggregation. Their results showed that disaggregation improves the inflation forecast only in the short run, while in the medium and in the long run the direct forecasting of inflation is better. Benatal et al. (2004) arrived at a similar conclusion based on the eurozone data. Demers and Champlain (2005), who used data on Canada, obtained more encouraging results: separate modeling of the CPI component significantly improved inflation forecasts for all terms, particularly for the short term. An alternative approach to the improvement of CPI inflation forecasting using VAR was proposed by Hendry and Hubrish (2006). They showed theoretically that the inclusion of lags of individual components in the equation of CPI inflation, in addition to the lags of CPI inflation itself, improves the forecast of CPI inflation. However, empirical examinations in the USA and in the Euro-Area provided partial support for their theoretical findings.

One important reason why these studies have failed to improve inflation forecasts through disaggregation can be connected to the fact that the CPI components were treated homogenously, meaning that each component was explained by the same block of variables. This leads to an "over fitting" phenomenon, eventually deteriorating out of sample inflation forecasts. Additionally, ignoring certain important unique variables explaining separate CPI components could detract from the forecast results.

In Israel, the need to disaggregate the CPI and model certain of its components separately was emphasized in empirical investigations. Bruno and Sussman (1979), Azoulay and Elkayam (1997) based their theoretical framework on the assumption that the CPI is an average of two components: tradable and non-tradable goods. Notwithstanding the recognition for this separation, they choose to model the CPI inflation directly, because of their practical and theoretical difficulties involving the separate component estimations. Suchoy and Rotberger (2006) separated the CPI into fourteen subgroups in the consumption basket and estimated each

of them using X-12-ARIMA method. Even though the authors did not discuss specifically the benefits of disaggregation, they mentioned the importance of the addition of certain economic variables in improving the accuracy of their model inflation forecasts.

It should be noted that the above mentioned literature has dealt with the two approaches of inflation forecasting based on pure statistical models. These statistical models, such as ARIMA and VAR, used a few economic variables influencing the CPI, and were characterized with many exogenous variables that are not determined within the model. Furthermore, with these statistical models it is unreasonable to forecast inflation for more than one or two periods ahead. This is because in real time the explanatory variables (for example, the exchange rate, interest rate, world prices etc.) that are not determined within the model, are known only ex-post. Assuming some exogenous path of these variables, in most cases, would not be consistent with the path of inflation and other variables. Moreover, in an inflation targeting regime, the assumption that the interest rate can be constant or exogenous during the forecast period is "inherently problematic and confusing..., it should be abandoned sooner rather than later" (Svensson [2006]). Undoubtedly, the lack of structural relationships within these statistical models is a key reason for their failure to providing comprehensive answers to the following questions: What is the derived long-run inflation solution? Is this solution consistent with the central bank inflation target? What should be the policy rate consistent with the monetary policy objective? Therefore, in order to produce improved short and medium term inflation forecasts, it is crucial to develop a structural model where the monetary policy instrument (the interest rate) and other economic variables are endogenous, and are determined simultaneously with inflation.

This study seeks to compare disaggregated and aggregated inflation forecasts in a novel structural model estimated monthly. Under this approach, other variables, such as interest rate, output gap and other economic variables, are determined simultaneously with the CPI inflation. The structural model, constructed under two versions, is based on the New-Keynesian theory, but emphasis is given to its empirical characteristics. Furthermore, the model enhances policy-makers' ability to analyze the impact of the incomplete information of some crucial variables on the central bank loss function⁶.

⁶ The monetary policy loss function is present below.

This paper also discusses the important possible bias in the relevance of some variables explaining CPI inflation when estimated aggregatively⁷. I show that the bias exists in some CPI components that are mainly characterized with unique explanatory variables, which could then lead to an unjustified exclusion of variables from the CPI inflation regression. Surprisingly, the bias in the relevance is also found in common factor variables that tend to affect all components in the CPI. I therefore propose in this paper an alternative technique in order to better identify the relevance of explanatory variables in the CPI equation, even though it may appear less statistical. The paper is organized as follows: Chapter 1 introduces the bias in the relevance of some variables explaining CPI inflation when estimated aggregatively. Chapter 2 compares the goodness of fit in the aggregated and disaggregated measures. Chapter 3 discusses the consequences of the erroneous assessment of unobserved variables and true inflation process in the economy on monetary policy loss function. Eventually, the main conclusions of this paper are presented in Chapter 4.

1. Heterogeneous Behavior of CPI Components - Consequences for Statistical Identification in Inflation Estimation, Performed Through the Aggregative Approach

This chapter discusses identification issues involved in estimating CPI inflation using the aggregative approach. I claim that the heterogeneous behavior of CPI components and the presence of a positive correlation between their shocks, may mistakenly decrease the significance level of certain explanatory variables included in the CPI inflation equation. These variables consist of common factor variables, affecting all CPI components, and unique variables, affecting only specific components. Specifically, I show that under some conditions, there is a tendency to accept the null hypothesis (and infer the lack of significance of the explanatory variables), when the null hypothesis is virtually false, resulting in a second order error.

⁷ It is important to note that the concept "bias in the relevance" here refers to the likelihood of unintentionally omitting important variables in the estimation of aggregated CPI equation. It doesn't deal with bias in distribution of errors in the aggregated equation resulting from real statistical issues in parameters. One main assumption in this research is that the errors in the aggregated equation are independent and identically distributed.

1.1 Describing CPI Components Behavior

CPI is defined as a weighted average of its components:

$$p = \sum_{i=1}^{n} \alpha_i p_i(\alpha_i)$$

Where $p_i(\alpha_i)$ is the price level of the *i* component and α_i is its weight, i = 1...n

For the sake of simplicity, we will assume that the CPI includes only two components, p_1 and p_2 , the CPI is then:

(1)
$$p_t = \alpha p_t^1 + (1 - \alpha) p_t^2 \implies \Delta p_t = \alpha \Delta p_t^1 + (1 - \alpha) \Delta p_t^2$$

where α is the first component weight.

I will now examine two cases. The first case examines the bias in the relevance of common factor variables and in the second case this issue is examined when different explanatory variables are included in each of the CPI components.

1.1.1 Case 1: Existence of a Common Factor

Assume that the true data generating process of these two components is described as follows:

(2) $\Delta p_t^1 = \beta_1 x_t + \gamma z_t + \varepsilon_t^1 \quad \varepsilon^1 \sim N(0, \sigma_1^2)$ (3) $\Delta p_t^2 = \beta_2 x_t + \varepsilon_t^2 \quad \varepsilon^2 \sim N(0, \sigma_2^2)$

where x is the common group of explanatory variables included in each component, and z is the unique group of explanatory variables included only in the first component. z includes factors affecting the relative price of specific components, where x includes factors reflecting inflationary pressures in the whole economy. I assume that x, z are weakly exogenous, making it possible to estimate the two components by means of the O.L.S. method. Substituting (2) and (3) into (1) gives the true data generating process of total CPI.

(4)
$$\Delta p_t = \alpha (\beta_l x_t + \gamma z_t + \varepsilon_t^l) + (1 - \alpha) (\beta_2 x_t + \varepsilon_t^2) = [\alpha \beta_l + (1 - \alpha) \beta_2] x_t + \alpha \gamma z_t + [\alpha \varepsilon_t^l + (1 - \alpha) \varepsilon_t^2]$$

After rearranging (4) the true CPI inflation behavior is:

(5)
$$\Delta p_t = \beta^* x_t + \gamma^* z_t + \varepsilon_t^*$$
$$[\beta^* = \alpha \beta_1 + (1 - \alpha) \beta_2; \gamma^* = \alpha \gamma; \varepsilon^* = \alpha \varepsilon^1 + (1 - \alpha) \varepsilon^2]$$

Practically, estimation of (5) in finite samples should provide consistent parameter estimators. However, these estimators may not be statistically significant and may therefore unjustifiably result in the exclusion of important variables explaining inflation, leading to a reduction in the equation's ability to accurately represent the inflation process and result in it producing failed forecasts. The factors affecting the statistical significance of the <u>unique group z</u> are: (1) The weight of component

1 in the CPI (α); (2) The variances ratio of shocks in components 1 and 2 $(\frac{\sigma_2^2}{\sigma_1^2})$; (3) The correlation

between shocks in components 1 and 2 (ρ_{12}). The factor affecting the statistical significance of common factor variables is the correlation between shocks in components 1 and 2 (ρ_{12}).

1.1.2 Case 2: Non-Existence of a Common Factor

Let us assume now that the true data generating process of the two components is as follows:

(6) $\Delta p_t^l = \beta_l x_t + \gamma z_t + \varepsilon_t^l \quad \varepsilon^l \sim N(0, \sigma_l^2)$ (7) $\Delta p_t^2 = \beta_2 y_t + \varepsilon_t^2 \quad \varepsilon^2 \sim N(0, \sigma_2^2)$

It can be seen that the explanatory variables included in the first component are completely different and independent from those included in the second component. If I substitute (6) and (7) into (1) the <u>true</u> CPI inflation process will be:

(8)
$$\Delta p_t = \alpha(\beta_l x_t + \gamma z_t + \varepsilon_t^l) + (1 - \alpha)(\beta_2 y_t + \varepsilon_t^2) = \alpha\beta_l x_t + \alpha\gamma z_t + (1 - \alpha)\beta_2 y_t + \alpha\varepsilon_t^l + (1 - \alpha)\varepsilon_t^2$$

Rewriting (8):

$$\begin{array}{l} (8') \quad \Delta p_t = \alpha^* x_t + \beta_1^* z_t + \beta_2^* y_t + \varepsilon_t^* \\ \alpha^* = \alpha \beta_1, \beta_1^* = \alpha \gamma, \beta_2^* = (1-\alpha)\beta_2, \varepsilon^* = \alpha \varepsilon_t^1 + (1-\alpha)\varepsilon_t^2 \end{array}$$

As in case 1, the estimated coefficients can be consistent but the possibility of unjustifiably dropping some important variables remains also here. <u>The proof we see later.</u>

1.2 Mathematical Proofs

1.2.1 Case 1: With Common Factors

A. The identification of unique variables (z)

I analyze now the factors that cause the bias in statistical relevance of unique variables (z). Assume that each group of the explanatory variables x, y includes only one element. In addition assume independence between x and z^8 . Define:

$$s=S-S \quad (S\subset x,z,y,\varDelta p_1,\varDelta p_2,\varDelta p \)$$

From the estimation of equation (2) I can obtain the estimated parameter and its variance:

(9)
$$\hat{\gamma}_{l} = \frac{\sum z_{i} \Delta p_{li}}{\sum z_{i}^{2}}, \quad var(\hat{\gamma}_{l}) = \frac{\sigma_{l}^{2}}{\sum z_{i}^{2}}.$$

The t-statistic is derived as follows:

(10)
$$t(\gamma_{I}) = \frac{\gamma_{I}}{S.E.(\gamma_{I})} = \frac{\sum z_{i} \Delta p_{Ii}}{\sum z_{i}^{2}} / \sqrt{\frac{\sigma_{I}^{2}}{\sum z_{i}^{2}}} = \frac{\sum z_{i} \Delta p_{Ii}}{\sigma_{I} \sqrt{\sum z_{i}^{2}}}$$

Equation (10) can be transformed to the following alternative form:

(11)
$$t(\gamma_1)\sigma_1 = \frac{\sum z_i \Delta p_{Ii}}{\sqrt{\sum z_i^2}}$$

The estimated parameter from estimating (5) is:

(12)
$$\hat{\gamma} = \frac{\sum \Delta p_i z_i}{\sum z_i^2} = \frac{\sum (\alpha \Delta p_{1i} + (1 - \alpha) \Delta p_{2i}) z_i}{\sum z_i^2} = \frac{\alpha \sum \Delta p_{1i} z_i}{\sum z_i^2} = \alpha \hat{\gamma}_1$$

Before I present the variance of γ , it is necessary to examine the statistical properties of shocks in the aggregated equation (5):

$$\varepsilon_t^* = \alpha \varepsilon_t^1 + (1 - \alpha) \varepsilon_t^2$$

Now the variance of the shock is:

$$var(\varepsilon_{t}^{*}) = \alpha^{2}\sigma_{1}^{2} + (1-\alpha)^{2}\sigma_{2}^{2} + 2\alpha(1-\alpha)\sigma_{12} \quad (\sigma_{12} = \rho_{12}\sigma_{1}\sigma_{2})$$

In the following step I calculate the variance of γ :

⁸ Violation of this condition will complicate calculations; the main conclusion remains.

(13)
$$\operatorname{var}(\hat{\gamma}) = \frac{\operatorname{var}(\varepsilon_t^*)}{\sum z_i^2} = \frac{\alpha^2 \sigma_1^2 + (1-\alpha)^2 \sigma_2^2 + 2\alpha (1-\alpha) \rho_{12} \sigma_1 \sigma_2}{\sum z_i^2}$$

Its t-statistic is:

(14)
$$\hat{t(\gamma)} = \frac{\alpha \sum \Delta p_{li} z_i}{\sum z_i^2} / \sqrt{\frac{\alpha^2 \sigma_l^2 + (1-\alpha)^2 \sigma_2^2 + 2\alpha (1-\alpha) \rho_{l2} \sigma_l \sigma_2}{\sum z_i^2}}$$

Rearranging (14):

(14')
$$\hat{t(\gamma)} = \frac{\alpha \sum \Delta p_{1i} z_i}{\sqrt{\sum z_i^2} \sqrt{\alpha^2 \sigma_1^2 + (1-\alpha)^2 \sigma_2^2 + 2\alpha (1-\alpha) \rho_{12} \sigma_1 \sigma_2}}$$

Substitute (11) into (14'):

(15)
$$t(\gamma) = \frac{\alpha \sigma_1}{\sqrt{\alpha^2 \sigma_1^2 + (1-\alpha)^2 \sigma_2^2 + 2\alpha(1-\alpha)\rho_{12}\sigma_1\sigma_2}} t(\gamma_1)$$

Divide both the numerator and denominator by σ_1 :

(16)
$$\frac{t(\gamma)}{t(\gamma_1)} = \frac{\alpha}{\sqrt{\alpha^2 + (1-\alpha)^2 k^2 + 2\alpha(1-\alpha)\rho_{12}k}}$$

Equation (16) is divided by the weight α :

(17)
$$\frac{\hat{t(\gamma)}}{\hat{t(\gamma_{1})}} = \frac{1}{\sqrt{\delta^{2}k^{2} + 2\delta k\rho_{12} + 1}}$$

Where $k = \frac{\sigma_2}{\sigma_1}$, $\delta = \frac{1-\alpha}{\alpha}$.

The t-statistic ratio in the first component equation and in the CPI equation depends on three factors: the weight of the first component in the CPI, the variance ratio between shocks in both components, and the correlation between these shocks.

Now I identify the conditions under which the $t(\gamma)$ has a downward bias:

$$(18) \quad \frac{1}{\sqrt{\delta^2 k^2 + 2\delta k \rho_{12} + 1}} < 1$$

Rearranging (18):

 $(19) \qquad \delta^2 k^2 + 2\delta k \rho_{12} > 0$

$$(19') \qquad \rho_{12} > -\frac{\delta k}{2}$$

Eventually, the condition for a bias in the statistical inference is:

(20)
$$\rho_{12} > \frac{(\alpha - 1)}{2\alpha}k$$

Condition (20) is necessarily valid if $\rho_{12} \ge 0$ (because the right hand side of the term is always negative). Additionally, in a special case when the weight α is sufficiently large and there is strong negative correlation between shocks, the t-statistic in the aggregated CPI equation is larger then in the equation of the first component.

B. The identification of common factor variables (x)

So far I have analyzed the identification problem of the unique variable z in estimating the CPI inflation equation (aggregative approach). Now, I perform similar analysis on common factor variables x. The estimated parameter of variable x and its variance can be calculated using equation (2):

(21)
$$\hat{\beta}_{I} = \frac{\sum x_{i} \Delta p_{Ii}}{\sum x_{i}^{2}} \operatorname{var}(\hat{\beta}_{I}) = \frac{\sigma_{I}^{2}}{\sum x_{i}^{2}}$$

Similarly, the estimated parameter of variable x and its variance can be calculated from (3):

(22)
$$\hat{\beta}_2 = \frac{\sum x_i \Delta p_{2i}}{\sum x_i^2} \operatorname{var}(\hat{\beta}_2) = \frac{\sigma_2^2}{\sum x_i^2}$$

The t-statistic of $\hat{\beta}_{l}$ (from equation 2) is:

$$(23) \quad t(\hat{\beta}_{1}) = \frac{\beta_{1}}{S.E.(\hat{\beta}_{1})} = \frac{\sum x_{i} \Delta p_{1i}}{\sum x_{i}^{2}} / \sqrt{\frac{\sigma_{1}^{2}}{\sum x_{i}^{2}}} = \frac{\sum x_{i} \Delta p_{1i}}{\sigma_{1} \sqrt{\sum x_{i}^{2}}} \qquad \Longrightarrow t(\hat{\beta}_{1}) \sigma_{1} = \frac{\sum x_{i} \Delta p_{1i}}{\sqrt{\sum x_{i}^{2}}}$$

The t-statistic of $\hat{\beta}_2$ (from equation 3) is:

$$(24) \ t(\hat{\beta}_{2}) = \frac{\hat{\beta}_{2}}{S.E.(\hat{\beta}_{2})} = \frac{\sum x_{i} \Delta p_{2i}}{\sum x_{i}^{2}} / \sqrt{\frac{\sigma_{2}^{2}}{\sum x_{i}^{2}}} = \frac{\sum x_{i} \Delta p_{2i}}{\sigma_{2} \sqrt{\sum x_{i}^{2}}} \qquad \Rightarrow t(\hat{\beta}_{2})\sigma_{2} = \frac{\sum x_{i} \Delta p_{2i}}{\sqrt{\sum x_{i}^{2}}}$$

The estimated parameter β and its variance in the CPI inflation equation (5) are:

(25)
$$\hat{\beta} = \frac{\sum \Delta p_i x_i}{\sum x_i^2} = \frac{\sum (\alpha \Delta p_{1i} + (1 - \alpha) \Delta p_{2i}) x_i}{\sum x_i^2} = \alpha \hat{\beta}_1 + (1 - \alpha) \hat{\beta}_2$$

(26) $var(\hat{\beta}) = \frac{var(\varepsilon_t^*)}{\sum x_i^2} = \frac{\alpha^2 \sigma_1^2 + (1 - \alpha)^2 \sigma_2^2 + 2\alpha (1 - \alpha) \rho_{12} \sigma_1 \sigma_2}{\sum x_i^2}$

The t-statistic can be immediately derived:

$$(27) \quad t(\hat{\beta}) = \frac{\alpha \sum \Delta p_{1i} x_i + (1-\alpha) \sum \Delta p_{2i} x_i}{\sum x_i^2} / \sqrt{\frac{\alpha^2 \sigma_1^2 + (1-\alpha)^2 \sigma_2^2 + 2\alpha (1-\alpha) \rho_{12} \sigma_1 \sigma_2}{\sum x_i^2}} \implies (27') \quad t(\hat{\beta}) = \frac{\alpha \sigma_1 t(\hat{\beta}_1) + (1-\alpha) \sigma_2 t(\hat{\beta}_2)}{\sqrt{\alpha^2 \sigma_1^2 + (1-\alpha)^2 \sigma_2^2 + 2\alpha (1-\alpha) \rho_{12} \sigma_1 \sigma_2}} \implies (27') \quad t(\hat{\beta}) = \frac{\alpha \sigma_1 t(\hat{\beta}_1) + (1-\alpha) \sigma_2 t(\hat{\beta}_2)}{\sqrt{\alpha^2 \sigma_1^2 + (1-\alpha)^2 \sigma_2^2 + 2\alpha (1-\alpha) \rho_{12} \sigma_1 \sigma_2}} \implies (27') \quad t(\hat{\beta}) = \frac{\alpha \sigma_1 t(\hat{\beta}_1) + (1-\alpha) \sigma_2 t(\hat{\beta}_2)}{\sqrt{\alpha^2 \sigma_1^2 + (1-\alpha)^2 \sigma_2^2 + 2\alpha (1-\alpha) \rho_{12} \sigma_1 \sigma_2}} \implies (27') \quad t(\hat{\beta}) = \frac{\alpha \sigma_1 t(\hat{\beta}_1) + (1-\alpha) \sigma_2 t(\hat{\beta}_2)}{\sqrt{\alpha^2 \sigma_1^2 + (1-\alpha)^2 \sigma_2^2 + 2\alpha (1-\alpha) \rho_{12} \sigma_1 \sigma_2}} \implies (27') \quad t(\hat{\beta}) = \frac{\alpha \sigma_1 t(\hat{\beta}_1) + (1-\alpha) \sigma_2 t(\hat{\beta}_2)}{\sqrt{\alpha^2 \sigma_1^2 + (1-\alpha)^2 \sigma_2^2 + 2\alpha (1-\alpha) \rho_{12} \sigma_1 \sigma_2}} \implies (27') \quad t(\hat{\beta}) = \frac{\alpha \sigma_1 t(\hat{\beta}_1) + (1-\alpha) \sigma_2 t(\hat{\beta}_2)}{\sqrt{\alpha^2 \sigma_1^2 + (1-\alpha)^2 \sigma_2^2 + 2\alpha (1-\alpha) \rho_{12} \sigma_1 \sigma_2}} \implies (27') \quad t(\hat{\beta}) = \frac{\alpha \sigma_1 t(\hat{\beta}_1) + (1-\alpha) \sigma_2 t(\hat{\beta}_2)}{\sqrt{\alpha^2 \sigma_1^2 + (1-\alpha)^2 \sigma_2^2 + 2\alpha (1-\alpha) \rho_{12} \sigma_1 \sigma_2}} \implies (27')$$

Equation (27') is divided now by σ_1 and by $(1-\alpha)$:

(28)
$$t(\hat{\beta}) = \frac{\frac{\alpha}{1-\alpha}t(\hat{\beta}_1) + kt(\hat{\beta}_2)}{\sqrt{(\frac{\alpha}{1-\alpha})^2 + k^2 + \frac{2\alpha}{1-\alpha}\rho_{12}k}}$$

where $k = \frac{\sigma_2}{\sigma_1}$.

As oppose to the case 1, here it is impossible to express explicitly the t-statistic ratio of parameter $\hat{\beta}$ derived from the three different estimations. As a result, the analysis is conducted as follows: As we saw before, estimating the two components (1 and 2) gave us the t-statistic for parameter of common factor variables. The main question now should be presented in this way: If is it possible

that the t-statistic of the common factor parameter in the direct estimation of CPI inflation equation might be smaller then the smallest t-statistic derived from estimating two separate components?

These two cases should be examined: the first case is $t(\hat{\beta}_1) > t(\hat{\beta}_2) > t(\hat{\beta})$ and the second one is $t(\hat{\beta}_2) > t(\hat{\beta}_1) > t(\hat{\beta})$.

The Analysis of Case 1

The question in review is under what conditions equation (29) can be valid:

$$(29) \quad t(\hat{\beta}) = \frac{\frac{\alpha}{1-\alpha}t(\hat{\beta}_{1}) + kt(\hat{\beta}_{2})}{\sqrt{(\frac{\alpha}{1-\alpha})^{2} + k^{2} + \frac{2\alpha}{1-\alpha}\rho_{12}k}} < t(\hat{\beta}_{2})$$

$$(30) \quad \delta t(\hat{\beta}_{1}) + kt(\hat{\beta}_{2}) < t(\hat{\beta}_{2})\pi \quad \text{when} \quad \pi = \sqrt{\delta^{2} + k^{2} + 2\delta\rho_{12}k}, \quad \delta = \frac{\alpha}{1-\alpha}$$

After some arrangements I get:

(31)
$$\hat{\delta t(\beta_1)} < t(\beta_2)[\pi-k]$$

The derivation of the t-ratio is given by:

(32)
$$\frac{\dot{t(\beta_1)}}{\dot{t(\beta_2)}} < \frac{\pi - k}{\delta}$$

Since in this case $t(\hat{\beta_1}) > t(\hat{\beta_2})$, the condition in (33) must hold:

$$(33) \quad \frac{\pi - k}{\delta} > 1 \implies \pi > \delta + k$$

In (34), π can be express by the original variables:

$$(34) \quad \sqrt{\delta^2 + k^2 + 2\delta\rho_{12}k} > \delta + k$$

With simple algebraic arrangements I obtain (35):

(35)
$$\delta^2 + k^2 + 2\delta\rho_{12}k > \delta^2 + k^2 + 2\delta k$$

And (36) is the condition for a bias in the statistical inference:

$$(36) \quad \rho_{12} > 0$$

The Analysis of Case 2:

The question here is under what conditions equation (37) can be valid:

$$(37) \quad t(\hat{\beta}) = \frac{\frac{\alpha}{1-\alpha}t(\hat{\beta}_{1}) + kt(\hat{\beta}_{2})}{\sqrt{(\frac{\alpha}{1-\alpha})^{2} + k^{2} + \frac{2\alpha}{1-\alpha}\rho_{12}k}} < t(\hat{\beta}_{1})$$

$$(38) \quad \delta t(\hat{\beta}_{1}) + kt(\hat{\beta}_{2}) < t(\hat{\beta}_{1})\pi \quad \text{when} \quad \pi = \sqrt{\delta^{2} + k^{2} + 2\delta\rho_{12}k}, \quad \delta = \frac{\alpha}{1-\alpha}$$

After arrangements (39) is derived:

(39)
$$kt(\hat{\beta}_2) < t(\hat{\beta}_1)[\pi - \delta]$$

The t-ratio is then:

(40)
$$\frac{t(\hat{\beta}_2)}{t(\hat{\beta}_1)} < \frac{\pi - \delta}{k}$$

Because in this case $t(\hat{\beta}_2) > t(\hat{\beta}_1)$, the condition in (41) must hold:

$$(41) \ \frac{\pi-\delta}{k} > 1 \Longrightarrow \pi > \delta + k$$

From this stage the mathematical development is similar to that implemented in case 1, and eventually (42) is the same condition of bias in the statistical inference:

(42)
$$\rho_{12} > 0$$

Case B – Analysis of Non-Existence of a Common Factor in CPI components

The estimators and their variances of equation (6) including one component are presented in (43):

(43)
$$\hat{\gamma} = \frac{\sum z_i \Delta p_{1i}}{\sum z_i^2}$$
, $var(\hat{\gamma}) = \frac{\sigma_l^2}{\sum z_i^2}$; $\hat{\beta}_l = \frac{\sum x_i \Delta p_{1i}}{\sum x_i^2}$, $var(\hat{\beta}_l) = \frac{\sigma_l^2}{\sum x_i^2}$

The estimators and their variances of equation (7) including two components are presented in (44):

(44)
$$\hat{\beta}_{2} = \frac{\sum x_{yi} \Delta p_{2i}}{\sum y_{i}^{2}}, \ var(\hat{\beta}_{2}) = \frac{\sigma_{2}^{2}}{\sum y_{i}^{2}}$$

The t-statistics for these three estimated parameters are as follows:

(45a)
$$t(\hat{\beta}_{1}) = \frac{\hat{\beta}_{1}}{S.E.(\hat{\beta}_{1})} = \frac{\sum x_{i} \Delta p_{1i}}{\sum x_{i}^{2}} / \sqrt{\frac{\sigma_{1}^{2}}{\sum x_{i}^{2}}} = \frac{\sum x_{i} \Delta p_{1i}}{\sigma_{1} \sqrt{\sum x_{i}^{2}}}$$

(45b)
$$t(\gamma) = \frac{\gamma}{S.E.(\gamma)} = \frac{\sum z_i \Delta p_{Ii}}{\sum z_i^2} / \sqrt{\frac{\sigma_I^2}{\sum z_i^2}} = \frac{\sum z_i \Delta p_{Ii}}{\sigma_I \sqrt{\sum z_i^2}}$$

(45c)
$$\hat{\beta}_{2} = \frac{\hat{\beta}_{2}}{S.E.(\hat{\beta}_{2})} = \frac{\sum y_{i} \Delta p_{2i}}{\sum y_{i}^{2}} / \sqrt{\frac{\sigma_{2}^{2}}{\sum y_{i}^{2}}} = \frac{\sum y_{i} \Delta p_{2i}}{\sigma_{2} \sqrt{\sum y_{i}^{2}}}$$

Similarly the t-statistics can be derived from the aggregated estimation (equation (8')):

(46a)
$$\operatorname{var}(\hat{\beta}_{1}^{*}) = \frac{\operatorname{var}(\varepsilon_{t}^{*})}{\sum x_{i}^{2}} = \frac{\alpha^{2}\sigma_{1}^{2} + (1-\alpha)^{2}\sigma_{2}^{2} + 2\alpha(1-\alpha)\rho_{12}\sigma_{1}\sigma_{2}}{\sum x_{i}^{2}}$$

(46b)
$$\operatorname{var}(\gamma^{*}) = \frac{\operatorname{var}(\varepsilon_{t}^{*})}{\sum z_{i}^{2}} = \frac{\alpha^{2}\sigma_{1}^{2} + (1-\alpha)^{2}\sigma_{2}^{2} + 2\alpha(1-\alpha)\rho_{12}\sigma_{1}\sigma_{2}}{\sum z_{i}^{2}}$$

(46c)
$$\operatorname{var}(\hat{\beta}_{2}^{*}) = \frac{\operatorname{var}(\varepsilon_{t}^{*})}{\sum y_{i}^{2}} = \frac{\alpha^{2}\sigma_{1}^{2} + (1-\alpha)^{2}\sigma_{2}^{2} + 2\alpha(1-\alpha)\rho_{12}\sigma_{1}\sigma_{2}}{\sum y_{i}^{2}}$$

The assumed independence between these three variables makes it possible to analyze them symmetrically. The proof of biasness is similar to case A for a unique variable z (and the inferring bias

is a function of a weight α , correlation between shocks ρ_{12} and variance ratio $\frac{\sigma_2^2}{\sigma_1^2}$).

Interim Conclusions

The discussion has so far examined the case where in the heterogeneous data generating process of CPI components, at least one explanatory variable is not a common-factor. One key finding suggests that the inferring bias regarding the estimated parameters appears when estimating CPI inflation equation directly. In other words, we tend to make a false inference about them regarding the null hypothesis despite the fact that the estimated parameters in the CPI inflation equation may be consistent and identified⁹. This leads to negative consequences when some important explanatory variables are unjustifiably dropped from the regression resulting in inaccurate forecasting and economic analysis.

Key Implication

In real life empirical work one key question is why explanatory variables in a CPI inflation equation may be insignificant. Is it because some variables are indeed irrelevant, or is it merely due to an error in the statistical inference such as the one mentioned above? The examination of the effect of explanatory variables effect **on separate components of the CPI** could help in this respect, if it is considered when the effects are estimated using a system of equations. Also, exercising tests such as stability of the parameters over different samples and consistency with economic theory, could increase the support for the inclusion of certain explanatory variables that were statistically irrelevant in the aggregate CPI equation. If these conditions hold, it is likely that the variables are important in the explanation of inflation and therefore should be included in the regression even though they are statistically insignificant.

⁹ Because it was assumed that explanatory variables are weakly exogenous and it is therefore appropriate to use the OLS method.

Simulation-Based Illustrations

Here, I use numerical simulations to demonstrate the bias-inference problem in estimating CPI inflation equation identified in the aggregated approach (aforementioned theoretically). The following process is implemented:

1) The true parameters β_1, β_2, γ are determined (by our assumptions) for each CPI component, followed by the population of explanatory variables x, z with normal distribution (zero mean and some variance) and the population of shocks $\varepsilon^1, \varepsilon^2$ (the population of explanatory variables and shocks include 100,000 observations) are simulated; thereafter the population of the CPI components $\Delta p_1, \Delta p_2$ (using equations (2) and (3)) can be created. Eventually, assumption of the value of the weight for the first component enables simulation of the CPI inflation variable using equation (5). 2) From the population created previously, 100 observations are randomly sampled, thereby

simulating finite samples in reality. Parameters of explanatory variables in equations (2) and (3) are estimated and their t-statistics in each equation are calculated. This experiment is repeated 1,000 times, enabling the creation of a distribution for the estimated parameters and their t-statistics.

3) Certain parameters are calibrated in order to examine the consequences on the extent of inference bias (and therefore on the probability for a second order error).

Two diagrams are presented for each of the five examined cases¹⁰: (1) The t-statistic derived from each experiment; (2) The distribution of the t-statistic over 1,000 experiments. For simplicity, these diagrams relate to a bias only in unique variable z and not for common factor variables. Moreover, the five cases differ in their initial assumptions on: the weight of the first CPI component, correlation between the shocks and the variance ratio.

In the first diagram the following assumptions are made: the weight of the first component is 10 percent, the correlation between the shocks is zero and variance ratio is 1.

¹⁰ For clarity reasons, the first diagram shows the results of 100 simulations only, where in the second diagram the distribution represents the whole 1000 experiments.

Diagrams 1 to 5

100 independent samples - The t-statistic values for the estimated parameter of unique variable (z) derived from 1'st component equation (blue line)

and from the CPI inflation equation (red line)

Diagram 1: $\sigma_2 / \sigma_1 = l$, $\rho_{12} = 0$, $\alpha = 0.1$



Diagram 2: $\sigma_2 / \sigma_1 = l$, $\rho_{12} = 0$, $\alpha = 0.8$



Diagram 3: $\sigma_2 / \sigma_1 = 10$, $\rho_{12} = 0$, $\alpha = 0.1$



Diagram 4: $\sigma_2 / \sigma_1 = 1$, $\rho_{12} = 0.9$, $\alpha = 0.1$





Diagram 5: $\sigma_2 / \sigma_1 = 1$, $\rho_{12} = -0.8$, $\alpha = 0.8$

As can be seen in diagram 1, the t-statistic values of the estimated parameter γ (derived from CPI equation) are systematically smaller than values of γ_1 (derived from 1'st component equation). When the weight of component 1 in the CPI is larger (diagram 2) this inferring bias disappears almost completely. When the variance ratio is bigger (diagram 3), the bias is larger relative to the first case. A similar result is obtained when the correlation between shocks is bigger (diagram 4). In the case where the weight (α) of the first CPI component is significantly large and correlation between shocks is negative and large – a case considered extremely exceptional - the t-statistic values in the aggregated CPI equation are larger than in the separated CPI component equation (diagram 5).

Part B

Aggregated and Disaggregated Monthly Inflation Structural Models

This part presents two monthly inflation structural models, based on Ilek (2006) and making explicit use of energy prices as an additional important crucial component in the CPI model. Below is a brief review of Ilek's model¹¹. Each model has five structural equations for the following economic variables: CPI inflation (under the two approaches); inflation expectations¹²; exchange rate; output gap; and monetary policy (Taylor rule). The two models differ in the representation of the CPI inflation equation and share the other four equations. In the first, CPI inflation is estimated aggregatively, and in the second, it is assessed by using estimates of its separate components. Moreover, the disaggregated inflation model includes equations to capture separate behavior of the following CPI components: housing, clothing and footwear, energy and CPI excluding housing, clothing and footwear, energy, fruit and vegetables¹³ (can be considered also as a proxy for core inflation measure). The forecast of total CPI inflation in the disaggregated model is then obtained by the forecast for each component averaged by its weight in the CP index.

Aggregated Inflation Equation (excl. fruit and vegetables)¹⁴

 $dp_t^{ex_f \& veg} = seas_t + \beta_l deimp_2_t + \beta_2 Expl_{t} + (1 - \beta_l - \beta_2)(dp_{t-l}^{ex_f \& veg} - seas_{t-l}) + \beta_3 ygap_3_{t-l} + \beta_5 rigap_3_{t-l} + \varepsilon_t^{w_p pir}$

 $deimp = de + dp _imp$

where:

 $dp^{ex_{-}f\&veg}$ - is the CPI inflation rate (excluding fruit and vegetables)

seas - seasonal factors

de - depreciation of shekel against dollar

¹¹ A detailed description regarding methods of estimation, pass-through channels from monetary policy to inflation and the equations can be found in Ilek (2006).

¹² There is an explicit modeling of inflation expectations in this model, as opposed to the study of Elkayam and Argov (2006), where inflation expectations are rational and are solved by the model.

¹³ The component of fruit and vegetables is excluded from the discussion because of its irregular behavior.

¹⁴ All changes are annual.

dp imp - the rate of change in import prices (in dollars) *Exp12* - inflation expectations for the next 12 months (derived from the capital market) *ygap* 3 - output gap (during the last 3 months¹⁵) *rigap* 3- gap between real interest rate and natural rate (during the last 3 months)

 $\varepsilon^{dp_excl_f\&veg}$ - shocks to inflation

The Disaggregated Inflation Model - Separated Equations

1. Inflation excl. fruit and vegetables, housing, energy, clothing and footwear

For simplicity of definition I call the inflation (excl. fruit and vegetables, housing, energy and clothing and footwear) core inflation ($dcp^{excl_f \& veg_housing_energy_clothing \& footwear} = dcp^{core}$)

The specification of core inflation is the same as in the aggregated model:

 $dcp_t^{core} = seas_t + \beta_1 deimp \quad 2_t + \beta_2 Exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}^{core} - seas_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}^{core} - seas_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}^{core} - seas_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}^{core} - seas_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}^{core} - seas_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}^{core} - seas_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}^{core} - seas_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}^{core} - seas_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}^{core} - seas_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}^{core} - seas_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}^{core} - seas_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}^{core} - seas_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}^{core} - seas_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}^{core} - seas_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}^{core} - seas_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}^{core} - seas_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}^{core} - seas_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}^{core} - seas_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}^{core} - seas_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}^{core} - seas_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}^{core} - seas_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}^{core} - seas_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}^{core} - seas_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}^{core} - seas_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}^{core} - seas_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}^{core} - seas_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}^{core} - seas_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}^{core} - seas_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}^{core} - seas_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}) + \beta_2 exp 12_t + (1 - \beta_1 - \beta_2)(dcp_{t-1}) + \beta_2 exp 12_t +$ + $\beta_3 ygap \quad \beta_{t-1} + \beta_4 rigap \quad \beta_{t-1} + \varepsilon_t^{dp_w}$

2. Housing prices

 $dcp^{housing}{}_{t} = seas_{t} + \beta_{1}de _ 2_{t} + \beta_{2}dbaald _ 2_{t-2} + \beta_{3}ygap _ 3_{t-1} + \beta_{4}rigap _ 3_$ $+(1-\beta_2)dpx_{t-1}^{excl_veg_housing} + \varepsilon_t^{diur}$

dbaald = dbaal - de dcp^{diur} - rate of change in housing prices seas - seasonal factors de - depreciation of shekel against dollar dbaal - rate of change of housing prices derived from a survey of apartments owners (in shekels)¹⁶ *dbaald* - rate of change of housing prices derived from a survey of apartments owners (in dollars) *ygap* 3 - output gap (during the last 3 months)

 ¹⁵The output gap is calculated using a Hodrick-Prescott filter.
 ¹⁶These prices are published by the Central Bureau of Statistics

rigap 3- gap between the real rate and natural rate (during the last 3 months)

dpx^{excl_f &veg_housing} - inflation rate (excluding fruit & vegetables and housing) during the

last 12 months.

 $\varepsilon^{hou \sin g}$ - shocks to housing prices

3) Clothing and footwear

 $dcp^{cl\&footwear}{}_{t} = seas_{t} + \beta_{l}dehalb \ 2^{f}_{t} + (1 - \beta_{l})dpx^{excl_{-}f\&veg_{-}cl\&footwear}_{t-l} + \varepsilon^{cl\&footwear}_{t} + \varepsilon^{cl\&footwea$ $dehalb^{f} = de + dcpf^{app \& footwear}$

Where:

dcp^{cl&footwear} - rate of change of clothing and footwear prices

de - depreciation of shekel against dollar

dcpf^{*cl&footwear*} - rate of change of clothing and footwear prices abroad (in dollars)

dpx^{excl_f &veg_cl& footwear} - inflation rate (excluding fruit &vegetables and clothing &footwear) during

the last 12 months

 $\varepsilon^{cl\&footwear}$ - shocks to clothing&footwear prices

4) Energy prices¹⁷

The energy component in the CPI consists of four sub-components: fuel and oils, kerosene and diesel oil, gas and electricity. Here I model these components as follows.

4.1 Fuel and oils prices¹⁸

$$dcp_{t}^{fuel_oils} = \beta_{1} + \beta_{2}(de_{t-1} + dcpf_{t-1}^{fuel_oils}) + \varepsilon_{t}^{fuel_oils}$$
$$\beta_{1} = 2(1 - \beta_{2})$$

where:

 ¹⁷ All the equations of energy prices are estimated using O.L.S. method.
 ¹⁸ As we can see, the price determination of the first three energy components is based on the changes in the exchange rate and in world prices in the previous month. This structure is not arbitrary, and is based on the manner in which the Israel Ministry of National Infrastructures sets energy prices (energy prices in Israel are controlled by the government).

 dcp^{fuel_oils} - the change in the domestic fuel and oils consumer prices $dcpf^{fuel_oils}$ - the change in foreign fuel and oils prices (in dollars) de - depreciation of the shekel against dollar

4.2 Kerosene and diesel oil prices:

 $dcp_{t}^{keros} = \alpha_{1} + \alpha_{2}(de_{t-1} + dcpf_{t-1}^{heat_oil}) + \alpha_{3}(de_{t-1} + dcpf_{t-1}^{keros}) + \varepsilon_{t}^{keros}$ $\alpha_{1} = 2(1 - \alpha_{2} - \alpha_{3})$

where:

 dcp^{keros} - the change in domestic kerosene and diesel consumer prices $dcpf^{heat}_{oil}$ - the change in foreign heat oil prices (in dollars) $dcpf^{keros}$ - the change in foreign kerosene prices (in dollars)

4.3 Gas prices

$$dcp_{t}^{gas} = \theta_{1} + \theta_{2}(de_{t-1} + dcpf^{gas}_{t-1}) + \varepsilon_{t}^{gas}$$
$$\theta_{1} = 2(1 - \theta_{2})$$
where:
$$dcp^{gas} - \text{the change in domestic gas consumer prices}$$
$$dcpf^{gas} - \text{the change in foreign gas prices}$$

4.4 Electricity prices

 $\begin{aligned} dcp_{t}^{electricity} &= \mathcal{G}_{1} + \mathcal{G}_{2}(de_{t-3} + dcpf_{t-3}^{coal}) + \mathcal{G}_{3}(de_{t-4} + dcpf_{t-4}^{coal}) + \mathcal{G}_{4}(de_{t-1} + dcpf_{t-1}^{crude_oil}) + \\ &+ \mathcal{G}_{5}(de_{t-2} + dcpf_{t-2}^{crude_oil}) + \mathcal{G}_{6}(de_{t-1} + dcpf_{t-1}^{fuel_oil}) + \mathcal{G}_{7}(dcp_3_{t}^{excl_fr\&veg}) + \mathcal{G}_{8}de_{t-2} + \varepsilon_{t}^{gas} \\ &\mathcal{G}_{1} = 2(1 - \mathcal{G}_{2} - \mathcal{G}_{3} - \mathcal{G}_{4} - \mathcal{G}_{5} - \mathcal{G}_{6} - \mathcal{G}_{7}) \end{aligned}$

where:

dcp ^{*electricity*} - the change in domestic electricity consumer prices

dcpf^{*coal*} - the change in foreign coal prices (in dollars)

dcpf^{*crude*_*oil*} - the change in foreign crude oil prices (in dollars)

dcpf ^{*fuel_oil*} - the change in foreign fuel oil prices (in dollars)

 $dcp_3^{excl_fr\&veg}$ - the changes in CPI (during the last 3 months)

The energy price forecast is therefore constructed by averaging four component forecasts by their weights in the energy index:

$$dcp^{energy} = \frac{m_{fuel}}{m_{energy}} dcp^{fuel_oils} + \frac{m_{keros}}{m_{energy}} dcp^{keros} + \frac{m_{gas}}{m_{energy}} dcp^{gas} + \frac{m_{electicity}}{m_{energy}} dcp^{electicity} + \frac{$$

where:

 $m_{energy} = m_{fuel} + m_{keros} + m_{gas} + m_{electicity}$ (the weight of energy component in the CPI is a simple sum of its sub-components weights)

After the explicit modeling of energy prices in the disaggregated model it is now possible to express the behavior of CPI inflation based on the equations of its main components:

$$dcp_{t}^{excl} - f^{\&veg} = \frac{m_{housing}}{(1000 - m_{f^{\&veg}})} dcp^{housing}_{t} + \frac{m_{cl\&footwear}}{(1000 - m_{f^{\&veg}})} dcp^{cl\&footwear}_{t} + \frac{m_{energy}}{(1000 - m_{f^{\&veg}})} dcp^{energy}_{t} + \left(1 - \frac{m_{housing} + m_{cl\&footwear} + m_{energy}}{1000 - m_{f^{\&veg}}}\right) dcp_{t}^{core}$$

where:

1000- sum of weights of all the component in CPI

 $m_{f\&veg}$ - weight of fruit and vegetables in CPI

 $m_{housing}$ - weight of housing in CPI

 $m_{cl\& footwear}$ - weight of clothing & footwear in CPI

 m_{energy} - weight of energy in CPI

The two models presented above have another four common equations for other endogenous variables. Importantly, the monthly model is designed to assure that inflation converges to its target¹⁹ in the long run by implementing a monetary policy based on an interest rate path aimed at meeting the inflation target. The following are the four additional equations closing the models.

Inflation Expectations Equation

 $Exp12_{t} = \beta_{1}Exp12_{t-1} + \beta_{2}deimp_{t} + \beta_{3}ygap_{t} + (1 - \beta_{1} - \beta_{2})dp_{t}^{ex_{f}dweg} + \varepsilon_{t}^{exp12}$

Where:

 $\begin{aligned} & Exp12 \text{ - inflation expectations for the next 12 months (derived from capital market)} \\ & deimp \text{ - depreciation of shekel against dollar and import prices} \\ & ygap \text{ - output gap} \\ & dp^{ex_{-}f\&veg} \text{ - CPI inflation rate (excluding .f&v)} \\ & \varepsilon_t^{exp12} \text{ - shocks to inflation expectations} \end{aligned}$

Taylor Rule

$$\begin{split} i_{t} &= \beta_{1}(r_{t}^{n} + dpt + \beta_{2}(Exp12_{t-1} - dpt) + \beta_{3}ygap_{3}_{t-1}) + (1 - \beta_{1})i_{t-1} + \\ &+ \beta_{4}(i_{t-1} - i_{t-2}) + \varepsilon_{t}^{ima} \end{split}$$

Where:

i - nominal central bank policy rate

 r^n - natural real interest rate²⁰

dpt - inflation target

*ygap*_3 - output gap (during the last 3 months)

Exp12 - inflation expectations (for the next 12 months)

 ε^{ima} - shocks of monetary policy

¹⁹ Through imposing restriction in each equation of the four CPI components.

²⁰ We adopt the approach proposed by Beenstock and Ilek (2005) to present the real natural rate by forward rate derived from indexed bonds traded in the capital market.

Exchange Rate Equation²¹

$$de_{t} = \frac{1 - \beta_{l}}{\beta_{l}} de_{t+l/t} + \frac{idolar_{t} - ima_{t} + prem_{t}}{\beta_{l}} - (idolar_{t-l} - i_{t-l} + prem_{t-l}) + \varepsilon_{t}^{de}$$

Where:

de -the change in the rate of depreciation of shekel against dollar ($de_{t+1/t}$ -expected) *idolar* -foreign interest rate (Libid) *prem* - the premium required on domestic assets ε^{de} - shocks to exchange rate

Output Gap Equation

 $ygap_{t} = \beta_{1}ygap_{t+1/t} + \beta_{2}ygap_{t-1} + \beta_{3}regap_{3}t_{t-1} + \beta_{4}rigap_{3}t_{t-1} + \beta_{5}ygapf_{3}t_{t-1} + \varepsilon_{t}^{ygap}t_{t-1} + \varepsilon_{t}^{yg$

Where:

- *ygap* output gap
- *regap* 3 gap between the real exchange rate and the potential (during the last 3 months)
- *rigap*_3- gap between real interest rate and natural rate (during the next 3 months)
- *ygapf* _3 output gap in the world (during the last 3 months)
- ε^{ygap} shocks to output gap

²¹ See Ilek (2006) for further information on how to develop this equation.

Part C: Goodness of Fit of the Aggregated and Disaggregated Models

This part examines the goodness of fit within the two version of the monthly model. I follow two approaches: (1) which is based on dynamic and static simulations of the inflation equation – an approach commonly used by many researchers; (2) which is based on stochastic simulation of the whole model²²- relatively new in the literature.

1) Dynamic and static simulations of inflation equation

I calculate the sum of squared residuals from inflation equation in each model $(SSE = \sum_{i=1}^{n} e(k)_i^2)^{23}$

derived from out of sample estimation (see Stock and Watson (1999)). In static simulation, all the explanatory variables including the lags of inflation are predetermined. In the dynamic simulation, however, inflation in the previous period is not predetermined, and is solved by the model. Table 1 presents numeric results of *SSE* criterion derived from static and dynamic simulations of aggregated and disaggregated models. This criterion enables us to compare between the two models in each period of time. In addition, in order to compare not only between models but also between periods in each model, I added (in brackets) the mean of the sum of squared residuals (actually normalizing the *SSE* by the number of observations in each period). Table 1 is a combination of static and dynamic simulation refers to the outcomes for longer periods of up to four months. The latter takes into account inflation solved in the model rather than actual inflation considered in the static case. Table 1 presents simulation results of the two models and diagrams 6-7 illustrate the forecasting residuals (of the change in CPI) for one month and four months ahead, accordingly.

²² Detailed explanation on these two approaches is given below.

²³ The sign k indicates the residuals from the inflation equation of model k.

Forecast	Aggregated	Disaggregated	Simulation period	
Horizon				
1	619.0	512.0		
1	(14.1)	(11.6)		
2	656.0	518.5	2003:01-2006:09	
	(14.9)	(11.8)		
3	712.3	541.0		
5	(16.2)	(12.3)		
4	713.1	544.7		
7	(16.2)	(12.4)		
			-	
1	558.4	472.1		
	(15.5)	(13.1)		
2	556.7	470.3	2003:01-2005:12	
	(15.5)	(13.1)		
3	503.5	444.7		
	(14.0)	(12.4)		
4	502.3	447.7		
	(14.0)	(12.4)		
			-	
1	370.9	362.4		
	(11.6)	(11.3)		
2	416.7	375.4	_ 2004:01-2006:09	
	(13.0)	(11.7)		
3	525.7	444.9		
-	(16.4)	(13.9)		
4	521.7	437.4		
	(16.3)	(13.7)		
	105 (2 (0) (
1	187.6	248.6	2005:01-2006:09	
	(9.4)	(12.4)		
2	232.2	224.1		
	(11.6)	(11.2)		
3	339.1	291.3		
	(17.0)	(14.6)	_	
4	323.2	297.1		
	(16.2)	(14.9)		

Table 1 – Goodness of fit within aggregated and disaggregated modelsStatic and Dynamic simulation, 2003:01-2006:09

The numbers present the calculated Sum Squared Errors

The numbers in brackets are the mean of SSE in each period



Diagram 6 - Forecasting Errors(one month ahead) from Static Simulation- Based on Aggregated and Disaggregated Models, 2003:01-2006:09

Diagram 7 - Forecasting Errors (four months ahead) from Dynamic Simulation- Based on Aggregated and Disaggregated Models, 2003:01-2006:09



Table 1 shows the superiority of the disaggregated model for almost all horizons, particularly during the entire period and in most of its sub-periods. It should be noted that the aggregated model performed slightly better during the last sub-sample (from 2005:01 up to 2006:09)²⁴. In sum, based

²⁴ Although in the economic literature there is a formal test that examines whether the difference between the MSE derived from different models are statistically significant (see Diebold and Mariano (1995), Harvey, Leybourne, Newbold [1997]), in this paper I do not use this test because: (1) by dividing the whole simulation period into sub-samples I indeed implicitly test the "significance" in priority of each model, that is I test the consistency of the SSE results over different samples; (2) this formal test is inaccurate in short samples.

on the results from dynamic and static simulation we obtain strong evidence in favor of disaggregated model for forecasting CPI inflation.

2) Goodness of fit based on stochastic simulation of the whole model

The sample performance comparison conducted above has two main drawbacks: (1) Because inflation and variables affecting the inflation are determined simultaneously, in sample forecasting must be limited to the very short run (1-3 months) only, where most explanatory variables can be known at the time of the forecast, while for a longer period (medium run - up to 12 months and long run - more than 12 months) forecasting is impossible. (2) The inclusion of certain forward looking variables in the models requires that their path be dependent on shocks which are explicitly ignored in the solution of the simple dynamic simulation of the whole model presented above. The stochastic simulation method performed by Smets and Wouters (2003), and Rotemberg and Woodford (1998), which simulate reality may be useful in dealing with these problems. Shocks are simulated, for every period, to every equation in the model (equations describing endogenous and exogenous variables)²⁵. In the second stage, I extract the theoretical moments (from the asymptotic sample) for each of the two models and compare them to the sample based moments²⁶. Obviously it is likely that the theoretical and sample based moments will not be identical, because actual moments were derived from a small and random sample of 100 observations (1998:01 2006:07). But a more important question is whether the sample based moments are not significantly distant from the theoretical moments. That is, are they inside the confidence interval? If the answer is positive, there is a strong indication that the structural model can be used for analysis and forecasting.

Figures 1 and 2 present the simulation results for the aggregated and the disaggregated models. The figures show sample based moments, the confidence interval and the median of theoretical moments of the following main variables in the models: CPI inflation (excl. fruit and vegetables) seasonally adjusted, inflation expectations, the rate of depreciation of the shekel against dollar, and the monetary interest rate. On the diagonal, the autoregressive correlations (up to 5 lags) are presented, and outside the diagonal the cross correlation (up to 5 lags) is shown. For illustration: dp & de(-t)presents the contemporaneous correlation, in lag 1, 2 until 5 between the CPI inflation and the rate of depreciation.

²⁵ The shocks to each equation are random and independent. The distribution of these shocks assumes zero mean and some variance. The variance is calculated from the residuals derived from each equation that was estimated in the sample mentioned. The residuals of exogenous variables derived from autoregressive equations.²⁶ These simulations were made in MATLAB.

It can be easily seen that in the aggregated model the sample-based moments are outside the confidence interval for the rate of depreciation and for inflation expectations (during the 1-3 first periods), while in the disaggregated model, they are inside. Moreover, impressively dp & de(-t) and dp & dp(-t) correlations are near the median of the confidence interval. Looking at other results we can see that the moments derived from the disaggregated model are superior to those calculated in the aggregated model: correlations between the monetary interest and inflation and the rate of depreciation (i & de(-t), i & dp(-t)) have better fit. In sum, the disaggregated model produces much convincing results based on stochastic simulations thus increasing the dominance of the disaggregated model over the aggregated one.

Actual and Theoretical Moments within the Aggregated and Disaggregated Models



fig (1) - Observed Cross Corr vs. Confidence Intervals -Aggregated Model



fig (2)- Observed Cross Corr vs. Confidence Intervals-Disaggregated Model

Part D

The Expected Loss Function of Monetary Policy under Incomplete Information in the Aggregated and Disaggregated Models

This part discusses the implications of incomplete information for the management of monetary policy. In the real world there are many important economic variables crucial for the successful implementation of monetary policy. Some of them are unobservable (for example the natural real interest rate, the potential output, the natural rate of unemployment, the risk premium required on domestic assets, etc.) and therefore should be estimated by different methods. Assuming that the central bank set its interest rate using the Taylor rule (see Taylor [1993]), which includes, *inter alia*, the natural interest rate and the output gap. Erroneous assessment of these two unobserved variables could cause a significant bias in monetary policy. As a result, inflation could deviate from its target for long period of time and thereby cause loss to the economy and eventually impair the credibility of monetary policy.

Each version of the model built by researchers in many central banks reflects the eventually presumed structure of the economy. Since many models exist, the same number of possible structures of the economy which they present might all prove to be wrong or at least one could resemble reality in a reasonable manner. Accordingly, the drawing of conclusions about the consequences of shocks in the economy, under complete and incomplete information, examination of loss function of central bank etc. will remain theoretical and conditioned on the presumed model. Beyond the difference in inflation specification in aggregated and disaggregated models, they eventually represent two different economies²⁷. We therefore cannot conclude unequivocally that monetary policy will be more efficient if it is managed by taking into account the inflation process and other variables in a disaggregated approach. In order to extract operative benefits from this analysis one should first of all be persuaded that the analyzed model captures the relevant characteristics of the economy to a reasonable extent. Based on this description I make the two following tests: in the first one, I measure the loss of monetary policy within the two versions of models under inaccurate assessments of crucial unobservable economic variables. In the second

²⁷ As reflected by the difference in their endogenous variables responses (IRF) to shocks in the economy. Not only would the CPI inflation process react differently, but the other endogenous variables would also differ in their responses because of direct and indirect effects, resulting from different inflation specification in aggregated and disaggregated models.

case, I examine the expected loss for monetary policy, when in addition to wrong assessment of economic variables, monetary policy also misidentifies the true structure of the economy. For these purposes I examine in the first case the two unobserved variables - the real natural interest rate and the exchange rate risk premium. In the second case, I assume that monetary policy inaccurately identifies the inflation process in the economy. Eventually, what can we learn from this analysis and how we can benefit from it for real use of these models depends primarily on previous analyses of the goodness of fit of the two models presented.

The loss function of the monetary policy is determined as follows²⁸:

$$\sum \left[(dpx - dpt)^2 + \beta_1 ygap^2 + \beta_2 \Delta i^2 \right]$$

Where $\beta_1 = 0.25$, $\beta_2 = 5$

and dpx is a CPI inflation rate during the last 12 months, dpt is the inflation target (2 percent), ygap is the output gap and Δi changes in the monetary interest rate.

The implicit assumption is that monetary policy prescribes a flexible inflation targeting regime and is also concerned with financial stability. (This structure of loss function is consistent with the specification of the Taylor rule shown above).

1. Consequences of the inaccurate assessment of the natural rate of interest

Assuming that a shock in the natural interest rate occurred and that its magnitude is a common knowledge (for central bank, households, private sector etc.). However, the central bank mistakenly perceives this shock as permanent. In other words, the central bank mistakenly perceives the D.G.P. of natural rate as:

1)
$$r_t^n = r_{t-1}^n + \varepsilon_t^{rn}$$

whereas the true process is :

2)
$$r_t^n = \beta_0 + \beta_1 r_{t-1}^n + \varepsilon_t^m$$

 $E(r^n) = \frac{\beta_0}{1 - \beta_1}$

Assuming that $\beta_0 = 0.3$, $\beta_1 = 0.9$, so in the long run the natural rate converges to 3 percent.

²⁸ Although the parameter values in the loss function cannot be observed, a reasonable assumption regarding monetary policy management in Israel can help in determining their values; see Argov (2005) for more detail.

Table 2 presents the monetary policy loss resulting from a 1 percent shock to the natural real rate level with its interpretation is shown below.

(2) Consequences of the inaccurate assessment of the exchange-rate risk premium

Let us assume that the shock in exchange-rate risk premium occurred and that its magnitude is common knowledge (for the central bank, households, the private sector etc.). However, the central bank mistakenly perceives this shock as permanent. To examine the expected loss incurred by monetary policy as a result of this mistaken assessment of the risk premium, it is important to distinguish between the true risk premium and the wrong one as conceived by monetary policy. Below I construct a Taylor rule version that allows the interest rate to change with fluctuations in the risk premium trough in the exchange rate. This version includes the lag and expected change in the exchange rate in the next period, in addition to the other variables as follows:

3)
$$i_t = \alpha_1 [r_t^n + dpt + \alpha_2 (Expl2 - dpt)_{t-1} + \alpha_3 Exp_t (de_{t+1}) + \alpha_4 de_{t-1} + \alpha_5 ygap3_{t-1}] + (1 - \alpha_1)i_{t-1} + \alpha_4 (i_{t-1} - i_{t-2}) + \varepsilon_t$$

Let us assume that the parameters $\alpha_3 = 0.2 \ \alpha_4 = 0.1^{29}$

As before, the exchange rate equation is constructed in the following manner:

4)
$$de_{t} = \frac{1-\beta}{\beta}de_{t+1} + \frac{idolar_{t} - ima_{t} + prem_{t}}{\beta} - (idolar_{t-1} - i_{t-1} + prem_{t-1})$$

Let us now assume that the true D.G.P. of the premium is:

5)
$$prem_t = \beta_0 + \beta_1 prem_{t-1} + \varepsilon_t$$

 $E(prem) = \frac{\beta_0}{1 - \beta_1}$

Assume that $\beta_0 = 0.05$, $\beta_1 = 0.9$, so that in the long-run the risk premium converges to 0.5 percent³⁰.

²⁹ The value of these parameters is supported by the empirical estimation of the Taylor rule in Israel.

³⁰ These parameters were obtained from estimation equation (5) based on the premium derived from shekel-dollar options (see Pumpeshko and Hecht [2005]).

Let us also assume that the central bank mistakenly perceives the stochastic behavior of the premium in the following manner (random walk):

6)
$$prem_e^* = prem_e^* + \varepsilon_t$$

The central bank therefore bases its monetary policy on exchange rate behavior (equation (7) that includes an inaccurate premium $(prem_e^*)$. Equations (4) and (7) are identical, except for the difference in the premium process.

7)
$$de_{t}^{*} = \frac{1-\beta}{\beta} de_{t+1}^{*} + \frac{idolar_{t} - i_{t} + prem_{e_{t}}^{*}}{\beta} - (idolar_{t-1} - i_{t-1} + prem_{e_{t-1}}^{*})$$

Discussion on Results Presented in Table 2

Table 2 presents the loss to the economy as the result of shocks to the natural real interest rate and the risk premium. Columns 2 and 3 present the value of the expected loss function in the aggregated and disaggregated models respectively. In order to identify the negative contribution to the loss incurred by monetary policy that stems from incomplete information about these two economic variables, a "natural" loss function for each model with <u>complete information</u> is calculated. This "natural" loss depends on presumptions of the economic structure reflected in the model and is presented in the first line of the table ($L^{natural}$). In the next step I calculate the value of the loss function where monetary policy is mistaken in its assessment of the process of these two variables, which is presented in the second line (L^{false}). Eventually, in order to derive the contribution to the loss function as a result of incomplete information, I subtract the value of the "natural" loss from the "mistaken assessment" loss ($L^{false} - L^{natural}$).

Results Interpretations: One can see that the loss to the economy is, not surprisingly, higher when monetary policy is mistaken in its assessments with respect to both the natural rate and the risk premium. In addition, the negative contribution to the loss function is significantly lower in the disaggregated model. Therefore, if the structure of the Israeli economy were to be described by the disaggregated model, rather than by the aggregated model, the loss to society in the incomplete information world would have been much smaller. This advantage of the disaggregated model³¹

³¹ We saw that disaggregated model emulates the real development of inflation and other economic variables far more effectively than the aggregated model.

comes in addition to its advantage mentioned in the previous chapter where it was far superior to the aggregated model with respect to the goodness of the fit. This provides support for the theory that the economy would benefit more if monetary policy were to be managed by means of the disaggregated approach.

Table 2 - Monetary Policy Expected Loss Resulting from a Mistaken Assessment ofUnobserved Variables (natural interest rate and premium) in the Economy in the Aggregated

Disaggregated Models					
Aggregated model	Disaggregated model	Unobserved variable			
$L^{natural} = 3.31$	$L^{natural} = 2.96$	Natural interest rate (r^n)			
$L^{false} = 20.33$	$L^{false} = 14.52$				
$\Delta L = 17.02$	$\Delta L = 11.57$				
$L^{natural} = 0.46$	$L^{natural} = 0.81$	Risk premium (prem_e)			
$L^{false} = 1.71$	$L^{false} = 1.31$				
$\Delta L = 1.25$	$\Delta L = 0.50$				
1	1	1			

and Disaggregated Models³²

I now examine the case where the central bank's mistaken assessment relates to the true structure of the inflation process in the economy. In other words, a mistaken assessment is made that CPI inflation is described by an aggregated model, when the true structure is a disaggregated one. In the alternative case I consider the opposite situation where the aggregated model is true. I calculate the loss function in these two alternatives within three structural shocks: shocks to monetary policy, the exchange rate and the output gap. Because inflation and other endogenous variables are influenced by the true inflation process, misidentifying the inflation process results in a higher loss function because it affects projected paths of endogenous variables in the model, in addition to the deterioration in future inflation developments. Monetary policy attributes its actions to an inappropriate model whereas the changes in the monetary interest rate impact "true" economic variables (inflation, output gap, exchange rate etc.).

³² The smaller loss of monetary policy revealed in the disaggregated model was robust in the face of different choices of parameters in the above mentioned loss function.

Discussion on Results Presented in Table 3:

Table 3 presents the value of the loss function when monetary policy misidentifies the true structure of the inflation process in the economy. In the first case, the aggregated process of inflation is considered true and in the second case the disaggregated process is considered true. I calculate the loss function under three different shocks: monetary interest rate, exchange rate and output gap. As in table 2 above, I first calculate the "natural" loss with complete information (line 1 for each model) and then calculate loss when the central bank misidentifies the true structure of the inflation process (the second line). The difference between these two calculations represents the negative contribution of mistaken assessments to the central bank's loss function.

Results Interpretations: Table 3 reveals that the contribution to the central bank loss is smaller if the true model is disaggregated for all three shocks, with the most significant contribution attributed to monetary shocks in these two models. There is no significant difference in contribution between the models and they are relatively small when shocks occur in the exchange rate or in the output gap. Although these findings cannot tell "which model is better", they further support to the argument that if the "true" model were to be the disaggregated model, the loss function associated with misunderstanding the structured inflation process in the economy will be smaller in comparison to the aggregated case.

Aggregated model is true	Disaggregated model is true	The type of structural shock
(disaggregated model is false)	(aggregated model is false)	
$L^{natural} = 17.42$	$L^{natural} = 13.16$	Shock of 1 percent to
$L^{false} = 25.98$	$L^{false} = 19.03$	monetary interest rate (ima)
$\Delta L = 8.56$	$\Delta L = 5.87$	
$L^{natural} = 1.62$	$L^{natural} = 1.75$	Shock of 10 percent to
$L^{false} = 4.53$	$L^{false} = 4.38$	exchange rate (de)
$\Delta L = 2.91$	$\Delta L = 2.63$	
$L^{natural} = 0.79$	$L^{natural} = 0.67$	Shock of 1 percent to output
$L^{false} = 0.84$	$L^{false} = 0.71$	gap (ygap)
$\Delta L = 0.05$	$\Delta L = 0.04$	

<u>Table 3- Expected Monetary Policy Loss Resulting from Mistaken Assessment of the "True"</u> <u>Inflationary Structure of the Economy</u>

Part E - Main conclusions:

This paper includes two main parts: in the first part, the existence of a bias regarding the possible relevance of explanatory variables in the CPI inflation estimated aggregately is introduced and explored. It was found that under certain reasonable conditions we tend to accept the null hypothesis of explanatory variables when estimating CPI inflation, directly leading to their exclusion from the regression. It is therefore important to be cautious about interpreting statistical t-tests regarding the parameters of explanatory variables in CPI inflation equation and rather emphasize the judgment of their relevance in explanatory inflation on the basis of two main criteria: consistency of these parameters with economic theory and their stability over different samples. However, the relevance of certain economic variables in explaining the development of the CPI should be based on an examination of the causal effect of these variables on individual CPI components, through disaggregated estimation.

The second part of this paper presents two alternative approaches for modeling the inflation structure of the economy. Under the first method, inflation forecasting is performed directly using an aggregated inflation equation. Under the alternative approach, the forecast of CPI inflation is made by aggregation of forecasts from separated CPI components. These two different specifications of inflation processes are included in a monthly structural model, where the monetary interest rate (consistent with the inflation target) and other key economic variables are determined simultaneously with inflation. I found that the disaggregated model performs much better than the aggregated model for CPI inflation forecasting purposes. Moreover, disaggregation of the CPI makes it possible to identify more accurately the sources and features of inflation pressures in the economy, thereby improving the efficiency of monetary response.

I also examined in this paper the consequences of the mistaken assessment of certain unobserved economic variables (such as the natural interest rate and the risk premium) and the misidentification of the true inflation structure with respect to the loss function of monetary policy. It appears that under incomplete information, if the economy is better described by the disaggregated model, then the loss to the economy is smaller. Moreover, if the central bank mistakenly perceives the aggregative structure of inflation process while the disaggregated process is the true process, then the loss to monetary policy is also smaller relative to the opposite case when the true process is the aggregated process.

Overall, combining these outcomes together with the goodness of the fit result (from part C), provides strong evidence for favoring the use of the disaggregated model introduced in this paper, in order to manage monetary policy, especially in the short and medium run.

Bibliography

Argov E.(2005), "Optimal policy rule for monetary model in Israel", Monetary Studies, Bank of Israel.

Barker T. and Pesaran M.H. (1990), "Disaggregation in econometric modelling", Routledge, London and New-York.

Beenstock M. and Ilek A. (2005), "Wicksell's Classical Dichotomy: is the Natural Rate of Interest Independent of the Money Rate of Interest, Monetary Studies, Bank of Israel

Benalal N. Diaz del Hoyo J.L., Landau B., Roma, M. Skudelny F. (2004), "To aggregate or not aggregate? Eurozone inflation forecasting, Working paper 374, European Central Bank

Bruno M. and Sussman Z. (1979), "Exchange Rate Flexibility Inflation and Structural Change: Israel Under Alternative Regimes", Journal of Development Economics, 6, No.4, 483-514.

Calvo G.A (1983), "Staggered pricing in a utility –maximizing framework", Journal of Monetary Economics 12, 383-398.

Chinn M.D., and Meredith G. (2004),"Monetary Policy and Long-Horizon Uncovered Interest Parity ", IMF Staff Papers, Vol.51, No.3, IMF.

Clarida R.J, Gali G., Gertler M., (1999), "The science of monetary policy – A New Keynesian perspective ", Journal of Economic Literature 37, 1661-1707.

Demers F. and Champlain A. (2005), "Forecasting Core Inflation in Canada: Should We forecast the Aggregate or the Components?", Working paper 44, Bank of Canada

Diebold F. and Mariano R.S. (1995), "Comparing Predictive Accuracy", Journal of Business and Economic Statistics, Vol. 13, No.3

Elkayam, D. and Argov, E. (2006). "An Estimated New-Keynesian Model for a Small Open Economy: an Application for Israel", Monetary Studies, 2, Bank of Israel (in Hebrew)

Elkayam, D. and Azoulay, E.(1997). "A short run effect of interest rate and exchange rate on inflation in Israel, 1990-1996, Israel Economic Review, 97 (in Hebrew)

Elkayam D. (2001), "A model for monetary policy under inflation targeting: the case of Israel, Monetary Studies, Bank of Israel. Fuhrer J.C. and Moore G.R. (1995), "Inflation persistence", Quarterly Journal of Economics 110, 127-159

Grunfeld Y. and Grilichez Z.(1960), "Is aggregation necessarily bad?", Review of Economics and Statistics (1): 1-13.

Harvey D., Leybourne S., Newbold P.(1997), "Testing the equality of prediction mean squared errors", International Journal of Forecasting 13, 281-291.

Hendry D.F. and Hubrich K. (2006), "Forecasting economic aggregates by disaggregates", working paper 589, European Central Bank

Hubrish. K. (2005), "Forecasting Euro area inflation: Does aggregating forecasts by HICP component improve forecast accuracy?", International Journal of Forecasting 21: 119-36

Ilek. A.(2006), "A monthly model for evaluation of inflation and monetary policy in Israel", 2006.04, discussion paper series, Monetary studies, Bank of Israel.

Laxton D., Epstein N., Karam P., Rose D.(2006), "A simple forecasting and policy analysis for Israel: Structure and Applications", IMF publications.

Laxton D., Karam P. and Berg A. (2006), "Practical Model-Based Monetary Policy Analysis – A How- To Guide", IMF working paper, wp/06/81.

Lutkepohl H. (1984), "Forecasting contemporaneously aggregated vector ARMA processes", Journal of Business and Economic Statistics 2 (3): 201-214.

Marcellino M., Stock J.H., Watson M.W. (2001), "Macroeconomic forecasting in the euro area: country specific versus area-wide information, European Economic Review

Melnick R. (2005), "A Peek into the Governor's Chamber: the Israeli Case", Israeli Economic Review, Vol.3, No.1

Pomposhko, H. and Hecht, Y.(2006). "RND", Monetary Studies, 1, Bank of Israel (in Hebrew)

Reijer A.H.J. and Vlaar P.J.G. (2003), "Forecasting Inflation: An art as well as a science !", Dutch Central Bank, mimeo.

Rotemberg J. and Woodford M. (1998), "An optimization-based econometric framework for the evaluation of monetary policy: expanded version, working paper 223, NBER

Smets F. and Wouters R. (2003), "An estimated stochastic dynamic general equilibrium model of the eurozone", Journal of European Economic Association.

Suchoy T., and Rotberger Y. (2006), "Enhancing the accuracy of the seasonal model for forecasting CPI in the short term", Discussion paper of research department, Bank of Israel

Stock J. and Watson M. (1999), "Forecasting Inflation", working paper 7023, NBER

Svensson L.E.O. (2000), "Open-economy inflation targeting", Journal of International Economics 50, 155-183.

Svensson L.E.O. (2006), "Open-economy inflation targeting: further developments of inflation targeting", NBER

Theil H. (1954), "Linear aggregation of economic relations, North Holland, Amsterdam.

Wallis K.F. (2004), "Comparing empirical models of the euro economy", Economic Modeling 21, 735-758

Woodford M.(2003), "Interest and Prices Foundations of a Theory of Monetary Policy", Princeton University Press, Princeton, NJ.

Zelner A. and Tobias J. (2000), "A note on Aggregation, Disaggregation and Forecasting Performance", Journal of Forecasting 19: 457-69

Monetary Studies

עיונים מוניטריים

אי אזולאי, די אלקיים – מודל לבחינת ההשפעה של המדיניות המוניטרית על האינפלעיה בנשראל 1988 עד 1996	1999.01
האינבעביה בישראל, 1960 ער 1990 די אלקיים, מי סוקולר – השערת הניטרליות של שיעור האבטלה ביחס לאינפלציה בישראל – בחינה אמפירית, 1990 עד 1998	1999.02
M. Beenstock, O. Sulla – The Shekel's Fundamental Real Value	2000.01
O. Sulla, M. Ben-Horin – Analysis of Casual Relations and Long and Short-term Correspondence between Share Indices in Israel and the	2000.02
Y. Elashvili, M. Sokoler, Z. Wiener, D. Yariv – A Guaranteed-return Contract for Pension Funds' Investments in the Capital Market	2000.03
יי אלאשווילי, צי וינר, די יריב, מי סוקולר – חוזה להבטחת תשואת רצפה לקופות פנסיה תוך כדי הפנייתן להשקעות בשוק ההון	2000.04
די אלקיים – יעד האינפלציה והמדיניות המוניטרית – מודל לניתוח ולחיזוי	2001.01
ע׳ אופנבכר, ס׳ ברק – דיסאינפלציה ויחס ההקרבה : מדינות מפותחות מול מדינות מתעוררות	2001.02
D. Elkayam – A Model for Monetary Policy Under Inflation Targeting: The Case of Israel	2001.03
די אלקיים, מי רגב, יי אלאשווילי – אמידת פער התוצר ובחינת השפעתו על האינפלציה בישראל בשנים האחרונות	2002.01
על שער Call רי שטיין – אמידת שער החליפין הצפוי באמצעות אופציות ה-Forward	2002.02
רי אלדור, שי האוזר, מי קהן, אי קמרה – מחיר אי-הסחירות של חוזים עתידיים (בשיתוף הרשות לניירות ערך)	2003.01
R. Stein - Estimation of Expected Exchange-Rate Change Using Forward Call Options	2003.02
רי שטיין, יי הכט – אמידת ההתפלגות הצפויה של שער החליפין שקל-דולר הגלומה במחירי האופציות	2003.03
D. Elkayam – The Long Road from Adjustable Peg to Flexible Exchange Rate Regimes: The Case of Israel	2003.04
R. Stein, Y. Hecht – Distribution of the Exchange Rate Implicit in Option Prices: Application to TASE	2003.05
א׳ ארגוב – מודל לחיזוי הגירעון המקומי של הממשלה	2004.01

יי הכט, וה׳ פומפושקו – נורמליות, רמת סיכון שכיחה ושינוי חריג בשער החליפין	2004.02
D.Elkayam ,A.Ilek – The Information Content of Inflationary Expectations Derived from Bond Prices in Israel	2004.03
ר. שטיין – ההתפלגות הצפויה של שער החליפין שקל-דולר, התפלגות א-פרמטרית הגלומה באופציות מטבע חוץ	2004.04
Y. Hecht, H. Pompushko – Normality, Modal Risk Level, and Exchange-Rate Jumps	2005.01
יי אלאשווילי, מי רגב – גזירת הציפיות לאינפלציה משוק ההון	2005.02
אי ארגוב – כלל ריבית אופטימלי למודל מוניטרי של המשק הישראלי	2005.03
M.Beenstock, A.Ilek – Wicksell's Classical Dichotomy: Is the Natural Rate of Interest Independent of the Money Rate of Interest ?	2005.04
יי הכט והי פומפושקו- RND	2006.01
די אלקיים, אי ארגוב – ניסוח ואמידה של מודל ניאו-קיינסיאני למשק קטן ופתוח, יישום למשק הישראלי	2006.02
Z.Wiener, H.Pompushko- The Estimation of Nominal and Real Yield Curves from Government Bonds in Israel	2006.03
אי אילק – המודל החודשי להערכת האינפלציה והמדיניות המוניטרית בישראל	2006.04
E.Azoulay, M.Brenner, Y.Landskroner –Inflation Expectations Derived from Foreign Exchange Options	2007.01
A.Ilek -Aggregation versus Disaggregation- What can we learn from it?	2007.02

Bank of Israel – Monetary Department בנק ישראל – המחלקה המוניטרית POB 780 91007 Jerusalem, Israel

ת״ד 780 ירושלים 91007