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Public Debt in a Long Term Discretionary Model

by

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Abstract

This paper combines two approaches to optimal monetary policy in a unified analytical framework One is the liquidity aspect, which usually deals with the Friedman rule in the context of a long term model, and the other focuses on the motivation to erode the public debt by surprise inflation, which is usually conducted in the framework of a short term model (from the point of view of the individuals). Contrary to the existing literature, we show that it is possible to have a steady state solution of the long term model with positive financial wealth in the discretionary regime, but this requires an extra monetary instrument in addition to base money. For this purpose we use interest on money. We show that the Friedman rule does not hold in the discretionary model when financial wealth is positive. We compare the results of the discretionary regime with an "honest government" model.

Key words: Optimal Money, Friedman Rule, Discretionary Regime, Financial Wealth

Introduction

The time consistency problem in monetary policy, in the context of the public debt, has been addressed mainly in the framework of one period models (from the point of view of the planning horizon of the private sector). This is in the tradition of Kydland and Prescott (1977) and Barro-Gordon (1983) models, which led to the important distinction between discretion and rules. These models highlight the role of surprise inflation as a major factor in creating the problem of dynamic consistency in monetary policy. Another branch of monetary theory deals with models where the public consists of individuals with an infinite horizon and with a benevolent government, which designs monetary policy so as to maximize the welfare of society.

The latter models, which highlight the liquidity motive and the validity of the Friedman rule, can be dynamically inconsistent, unless the initial non-indexed public debt (in which we include base money) is zero. This is because there is a motivation to erode the real value of the debt by surprise inflation, which is usually ignored in the context of the long-term models. This problem motivated authors of long term models to employ the assumption of the "honest government" which (by assumption) does not engage in surprise inflation tactics (as for example in Vegh 1987). In other cases the problem of dynamic inconsistency is disposed of by assuming (unrealistically) that initial, non-indexed, financial wealth is zero [as in Lucas and Stokey (1983) or Chari et. al. (1996)], or that the initial price level is arbitrarily large so as to wipe out the real value of initial financial wealth, as in Correia-Teles (1999). In the above cases, steady state inflation will be zero (if interest on reserves is set at the real interest rate) or minus the real interest rate (to conform with a zero nominal interest rate). The latter solutions are clearly inconsistent with reality, since usually we observe that the public carries positive financial wealth year in and year out.

The problem of time consistent monetary policy in the presence of non-indexed debt has been analyzed by Barro (1983) and by Poterba and Rotemberg (1990), in the context of a one period model (with respect to the individuals)[1]. These models arrive at the conclusion that in the discretionary equilibrium inflation must be positive in order to conform to the adverse expectations by the public, in view of the policymaker's incentive to erode the public debt. However, the one period models ignore the dual role of monetary policy in the discretionary regime- the erosion of the public debt <u>and</u> the provision of the desired liquidity for the economy. As long as these models use a single monetary instrument (base money or, directly, the inflation rate) they devote it to the aim of erosion of the public debt, ignoring the liquidity target.

The long term (representative agent) models suffer from the same problem, being based on a single monetary instrument (base money). The issue of the dual target is then resolved by treating only the liquidity consideration, ignoring the aim of eroding the public debt. The purpose of our paper is to extend the discretionary features of the one period models, to the infinite horizon agents, and to deal jointly with <u>both</u> of the above problems (the liquidity and the erosion motives) in the context of the public debt. However, as explained below, this requires the introduction of an additional monetary instrument.

In the attempt to introduce time consistency considerations in the long-term models we join recent papers, which proceeded in the same direction, like Obstfeld (1997) and Nicolini (1998). We focus, like Obstfeld on the motivation to create surprise inflation in order to increase fiscal revenues. In that model the steady state equilibrium is one of zero inflation, with initial financial wealth which does not constitute a fiscal burden for the government (in the sense that its shadow price is zero). <u>Unlike</u> the above paper, we show that we can have steady state equilibriums, which are dynamically consistent, along with

[1] The <u>government,</u> as opposed to individuals, in these models has an infinite horizon.

positive inflation and positive initial financial wealth, even though the latter constitutes a fiscal burden for the government.

The results of the Obsfeld model, like other representative agent models, are due the fact that they are based solely on a single monetary instrument (base money) which confines the possible set of steady state solutions to the zero or negative inflation. By contrast, we allow the policymaker to use another monetary instrument, and this enables us to consider alternative steady states, which correspond to different levels of the public debt and to different rates of inflation. In our model the additional monetary policy instrument takes the form of interest on commercial banks' reserves. This policy was common practice in the so-called "chronic inflation economies"[2], but it has not been implemented in the industrial world (these countries deserve a special treatment, as we shall explain later).

The coexistence of the motivation to erode financial wealth and the fact that the latter continues to be held by the public, must be explained by the cost of creating surprise inflation. In the present paper we follow Obstfeld (1997) in assuming that the cost is in the form of a negative effect of inflation on output, in addition to the usual cost of a reduction in the demand for money. In equilibrium, the benefit of erosion of financial wealth must be balanced by the damage to output. The public, who is fully aware of the policymaker's problem, raises its inflationary expectations to match the policymaker's optimal inflation, so that in equilibrium the policymaker cannot gain from surprise inflation, because there is none. However, unlike the Obstfeld model, we allow the policymaker to offset some of the effects of adverse expectations on the demand for money by paying interest on reserves (this is consistent with a second best optimum). In fact we show that the payment of interest on reserves (as an additional monetary

[2] For example, in Israel these payments amounted to 0.8 percent o GDP at the height of the inflationary process (in 1984).

<u>instrument) is necessary for the existence of a steady state solution with positive financial wealth.</u>

The incentive to pay interest on commercial banks' reserves was prevalent in chronic high inflation economies. The motivation was to protect the real liquidity in the economy from erosion by inflation. Recent cash in advance models, such as Nicolini (1998) and Rankin (2002), assume that the real money balances are equal to consumption and thus do not enable the real balances ratio to consumption to be affected by inflation. The payment of interest on reserves is useless in these cases. However, it is well known that real money balances (in relation to consumption) decrease dramatically when inflation increases. Thus in Israel the ratio of M_1 to GDP declined from fifteen percent at the beginning of the inflationary process in the early seventies to a mere three percent towards its end in 1985, while the ratio of consumption to GDP was relatively stable. We therefore enable the ratio of money to consumption to be flexible.

Since our version of the representative agent model is more general, it allows some modifications in the standard results which have been derived previously. Specifically, we will show that the Friedman rule (which requires that the cost of holding money should be brought down to zero) does not hold in the discretionary regime, when initial financial wealth is positive and when the payment of interest on reserves is allowed. In particular, the quantity of money will fall short of the satiation level. This leads to a resurrection of the trade-off between ordinary taxes and the inflation tax [as in Poterba-Rotemberg <u>op.cit.</u> and Mankiw (1987)], which is eliminated when the Friedman rule holds. Indeed, as noted, paying interest on banks' reserves (in excess of that required by the Friedman rule when inflation is zero)[3] emerges as an essential part of the solution in the discretionary regime [unlike the conclusion of Sargent-Wallace (1985)], which is not the case under full

[3] See review of this issue in Woodford (1990).

commitment, or when there is no incentive to erode the real financial wealth (for example, when it is zero).

Since in industrial countries it is uncommon to find that central banks pay interest on reserves, and yet they usually have positive (unindexed) public debt, which is maintained over time, we compared the foregoing analysis with the honest government model that does not necessarily require the payment of interest on reserves. This model relinquishes (credibly) the option of using surprise inflation tactics (this type corresponds to the "rules regime"). The latter model is dynamically inconsistent by the usual standards, but can be evaluated differently by alternative standards. By that we mean that in the latter case there are probably severe sanctions against using surprise inflation tactics (we abstract here from considerations of reputation). We compare the performance of the economy under the alternative economic systems with the aid of computer simulations.

In these simulations we solve the model numerically under steady state conditions, and compute the comparative statics. Our main finding in the simulations is that inflation in the discretionary regime is extremely sensitive to the reduction of its negative effect on output. Specifically, a reduction in the output cost of inflation entails a tremendous increase in the equilibrium level of inflation without causing any significant disruption in the real economy. Moreover, the increase in inflation for the above reason does not cause an increase in the inflation tax (a feature which has been observed in many inflation processes). All this is in sharp contrast to the honest government model, which is rather insensitive to the reduction in the output cost of inflation. This finding can be interpreted as a reflection of the extreme effect of indexation on inflation in an inflationary environment (the reverse causality is well-known).

Since our model permits the increase in financial wealth under steady state conditions, we can use the simulations to analyze the effect of the public debt, in the discretionary regime, on such macro economic variables as inflation, output, consumption, real balances and the rate of taxation. As one may expect, since an increase in financial wealth is associated with higher inflation, its effect on the economy is negative.

The role of interest on reserves in the discretionary equilibrium.

In the discretionary equilibrium there is an essential role for an additional (independent) policy variable, which in our model takes the form of the interest rate on money (v_t). As this is a novel feature in monetary policy models, it requires some intuitive explanation.

Since any discretionary equilibrium is also an equilibrium of the private sector given the market variables, it must satisfy the following conditions:

(a) MRS (c_{t+1}, c_t)= real interest in period t.

(b) MRS (m_t, c_t) = alternative cost of holding money in period t.

where MRS stands for Marginal Rate of Substitution, c_t is consumption and m_t and are real money balances (=M_t/P_t where M denotes nominal balances and P the price level) in period t. Thus MRS (c_{t+1}, c_t) is the MRS between c_{t+1} and c_t. In addition, the time consistent policy has to satisfy

(c) P_t is set so as to achieve the optimal erosion of initial wealth W_{t-1}.

W_{t-1} consists of nominal money balances and nominal bonds (including interest on these assets) carried over from the past. If W_{t-1} =0 then (c) is redundant. In this case, conditions (a) and (b) are satisfied by inflation π_{t+1} [=(P_{t+1}/P_t)-1] and the nominal interest rate i_t (between t and t+1), for an arbitrary value of v_t, as follows:

(a) MRS (c_{t+1}, c_t)= $(1+ i_t)/(1+ \pi_{t+1})$

(b) MRS (m_t, c_t) = $(i_t - v_t) / (1+ i_t)$

where m_t , c_{t+1} and c_t correspond to the equilibrium path. If, for example, the Friedman rule holds then i_t is set equal to v_t so that MRS(m_t, c_t)=0.

However, when $W_{t-1} > 0$ then P_t in every period (and hence also π_{t+1}) has to reflect the optimal erosion of the real value of W_{t-1} , namely condition (c). Under these circumstances, the values of π_{t+1} and i_t alone (as endogenous variables) are not sufficient to support the discretionary equilibrium. In this case we need to have v_t determined endogenously to satisfy (a) and (b), since π_{t+1} is tied by condition (c). It follows that v becomes an essential policy variable in the discretionary equilibrium.

Note that if there are no output costs of inflation, and $w_{t-1}(= W_{t-1}/P_{t-1})$ is positive, then it is optimal to set P_t at an arbitrarily large value (as in Correia-Teles op.cit.). However, in the presence of output costs of inflation, P_t has to be set so as to strike the right balance between the fiscal benefits of eroding financial wealth and the above costs.

__When we speak of the policymaker "setting P_t" , it does not necessarily mean that he sets it directly. What we mean is that he sets his monetary policy instruments so that they are consistent with the specified P_t.

The model

We start with a basic identity, which connects every two consecutive periods:

$$M_t+B_t+P_tc_t=(1-h_t)(1-\tau_t)(1-f_t)\, P_t+(1+i_{t-1})B_{t-1}+M_{t-1}(1+v_{t-1}) \qquad (1)$$

where B denotes nominal government (one period) bonds, i is the nominal interest rate on bonds and v is the nominal interest rate on money (we do not distinguish between base money and means of payment). The tax rate on labor income is denoted τ, and it is assumed to be a positive fraction.

The need for introducing v was clarified earlier. f_t denotes the percentage of reduction in output caused by inflation. Here we use two alternative models. For most of the paper we use Model A which assumes that the output costs are associated with deviations of inflation from zero, so that $f_t = f(\pi_t)$ and $f(0)=f'(0)=0$. We assume that $f'(\pi_t)$ has the same sign as π_t , and $f''(\pi_t)>0$ (primes denote derivatives). In Model B, which will be analyzed in the end of the paper, we assume that the output cost is associated with deviations of inflation from minus the discounted real interest rate ($-\rho/(1+\rho)$), which is the optimal rate of inflation under the Friedman rule, when there is no objection to negative inflation. Model A is more realistic, but we also use Model B in order to highlight the role of interest on reserves in the discretionary model, over and above its role in the Friedman model. In both models $(1-h_t)(1-\tau_t)(1-f_t)$ is the after-tax and after-inflation output, and $(1-h_t)$ is the time spent on production activity, which results in output of the same amount. In the following we focus on Model A.

To simplify the notation, it is convenient to define ("dissaving")

$$s_t \equiv c_t-(1-h_t)(1-\tau_t)(1-f_t) \tag{2}$$

so that we can write (1) as

$$M_t+B_t+P_t\, s_t=(1+i_{t-1})B_{t-1}+M_{t-1}(1+v_{t-1})\equiv W_{t-1} \tag{3}$$

which means that $M_t+B_t+P_tc_t$ is financed by current disposable income plus the financial wealth carried over from the previous period. Moving (3) one period forward and solving for B_t we obtain

$$B_t=(1+i_t)^{-1}\,[\,M_{t+1}+B_{t+1}+P_{t+1}\, s_{t+1}-M_t(1+v_t)] \tag{4}$$

Substituting (4) in (3) repeatedly and employing the non-Ponzi-game condition so as to eliminate B_{t+j}, we obtain the intertemporal budget constraint[4] of the individuals for t:

$$\Sigma m_{t+j}\, I_{t+j}\,(P_{t+j}/P_t)Q_{t+j-1}+\Sigma s_{t+j}\,(P_{t+j}/P_t)Q_{t+j-1}=(W_{t-1}/P_{t-1})(1+\pi_t)^{-1} \tag{5}$$

$$I_{t+j}\equiv(i_{t+j}-v_{t+j})(i_{t+j})^{-1},\quad Q_{t+j}\equiv[(1+i_t)(1+i_{t+1})\dots(1+i_{t+j})]^{-1},\quad Q_{t-1}=1.$$

where the summation is over j=0,1,2,…to ∞. The initial financial wealth W_{t-1}, defined in (3), is assumed to be positive.

[4] The intertemporal budget constraint of the government is $\Sigma m_{t+j}\, I_{t+j}\,(P_{t+j}/P_t)Q_{t+j-1}+\Sigma T_{t+j}\,(P_{t+j}/P_t)Q_{t+j-1}=(W_{t-1}/P_{t-1})(1+\pi_t)^{-1}+g\,\Sigma\,(P_{t+j}/P_t)Q_{t+j-1}$, where T denotes taxes. In steady states this reduces to $mI+T-g=[\rho/(1+\rho)]w_{-1}(1+\pi)^{-1}$. Adding the public and private budget constraints yields the resource constraint of the economy.

For simplicity, we use the approach of "money in the utility function", noting that there is a well known correspondence between this approach and the cost of transaction approach[5]. The discounted sum of utilities is given by

$$U_t= \Sigma\beta^j u(c_{t+j}, h_{t+j,} m_{t+j}) \qquad (6)$$

where β is the discount factor (a positive fraction)[6] and the summation over j is from zero to infinity. We assume that u has decreasing marginal utilities in all arguments and is separable in (m,c) and h. In some later applications we shall assume that the utility function is homothetic in m and c. The maximization of U_t at any t subject to the budget constraint (5) yields the following first order conditions for the private sector at t:

$$\beta u_c(t+j+1)/u_c(t+j)=(P_{t+j+1}/P_{t+j})(1+i_{\,t+j})^{-1} \qquad (7)$$

$$u_h(t+j)= u_c(t+j)(1-\tau_{t+j})(1-f_{t+j}) \qquad (8)$$

$$u_m(t+j)= u_c(t+j)\ I_{t+j} \qquad (9)$$

for j running from zero to infinity. (P_{t+j} should be interpreted as expectations, but in equilibrium these prices must be equal to the actual ones). Here $u_y(t+j)$ denotes the partial derivative of u with respect to y at period t+j. Inserting these first order conditions in (5) yields the implementability condition (IC)

$$IC_t= \Sigma\beta^j\,[m_{t+j}u_m(t+j)+s_{t+j}\ u_c\,(t+j)]\ -u_c\,(t)w_{t-1}\,(1+\pi_t)^{-1} =0, \qquad (10)$$

$$s_{t+j} \equiv c_{t+j}- (1-h_{t+j})u_h(t+j)/u_c(t+j).\ , \qquad w_{t-1} \equiv (W_{t-1}/P_{t-1})$$

where the summation is again over j from zero to infinity. The resource constraint (RC) of the economy is given by

$$RC_{t+j}=c_{t+j} +g -(1-h_{t+j})(1-f_{t+j})=0, \quad \text{all } j, \qquad (11)$$

where g is treated as a constant.

The optimization of the government in the discretionary model can be thought of as taking place given the public's expectations of the price series $\{P^e_{t+j}\}$which equals the

[5] See Correia-Teles (1999) for details.
[6] As required by Strotz (1956) for dynamic consistency.

equilibrium price series. Then the equilibrium series $\{m_{t+j}\}$ and the price series determine the series of nominal balances $\{ M_{t+j} \}$. It can be shown that these series determine the series $\{ v_{t+j} \}$ as a residual[7]. At a given t, an optimal plan of the government consists of a path of c_{t+j}, h_{t+j}, m_{t+j} and π_t[8] which maximizes U_t under the constraints (10) and (11) for a given w_{t-1}. We form accordingly the Lagrangian function

$$L_t= U_t +\psi_t IC_t -\Sigma\lambda_{t+j} RC_{t+j} \qquad (12)$$

where the summation is over j. The Lagrange multipliers ψ_t and λ_{t+j} are positive by the nature of the problem. The first order conditions at period t (setting j=0), omitting the subscript t, are as follows:

$$\partial L/\partial m =u_m + \psi [u_m +(m u_{mm} +c u_{cm}) - w_{-1} (1+\pi)^{-1}u_{cm}]=0 \qquad (13)$$

$$\partial L/\partial c = u_c + \psi [u_c +(cu_{cc} +mu_{mc})- w_{-1} (1+\pi)^{-1}u_{cc}] -\lambda =0 \qquad (14)$$

$$\partial L/\partial h = u_h + \psi [u_h -(1-h)u_{hh}] -\lambda(1-f) =0 \qquad (15)$$

$$\partial L/\partial \pi = \psi w_{-1} (1+\pi)^{-2}u_c -\lambda(1-h)f'=0 \qquad (16)$$

$$\partial L/\partial \lambda= RC=0 \qquad (17)$$

$$\partial L/\partial\psi= IC=0 \qquad (18)$$

These six equations determine the six unknowns m, c, h, π, λ and ψ for a given value of w_{-1} (assuming the solution exists). Denoting the vector of the six variables by z, we may consider them as functions of w_{-1}, say $z=\sigma(w_{-1})$. It can be seen that these function are time invariant. In a stationary equilibrium, which we shall analyze later, w_{-1} is constant over time and so is z.

[7] The optimal path determines the series $\{m_{t+j} \}$, $\{(P_{t+j+1}/P_{t+j})(1+i_{t+j})^{-1} \}=\{Z_{t+j} \}$ where $Z_{t+j}\equiv \beta u_c(t+j+1)/u_c(t+j)$ and $\{I_{t+j} \}\equiv\{u_m(t+j)/u_c(t+j)\}=\{(i_{t+j}-v_{t+j})(1+i_{t+j})^{-1}\}$. From this we can derive the expression $M_{t+1}/M_t = Z_t (m_{t+1}/m_t) (1+v_t)(1- I_t)^{-1}$, which shows that once the optimal path is determined then the series $\{M_{t+j} \}$ determines $\{v_{t+j}\}$.

[8] The last equation in the previous footnote shows that we have a degree of freedom to set P_t or π_t optimally for the initial period independently from $\{m_{t+j} \}$. Alternatively, it enables to set M_t consistent with P_t, leaving m_t constant, where P_t is chosen optimally. The equality is maintained by an offsetting change in v_t.

We assume that the utility function based on m and c is homothetic which implies that $(mu_{mm} + cu_{cm}) = 0$, with $u_{cm} > 0$, as in Correia-Teles (1999)[9]. It can then be seen from (13) that $u_m = 0$[10] if $w_{-1} = 0$ (as in the recent papers). However, $u_m > 0$ if $w_{-1} > 0$. Thus the Friedman rule holds if the possibility of $w_{-1} > 0$ is assumed away, but it does not hold in the discretionary equilibrium with $w_{-1} > 0$. This is also consistent with (16) which shows that if there are no costs of inflation ($f \equiv 0$) and $w_{-1} > 0$, then it is optimal to set inflation at an infinite rate, so as to erode completely the real value of initial financial assets. However, given the properties of f which we assumed previously, it can be seen from (16) that π_t must be positive if $w_{-1} > 0$, and the same must be true for u_m.

The intuition for positive values for u_m and π_t when $w_{-1} > 0$, is that the latter state induces the policymaker to take steps in order to stay within the limits of the resource constraint of the economy. This can be done by raising the income tax rate and by creating surprise inflation so as to erode w_{-1}, both of which involve a cost in terms of output. The public internalizes the inflationary intentions of the government and raises its inflationary expectations. This reduces m and increases u_m. (This argument is based on $u_{cm} > 0$).

The intuition can be formulated, alternatively, in terms of the transaction cost approach. As w_{-1} increases, the demand for consumption tends to increase by the wealth effect. In order to contain this tendency the price of consumption has to increase. This is performed by a reduction in m, which raises the transaction cost of consumption. In fact, all the results for the discretionary equilibrium can be derived by the transaction function approach instead of the "money in the utility function" approach. In the former approach money does not appear explicitly in the utility function. Instead it enters through the (constant returns to scale) transaction cost function $l = l(c,m)$, which is affected positively

[9] In that paper the above properties are assumed to hold at the satiation point of real balances.
[10] The satiation level m^* at $u_m = 0$ is assumed to be finite.

by c and negatively by m (for m smaller than the satiation level). As noted, this replicates the previous results[11].

Since all the functions relating the endogenous variables z to w_{-1} are time invariant, we may consider the above solution as pertaining to a steady state for a given value of w_{-1}, which stays constant over time. To determine i and v we use the first order conditions (7) and (9) in steady states:

$$u_m/u_c = (i-v)/(1+i) \tag{19}$$

$$(1+i) = (1+\pi)/\beta \tag{20}$$

where u_m/u_c and π can also be viewed as functions of w_{-1}[12], given by the solution of equations (13)-(18). It can be seen that we cannot set v arbitrarily, because then i cannot generally satisfy both (19) and (20). If $w_{-1}=0$ then $u_m=0$ by (13) and hence $i=v$ in (19). We also have $f'=0$ from (16), which implies $\pi=0$ by our assumptions. It then follows from (20) that $1+i=1+v=1/\beta>1$. An equilibrium solution, with $w_{-1}=0$, requires the payment of interest on reserves, which equals the nominal and real interest rate on bonds, as in Friedman's suggestion. However, if $w_{-1}>0$, then $\pi>0$ and $i>v$.

Differentiating Model A at $w_{-1}=0$.

Suppose that we take as a starting point $w_{-1}=0$ in Model A. By (13) and (16) $\pi=u_m=0$ at this point. If the utility function is homothetic in m and c, then the condition $u_m=0$ determines the ratio m/c. Suppose that we increase w_{-1} to a small positive value. Then π and u_m change from zero to positive values, and i-v changes from zero to $i>v$. These changes, evaluated at $w_{-1}=0$, can be described by the following derivatives:

$d\pi/dw_{-1}>0$, $di/d\pi=1/\beta>1$ and $dv/di<1$[13]. The latter does not imply that dv/di is positive, but we specify below the conditions under which this is true.

[11] For the equivalence of the two approaches see Correia-Teles op.cit.

[12] We will show later that in the framework of a steady state, w_{-1} has to be supported by an appropriate value of public debt b.

[13] Note that the left hand side of (19) increases as w_{-1} is raised above zero, since u_m becomes positive.

As noted we may consider u_m/u_c as a function of w_{-1} , say $\varphi(w_{-1})$. In addition, we may consider π as a function of w_{-1}, say $\pi(w_{-1})$. It follows from our assumptions that $\varphi'(0)>0$ and $\pi'(0)>0$ and that $di/dw_{-1} = (1/\beta)\pi'(0)$. Now if as a result of the increase in w_{-1}, $i/(1+i)$ rises by more than u_m/u_c then, by (19), v must increase. To show the conditions when this is true, note that $\pi(w_{-1})$ is determined by (16) which involves $f'(\pi)$, which is not the case with $\varphi(w_{-1})$. Equation (16) can be written as $w_{-1}= [\lambda(1-h)/(u_c\psi)]\ [(1+\pi)^2f'(\pi)]\equiv HD$ which equals zero at the point $w_{-1}=0$, since $[(1+\pi)^2f'(\pi)]\equiv D=0$ at $\pi=0$. Differentiating by w_{-1} and π, at $w_{-1}=0$, using $D(\pi=0)=0$, we obtain $d\pi/dw_{-1} =[Hf''(0)]^{-1}$. It follows that when f'' is very small we obtain a very large increase in i as result of an increase in w_{-1}. Since u_m/u_c does not depend on f, we may always obtain that the increase in i warrants an increase in v, under the foregoing conditions, provided $f''(0)$ is sufficiently small (this is confirmed in the simulations below).

To summarize: an increase in initial wealth (w_{-1}) from zero to positive values leads in model A to increases in inflation, nominal interest rate, interest on reserves (if f'' is sufficiently small) and the cost of holding money. All these features relate to the discretionary regime where initial wealth is positive. Under a regime where initial wealth is assumed to be zero, the Friedman rule prevails, so that the cost of holding money is reduced to zero.

To illustrate these and additional results, let us take a specific example for the utility function of the form

$$u= \eta(m, c)+q(h),.\quad q'>0,\ q''<0 \qquad (21)$$

$$\eta\equiv [m^{\alpha}c^{(1-\alpha)}-am-ec],\quad 0<\alpha<1 \text{ and } a,e>0.$$

Let us define $m/c\equiv k$. If we differentiate the system (13)-(18) at $w_{-1}=0$ we obtain the following results:

$$dk/dw_{-1} = -[\psi/(1+\psi)](k/c)<0$$

$$du_m/dw_{-1} = [\psi/(1+\psi)](1-\alpha)(a/c)>0$$

$d\pi/dw_{-1} = \{[\lambda(1-h)/\psi u_c]f'(0)\}^{-1} > 0$, $u_c = (1-\alpha)k^{\alpha} - e$

$du/dw_{-1} = (u_c - q_h)(dc/dw_{-1}) < 0$

$dc/dw_{-1} < 0$

$dh/dw_{-1} = - dc/dw_{-1} > 0$

The last three results deserve some comment. First, $(u_c - q_h)$ is positive by the first order optimality condition of the individuals. It can be seen intuitively that du/dw_{-1} must be negative, since an increase in financial wealth does not increase the real resources of the economy and its effect on demands must be sterilized by the government by raising the tax rate and the inflation rate, which are both undesirable. This implies $dc/dw_{-1} < 0$. Formally, if we differentiate the Lagrange function in (12) partially w.r.t. w_{-1} (at $w_{-1}=0$) we obtain $-\psi u_c < 0$. By using the envelope theorem we infer that, to a first approximation, the overall effect of w_{-1} on u is also negative. This confirms that $dc/dw_{-1} < 0$ and, by the resource constraint of the economy, $dh/dw_{-1} > 0$.

Simulations

To obtain some indications of what the model implies when initial wealth is positive, we ran some simulations of Model A where we increased w_{-1}, the cost of inflation (f) and the magnitude of government consumption (g). For this purpose we used the utility function (21) with $q(h)=h^{\delta}$, $0<\delta<1$. For $f(\pi)$ we assumed the form $f=\gamma\pi^2$, $\gamma>0$, which satisfies our previous assumptions[14]. All the simulations are solutions of (13)-(20) for $w_{-1} \geq 0$, with positive values of all variables (including the Lagrange multipliers). The results are presented in tables 1-3 in appendix 1 and in figures 1-3. It is clear that when $w_{-1}=0$, we can attain the first best optimum, but the situation is quite different when $w_{-1}>0$.

[14] The parameter values we used in the simulation are as follows: $\alpha=0.5$, $\beta=0.9$, $\gamma=1$, $\delta=0.5$, $w_{-1}=0.5$, $g=0.1$, $a=b=0.2$.

Figure 1 shows that inflation increases with initial financial wealth (w_{-1}), as one would expect, since the government has a greater incentive of eroding the real value of w_{-1} by inflation. The increase in w_{-1} induces also an increase in the tax rate, as we may expect by the intertemporal budget constraint of the government [this is consistent with the conclusion of the Poterba and Rotemberg (1990) model and Mankiw (1987)]. We can also see in figure 1 that the rate of interest on money (v) increases with w_{-1}. Note that according to the Friedman rule, if deflation is to be avoided, the interest on money should equal the real rate of interest, $\rho=(1/\beta)-1$, which in our simulation equals 11%. However, in the discretionary equilibrium, v increases with w_{-1} to levels far above the real interest rate, which may explain why inflationary economies (which are presumably closer to the discretionary regime) usually pay interest on banks' reserves in excess of the real interest rate.

Figure 2 shows the dramatic effect of a reduction in the output cost of inflation (as reflected by γ) on inflation and on interest on money. This is consistent with our previous analysis, noting that $f''=2\gamma$. It seems plausible to associate the decrease in γ with an increase in indexation in the economy. As long as $w_{-1}=0$ this factor does not come into play. However, in a discretionary regime, with $w_{-1}>0$[15], this has a dramatic effect on inflation, even though g does not change. In fact, the reduction in γ has the most conspicuous effect on inflation among all our experiments, and this happens without a corresponding effect on the rest of the system. This suggests that in the discretionary regime inflation may attain fantastic rates once the economy gets adjusted to "living with inflation", without any appreciable effect to the real part of the economy. This conforms with the notion that in chronic inflation economies, "inflation has a life of its own". Table 2 appendix 1shows that inflation may increase along with a <u>reduction</u> in the inflation tax, as the economy gets more adjusted to inflation.

[15] Whenever the simulations are based on a given positive value of w_{-1} we set it at $w_{-1}=0.5$.

On the positive side, we see that the reduction in γ has a favorable effect on output, consumption and utility. This may indicate the favorable side of an increase in indexation in the discretionary regime, and explain the motivation for its implementation by policymakers. The model is not capable, however, of showing the downside of indexation, since it does not contain the negative long-term effect of inflation on growth.

It is interesting to examine the effect of the increase in government expenditures (g) on the equilibrium values of the various variables (for a positive w_{-1}). We can see in figure 3 (see also appendix 1 table 3) that both inflation and the tax rate increase with g, reflecting the need to finance the increase in government expenditures. Note that when the Friedman rule prevails (when $w_{-1}=0$), it will not be optimal to use the inflation tax for this purpose. However, in the discretionay regime, with $w_{-1}>0$, there is no qualitative difference between the two sources of finance. What is less obvious is that the increase in g entails an increase in the interest rate on money (v). This is due to the negative effect of the rise in g on real money balances (via the rise in inflation), which the policymaker tries to obviate. The negative influence of the rise in g on the economy, is also reflected in the decrease in net output (1-h)(1-f) and utility (appendix 1 table 3).

Note that the effect of an increase in g on inflation is much milder than the effect of a reduction in γ, which suggests that the main reason for the increase in inflation in inflationary economies was related to the inflation-mitigation technologies rather to the increase in government expenditures or its deficit (as reflected by the inflation tax in our model).

<u>The complete steady state</u>

The above equations [(13)-(20)] have been derived for an arbitrary value of w_{-1}. However, in a full steady state equilibrium $w_{-1}=w$, hence

$$w_{-1}=w=m(1+v)+b(1+i) \qquad (22)$$

Given the value of w_{-1}, which was used to solve the latter system, we can solve (19), (20) for i and v and use (22) to determine b[16]. Thus the steady state value of w has to be supported by an appropriate value of b. Note that the optimal path derived earlier on the basis of (12) determines a path of w_{t+j} for all j. In a stationary equilibrium the values of w_{t+j} are constant over time. We may also say that in the stationary equilibrium, in every t, w_{t-1} determines $w_t = w_{t-1}$.

In a stationary equilibrium the rate of inflation is constant over time and the economy faces each period the same w_{-1}. Agents, including the government, use in every period the same decision rules based on the time invariant functions of w_{-1}, derived earlier. This assures time consistency. Indeed if people expect the foregoing steady state solution, then they will never be disappointed and their future plans will be realized. In fact there is no reason for anybody to deviate from the steady state solution since everyone is optimizing every period, taking into account the government's incentive to erode the financial wealth.

The steady state solution determines the rates of growth of M and B which are both equal to π, as well as the rate of v. Note that the above rates of growth are consistent with the optimal erosion of financial wealth as given by (16). When the latter is positive, the cost of holding money will also be positive.

A diagrammatic interpretation

The basic economic considerations of our analysis can be illustrated by a graphical presentation of a simplified version of our model[17]. There are two channels which connect π to w_{-1} – the liquidity channel and the erosion channel. In equilibrium, π derived from both channels must be equal. The liquidity channel involves three steps: the optimal

[16] The value of b as determined in (22) is consistent with the value of b implied by equation (3). By (13)-(18) the value of b in (3) is a function of w_{-1} , which is constant over time in steady states. This value of w_{-1} , combined with i and v, as determined by (19) and (20), ensure that the value of b in (22) correspond to the same steady state, which implies the consistency of b from the two calculations. This equality can also be verified from our simulations.

[17] In this simplified presentation we ignore the role of the function f(.) in equations (15) and (17).

liquidity, equation (13), the demand for money, equation (19), and the Fisher equation (20). The optimal liquidity step is described in figure 4, which shows that in the absence of any cost of increasing m the policymaker would reach the satiation level (u_m=0) at m*. However, the increase in m raises the utility value of w_{-1}, which is a disadvantage to the government, and this keeps u_m >0. An increase in w_{-1} will raise u_m and reduce m/c. Through its effect on the demand for money, (19), an increase in w_{-1} will raise i (treating for the moment v as a fixed parameter), which will raise π through the Fisher equation. Thus π is an increasing function of w_{-1}, and it can be seen that it is also increasing in v. This relationship is depicted in figure 5. Note that at w_{-1} =0 we have i=v and hence π=0 when v=ρ and π>0 when v>ρ. We denote the connection illustrated by this channel by the function $\pi=\theta(w_{-1},v)$.

The erosion channel reflects the balancing of the benefits from the erosion of w_{-1} and the cost of inflation in terms of output (figure 6). It can be seen that an increase in w_{-1} increases π as does a reduction in γ. We denote this function by $\pi=\varphi(w_{-1}, \gamma)$, which is represented by BB in figure 7[18].

For a given w_{-1}^0 the equilibrium is depicted by the intersection of BB and AA, which represents the liquidity channel θ, at the point E. That is, in equilibrium we have an equality $\pi=\theta(w_{-1},v)= \varphi(w_{-1}, \gamma)$. Essentially, AA has to adjust through v in order to cross through the point E, which is determined by BB and w_{-1}^0. Thus v is determined endogenously to conform with the steady state equilibrium. <u>If v would not adjust endogenously, then the steady state solution would not be possible when w_{-1} is positive</u>. Note also that a reduction in γ for a given w_{-1} will raise both π and v.

<u>The honest government model</u>

Industrial countries do not pay interest on reserves, and yet they are presumably capable of attaining a steady state equilibrium with positive financial wealth. This calls

[18] Note that the slope of this curve can be made arbitrarily large with the reduction of γ.

for an explanation in view of the fact that we stressed the essential role of interest on reserves in securing a steady state equilibrium when financial wealth is positive. In order to explain this phenomenon we have to assume that these economies face some sanctions, that are not treated explicitly in our model, for engaging in surprise inflation tactics. Thus if the sanctions of reneging on a promise are very severe politically, the public may be inclined to expect no intentional surprises on the part of the policymakers. In that case we may say that the policymakers are acting as in the "honest government model", due to Auernheimer (1974). How is this expressed in the framework of our analysis?

A natural way of formulating the latter regime in terms of our model is to assume that the policymaker has a way of giving up credibly the option of using surprise inflation tactics. Thus the honest government model is a version of the "commitment (or rules) regime". The formal way of doing this is to eliminate equation (16) along with the variable v from the model. This leaves us with seven equations and seven variables, which enable us to solve the system without requiring that the policymaker should set the inflation with the aim of eroding initial financial wealth, as stated by equation (16). In order to analyze the implications of this regime we used the system of equations (13)-(20), excluding (16), to solve for all endogenous variables, excluding v. We simulated this model for alternative values of w_{-1}.

The results of the simulation when v is set at zero, are given in appendix 2 table 1. It is shown that, in general, the reaction of the endogenous variables to an increase in w_{-1}, is similar to that of the discretionary regime. It is, however, somewhat surprising that the regime of the honest government performs worse, in terms of output and utility, for the same parameters, than the discretionary regime. This result can be traced to the large absolute value of inflation (which is negative in this simulation), which causes a big reduction in output through the function $f(\pi)$.

Basically, the optimal value of m is the one corresponding to $u_m=0$, which implies, when v=0, that the nominal interest rate is zero, so that $\pi= -\rho/(1+ \rho)$, while the optimal value of π is $\pi=0$, according to $f(\pi)$. Without the use of v, inflation may assume negative values which according to $f(\pi)$, may reduce the level of utility through equations (15) and (17). This can result in the honest government model being outperformed by the discretionary model, which is free to use v.

In order to investigate this point further, we ran the simulation setting v, arbitrarily, at a constant value equal to the real interest rate ρ $(=\beta^{-1}-1)$, as under the Friedman rule when inflation is required to be zero (see appendix 2 table 2). When $w_{-1}=0$ we can attain both the optimal quantity of money and the optimal inflation. However, for $w_{-1}>0$, we have $u_m>0$ so that we cannot attain the optimal quantity of money. The simulation results (appendix 2, table 2) show low positive values of inflation for $w_{-1}>0$, which still enable this version of the honest government regime to perform slightly “better” than the discretionary regime with the same parameter values (as judged by the utility value). Our conclusion is therefore that the honest government regime does not necessarily dominate the discretionary regime under all circumstances. Especially, if the honest government regime does not allow the payment of interest on reserves, when the optimal inflation is zero, then we may obtain in some cases that the discretionary regime is superior. The ambiguity of the results for $w_{-1}>0$ stems from two opposing considerations. On the one hand, the discretionary regime enjoys the benefit of the ability to use an additional monetary instrument (v), but on the other hand it has to face adverse expectations because it retains the option of using surprise inflation tactics.

Model B

In this case $f_t= f(\pi_t+\rho/(1+ \rho))$, where f has the same properties as those stated above. To simplify we denote $\pi_t+\rho/(1+ \rho)=x_t$. In the model B version of the discretionary regime, $w_{-1}=0$ implies x=0 in (16) and hence i=0 by (20), which means that $u_m=0$ can be satisfied in (19) with v=0. Thus when w_{-1}=0, we can attain the first best optimum in Model B in the discretionary regime without the use of v. However, when w_{-1}>0 the incentive to use surprise inflation arises in the same way as in Model A and consequently v becomes positive (appendix 3 table 1). Since in this case π is still determined as a function of w_{-1} we can see that (19) and (20) cannot be satisfied by i alone, and this gives rise to the use of v for positive w_{-1}.

In the version of Model B the honest government can also achieve the first best optimum when w_{-1} =0, even without the use of v. However, it cannot attain this result when w_{-1} >0 since then u_m>0. Note that this version of the honest government is very similar to the Model A version of the honest government where v is set equal to ρ. In both cases the government can attain the first best optimum when w_{-1} =0; still when w_{-1} >0 the latter version of the honest government performs better in our simulations than the former one (compare appendix 2 table 2with appendix 3 table 2).

A diagrammatic comparison

Figure 8 compares the determination of equilibrium in the Model B version of the discretionary and honest government regimes. It can be seen that with w_{-1} =0 both regimes attain the first best optimum with π= -ρ/(1+ ρ) without the use of v. However, when w_{-1} >0, the equilibrium rate of inflation in the discretionary regime rises along with v (which becomes positive).

Concluding remarks

The previous analysis indicates that we can attain the first best optimum when w_{-1} =0 in the discretionary model when the government uses v in Model A, or without the use of v in Model B. In the honest government model, the government can attain the first best optimum when w_{-1} =0, in the framework of Model A only if it sets v=ρ, or in the framework of Model B even without the use of v. However, when w_{-1}>0, none of the models can achieve the first best optimum and this gives rise to the incentive to erode the value of initial financial wealth in the discretionary model. This gives some advantage to the discretionary regime when $w_{-1}>0$, since this regime is free to use v as compared with the honest government, insofar as it is constrained in the use of v.

When $w_{-1}>0$, our simulations result in the following ordering of the above four cases in terms of the utility function: the best performance is related to the honest government in Model A with $v=\rho$. Next is the discretionary Model A with v determined endogenously. Then is the honest government Model B with v=0, and finally is the discretionary Model B with v=0. It must be stated, however, that the differences, as derived in the simulations, are rather small and may not be significant for policy evaluations. What is perhaps more significant is that the payment of interest on money enables the discretionary model to perform quite well with a rather mild increase in inflation. These results are presented in a concise manner in the summary table below.

The main difference between the discretionary regime and the honest government is not to be found in the reaction to changes in initial wealth, but rather in the reduction in the cost of inflation (γ). While we found that a decrease in γ has a dramatic effect on inflation in the discretionary model it has practically no effect on inflation in the honest government model because of the omission of (16). Although γ affects the solution of the honest government model through equation (15) and (17), this effect, being based on f(.) rather than on its derivative, is negligible (appendix 3 table 4). This leads to the conclusion that when the sanction for using surprise inflation is based solely on its damage to output, rather on social norms, then the inflationary situation in the economy may deteriorate to fantastic rates.

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Summary Table

$w_{-1}=0$ (All first best)

	Model A	Model B
Discretion	$u_m=\pi=0$,	$u_m=x=0$,
	v-endogenous, $v=\rho$	$v\equiv 0$
Honest Gov.	$u_m=\pi=0$,	$u_m=x=0$,
	v-exogenous, $v=\rho$	$v\equiv 0$

$w_{-1}>0$

	Model A	Model B
Discretion	u_m, $\pi>0$ u=second	u_m, $x>0$, u=fourth
	$v>\rho$	$v>0$
Honest Gov.	u_m, $\pi>0$, u=first	u_m, $x>0$, u=third
	v-exogenous, $v=\rho$	$v\equiv 0$

$f=f(\pi)$, $f(0)=0$

$f=f(x)$, $f(0)=0$, $x=\pi+\rho/(1+\rho)$

Appendix 1 - The simulation results – model A:

Table 1: The Initial Wealth effect on the economy:

Initial Wealth W_{-1}	Output $(1-h)(1-f)$	Consumption C	Leisure h	Tax rate τ	Tax revenue $\tau(1-h)(1-f)$	Inflation π	Interest on Money V	Cost of Money (i-v)/(1+i)	Quantity of Money m	Inflation Tax m(i-v)/(1+i)	Utility $\eta(c,m)+q(h)$	Lagrange Multiplier λ	Lagrange Multiplier ψ
0.00	0.690	0.590	0.31	14.5%	0.100	0.0%	11%	0.00%	3.686	0.000	2.032	1.24	0.18
0.05	0.683	0.583	0.32	14.7%	0.100	0.6%	12%	0.12%	3.600	0.004	2.012	1.22	0.18
0.10	0.677	0.577	0.32	14.9%	0.101	1.1%	12%	0.25%	3.512	0.009	1.992	1.20	0.18
0.15	0.669	0.569	0.33	15.2%	0.101	1.7%	13%	0.39%	3.422	0.013	1.971	1.18	0.18
0.20	0.662	0.562	0.34	15.4%	0.102	2.2%	13%	0.53%	3.329	0.018	1.949	1.16	0.18
0.25	0.653	0.553	0.35	15.7%	0.102	2.8%	13%	0.68%	3.234	0.022	1.926	1.14	0.18
0.30	0.644	0.544	0.35	15.9%	0.103	3.4%	14%	0.84%	3.134	0.026	1.902	1.13	0.18
0.35	0.635	0.535	0.36	16.2%	0.103	4.1%	14%	1.01%	3.029	0.031	1.876	1.11	0.18
0.40	0.624	0.524	0.38	16.5%	0.103	4.7%	15%	1.21%	2.917	0.035	1.848	1.09	0.18
0.45	0.611	0.511	0.39	16.8%	0.103	5.5%	16%	1.43%	2.795	0.040	1.817	1.07	0.18
0.50	0.597	0.497	0.40	17.1%	0.102	6.3%	16%	1.68%	2.660	0.045	1.783	1.05	0.19

Table 2: The output cost of inflation (Gamma) effect on the economy ($f(\pi)=\gamma\pi^2$):

Cost of inflation	Output	Consumption	Leisure	Tax rate	Tax revenue	Inflation	Interest on Money	Cost of Money	Quantity of Money	Inflation Tax	Utility	λ	ψ
0.02	0.62	0.52	0.37	16.2%	0.100	93%	112%	0.87%	2.99	0.03	1.85	1.15	0.20
0.05	0.61	0.51	0.38	16.5%	0.100	57%	73%	1.11%	2.86	0.03	1.82	1.12	0.20
0.10	0.60	0.50	0.39	16.7%	0.100	38%	51%	1.29%	2.78	0.04	1.81	1.10	0.20
0.20	0.60	0.50	0.39	16.9%	0.101	23%	35%	1.45%	2.72	0.04	1.79	1.08	0.19
0.50	0.60	0.50	0.40	17.1%	0.102	11%	22%	1.61%	2.67	0.04	1.78	1.06	0.19
1.00	0.60	0.50	0.40	17.1%	0.102	6%	16%	1.68%	2.66	0.04	1.78	1.05	0.19
2.00	0.60	0.50	0.40	17.2%	0.103	3%	13%	1.72%	2.65	0.05	1.78	1.05	0.19
5.00	0.60	0.50	0.40	17.2%	0.103	1%	11%	1.75%	2.65	0.05	1.78	1.04	0.18
7.00	0.60	0.50	0.40	17.2%	0.103	1%	10%	1.76%	2.65	0.05	1.78	1.04	0.18

Table 3: The Government spending (g) effect on the economy:

Government g	Output (1-h)(1-f)	Consumption C	Leisure h	Tax rate τ	Tax revenue τ(1-h)(1-f)	Inflation Π	Interest on Money V	Cost of Money (i-v)/(1+i)	Quantity of Money m	Inflation Tax m(i-v)/(1+i)	Utility η(c,m)+ q(h)	Lagrange Multiplier λ	Lagrange Multiplier ψ
0.00	0.74	0.74	0.26	5.5%	0.041	1.0%	12%	0.2%	4.53	0.01	2.34	1.05	0.03
0.01	0.73	0.72	0.27	6.6%	0.048	1.3%	12%	0.3%	4.40	0.01	2.30	1.05	0.04
0.02	0.72	0.70	0.28	7.7%	0.056	1.6%	12%	0.3%	4.25	0.01	2.26	1.06	0.05
0.03	0.71	0.68	0.29	8.8%	0.063	1.9%	13%	0.4%	4.11	0.02	2.21	1.06	0.06
0.04	0.70	0.66	0.30	10.0%	0.070	2.2%	13%	0.5%	3.95	0.02	2.16	1.06	0.07
0.05	0.69	0.64	0.31	11.1%	0.077	2.7%	13%	0.6%	3.79	0.02	2.11	1.06	0.08
0.06	0.68	0.62	0.32	12.3%	0.083	3.1%	14%	0.7%	3.61	0.03	2.06	1.06	0.09
0.07	0.66	0.59	0.34	13.4%	0.089	3.7%	14%	0.8%	3.42	0.03	2.00	1.06	0.11
0.08	0.65	0.57	0.35	14.6%	0.095	4.3%	15%	1.0%	3.20	0.03	1.94	1.06	0.13
0.09	0.63	0.54	0.37	15.9%	0.099	5.2%	15%	1.3%	2.96	0.04	1.87	1.05	0.15

Appendix 2 - The honest Government – model A:

Table 1: The Initial Wealth effect on the economy with v=0:

Initial Wealth W-1	Output (1-h)(1-f)	Consumption C	Leisure h	Tax rate T	Tax revenue τ(1-h)(1-f)	Inflation Π	Interest on Money V	Cost of Money (i-v)/(1+i)	Quantity of Money m	Inflation Tax m(i-v)/(1+i)	Utility η(c,m)+ q(h)	Lagrange Multiplier λ	Lagrange Multiplier ψ
0.00	0.674	0.574	0.32	14.8%	0.100	-10.0%	0%	0.00%	3.589	0.000	2.000	1.25	0.19
0.05	0.667	0.567	0.33	15.0%	0.100	-9.9%	0%	0.15%	3.489	0.005	1.978	1.23	0.19
0.10	0.660	0.560	0.33	15.2%	0.101	-9.7%	0%	0.31%	3.388	0.011	1.955	1.20	0.19
0.15	0.651	0.551	0.34	15.5%	0.101	-9.6%	0%	0.48%	3.284	0.016	1.931	1.18	0.19
0.20	0.643	0.543	0.35	15.7%	0.101	-9.4%	0%	0.66%	3.178	0.021	1.906	1.16	0.19
0.25	0.633	0.533	0.36	16.0%	0.102	-9.2%	0%	0.85%	3.067	0.026	1.880	1.13	0.19
0.30	0.623	0.523	0.37	16.3%	0.102	-9.0%	0%	1.06%	2.951	0.031	1.852	1.11	0.19
0.35	0.611	0.511	0.38	16.7%	0.102	-8.8%	0%	1.29%	2.827	0.037	1.822	1.09	0.19
0.40	0.598	0.498	0.40	17.0%	0.102	-8.6%	0%	1.56%	2.693	0.042	1.789	1.07	0.19
0.45	0.583	0.483	0.41	17.4%	0.101	-8.3%	0%	1.88%	2.542	0.048	1.751	1.04	0.19
0.50	0.563	0.463	0.43	17.8%	0.100	-7.9%	0%	2.29%	2.365	0.054	1.705	1.01	0.20

Table 2: The Initial Wealth effect on the economy with v=real interest rate (β^{-1}-1):

Initial Wealth W-1	Output (1-h)(1-f)	Consumption C	Leisure h	Tax rate T	Tax revenue τ(1-h)(1-f)	Inflation π	Interest on Money v	Cost of Money (i-v)/(1+i)	Quantity of Money m	Inflation Tax m(i-v)/(1+i)	Utility η(c,m)+ q(h)	Lagrange Multiplier λ	Lagrange Multiplier ψ
0.00	0.690	0.590	0.31	14.5%	0.100	0.0%	11%	0.00%	3.686	0.000	2.032	1.24	0.18
0.05	0.683	0.583	0.32	14.7%	0.100	0.1%	11%	0.13%	3.600	0.005	2.012	1.22	0.18
0.10	0.677	0.577	0.32	14.9%	0.101	0.3%	11%	0.26%	3.511	0.009	1.992	1.20	0.18
0.15	0.670	0.570	0.33	15.2%	0.102	0.4%	11%	0.39%	3.421	0.013	1.971	1.18	0.18
0.20	0.662	0.562	0.34	15.4%	0.102	0.5%	11%	0.54%	3.329	0.018	1.949	1.16	0.18
0.25	0.654	0.554	0.35	15.7%	0.103	0.7%	11%	0.69%	3.233	0.022	1.926	1.14	0.18
0.30	0.645	0.545	0.36	16.0%	0.103	0.9%	11%	0.86%	3.133	0.027	1.903	1.12	0.18
0.35	0.636	0.536	0.36	16.2%	0.103	1.0%	11%	1.04%	3.028	0.031	1.877	1.10	0.18
0.40	0.625	0.525	0.38	16.5%	0.103	1.3%	11%	1.24%	2.917	0.036	1.850	1.08	0.18
0.45	0.613	0.513	0.39	16.9%	0.103	1.5%	11%	1.46%	2.797	0.041	1.820	1.06	0.18
0.50	0.600	0.500	0.40	17.2%	0.103	1.8%	11%	1.73%	2.665	0.046	1.787	1.04	0.18

Table 3: The output cost of inflation (Gamma) effect on the economy (f(π)=γπ2) with v=real interest rate (β^{-1}-1):

Gamma	Output (1-h)(1-f)	Consumption C	Leisure h	Tax rate T	Tax revenue τ(1-h)(1-f)	Inflation π	Interest on Money v	Cost of Money (i-v)/(1+i)	Quantity of Money m	Inflation Tax m(i-v)/(1+i)	Utility η(c,m)+ q(h)	Lagrange Multiplier λ	Lagrange Multiplier ψ
0.02	0.600	0.500	0.40	17.2%	0.103	1.75%	11.1%	1.72%	2.671	0.046	1.788	1.04	0.18
0.05	0.600	0.500	0.40	17.2%	0.103	1.75%	11.1%	1.72%	2.671	0.046	1.788	1.04	0.18
0.10	0.600	0.500	0.40	17.2%	0.103	1.75%	11.1%	1.72%	2.670	0.046	1.788	1.04	0.18
0.20	0.600	0.500	0.40	17.2%	0.103	1.75%	11.1%	1.72%	2.670	0.046	1.788	1.04	0.18
0.50	0.600	0.500	0.40	17.2%	0.103	1.75%	11.1%	1.72%	2.668	0.046	1.787	1.04	0.18
1.00	0.600	0.500	0.40	17.2%	0.103	1.76%	11.1%	1.73%	2.665	0.046	1.787	1.04	0.18
2.00	0.599	0.499	0.40	17.2%	0.103	1.76%	11.1%	1.73%	2.660	0.046	1.785	1.04	0.18
5.00	0.596	0.496	0.40	17.2%	0.103	1.79%	11.1%	1.75%	2.642	0.046	1.780	1.04	0.18

Appendix 3 – model B:

Table 1: The Discretionary Regime - The Initial Wealth effect on the economy:

Initial Wealth W_{-1}	Output (1-h)(1-f)	Consumption C	Leisure h	Tax rate τ	Tax revenue τ(1-h)(1-f)	Inflation Π	Interest on Money V	Cost of Money (i-v)/(1+i)	Quantity of Money m	Inflation Tax m(i-v)/(1+i)	Utility η(c,m)+ q(h)	Lagrange Multiplier λ	Lagrange Multiplier ψ
0.00	0.690	0.590	0.31	14.5%	0.100	-10.0%	0.0%	0.00%	3.686	0.000	2.032	1.24	0.18
0.05	0.683	0.583	0.32	14.7%	0.101	-9.3%	0.6%	0.14%	3.590	0.005	2.010	1.22	0.18
0.10	0.675	0.575	0.32	15.0%	0.101	-8.6%	1.2%	0.28%	3.492	0.010	1.987	1.20	0.18
0.15	0.667	0.567	0.33	15.2%	0.102	-8.0%	1.8%	0.43%	3.392	0.015	1.964	1.17	0.18
0.20	0.658	0.558	0.34	15.5%	0.102	-7.3%	2.4%	0.59%	3.288	0.019	1.939	1.15	0.18
0.25	0.648	0.548	0.35	15.8%	0.103	-6.5%	3.1%	0.76%	3.180	0.024	1.913	1.13	0.18
0.30	0.638	0.538	0.36	16.1%	0.103	-5.8%	3.7%	0.95%	3.066	0.029	1.885	1.12	0.18
0.35	0.626	0.526	0.37	16.4%	0.103	-5.0%	4.4%	1.15%	2.944	0.034	1.855	1.10	0.18
0.40	0.613	0.513	0.39	16.8%	0.103	-4.1%	5.0%	1.39%	2.812	0.039	1.821	1.07	0.18
0.45	0.597	0.497	0.40	17.1%	0.102	-3.2%	5.8%	1.66%	2.663	0.044	1.783	1.05	0.19
0.50	0.577	0.477	0.42	17.5%	0.101	-2.1%	6.6%	2.01%	2.486	0.050	1.737	1.03	0.19

Table 2: The honest Government The Initial Wealth effect on the economy - with v=0:

Initial Wealth W_{-1}	Output (1-h)(1-f)	Consumption C	Leisure h	Tax rate τ	Tax revenue τ(1-h)(1-f)	Inflation Π	Interest on Money V	Cost of Money (i-v)/(1+i)	Quantity of Money m	Inflation Tax m(i-v)/(1+i)	Utility η(c,m)+ q(h)	Lagrange Multiplier λ	Lagrange Multiplier ψ
0.00	0.690	0.590	0.31	14.5%	0.100	-10.0%	0.0%	0.00%	3.686	0.000	2.032	1.24	0.18
0.05	0.683	0.583	0.32	14.7%	0.101	-9.9%	0.0%	0.14%	3.590	0.005	2.010	1.22	0.18
0.10	0.675	0.575	0.32	15.0%	0.101	-9.7%	0.0%	0.29%	3.491	0.010	1.987	1.19	0.18
0.15	0.667	0.567	0.33	15.2%	0.102	-9.6%	0.0%	0.44%	3.391	0.015	1.964	1.17	0.18
0.20	0.658	0.558	0.34	15.5%	0.102	-9.5%	0.0%	0.60%	3.287	0.020	1.939	1.15	0.18
0.25	0.649	0.549	0.35	15.8%	0.103	-9.3%	0.0%	0.78%	3.178	0.025	1.913	1.13	0.18
0.30	0.639	0.539	0.36	16.2%	0.103	-9.1%	0.0%	0.97%	3.064	0.030	1.886	1.11	0.18
0.35	0.627	0.527	0.37	16.5%	0.103	-8.9%	0.0%	1.19%	2.943	0.035	1.856	1.09	0.18
0.40	0.615	0.515	0.39	16.8%	0.103	-8.7%	0.0%	1.44%	2.812	0.040	1.824	1.07	0.18
0.45	0.600	0.500	0.40	17.2%	0.103	-8.4%	0.0%	1.72%	2.666	0.046	1.787	1.04	0.18
0.50	0.582	0.482	0.42	17.6%	0.102	-8.1%	0.0%	2.08%	2.498	0.052	1.743	1.02	0.19

Table 3: The honest Government The Initial Wealth effect on the economy - v=real interest rate (β^{-1}-1) - :

Initial Wealth W_{-1}	Output (1-h)(1-f)	Consumption C	Leisure h	Tax rate T	Tax revenue τ(1-h)(1-f)	Inflation π	Interest on Money V	Cost of Money (i-v)/(1+i)	Quantity of Money m	Inflation Tax m(i-v)/(1+i)	Utility η(c,m)+ q(h)	Lagrange Multiplier λ	Lagrange Multiplier ψ
0.00	0.674	0.574	0.32	14.8%	0.100	0.0%	11.1%	0.00%	3.589	0.000	2.000	1.25	0.19
0.05	0.667	0.567	0.33	15.0%	0.100	0.1%	11.1%	0.14%	3.494	0.005	1.978	1.23	0.19
0.10	0.659	0.559	0.33	15.2%	0.100	0.3%	11.1%	0.28%	3.396	0.010	1.956	1.21	0.19
0.15	0.651	0.551	0.34	15.5%	0.101	0.4%	11.1%	0.43%	3.297	0.014	1.932	1.19	0.19
0.20	0.642	0.542	0.35	15.7%	0.101	0.6%	11.1%	0.60%	3.193	0.019	1.908	1.17	0.19
0.25	0.633	0.533	0.36	16.0%	0.101	0.8%	11.1%	0.77%	3.085	0.024	1.882	1.15	0.19
0.30	0.622	0.522	0.37	16.2%	0.101	1.0%	11.1%	0.97%	2.971	0.029	1.854	1.13	0.19
0.35	0.610	0.510	0.38	16.5%	0.101	1.2%	11.1%	1.18%	2.848	0.034	1.823	1.11	0.19
0.40	0.596	0.496	0.40	16.8%	0.101	1.5%	11.1%	1.43%	2.713	0.039	1.790	1.08	0.20
0.45	0.580	0.480	0.41	17.2%	0.100	1.8%	11.1%	1.74%	2.559	0.044	1.750	1.06	0.20
0.50	0.559	0.459	0.43	17.6%	0.098	2.2%	11.1%	2.14%	2.371	0.051	1.701	1.03	0.21

Table 4: The honest Government - The output cost of inflation (Gamma) effect on the economy with v=0:

Gamma	Output (1-h)(1-f)	Consumption C	Leisure h	Tax rate T	Tax revenue τ(1-h)(1-f)	Inflation π	Interest on Money V	Cost of Money (i-v)/(1+i)	Quantity of Money m	Inflation Tax m(i-v)/(1+i)	Utility η(c,m)+ q(h)	Lagrange Multiplier λ	Lagrange Multiplier ψ
0.02	0.583	0.483	0.42	41.7%	0.176	-8.10%	0.0%	2.07%	2.505	0.052	1.746	1.02	0.19
0.05	0.583	0.483	0.42	41.7%	0.176	-8.10%	0.0%	2.07%	2.505	0.052	1.746	1.02	0.19
0.10	0.583	0.483	0.42	41.7%	0.176	-8.10%	0.0%	2.07%	2.505	0.052	1.745	1.02	0.19
0.20	0.582	0.482	0.42	41.8%	0.176	-8.10%	0.0%	2.07%	2.504	0.052	1.745	1.02	0.19
0.50	0.582	0.482	0.42	41.8%	0.176	-8.09%	0.0%	2.08%	2.502	0.052	1.745	1.02	0.19
1.00	0.582	0.482	0.42	41.8%	0.176	-8.09%	0.0%	2.08%	2.498	0.052	1.743	1.02	0.19
2.00	0.580	0.480	0.42	41.9%	0.176	-8.08%	0.0%	2.09%	2.490	0.052	1.741	1.02	0.19
5.00	0.577	0.477	0.42	42.2%	0.176	-8.04%	0.0%	2.13%	2.464	0.053	1.734	1.02	0.19

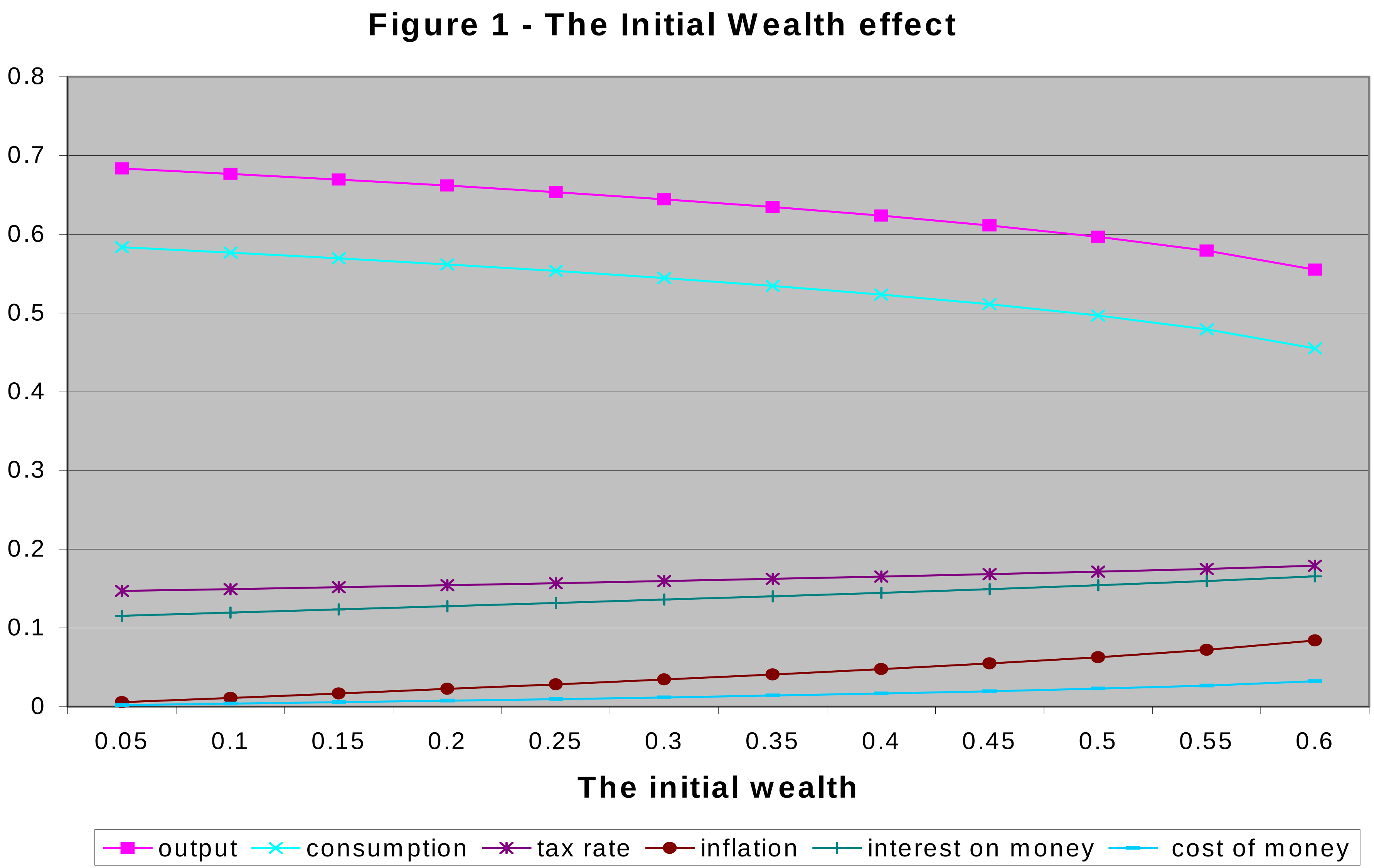
Figure 1 - The Initial Wealth effect
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
0.6
The initial wealth
output
consumption
tax rate
inflation
interest on money
cost of money

Figure 2 - The output cost of inflation

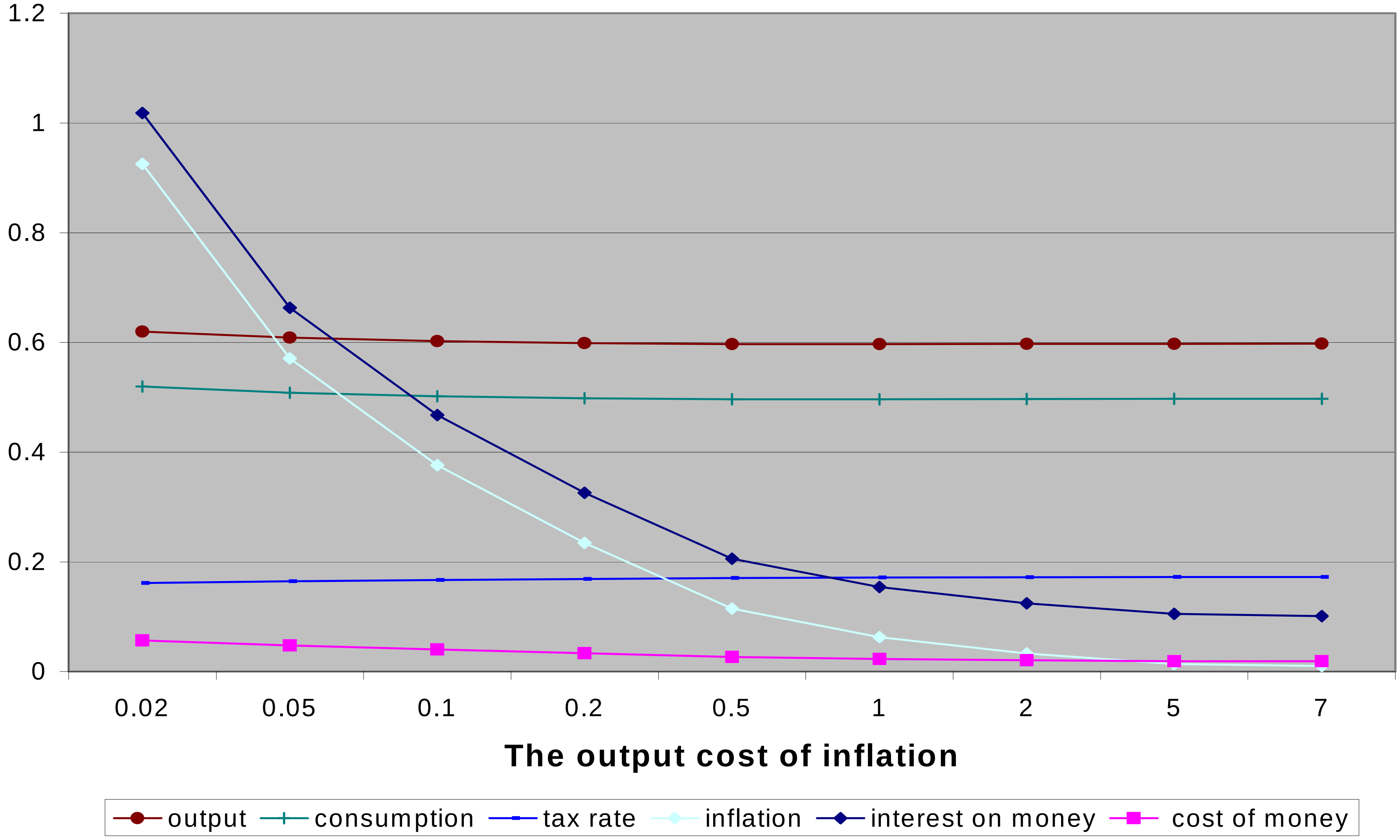

Figure 3 - The Governmet expeditures

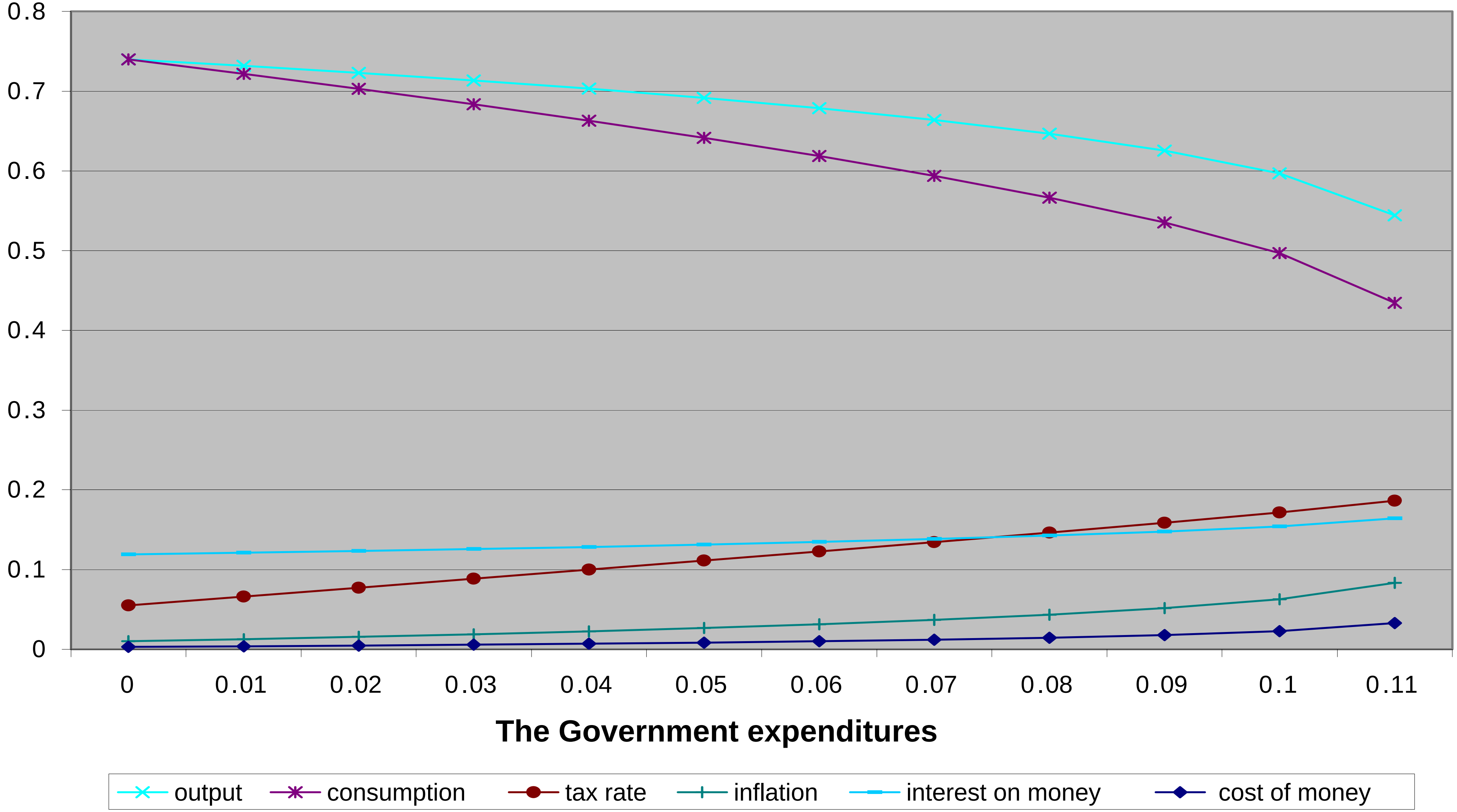

Figure 4 – Optimal Liquidity (Eq. 13)

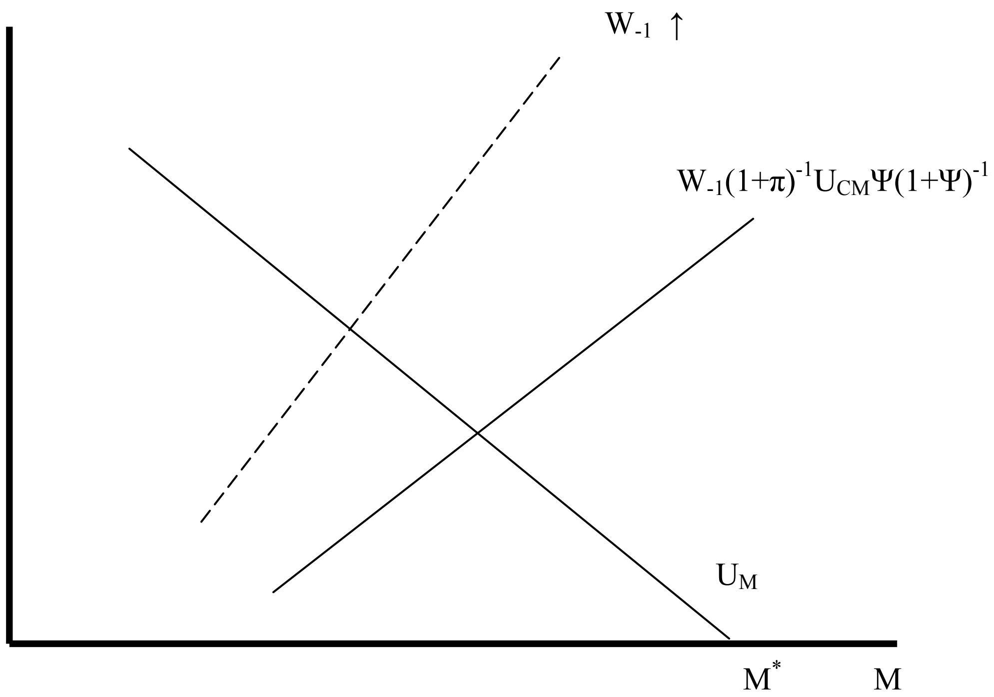

Figure 5 – The Liquidity Channel

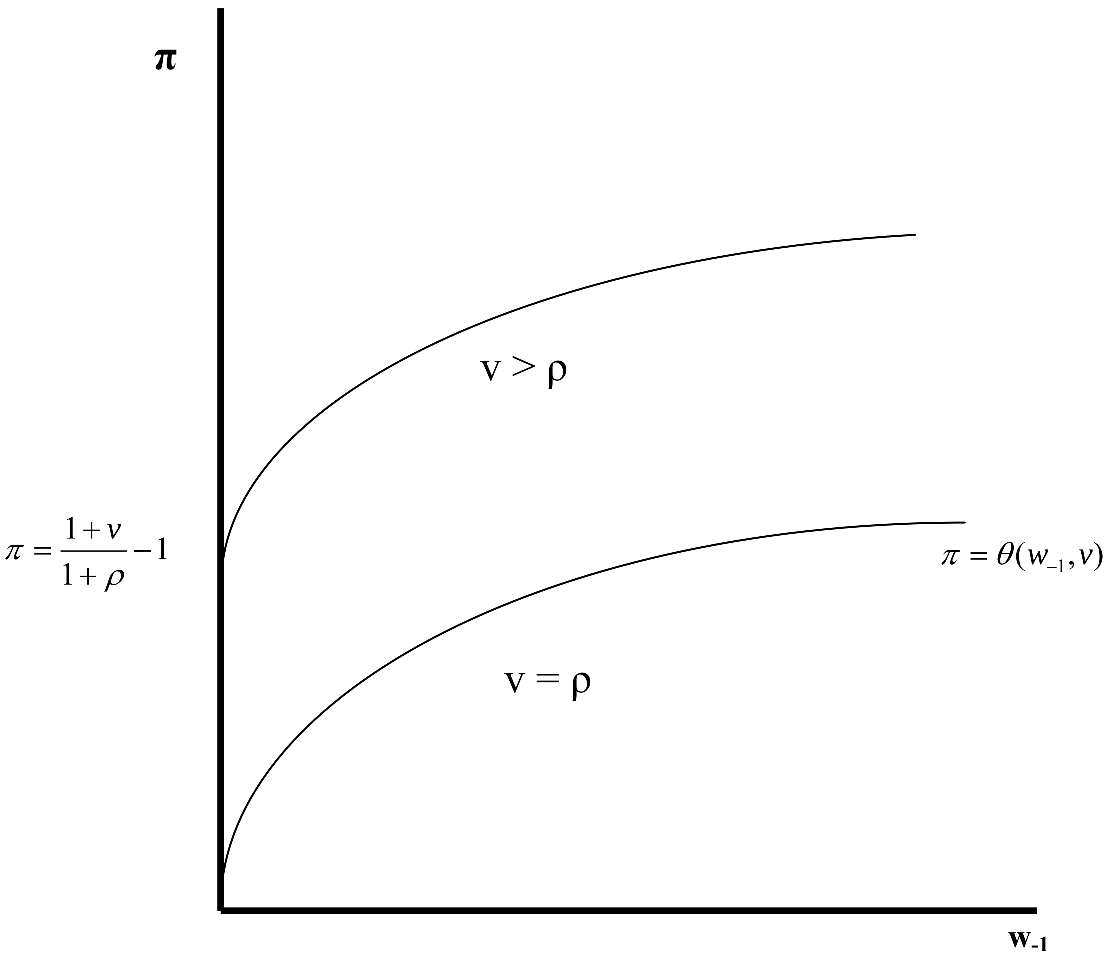

Figure 6 – The Erosion Channel (Eq. 16)

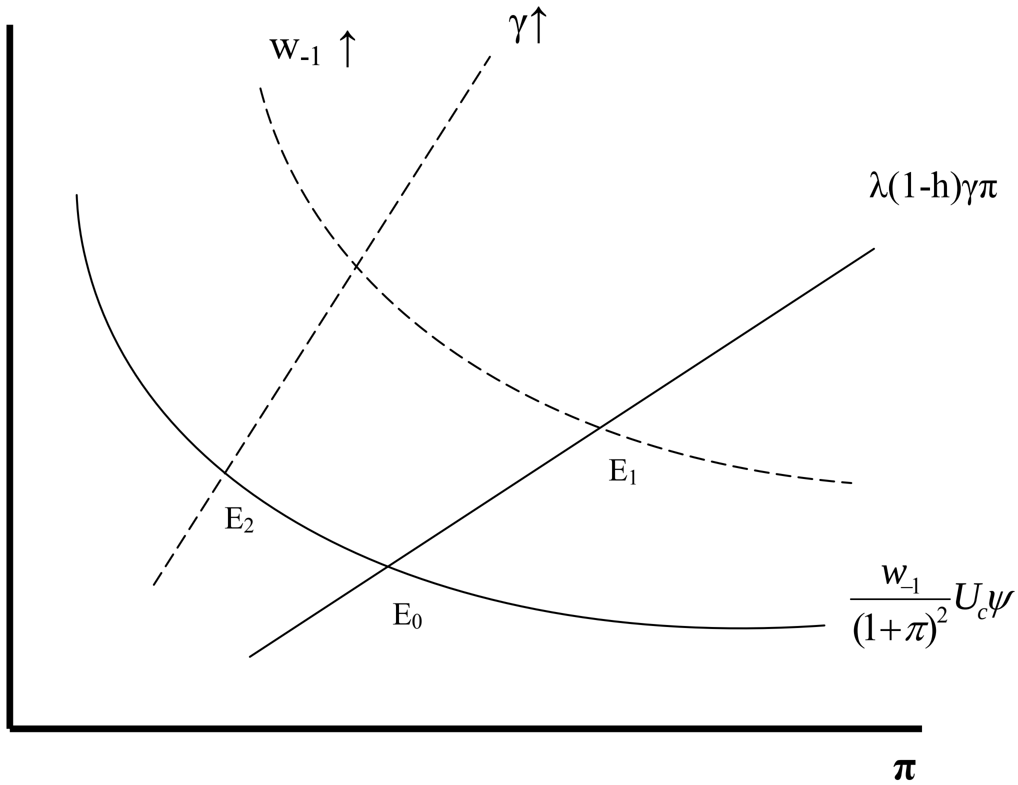

Figure 7 – Equilibrium

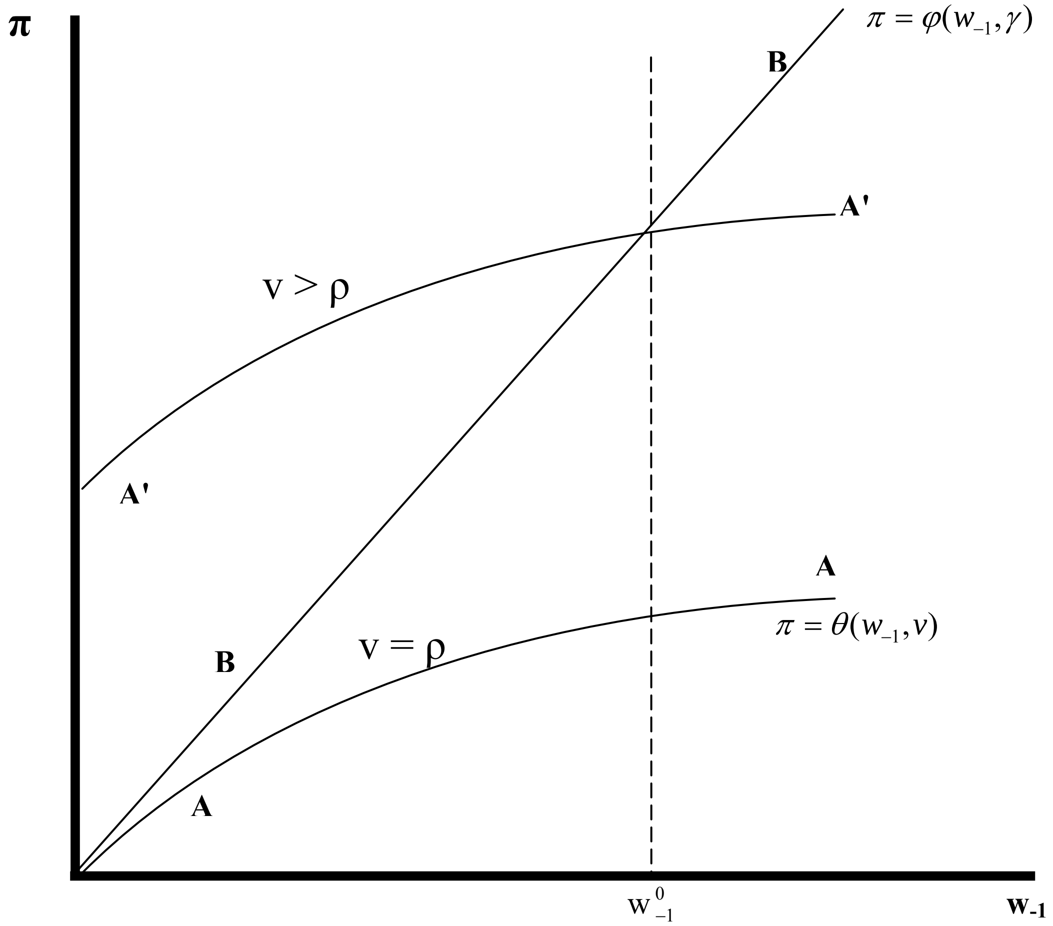

Figure 8 – Comparison of Regimes