

Bank of Israel



Research Department

**Does the Israeli Yield Spread Contain
Leading Signals? A Trial of Dynamic
Ordered Probit**

by

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Abstract

This work aims to predict downturns in real economic activity, by exploring a well-described theoretical linkage between the yield spread and a coming recession. Since the Israeli data are of short span and do not provide convincing evidence of forward-looking yield spread, I introduce the "standardized" yield spread, adjusted for monetary shocks, which are not found to be connected directly to real output. This prior adjustment factor is captured as a latent variable, which can be identified by a regime of conditional volatility. For this purpose, the two-regime Markov chain with switching mean and variance is applied.

Downturns are evaluated in respect of their onsets, but not their deepness or duration. The ex ante probability of decline, calculated six months ahead, is kept conditional on the current economic growth. To account for the serial correlation effect, a dynamic ordered method has been applied. The parameters are estimated via the Gibbs sampler.

The suggested approach seems to overcome the lack of linearity between the yield spread and subsequent real output. The main recessionary episodes of 1996, 1998 and late 2000 could be anticipated, by simulation results. Since the model parameters are self-updating upon the new sample data, this method can apparently be used in a current projection.

In the upward direction, only two rightly predicted upturns have been accepted. Because of a short sample, one cannot conclude whether the well-known asymmetry of yield-spread predictive power (in favor of recessionary signals) really holds.

1. Introduction

"Predicting the future is a tricky business", - sigh A. Estrella and F. Mishkin in one of their studies in the field. We agree wholeheartedly with regard to the case of Israel. The samples are short and volatile; preliminary released CBS estimates concerning current GDP and industrial production arrive after a delay and are then revised. Nevertheless, we seek to maintain a set of leading indicators for business-cycle monitoring.

Frequently, leading information is extracted from financial behavior, because investors try to foresee recessions and reduce their exposure ahead of time. Many economic institutions exploit variables such as interest rates, stock prices and monetary aggregates as leading business-cycle indicators. An exhaustive review of the latest econometric attempts is made by Stock and Watson [2001].

This study addresses the predictive content of the yield curve with regard to Israel's cyclical downturns.

A yield curve (spread), i.e., the difference between nominal long-term and short-term yields to maturity of government indexed bonds, has been documented in the econometric literature as a significant predictor of future economic activity. However, most empirical studies emphasized recessionary signals contained in the yield curve rather than expansionary ones. Although rising long-term and short-term nominal rates both hurt real economic activity – long, on the investment side and short, on the current activity side (consumption, imports-exports through real exchange rate) - the hidden assumption behind the focus on the spread is that short-term rates affect real economic activity more strongly than do long ones. In this context long-term rates are considered as the financial market's anticipation of future short-term rates. Estrella and Mishkin [1995, 1996] suggest that this forward interest rate can be decomposed into the expected real interest rate and expected inflation, both informative about future real growth.

Theory [Harvey, 1997] says that the transmission of demand between short-term and long-term bonds constitutes a hedging mechanism, in which the price of bonds is set in advance of possible downturns. If investors expect the economy to slide into recession, they will shift funds from a short-term investment to a longer one. As demand for longer maturity securities rises, their price is bid up and their yield moves down. The yield curve becomes flat or inverted. Another possible reason for an inverted yield curve is that a tight monetary policy may precede a recession [Friedman and Kuttner, 1989; Chauvet and Potter, 2001].

The linkage between the shape of the yield curve and the future position of the business cycle was first set as linear [Estrella and Hardouvelis, 1991; Stock and Watson, 1989]. Their works provide empirical evidence that a linear regression based on the term structure outperforms autoregression in real growth forecasting, just because the first uses extra information about future real activity beyond that contained in past economic activity or monetary policy. Hamilton and Kim [1999] have also shown how this incremental effect could be decomposed into two factors: an expectation effect and a term premium effect, both statistically significant.

Unfortunately, most linear models failed to anticipate the 1990-1991 USA recession. From this point, researchers have documented that the predictive power of the yield curve is not stable over time. Chauvet and Potter [2001] explain the parameter instability of models using the yield spread by the different response of the economy to real or monetary shocks. It is conceivable that the yield curve indicates future real activity only when the monetary authorities react to a deviation of actual output from its potential level [Paya et al., 2002], i.e., they do not only aim to control inflation. Another possible reason is structural changes in the stock market and changes in its volatility. A main issue was that the forecasting ability of linear models using the yield spread as exogenous for future output growth has decreased since the mid-1980s.

Israel's yield spread, available from 1988, gives the impression of being backward looking rather than forward looking. Empirically, the yield curve dynamic seems to be driven by the past Bank's interest rate¹ (in real terms) and exhibits very weak positive correlation with subsequent growth of real activity. Actually, the Bank of Israel first set an inflationary target in 1992 and has since managed a disinflation policy, mostly by maintaining interest rates high. As a result, the yield curve is mostly inverted throughout the period and this fact alone cannot explain any particular downturn.

This work aims to test which hypothesis is more plausible in Israel's case: the first one, that the tight monetary policy suppresses the connection between the yield curve and future real activity, or the second one, that the leading signals exist, but could not be extracted by linear technique.

Among the latest non-linear developments using the yield curve I would single out some that look promising from a practical point of view and gave rise to some of the ideas of this study.

The first is the threshold autoregression (TAR) model, applied by Paya and others [2001] for forecasting of industrial production in the USA and UK and based on yield curve content. They suggest an asymmetry and varying strength in the "yield spread - real growth" relationship, becoming weaker (or non-existent) after the adoption of inflation-oriented policies.

The second is regime-shift implementations: Markov-switching variance [Dueker, 1997] and Markov-switching mean [Bansal and Zhou, 2002]. The last work, for example, argues that the regimes related to the short interest-rate dynamics are intimately related to business cycle phases.

Facing the lack of linearity, this study exploits probit direction, that is it concentrates on the onsets of the troughs and not their deepness. I attempt to enhance the probit performance via two channels: first, by accounting for monetary shocks accompanying the business cycle, which may cause parameter weakness or instability. Second, I allow serial correlation of the produced forecasts.

I explore the idea that the financial market conditions are more volatile than the real economic state. Sectoral and short-living fluctuations, reflecting in investors' reaction

¹ The linear regression of the spread between the 7-year and 1/2-year indexed government bonds rates (ys) on the Bank's interest rate (r) in the previous month and adjusted for inflationary expectations, presented below, explains 75% of its variation (t-statistics are in parentheses):

$$ys_t = -1.51 + 0.51ys_{t-1} - 0.24r_{t-1} - 0.35\varepsilon_{t-1}, \text{ where } \varepsilon_{t-1} \text{ is a moving average term.}$$

(-6.9**) (5.0**) (-2.9**) (-3.3**)

to market-risk factors, do not necessarily generate recessionary expectations. The bear markets do not implicitly indicate an imminent recession. Chauvet and Potter [2000] argue that investors' behavior toward risk is asymmetric and generates financial states with higher volatility, amplitude and frequency than the business cycle has.

Since the yield returns are subject to financial shocks, the forward-looking coefficients of the spread relative to predicted business activity are time-varying. Obviously, they weaken with strong shocks. In this study these coefficients are assumed to switch between the lower and higher values, according to the variance of noise, coming from financial markets.

Chauvet and Potter [2001] suggested a "business-cycle-specific probit", using the yield spread, which allows the variance of disturbances to change every time, when each new cycle has started, i.e., since each recession onset. This study assumes that the location of breaks between the volatility states is unknown a priori and could not be directly related to the change of business-cycle phases. I try to evaluate their location by comparing the yield curve with the contemporaneous output (here – the industrial production rate), assuming that the conditional expectation and conditional variance of the spread are driven by a two-state Markov chain. The states of higher mean and lower volatility can be associated with bull markets, while the states of lower mean and high volatility indicate bear markets, but not necessarily the recession in real activity.

As soon as the financial states are identified, the yield spread may be standardized, i.e., adjusted for breakpoints of high volatility. Hence, the forecast equation views the spread lags with a fixed elasticities.

Next, the forecasts of downturns are calculated conditionally on the current phase of the business cycle. In contrast, while produced by simple probit, the recession forecasts depend only on the spread lags and are captured as timely independent.

Actually, introducing serial correlation in the probit is not straightforward, because the dependant variable is noncontinuous. Eichengreen, Watson and Grossman [1985] undertook a dynamic generalization of the simple probit, applied for ordered data. They offered a maximum-likelihood estimation inside each interval that the dependant variable belongs to. Due to Chib and Albert [1993, 1995] the inferring of parameters was greatly simplified through the Monte-Carlo simulation. This tool, named Gibbs sampling, was quickly appreciated and was applied in forecasting by Dueker [1997,1998], Chauvet and Potter [2001] and the present work as well.

The rest of the paper is organized as follows. Section 2 suggests the way to identify financial states of high and low volatility. Eventually this enables prior adjustment of the yield spread across the sample and stabilizes forward-looking elasticities. Section 3 evaluates the prediction parameters, as well as probability of downturn, whether it occurs in the six-month forecast horizon. Section 4 deals with simulation results. Section 5 tests the prediction power of the standardized spread against the original one via Bayes factors. Section 6 concludes.

2. The model

I consider the composite State-of-the-Economy Index as a reference series for the business cycle². We have shown in Marom et al. [2003] the convenience of this variable in real-time projection, like high frequency and early availability. The composite index has an additional advantage over the GDP rate of being smoother than its components and robust to the end-sample updates.

Without loss of generality, I assume that the six-month changes of the Index can be divided into J categories, ordered by their values from the highest positive (expansionary) changes to the most negative (recessionary) ones. For example, like those in Table 1, classified accordingly to rising probability of recession.³

Table 1. Six-month State-of-the-Economy Index changes, classified by probability of recession intervals (Sample: 1988-2004)

Probability of Recession (ex post)	Frequency	Average	Median	Standard deviation
< 0.1	31	2.31	2.46	0.87
0.1 – 0.3	36	2.09	2.28	0.98
0.3 - 0.5	27	1.38	1.24	1.16
0.5 – 0.7	27	0.39	0.20	1.06
0.7 – 0.9	29	-0.27	-0.55	1.03
> 0.9	40	-1.18	-1.25	0.79

Empirically, there is no linear correlation between the yield spread dynamic and subsequent Index change, in comparison to the long-run rate, so the phase transmission cannot be predicted. Instead, we can try a more modest task: to predict the Index fall outside the range of its current growth, i.e., its move to the lower category of growth.

Let us introduce J unobservable variables z_j ($j = 1, \dots, J$), which change around the last known State index I_{t-6} in the range $[I_{t-6} + \varpi^{(j)}, I_{t-6} + \varpi^{(j-1)}]$, $j(j = 1, \dots, J)$. Suppose that each z -variable follows linearly in this range after the yield spread lags, as follows:

$$\Delta z_t^{(j)} = \beta_{0t}^{(j)} + \sum_l \beta_{lt}^{(j)} x_{t-l} + \varepsilon_t^{(j)}, \quad \varepsilon_t^{(j)} \sim i.i.d$$

$$I_{t-6} + \varpi^{(j)} \leq z_{jt} \leq I_{t-6} + \varpi^{(j-1)}, \quad j = 1, \dots, J \quad (1)$$

where x_{t-l} - the yield spread, registered l months ago,

I_{t-6} - the State Index level (in log), known at the projection time,

$\beta_{lt}^{(j)}$ - time-varying elasticities in the range $[\varpi^{(j)}, \varpi^{(j-1)}]$.

² The composite State-of-the-Economy Index includes five components, calculated in terms of percent monthly change: industrial production, imports (excluding capital goods), revenue of trade and services, business-sector employee posts and exports goods(excluding agriculture).

³ Since January 2003 the ex post probability of recession has been computed monthly by the Bank of Israel alongside the State-of-the-Economy Index and used for classification of recessionary periods in Israel. See Bank of Israel Annual Report, 2003, p.p. 7

It can be easily seen from (1) the way that the model allows for serial correlation : the past observed index I_{t-6} exerts pressure on the z_j values, which give predictions for month t , so the latest are conditional on the previous cycle position.

One can calculate the expected six-month Index change ΔI_t^* from each range $[\varpi^{(j)}, \varpi^{(j-1)}]$ ($j = 1, \dots, J$) as :

$$\Delta I_t^* = z_{jt} - I_{t-6}$$

If ΔI_t^* , evaluated upon the range $[\varpi^{(j)}, \varpi^{(j-1)}]$, is distributed normally over this range, that is $\Delta I_t^* \sim N(\mu^{(j)}, \delta^{(j)})$, then the high values of probability

$$P^{(j)} = 1 - \Phi\left(\frac{\Delta I_t^* - \mu^{(j)}}{\delta^{(j)}}\right), \quad j = 1, \dots, J \quad (2)$$

where $\Phi(\cdot)$ is the cumulative distribution function, will indicate that the expected Index rate does not lie in the same range. It means, in other words, that the expected six-month change belongs rather to another category of growth, lower than the current one. Section 4 reports the simulation results, while the crucial $P^{(j)}$ value is assigned at 0.9. The dates having $P^{(j)} > 0.9$ were qualified as potential downturns.

In fact, some uncertainty remains in real-time projection about the true range of the Index change, mainly because of the next updates. Given the ex ante probabilities $\{P_t^{(1)}, \dots, P_t^{(J)}\}$ obtained from different ranges $j = 1, \dots, J$, the $\hat{P}_t = \max\{P_t^{(1)}, \dots, P_t^{(J)}\}$ should be accepted as a recessionary signal if $\hat{P}_t > 0.9$. In practice this means that the earliest value $P_t^{(j)} > 0.9$ ($j = 1, \dots, J$) signals downturn t .

The specification (1) requires simultaneous estimation of time-varying β -elasticities, as well as unobservable z_j -variables inside each j -th category ($j = 1, \dots, J$).

The next section suggests such a transformation of the yield spread, which reduces the number of parameters in (1) evaluated simultaneously and simplifies it to the form (3) with time-invariant coefficients:

$$\begin{aligned} \Delta z_t^{(j)} &= \hat{\beta}_0^{(j)} + \sum_l \hat{\beta}_l^{(j)} \tilde{x}_{t-l} + \xi_t^{(j)}, \quad \xi_t^{(j)} \sim i.i.d \\ I_{t-6} + \varpi^{(j)} &\leq z_{jt} \leq I_{t-6} + \varpi^{(j-1)}, \quad j = 1, \dots, J \end{aligned} \quad (3)$$

where \tilde{x}_{t-l} is the standardized yield spread at l -th lag,

and the $\hat{\beta}^{(j)}$ -elasticities are time-invariant and depend properly on the range of growth $[\varpi^{(j)}, \varpi^{(j-1)}]$ ($j = 1, \dots, J$).

Since the Δz_j variables are noncontinuous, their joint distribution with the spread is hard to evaluate. I implement the Gibbs sampler for this purpose. This technique aims to approximate an unknown joint distribution by sampling from a conditional distribution. All unknown parameters are treated as random variables, whose values are drawn from appropriate distributions. Prior distributions of all system parameters are assumed to be flat (noninformative). The sampling scheme is organized as iterative process, which

generates parameters values conditionally on other parameters, drawn earlier, as well as on the observed data.

This approach leads to data augmentation, while the in-sample information is combined with simulated parameters. When evaluating the conditional densities of $\Delta z_j (j = 1, \dots, J)$, the method treats data, augmented by a sequence of draws, as observed data.

The convention is that the posterior parameter distribution, obtained after a large number of draws from the respective conditional distribution, converges to the joint distribution. To obtain posterior distributions, the first portion of draws is discarded, as a rule, in order to eliminate initial settings effect. The expected parameter value is assigned its average by draws. Similarly, the standard error is set equal to the standard deviation by draws.

The case of ordered data was fully argued by Albert and Chib [1993]. Appendix A describes the main steps.

In total 4,000 iterations were carried out in each range, the first 2,000 of which were discarded and the subsequent 2,000 were used to calculate expected values of parameters and their standard errors. The results are summarized in Section 4.

3. Prior adjustment factor

Financial market conditions cause the β - elasticities of (1) to vary in time. Rising risk factor and monetary shocks increase the volatility of yield returns, which, in turn, weakens the connection between the current term structure and future real growth. Under the low noise, the forward elasticity strengthens.

This section aims to detect levels of high financial volatility and hence to eliminate elasticity breaks in (1).

By this means, we disconnect the identification step from the forecast. We are interested first to evaluate which spread corresponds to current real growth and what is the noise magnitude in this relationship. We assume that investors' behavior toward the rising risk or beliefs in excess returns produce financial states of higher frequency than real activity cycle. It affects both the spread mean and its conditional volatility: the latter rises, when the mean should rather fall.

The "real pattern" within the yield spread dynamics can be recognized, for instance, by regressing it on the contemporaneous industrial production rate. This regression should include an additional latent factor, whose mean and conditional variance switch between two states, later interpreted as bull and bear markets.

In the particular framework, the industrial production rate as a regressor has the advantage over the State Index of being a raw indicator. Indeed, the composite State Index, by construction, filters out a large portion of the variance, contained in its components but not recognized as a "common pattern". It is the way that the composite index ensures smoothness of the business cycle fit. The danger of over-smoothing in an early stage, like this, is that the filtered variance affects residuals and may distort estimates of financial states volatility. At the same time, a three-month moving average of the industrial production rate is helpful to preserve autocorrelation of the spread dynamic.

Thus, I relate the yield spread dynamic to the contemporaneous real growth by following way:

$$\begin{aligned}
x_t &= \mu_{S_t} + \gamma_i + \eta_t, \quad \eta_t \sim N(0, \sigma_{S_t}^2) \\
\mu_{S_t} &= \mu_0 + \mu_1 S_t \\
\sigma_{S_t}^2 &= \sigma_0(1 + h S_t)
\end{aligned} \tag{4}$$

where x_t is the current yield curve,

i_t is the three-month moving average of the industrial production rate,

η_t is the noise coming from monetary shocks.

By this, the expected yield spread, $\bar{x}_t = \mu_{S_t} + \gamma_i$ depends, on the one hand, on the current economic growth (i_t), and on the other, on the financial market conditions, uncorrelated with i_t . Being unobservable, they are captured by state variable S_t , taking on two values, 0 and 1, corresponding to distinct volatility levels. The transition probabilities between the states are governed by the Markov chain.

The variable S_t is random, its value at each moment t is drawn from a uniform distribution between 0 and 1. If the generated number is less or equal to the Markov probability $\Pr[S_t = 1 | \eta_t]$, the value of 1 is assigned. Otherwise, the value $S_t = 0$ is set. The Markov probabilities are calculated recursively through a Hamilton filter and depend on the remaining parameters of the model. The algorithm is fully described in Kim and Nelson [1998].

In terms of results, consider the case of $h < 0$. Thus, the state value $S_t = 0$ will indicate a state of higher volatility, having $\sigma_0^2 > \sigma_1^2$. We can associate this state with a bear market, which reflects investors' reaction to rising risk. If, in addition, $\mu_1 < 0$, the expected yield spread in the case of $S_t = 0$ is lower, which supports the interpretation. In a similar way, the value $S_t = 1$ will indicate a state of a low volatility, i.e. a bull market.

Note, however, that the interpretation concerning the distinction between bull and bear markets arises here from the framework, without being affected by exogenous information. For this reason, it does not compete with market analysts' assessments, made daily in real-time.

Since the financial states S_t have been fully identified, according to (4), for each observation $t=1, \dots, T$, we can standardize the yield spread to the homoscedastic form

$\tilde{x}_t = \frac{x_t}{f_t}$, where f_t is the prior adjustment factor, calculated by:

$$f_t = \begin{cases} \frac{1}{\sigma_0} & , \text{ iff } S_t = 0 \\ \frac{1}{\sigma_0(1+h)} & , \text{ iff } S_t = 1 \end{cases} \tag{5}$$

While the financial volatility breaks are captured by the prior adjustment factor applied to the spread, we can view the elasticities in (1) as time-invariant and proceed with (3).

I apply the Bayesian method of parameters inference. For details I refer to Appendix B.

3. The yield spread: original and standardized data

The empirical base for this study is narrow in comparison to the USA data. The yield returns on Israeli government indexed bonds have been precisely registered, according to our database, only for the last 16 years, while the USA data cover more than 40 years (Figures 1a – 1b).

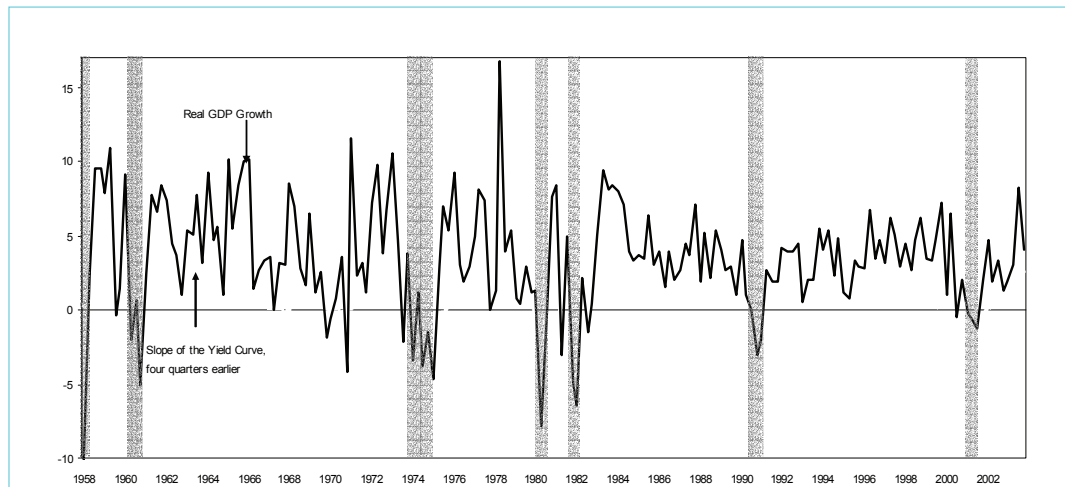


Figure 1a. The USA data for 1958-2003: real GDP growth in annual terms and the yield spread between the 10-year Treasury bond rate and the 3-month Treasury bill rate, 4 quarters earlier. The shaded areas denote NBER-dated recessions.

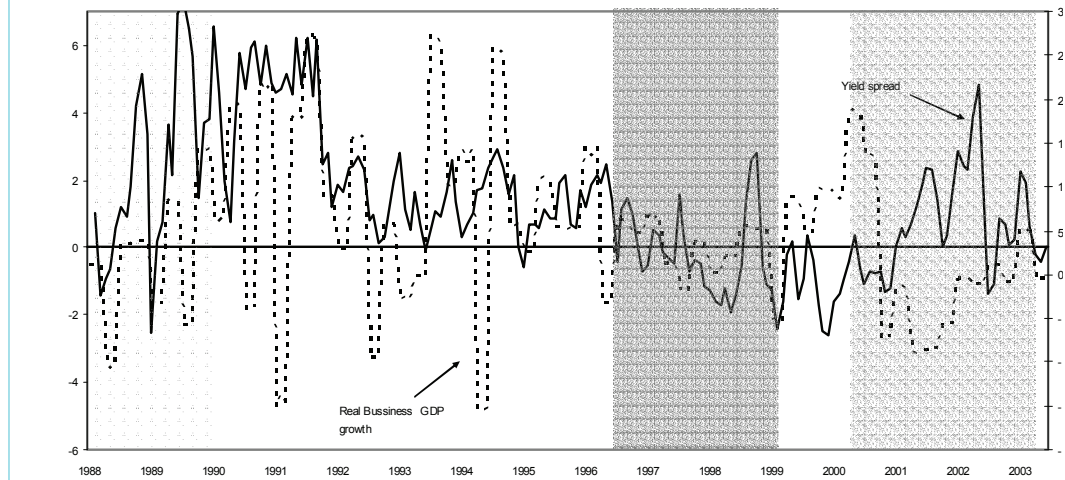


Figure 1b. The Israeli data for 1988-2003: real GDP growth in annual terms and the yield spread between the 10-year indexed government bonds rate and the 3-month Treasury bills rate, adjusted by inflationary expectations, six months earlier. The shaded areas denote Bank of Israel-dated recessions.

The term structure was strongly affected, during the considered period, by high inflation rates and disinflation policy steps. Thus, only indexed government bonds should be considered. The problem is that no yield returns of indexed government bonds having maturity period shorter than 1/2 year are available, so that a sufficiently wide spread cannot be calculated on the compatible base. The convention in literature stands on the spread between the 10-year government bonds rate and the 3-month Treasury Bills rate. In order to get proxy for the Israeli spread, as large as possible, I used the 3-month Treasury bills, adjusted by inflationary expectations.

Under the tight monetary policy since 1992 the yield curve is rather flat or inverted (Figure 1-b), the spread between the 10-year and the 1-year indexed government bonds rates stood at 2.23% for 1987-1991 and since 1992 has fallen to -0.17% on average. The slope of the spread between the 7-year and 1/2-year indexed government bonds rates was mostly negative through the disinflation period – (-1.5%) on average. The spread between the 10-year indexed government bonds rate and the 3-months Israel Treasury bills rate, adjusted by inflationary expectations has been likely flat since 1992 – about 0.5%.

The volatility of the spread differs between subperiods: the corresponding standard deviation for the span of 1988-1994 is 2.3%, of which only 0.8% in 1992-1994. During 1996-1999 it rises back to 1.43%

Table 2 reports the estimates of the parameters, obtained from (4). The yield spread refers to the spread between the 10-year indexed government bonds rate and the 3-month Treasury bills rate. The industrial production rate is calculated upon seasonally adjusted data and smoothed by a 3-month moving average.

Table 2. Estimated parameters* for the contemporaneous link between the yield spread and industrial production growth: sample 1988-2003

Parameter	Switching intercept and variance	Fixed intercept and switching variance
μ_0	0.57 (0.15) **	0.74 (0.10) **
μ_1	0.11 (0.10)	-
γ	75.8 (14.6) **	44.3 (13.6) **
σ_0^2	9.2 (1.75) ***	9.3 (1.72) ***
h	-0.88 (0.03) ***	-0.91 (0.02) ***

*The parameter standard error is in parentheses

It can be seen that the spread depends on contemporaneous real growth, and this connection is strongly heteroscedastic: two levels of noise, σ_0^2 and $\sigma_1^2 = \sigma_0^2(1+h)$ are extremely significant. Moreover, the derived variances differ almost 10 times: by the first version, the corresponding standard deviations are $\sigma_0 = \sqrt{9.2} = 3\%$ and $\sigma_1 = \sqrt{\sigma_0^2(1+h)} = \sqrt{9.2(1-0.88)} = 1\%$; by the second (with fixed intercept) – 3% and 0.9% accordingly.

One can learn from Table 2 that the intercept switch has the expected sign: it moves upward at the same time as volatility decreases. For illustration, this state is called "bull

market". The opposite case relates to "bear market". The previous section emphasizes, that this interpretation is model based and, concerning specific month, may differ from definitions by market analysts.

Further, the μ_1 -value is not found statistically significant. It means, in practice, that the "bull market" state does not guarantee an excess return in favor of the long-term bonds. Hence, equation (4) was recomputed with fixed intercept and switching variance (right column results). Finally, the prior adjustment of the spread was processed according to the revised version.

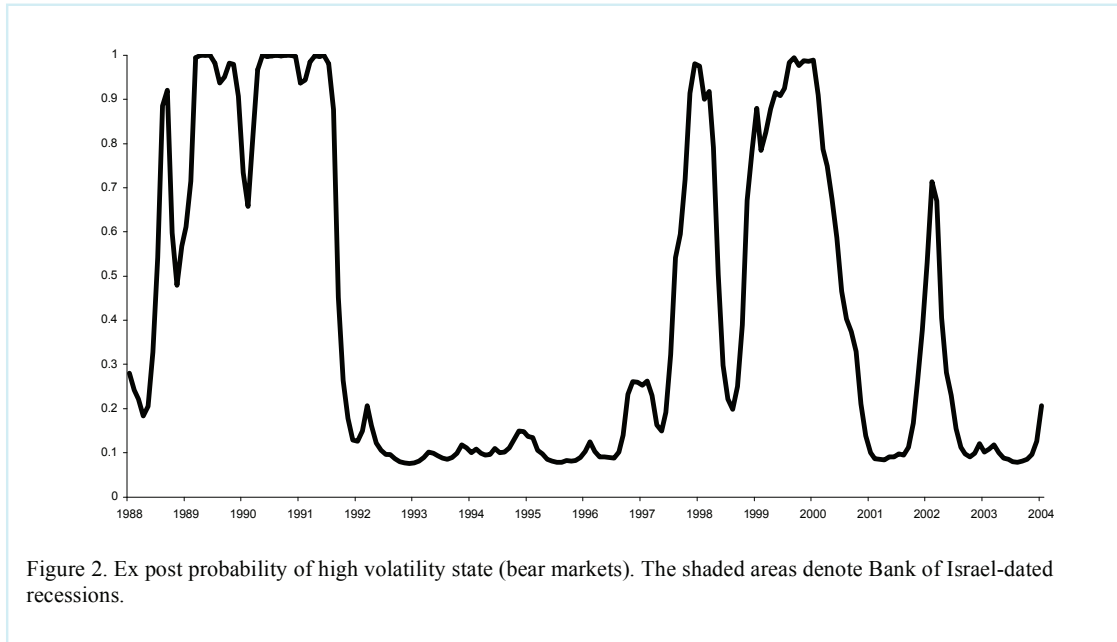


Figure 2. Ex post probability of high volatility state (bear markets). The shaded areas denote Bank of Israel-dated recessions.

Figure 2 demonstrates the probability of the high volatility state, corresponding to bear market conditions.

Table 3. Expected yield spread between the 10-year government indexed bonds rate and the 3-month Treasury bills rate (adjusted by inflationary expectations), by financial state and business-cycle phase*

Business cycle phase Financial state	Expansion ($\bar{i} = 0.78\%$)	Recession ($\bar{i} = -0.37\%$)	Long-run ($\bar{i} = 0.29\%$)
<i>By switching-intercept version</i>			
Bull markets	1.27%	0.4%	0.90%
Bear markets	1.16%	0.29%	0.79%
<i>By fixed-intercept version</i>			
Bear or Bull markets	1.1%	0.58%	0.87%

*The corresponding mean industrial production rate (per month) is given in parentheses.

The estimated link is helpful to in deriving an expected spread, corresponding to different financial and business-cycle states (Table 3). One can conclude that even if the switching intercept is introduced, the expected spread varies more due to business-cycle phases than by financial states. Recall, that the main difference between the financial states lies in the volatility domain.

Table 4. Linear projection coefficients* of the industrial production rate for 6 months, by yield spread, original and prior adjusted**

Spread lags	Original spread	Standardized spread
6	-0.40 (0.48)	0.17 (0.58)
7	-0.15 (0.64)	0.12 (0.68)
8	1.25 (0.64)**	0.86 (0.68)
9	0.34 (0.63)	0.26 (0.67)
10	0.52 (0.81)	0.01 (0.68)
11	1.00 (0.64)*	0.90 (0.68)
12	-0.52(0.48)	0.67 (0.58)
R^2	0.16	0.18
6-12 lags' average	1.46 (0.25)**	3.1 (0.5)**
R^2	0.15	0.17

* Multiplied by 10

** 3-month moving average, in %

As Table 4 shows, the yield spread can explain linearly 16-18% of variance of the industrial production rate six months ahead. At the same time, different forecasts may occur, depending on whether the original or the standardized spread is used as the predictor. In the first case, the lag structure of coefficients is crucial for specific prediction, because of the instability of their signs. When the adjusted spread is used, the spread has expected (positive) sign at all lags.

In this context, it is worth recalling the interesting results from Chauvet and Potter's [2001] predictive model. Using the yield spread, they simulated forecasts of recession in December 2001 upon different locations of structural breaks. Under the assumption of "no breaks" the ex ante probability of recession obtained was 45% ; when the breakpoint was assumed to be in 1984, the probability of recession rose to 90%; evaluating multiple breakpoints endogenously, the probability of recession stood at 32% only.

I would prefer an endogenous estimation from the following simple reason. The suggested model is oriented toward an operative projection. Thus, the presence of the breakpoints, as well as their location, is subject to change according to new data in the sample. Introducing (4), I ensure self-update of the parameters responsible for change in the financial state. In this way, the prior adjustment of the spread is carried out as well. In contrast, on-hand judgment requires repeated checks and is time consuming.

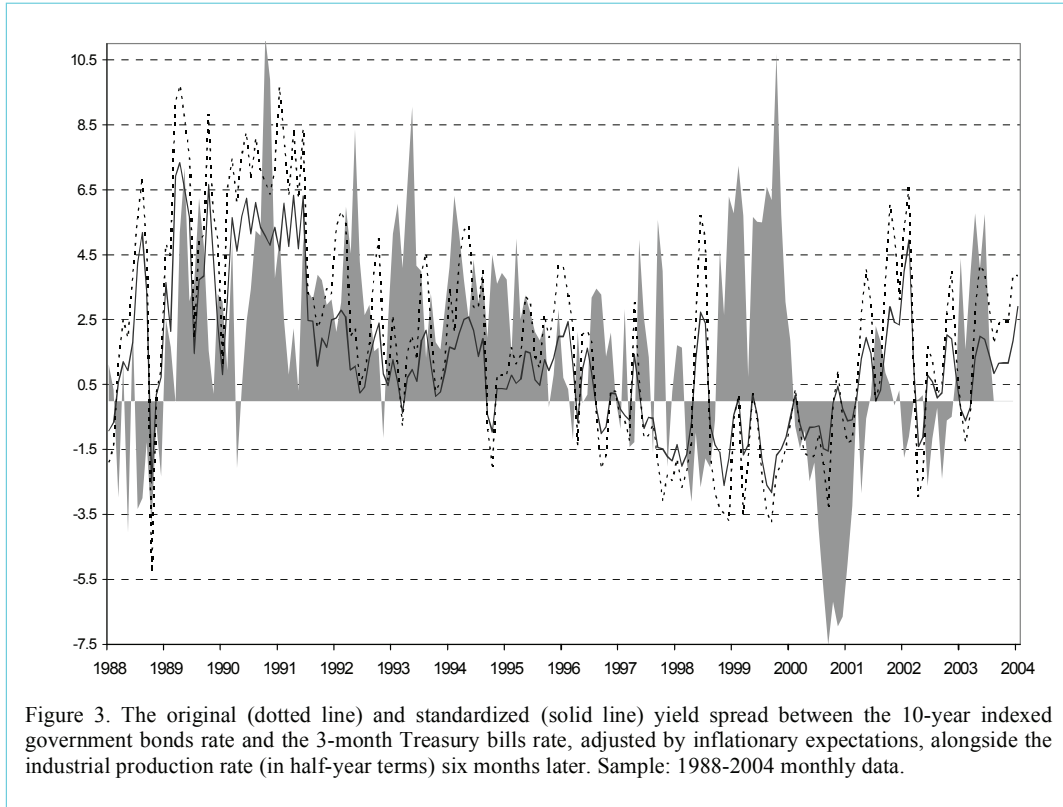


Figure 3 demonstrates the original and standardized yield spreads, in comparison to the industrial production rate (in half-year terms) six months later.

The next section describes the use of the standardized spread in prediction equation (3).

4. Dynamic ordered model: estimation results

I proceed now with an ordered model (3), which is supposed to provide ex ante probabilities that future real growth lies in a lower range than does the current growth. In general, different solutions may be obtained for the same month; first, because the elasticities vary from range to range; second, because the real growth (here – the State-of-the-Economy Index) does not have the same dynamic pattern during the recessionary phase as during the recovery phase. Third, the standard error of expected growth depends, among other things, on the width of the range, which are not equal. This, in turn, affects the derived probabilities.

Table 5 reports $\hat{\beta}^i$ -elasticities for the six-month forecast horizon, obtained from different ranges of change in the Index level. Concerning the last row in Table 5, notice, that the reported $\hat{\beta}^i$ -elasticities* were evaluated independently, after only one explainable variable was specified, constructed as a mean over 6-12 lags of standardized spread.

The Shapiro-Wilk statistics shows, that all estimated ΔI_t^* (expected six-month change) are normally distributed inside five appropriate intervals.

Table 5. The $\hat{\beta}^j$ -elasticities* of the forward six-month change in the State-of-the-Economy Index with respect to the standardized spread, by range of current growth

<i>Range Parameter</i>	> 2.28%	1.24% to 2.28%	0.20% to 1.24%	-0.55% to 0.20%	< -0.55%
<i>Intercept</i>	0.67 ** (0.06)	0.61 ** (0.07)	0.60 ** (0.07)	0.60 ** (0.06)	0.56 ** (0.07)
<i>lag 6</i>	0.07 * (0.05)	0.04 * (0.03)	0.00 (0.03)	0.01 (0.02)	0.01 (0.02)
<i>lag 7</i>	0.00 (0.05)	0.06 (0.06)	0.00 (0.06)	0.05 (0.05)	0.05 (0.05)
<i>lag 8</i>	0.07 (0.05)	0.06 ** (0.04)	0.05 ** (0.03)	0.03 * (0.02)	0.03 * (0.02)
<i>lag 9</i>	0.04 (0.04)	0.06 (0.06)	0.07 (0.06)	0.07 (0.06)	0.09 ** (0.06)
<i>lag 10</i>	0.05 (0.05)	0.07 * (0.05)	0.08 * (0.06)	0.07 * (0.05)	0.09 * (0.07)
<i>lag 11</i>	0.02 (0.03)	0.04 (0.05)	0.04 (0.05)	0.04 (0.04)	0.05 (0.05)
<i>lag 12</i>	0.05 * (0.03)	0.08 * (0.06)	0.08 * (0.05)	0.09 ** (0.05)	0.10 ** (0.07)
<i>6-12 lags` average</i>	0.288 ** (0.061)	0.250 ** (0.068)	0.283 ** (0.065)	0.257 ** (0.062)	0.251 ** (0.068)

* Multiplied by 10, excluding intercept

Figure 4 demonstrates early warnings of recessions, obtained six-month ahead upon different ranges. The bars indicate high levels of probability, that the rate of growth of the State Index, expected in the six-month forecast horizon, lies in a range, lower than the current one. These signals, simulated forward, have been compared with the ex post probability of recession (solid line). By this, the actual recessionary periods are recognizable by high and persistent ex post probability levels.

The simulation shows, that the produced signals do not provide alarm for all downturns, but do point toward the start of main recessions.

Unfortunately, we have no data to simulate the probability corresponding to the onset of the recession of 1987-1990. Obviously, the spikes of 1989-1990 at the end of the recession are not extremely helpful, because this recession had just started, unless they are interpreted as early warnings, which reflected concern about the Gulf War and the subsequent temporary decline of industrial production in 1991.

The recession of 1996 is caught quite timely. The first signals could be produced in June-December 1995 and related to the period December 1995-May 1996. It thus corresponds to the actual start of recession in 1996. The spike in August 1998 in the middle of the recession is superfluous. Note, however, that this signal would be produced in early 1998, at the end of temporary retreat of 1997, and would make sense in real-time monitoring in the face of the aggravation of the recession.

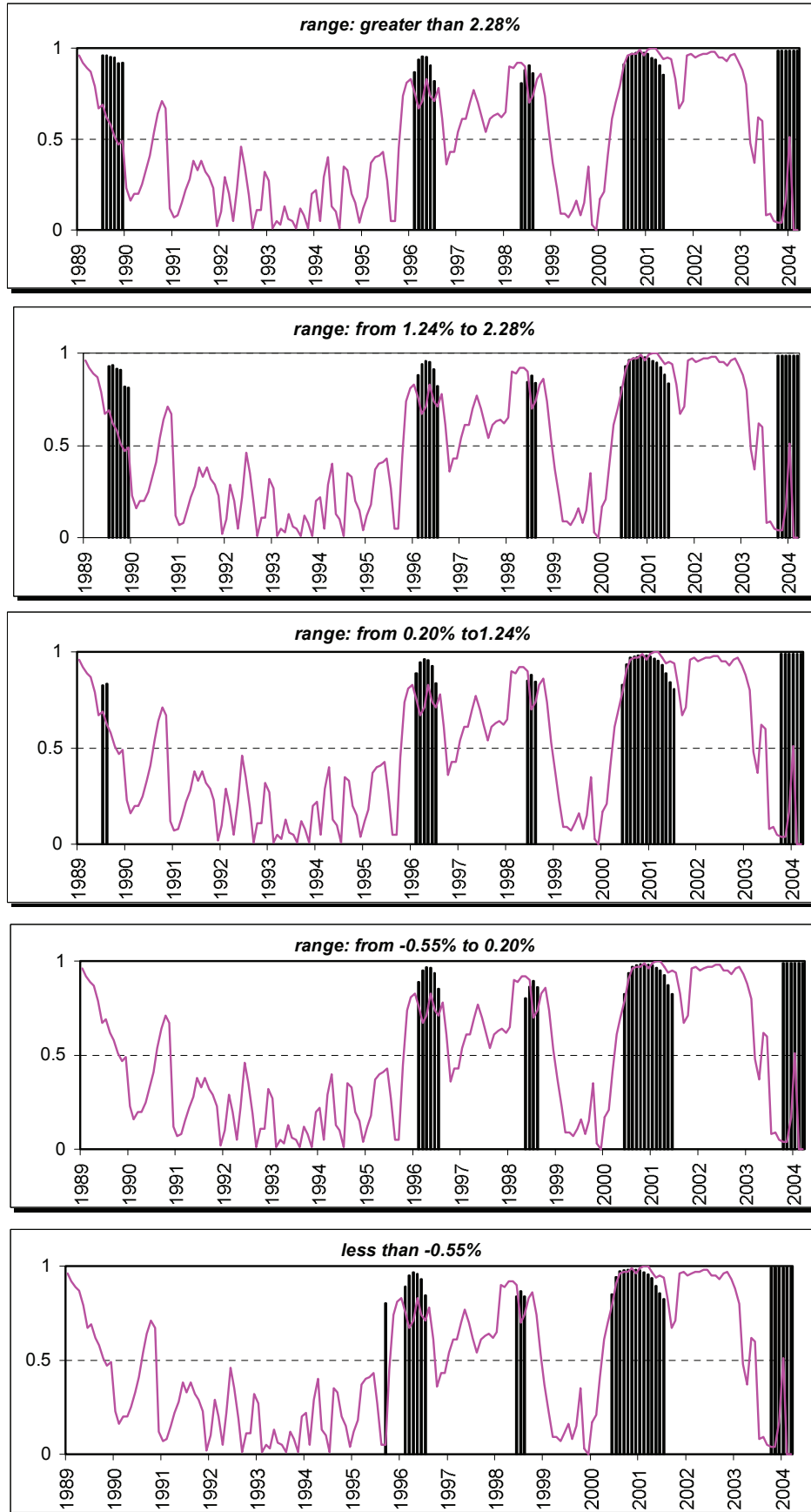


Figure 4. Hitting probabilities of downturn (bars), obtained upon the five ranges of the State Index rate, alongside the realized probability of recession (solid line).

The last documented recession, that of September 2000, could be signaled quite satisfactorily from all ranges in March-April 2000, pointing to its start in September-October 2000.

An additional significant recessionary signal was received about the second quarter of 2004, after the end of the last recession has been widely declared in November 2003. It will take time to conclude definitely whether this was a false alarm. The signal obtained gives evidence of the rising probability of recession since April 2004 and slowing down of the recovery documented by the CBS in October 2004.

In the upward direction, the model indicates accelerated growth in May-August 1992. This would not be dramatic news in the middle of the sharp recovery after the Gulf crisis. The next upward spike could be seen in December 1998-February 1999 and relates to May-August 1999. It provides timely indication of the high-tech boom that started in April 1999. The last recovery signal points to October 2003, the end of the long recessionary period during 2000-2003.

Because only two right recovery signals have been produced in advance, it is hard to conclude whether the well-known predictive asymmetry of yield spread (in favor of recessionary signals) really holds.

5. The Bayes factor for the standardized spread

I apply the Bayes factors to compare the performance of the model, using the standardized yield spread against the null hypothesis, when the original spread is used. The method fully corresponds to that in Chib [1995] and exploits the marginal likelihood ratio of the two alternative models.

This section aims to test the relative contribution of the standardized spread in predictive equations (3), relative to the original spread.

Since the ex ante probabilities of a downturn $P_t^{(j)}$ in (3) are evaluated for five different categories of growth ($j=1, \dots, 5$), five pairs of marginal likelihood values may be compared.

Denote the realized recessionary patterns by the binary variable $\text{Pr}_0 = 1$ if the ex post probability of recession is greater than 0.6, and $\text{Pr}_0 = 0$ otherwise. Then the marginal likelihood for model (3) estimated by Gibbs sampling is expressed in following way (for simplicity, the upper range notation (j) is omitted):

$$\ln m(\text{Pr}_0) = \ln f(\text{Pr}_0 | \hat{\beta}) + \ln \pi(\hat{\beta} | \bar{\beta}^{ols}, \delta_{\beta}^{ols}) - \ln \frac{1}{2000} \sum_{i=1}^{2000} \pi(\hat{\beta} | \beta_i, \delta_i) \quad (6)$$

where $m(\text{Pr}_0)$ - marginal likelihood, computed separately for each range;

$f(\text{Pr}_0 | \hat{\beta})$ - the likelihood function value, calculated under $\hat{\beta}$ -coefficients for each range according to

$$f(\text{Pr}_0 | \hat{\beta}) = \prod_{t=1}^T P^{\text{Pr}_0} (1 - P)^{1 - \text{Pr}_0} \quad ,$$

$\pi(\hat{\beta} | \bar{\beta}^{ols}, \delta_{\beta}^{ols})$ - the prior density of $\hat{\beta}$ -coefficients, having $\bar{\beta}^{ols}$ mean and δ_{β}^{ols} standard error, both obtained by OLS.

$\pi(\hat{\beta} | \beta_i, \delta_i)$ - the densities of $\hat{\beta}$ -coefficients, evaluated at each draw (i) of the Gibbs-sampler, relative to expectation β_i and standard deviation δ_i , both obtained in the i -th run of the sampler.

In the latest as well as in the previous expression, the density of the 8-dimensional normal random vector of elasticities at the point $\hat{\beta}$ is evaluated by:

$$(2\pi)^{\frac{8}{2}} \det(\Omega)^{\frac{1}{2}} \exp\left(-\frac{1}{2}(\hat{\beta} - \bar{\beta})\Omega^{-1}(\hat{\beta} - \bar{\beta})\right),$$

depending on which mean $\bar{\beta}$ and covariance Ω it relates to.

Notice also that the posterior density of $\hat{\beta}$ -coefficients in (6) is approximated by averaging the densities values across the draws.

Thus, marginal likelihood for the model estimated by Gibbs sampling measures the height of the maximized likelihood of the observed data, corrected by the ratio of the prior and posterior parameter densities.

The Bayes factor (in log terms) for Model A versus Model B is then:

$$\ln BF_{AB} = \ln m(\text{Pr}_0 | A) - \ln m(\text{Pr}_0 | B)$$

Recall that in our case Model A is based on the standardized yield spread, while Model B uses the original one.

Table 6 reports the results. To assess it, the following intervals of $\ln BF$ have been used, accordingly to recommendations of Jeffrey [1961]:

Table 6. Bayes factors* of standardized-spread-based model against the null (original-spread-based model), by range of the State-of-the-Economy Index growth, calculated for the six-month forecast horizon.**

Range	Likelihood*		Marginal likelihood*		Bayes factor*
	Original spread	Standardized spread	Original spread	Standardized spread	
> 2.28%	-121.75	-120.53	-123.53	-122.15	1.38
1.24% to 2.28%	-118.46	-112.67	-120.3	-114.28	6.02
0.20% to 1.24%	-115.93	-114.46	-118.26	-115.92	2.34
-0.55% to 0.20%	-115.54	-113.71	-119.08	-115.36	3.72
< -0.55%	-114.56	-112.67	-118.58	-114.28	4.30

* All statistics in this table are in log terms

**Original spread based model

$\ln BF_{AB} < 0$ - evidence supports the null;

$0 < \ln BF_{AB} < 1.15$ - very slight evidence against the null;

$1.15 < \ln BF_{AB} < 2.3$ - slight evidence against the null;

$2.3 < \ln BF_{AB} < 4.6$ - strong to very strong evidence against the null;

$\ln BF_{AB} > 4.6$ - very strong evidence against the null.

As can be seen from Table 6, the calculated Bayes factors provide strong to very strong evidence in favor of the standardized spread based model in four ranges of five, and once (for an upper range of growth) provide slight evidence against original spread use.

6. Conclusions

1. The suggested model builds the probability of downturn, which may occur in the next six months. It extracts leading recessionary signals from the yield-spread dynamic, which, in turn, captures the financial market reaction to the very first signs of economic decline.

2. There is no evidence of linear correlation between the yield-spread dynamic and the subsequent State-of-the-Economy Index rate throughout available data. The model exploits linearity only inside five local intervals: it signals decline to the lower range of growth, conditional on the current one.

3. The yield spread is cycle dependant. However, this connection is heteroscedastic, noised by financial markets' shocks, which cannot be related to real activity. The high and low noise levels differ almost 10 times, in terms of residual volatility. The expected spread varies more due to business cycle phases (from 0.3 % in recession to 1.3 % in expansion), than it does by financial states (from 0.79% to 0.9% in the long run). The only way to identify financial states, interpreted as bear and bull markets, is by the volatility level.

4. Since the yield spread is not variance stationary, its predictive capacity is unstable. In order to stabilize forward-looking elasticities, it is convenient to adjust the spread to the unique volatility regime. In the predictive equations a standardized spread, having unit variance, has been applied. The test shows that this transformation is approved.

5. The standardized spread can be considered as a weak (although significant) predictor, which effect is amplified by the high persistence of recessionary changes. The simulation shows that the recessions of 1996, 1998 and 2000 could have been timely foreseen.

6. The model is oriented toward an operative projection, hence it ensures self-updating of all the parameters, upon the receipt of new data in the sample.

7. The small number of early signals in the downward and upward directions does not enable us to conclude decisively whether the well-known predictive asymmetry of the yield spread (in favor of recessionary signals) really holds.

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Appendix A. Dynamic ordered probit: Gibbs-sampling estimation

The prediction equation (3) describes dynamics of J unobservable variables z_{jt} ($j = 1, \dots, J$), changing linearly after the standardized spread with time-invarying β -elasticities coefficients in the following way:

$$\Delta z^{(j)} = \hat{\beta}_0^{(j)} + \sum_l \hat{\beta}_l^{(j)} \tilde{x}_{t-l} + \xi_t^{(j)}, \quad \xi_t^{(j)} \sim i.i.d. ,$$

$$I_{t-6} + \varpi^{(j)} \leq z_{jt} \leq I_{t-6} + \varpi^{(j-1)}, \quad j = 1, \dots, J ,$$

where \tilde{x}_{t-l} – the standardized yield spread at the l -th lag,

I_t - the level (in log terms) of the State-of-the-Economy Index in month t ,

z_{jt} - the latent variable, developing in range $[I_{t-6} + \varpi^{(j)}, I_{t-6} + \varpi^{(j-1)}]$, $j = 1 \dots J$,

$\hat{\beta}^{(j)}$ - the coefficients vector appropriate to the range $[\varpi^{(j)}, \varpi^{(j-1)}]$ ($j = 1, \dots, J$)

Two main features of the prediction equation are of importance. First, neither β -coefficients not the dependent variables Z_j are known. Second, the Z_j as well as $\Delta z^{(j)}$ are noncontinuous. For this reason an iterative process of Gibbs sampling has been applied, which generates both β -coefficients and the unobservable Z_j values, by way of optimization.

An initial guess about β -coefficients is the ordinary least square estimate. It can be obtained by regressing the six-month change in the State-of-the-Economy Index on the standardized spread lags. Hence, the prior distribution of $\hat{\beta}$ is multivariate normal,

$$\hat{\beta} \approx N(X'X)^{-1} X' \Delta I, (X'X)^{-1} ,$$

where X is the 8-dimensional vector, including the unit vector and from the 6-th to the 12-th standardized spread lags,

ΔI - the six-month Index changes.

Notice, that the prior distribution of $\hat{\beta}$ is the same over the ranges, while the posterior distributions of $\hat{\beta}$, including their expectation and variance, differ by ranges.

The posterior distribution of $\hat{\beta}$ on the range $[\varpi^{(j)}, \varpi^{(j-1)}]$ is also multivariate normal:

$$\hat{\beta}^{(j)} \approx N(X'X)^{-1} X' \Delta Z^{(j)}, (X'X)^{-1}$$

depending on the changes in the latent variables, that is on the $\Delta z_t^{(j)} = z_{jt} - z_{jt-6}$, $t = 1, \dots, T$.

Given a previous value of $\hat{\beta}^{(j)}$, the unobservable Z_j ($j = 1, \dots, J$) are produced as independent variables having properties:

$Z_j | I, \hat{\beta}$ distributed $N(X\hat{\beta}^{(j)}, \delta^{(j)})$ truncated at the left by $[I_{t-6} + \varpi^{(j)}]$,
and at the right by $[I_{t-6} + \varpi^{(j-1)}]$.

In turn, the residuals' variance $(\delta^{(j)})^2$ is conditional on the $\hat{\beta}^{(j)}$ -coefficients and the evaluated latent variables Z_j . Since the residuals $\xi_t^{(j)}$ on the j-th range are given by

$$\xi_t^{(j)} = z_{jt} - z_{jt-6} - \sum_{l=6}^{12} \beta_l^{(j)} x_{t-l} \quad ,$$

the variance $(\delta^{(j)})^2$, assumed to be distributed as inverted Gamma, can be drawn from

$$(\delta^{(j)})^2 | Z_j, \hat{\beta}^{(j)} \sim IG\left(\frac{T}{2}, \frac{\xi^{(j)'} \xi^{(j)}}{2}\right)$$

The next iteration proceeds with the new draw of $\hat{\beta}$ -coefficients, conditional on the latest draw of Z_j -values and on the latest variance estimate $(\delta^{(j)})^2$.

Appendix B. Volatility states evaluation

Two states of distinct volatility levels are set by equations (4), rewritten as:

$$\begin{aligned} x_t &= \mu_0 + \mu_1 S_t + \nu_t + \eta_t, \quad \eta_t \sim N(0, \delta_{S_t}^2) \quad , \\ S_t &= 0, 1 \quad t = 1, \dots, T \end{aligned}$$

where x and i are observed and refer to the yield spread and industrial production rate, respectively. The remaining parameters have to be evaluated.

All unknown parameters are treated as random variables, whose values are drawn from the appropriate distribution. The sampling scheme is organized as repeated sequences of draws from appropriate distributions. Each cycle of draws gets through six groups of parameters, given below, and generates values, conditional on sample data and on the other parameters values, drawn during the current cycle or the preceding one. Such a process is run separately for each range of growth j ($j=1, \dots, J$).

Denoting by (k) the current draw, we process the following groups, from $g1$ to $g6$:

$$\begin{aligned} g1 &= \{S_t^{(k)}, t = 1, \dots, T | p^{(k-1)}, q^{(k-1)}, \sigma_0^{2(k-1)}, h^{(k)}, \mu_0^{(k-1)}, \mu_1^{(k-1)}, \gamma^{(k-1)}, x_T, i_T\} \\ g2 &= \{p^{(k)}, q^{(k)} | S_T^{(k)}\} \\ g3 &= \{\sigma_0^{2(k)} | h^{(k-1)}, \mu_0^{(k-1)}, \mu_1^{(k-1)}, \gamma^{(k-1)}, S_T^{(k)}, x_T, i_T\} \\ g4 &= \{h^{(k)} | \sigma_0^{2(k)}, \mu_0^{(k-1)}, \mu_1^{(k-1)}, \gamma^{(k-1)}, S_T^{(k)}, x_T, i_T\} \\ g5 &= \{\gamma^{(k)} | h^{(k)}, \sigma_0^{2(k)}, \mu_0^{(k-1)}, \mu_1^{(k-1)}, S_T^{(k)}, x_T, i_T\} \\ g6 &= \{\mu_0^{(k)}, \mu_1^{(k-1)} | h^{(k)}, \sigma_0^{2(k)}, \gamma^{(k)}, S_T^{(k)}, x_T, i_T\}. \end{aligned}$$

To start an iterative process, initial values of μ_0 and γ -coefficients are required, as well as an initial variance σ_0^2 . By regressing the yield spread on the contemporaneous

industrial production rate, the OLS estimates can be easily obtained and used as initial values for further iterations. Initially no intercept excess is assumed, i.e., $\mu_1 = 0$. The main estimation steps are given below by groups of parameters.

1. States values:

$$g1 = \{S_t^{(k)}, t = 1, \dots, T \mid p^{(k-1)}, q^{(k-1)}, \sigma_0^{2(k-1)}, h^{(k)}, \mu_0^{(k-1)}, \mu_1^{(k-1)}, \gamma^{(k-1)}, x_T, i_T\}.$$

The volatility states evaluation is conditional on the vector of residuals

$$\eta_t = x_t - \mu_0^{(k-1)} - \mu_1^{(k-1)} S_t^{(k-1)} - \gamma^{(k-1)} i_t, \quad t = 1, \dots, T,$$

which in turn depends on parameters estimated during the preceding sampler run.

The volatility state $S_t, t = 1, \dots, T$ is unobservable at each moment and evolves as a Markov chain with transition probabilities

$$\Pr\{S_t = 1 \mid S_{t-1} = 1\} = p, \quad \Pr\{S_t = 0 \mid S_{t-1} = 0\} = q.$$

The conditional filtered probabilities of the s -th state ($s=0, 1$) is computed according to Bayes' rule:

$$\Pr\{S_t = s \mid S_{t+1}, \eta_T\} = \frac{\Pr\{S_{t+1} \mid S_t = s\} \Pr\{S_t = s \mid \eta_T\}}{\sum_{s=0}^1 \Pr\{S_{t+1} \mid S_t = s\} \Pr\{S_t = s \mid \eta_T\}},$$

where $\Pr\{S_t \mid \eta_T\} = \Pr(S_t \mid S_{t-1}) \Pr\{\eta_t \mid S_t\} \Pr\{S_{t+1} \mid S_t\}$ is evaluated recursively, and the residuals are assumed to be normally distributed in any state, so that:

$$\Pr\{\eta_t \mid S_t\} = \frac{1}{\sqrt{2\pi\sigma_{s_t}^2}} \exp\left[-\frac{(x_t - \mu_{s_t} - \gamma i_t)^2}{2\pi\sigma_{s_t}^2}\right].$$

Once conditional probabilities of states are calculated, we can draw S_t from a uniform distribution on the $[0, 1]$ interval. If the generated random number is less than or equal to the calculated value of $\Pr\{S_t = 1 \mid S_{t+1}, \eta_t\}$, the state is identified as $S_t = 1$. Otherwise, $S_t = 0$.

2. Regime persistence : $g2 = \{p^{(k)}, q^{(k)} \mid S_T^{(k)}\}$

The transition probabilities $\{p, q\}$ can be defined as unconditional probabilities $\Pr\{S_t = s \mid S_{t-1} = k\}$ that the state k is followed by state s ($k, s=0, 1$), and populate the transition matrix:

$$\begin{pmatrix} p & 1-q \\ 1-p & q \end{pmatrix}$$

Values p and q are conditional on the states $S_t (t=1, \dots, T)$ and independent of the data set $x_t, i_t (t=1, \dots, T)$ and model parameters. The transition probabilities are assumed

to follow beta-distributions:

$$\begin{aligned} p &\approx \text{beta}(u_{11}, u_{10}), \\ q &\approx \text{beta}(u_{00}, u_{01}), \end{aligned}$$

where u_{ks} is the prior guess about the number of transitions between $S_{t-1} = k$ and $S_t = s$. After the states $S_t (t = 1, \dots, T)$ have been evaluated, the actual number of transitions n_{ks} may be counted, which, in turn, enables us to get posterior distributions

$$\begin{aligned} p | S &\approx \text{beta}(u_{11} + n_{11}, u_{10} + n_{10}) \\ q | S &\approx \text{beta}(u_{00} + n_{00}, u_{01} + n_{01}) \end{aligned}$$

The chosen initial u_{ks} are small relative to the sample size: $u_{00} = 10$, $u_{10} = 1$, $u_{11} = 10$, $u_{01} = 1$.

3. Basic volatility level: $g3 = \{\sigma_0^{2(k)} | h^{(k-1)}, \mu_0^{(k-1)}, \mu_1^{(k-1)}, \gamma^{(k-1)}, S_T^{(k)}, x_T, i_T\}$

Given that residuals in (4) are heteroscedastic, we first generate $\sigma_0^{2(k)}$, conditional on extra-variance $h^{(k-1)}$, by homoscedastic transformation:

$$\eta_t^* = \frac{\eta_t^{(k-1)}}{\sqrt{(1 + h^{(k-1)} S_t^{(k-1)})}}, \quad t = 1, \dots, T \quad ,$$

where the residuals $\eta_t^{(k-1)} = (x_t - \mu_0^{(k-1)} - \mu_1^{(k-1)} S_t^{(k-1)} - \gamma^{(k-1)} i_t)^2$ ($t = 1, \dots, T$) are calculated with parameters drawn during the preceding sampler cycle, as well as with extra-variance $h^{(k-1)}$.

To draw $\sigma_0^{2(k)}$, an inverted Gamma distribution is employed with parameters

$$\sigma_0^{2(k)} \approx IG\left(\frac{T}{2}, \frac{1}{2} \sum_t (\eta_t^*)^2\right) \quad ,$$

where the squares of residuals are summed up over all observations.

5. Extra-volatility: $g4 = \{h^{(k)} | \sigma_0^{2(k)}, \mu_0^{(k-1)}, \mu_1^{(k-1)}, \gamma^{(k-1)}, S_T^{(k)}, x_T, i_T\}$

To generate $h_1 = (1 + h^{(k)})$, conditional on $\sigma_0^{2(k)}$, we divide residuals in (4) by $\sigma_0^{(k)}$ and get:

$$\eta_t^{**} = \frac{\eta_t^{(k-1)}}{\sigma_0^{(k)}}, \quad t = 1, \dots, T \quad ,$$

where the residuals $\eta_t^{(k-1)} = (x_t - \mu_0^{(k-1)} - \mu_1^{(k-1)} S_t^{(k-1)} - \gamma^{(k-1)} i_t)^2$ ($t = 1, \dots, T$) depend on the parameters drawn during the preceding sampler cycle, and on the standard deviation $\sigma_0^{(k)}$ drawn in the current iteration.

The h_1 -value can be drawn from an inverted Gamma distribution, having parameters

$$h_1 \approx IG\left(\frac{T}{2}, \frac{1}{2} \sum_{t \in N_1} (\eta_t^{**})^2\right) \quad ,$$

where the squares of residuals are summing up only over the domain N_1 including the cases $S_t^{(k)} = 1$.

4. Elasticity by industrial production rate:

$$g5 = \{\gamma^{(k)} \mid \sigma_0^{2(k)}, h^{(k)}, \mu_0^{(k-1)}, \mu_1^{(k-1)}, S_T^{(k)}, x_T, i_T\}$$

First, we eliminate switching-intercept effect by subtraction:

$$x_t^{***} = x_t - \mu_0^{(k-1)} + \mu_1^{(k-1)} S_t^{(k)} \quad (t = 1, ..T) \quad .$$

Thus, the γ -elasticity, assumed normally distributed, can be drawn from

$$\gamma^{(k)} \approx N(i \Omega^{-1} i)^{-1} i \Omega^{-1} x^{***}, (i \Omega^{-1} i)^{-1} \quad ,$$

where i is the observed industrial production rate,

Ω refers to the $T \times T$ diagonal covariance matrix, with $\Omega_{tt} = \sigma_0^{2(k)}$ if $S_t^{(k)} = 0$ and $\Omega_{tt} = \sigma_0^{2(k)}(1+h)$ otherwise.

6. Switching intercept: $g6 = \{\mu_0^{(k)}, \mu_1^{(k-1)} \mid h^{(k)}, \sigma_0^{2(k)}, \gamma^{(k)}, S_T^{(k)}, x_T, i_T\}$.

To derive the lower and upper intercepts, the dependent spread is transformed as:

$$\dot{x}_t = x_t - \gamma^{(k)} i_t \quad (t = 1, ..T)$$

and the regression becomes

$$\dot{x}_t = \mu_0 + \mu_1 S_t^{(k)} \quad (t = 1, ..T) \quad .$$

Denoting by Y a $(T \times 2)$ -matrix, constructed from the unit vector and the vector of states $S_t^{(k)}$ ($t = 1, ..T$), we can draw a two-variate intercept from:

$$\mu^{(k)} \approx N(Y \Omega^{-1} Y)^{-1} Y \Omega^{-1} \dot{x}, (Y \Omega^{-1} Y)^{-1} \quad ,$$

where $\mu^{(k)} = (\mu_0^{(k)}, \mu_1^{(k)})$.