עיונים מוניטריים

Monetary Studies

Distribution of the Exchange Rate Implicit in Option Prices[1]: Application to TASE

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2003.05

October 2003

Discussion Papers מאמרים לדיון

Bank of Israel
Monetary
Department

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The views expressed in this paper are those of the authors only, and do not necessarily represent those of the Bank of Israel.

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[1] The authors would like to thank Elena Pompushko of the Bank of Israel's IT Department for her help. We also thank the participants of the seminars of the Research, Monetary, and Foreign Exchange Activity Departments for their helpful comments. Any mistakes are the authors' alone.

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Abstract

This study presents a method of estimating the implied distribution of the future NIS/$ exchange rate implicit in the NIS/$ options traded on the TASE (Tel Aviv Stock Exchange), using options with the same maturity but different strike prices. The distribution of the exchange rate as perceived by the markets is reflected by the option prices with different strike prices, because of the high sensitivity of the prices to expected developments in the financial markets.

The distribution estimated here is a double-log-normal distribution of the exchange rate that combines two separate log-normal distributions of the exchange rate.

The estimation used here has an advantage over the usual log-normal distribution in that it identifies market expectations regarding the future course of the exchange rate that incorporate the possibility that this will not be continuous, thus enabling more extensive information to be derived from the markets. In addition to expected changes in the exchange rate and the risk level, the model also enables us calculate the possibility of a jump in the exchange rate, and of leptokurtosis and skewness in the distribution.

A non-technical summary of the study is given in Appendix A.

Keywords: exchange rate, exchange-rate course, options, double-log-normal distribution, expectations, risk indicators

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I. INTRODUCTION

In the last few years, the information implicit in the prices of sophisticated financial assets, and in particular of options, has been of focal interest to central and commercial banks throughout the world as well as to the academic community and the private sector. The information contained in the price of options reflects market expectations regarding the future price of the underlying asset and the volatility of this price. In addition, various analytical methods enable the entire distribution of changes in the prices of underlying assets to be derived from those options. For example, options on the NIS/$ exchange rate at different strike prices can reveal not just the expected exchange rate and its volatility, but also the probabilities of different levels of change in the exchange rate.

The indicators derived from the capital and money markets, including expectations regarding inflation changes in the central bank's interest rate, and changes in the exchange rate, are used by policy maker in reaching monetary policy decisions. The distribution of the exchange rate, which incorporates investors' expectations, thus provides information that is important for decisions about monetary policy, and is also relevant for the estimation of the distribution by the markets. This is particularly the case in small and open economies like Israel's, where there is a statistical correlation between inflation and exchange-rate movements.

Many methods of estimating the distribution of the exchange rate on the basis of option prices at different strike prices are described in the literature; most are based on the assumption of investors' risk neutrality density (RND). This paper also makes that assumption.

Chang and Melick (1999) divided the estimation methods into two main categories:

The first estimates the price of options directly, by assuming any distribution function, dictated by parameters that have to be estimated.

(1) $c(X) = e^{-rt}\int_{X}^{\infty}(S - X)f(S)ds$.

The other estimates the distribution by using the second derivative of the option price relative to the strike price.

(2) $\frac{\partial^2 c(X)}{\partial X^2} = e^{-rt} f(S)$,

where

$c(X)$ = the value of a call option with price X.

r = the domestic risk-free interest rate.

S = the spot price.

$f(S)$ = the density function of the actual exchange rate.

t = time to maturity of the option.

Studies based on the first method generally differ from each other in their assumption regarding the form of the function *f(S)* and in the number of parameters determining that form (e.g., Jackwerth and Rubinstein, 1995; Melick and Thomas, 1994; and Rubinstein, 1994). The distribution function is estimated by applying the best fit method to the actual vis-à-vis the theoretical option prices. The theoretical prices are calculated from an option price equation based on certain assumptions about the distribution function.

Studies based on the second method calculate the density function, *f(S)*, using the second derivative of option prices relative to strike prices. This method is known as the non-parametric estimation of the distribution, as it embodies no underlying assumption regarding the shape of the distribution (e.g., Jarrow and Rudd, 1982; Longstaff, 1992, 1995: Shimko, 1993; Ait-Sahalia and Lo, 1995; and Malz, 1995). In order to calculate the second derivative, these studies smooth the option prices in different ways. The idea underlying the use of a non-parametric estimate derives from the economic significance of the price of an underlying asset, known as an elementary asset (or an Arrow-Debreu security). This asset yields an income of one shekel only if the exchange rate has a particular, predetermined value on a particular, predetermined date; otherwise it yields nothing. The price of the asset depends on investors’ expectations that the exchange rate will in fact be at the predetermined level on the appropriate date. These expectations are expressed in terms of probabilities. The second method of estimation enables the probability of discrete events to be calculated; its use to calculate indicators that reflect the distribution is very restricted, and that is its greatest drawback.

In this study, we estimate the distribution of the NIS/$ exchange rate via equation (1), and apply the method of the double-log-normal distribution because this satisfies the following two distinct assumptions, that are specially appropriate for the Israeli market.

The first is that the path followed by the underlying asset consists of random changes with jumps (e.g., Ball and Torous, 1983, 1985; and Bates, 1991). Ball and Torous (1983) applied a model of the Bernoulli jump process, based on this assumption to 47 shares traded on the NYSE over 500 trading days, and found that 78 percent of the shares indicated price jumps at a one percent level of significance. Ball and Torous (1985) compared two option-pricing models: that of Black and Scholes that assumes that the underlying asset follows a path of continuous random changes, so that the distribution is log-normal; and that of Merton (1976), that assumes that the underlying asset follows a random walk together with jumps. Ball and Torous used Bernoulli's version of the jump diffusion model where the size of the jump is a non-stochastic variable. In that case a maximum of only one jump is possible throughout the life of the option, and the distribution function can therefore be described as the mixture of a double log-normal distribution. They found that the differences between the two models in the distribution of the price changes of shares traded in the NYSE were not significant, but they noted that Merton's model was more appropriate for other assets—whose price jumps are infrequent but larger—such as foreign currency.

The second assumption is that the true distribution of the underlying asset comprises many log-normal distributions (e.g., Ritchey, 1990). This approach to the estimation of the true distribution is broader than the assumption regarding the stochastic process of the underlying asset. Melick and Thomas (1997) applied this approach and estimated the distribution via a combination of three log-normal distributions. Their method requires many degrees of freedom, and is therefore unsuitable for the TASE (Tel Aviv Stock Exchange), where options are traded at a limited number of strike prices (section III). Thus, in order to preserve a sufficiently large number of degrees of freedom, the authors made do with a combination of two log-normal distributions and estimated only five parameters that determined the shape of the distribution, similar to Bhara's (1997) model.

The estimation of the distribution via the double-log-normal method is simple to apply, and provides a range of parameters that yield information about the path the underlying asset price might follow. Furthermore, as Aguilar and Hordahl (1991) point out, this method is flexible, enabling the derivation of a wide range of distributions, of which the log-normal distribution is one specific case. For a double-log-normal distribution the method enables us to calculate four first moments that characterize expectations: the expected value, the standard deviation, the kurtosis of the distribution, and its skewness. Thus a distribution estimated via the double-log-normal method is more realistic than one derived via the log-normal method.

In addition to the double-log-normal distribution, there are other functions that allow flexibility in describing the expected changes in the prices of underlying assets, e.g., the Hermite polynomials first described in Madan and Milne's (1994), and Rubinstein's (1994) model based on the binomial model.

Campa, Chang and Reider (1998) apply and compare three different models: Shimko's (1993) model that uses the second derivative of option prices relative to strike prices; Rubinstein's (1994) model that uses an implied binomial tree to derive the distribution; and Melick and Thomas's (1994) model that uses a combination of log-normal distributions. Campa, Chang and Reider compared the three methods, using data on foreign-currency options traded OTC for several dates and for two periods to maturity—one month and three months. They found that the three methods produced similar distributions[2], and list the advantages and shortfalls of each of the methods. They make the following points:

1. When using method of the weighted average of a log-normal combination, the parameters obtained from the estimated distributions have pure economic interpretations.
2. In Shimko's non-parametric method, unlike the other two, the tails of the distribution cannot be elicited directly.
3. The tails of the distribution are smoothed by a log-normal combination method—the probability of the tails of the distribution declines monotonically and quickly

[2] The same finding as that of Jondeau and Rockinger (2002).

enough to prevent unreasonably fat tails—unlike the tails of the distributions obtained from the other models, which can rise inexplicably quickly.

In this study we describe the model based on the combination of two log-normal distributions incorporating several changes to Bhara's (1997) model. The changes improve both the estimation equations and the objective function, i.e., the differences between option prices obtained from the equations and their actual prices are at a minimum. We apply the model to the data on call and put NIS/$ options traded on the TASE, and give several estimates that reflect the level of expectations of changes in the NIS/$ exchange rate and the level of uncertainty and asymmetry of those expectations for periods of between one and two months.

Section II describes the model. Section III presents the data and focuses on the problems they entail and ways of dealing with them. Section IV applies the model and derives several estimates that reflect the shape of the distribution obtained from the model. Section V concludes.

II. THE MODEL

1. General background

The initial assumption is that of RND, i.e., that the price of an option that can be realized at a predetermined time equals the capitalized value at a risk-free interest rate of the total possible payments multiplied by their respective probabilities. Based on this working hypothesis of RND[3], the general price equation can be estimated; equation (3) shows the equation for call options, and equation (4) that for put options.

$$(3)\quad c(S,t) = e^{-it}\int_{X}^{\infty} q(S_T)(S_T - X)dS_T \ ,$$

$$(4)\quad p(S,t) = e^{-it}\int_{0}^{X} q(S_T)(X - S_T)dS_T \ ,$$

[3] Hull (2000) maintains that this assumption is not essential but is used for simplification. He argues that this is because even assuming that investors are not risk-indifferent, both the future payments in the general price equation and the interest rate used to capitalize the payments will change, and these two effects completely offset one another. In the present study, and despite Hull's contention, we assume RND because, as Hull states, supposedly as an alternative, it would be possible to discuss extrapolating

where

$c(S,t)$ - the price of a call option

$p(S,t)$ - the price of a put option

S_T - the expected spot price at time T

X - the option's strike price

$q(S_T)$ - the general density function of S_T

i - the domestic interest rate.

Theoretically, every density function $q(S_T)$ could fit the price equation, provided the parameters that determine its shape—obtained by matching the theoretical price with the actual price—can be derived from it. Ritchy (1990) assumed that $q(S_T)$ is a density function comprising K log-normal distributions. The assumption in the current study is that $q(S_T)$ is a density function composed of a combination of two log-normal distributions, as in Bhara's (1997) model, so that the price equation can be written as:

$$(5)\quad c(S,t) = e^{-it}\int_X^{\infty}\left[\theta f(\mu_1,\sigma_1;S_T) + (1-\theta) f(\mu_2,\sigma_2;S_T)\right](S_T - X)dS_T \ ,$$

$$(6)\quad p(S,t) = e^{-it}\int_0^{X}\left[\theta f(\mu_1,\sigma_1;S_T) + (1-\theta) f(\mu_2,\sigma_2;S_T)\right](X - S_T)dS_T \ ,$$

where

$\theta \in [0,1]$ is the coefficient

μ_1,σ_1 are the expectations and standard deviation in normal distribution (1)

μ_2,σ_2 are the expectations and standard deviation in normal distribution (2)

The assumption of two distributions yields richer data than is the case assuming a log-normal distribution, because the former contains a broader range of parameters. Among other things, this distribution enables several indices to be examined that reflect expectations of changes in the exchange rate, the market level of uncertainty regarding expectations, the probability of a jump in the exchange rate, and the degree of leptokurtosis and skewness of the distributions.

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from the distribution without assuming RND, but in our view this would make matters needlessly complicated.

The model here is based on Garman-Kohlhagen's (1983) (henceforth G&K) equation for pricing exchange-rate options. G&K adapted Black and Schole's equation to the foreign-currency market while maintaining the assumption that the expected exchange rate has a log-normal distribution. Thus, the values of call or put options are dictated by the combination of two log-normal distributions, assigning a weight to each one as follows:

(7) $c(S,t) = e^{-it}[\theta[Se^{\mu_1 t}N(d_1) - XN(d_2)] + (1-\theta)[Se^{\mu_2 t}N(d_3) - XN(d_4)]]$,

(8) $p(S,t) = e^{-it}[\theta[Se^{\mu_1 t}N(-d_2) - XN(-d_1)] + (1-\theta)[Se^{\mu_2 t}N(-d_4) - XN(-d_3)]]$,

where

$$d_1 = \frac{\ln(X/S) + (\mu_1 + \frac{1}{2}\sigma_1^2)t}{\sigma_1\sqrt{t}}$$

$$d_2 = d_1 - \sigma_1\sqrt{t}$$

$$d_3 = \frac{\ln(X/S) + (\mu_2 + \frac{1}{2}\sigma_2^2)t}{\sigma_2\sqrt{t}}$$

$$d_4 = d_3 - \sigma_2\sqrt{t}$$

$c(S,t)$	=	the price of a call option
$p(S,t)$	=	the price of a put option
$N(d)$	=	the cumulative distribution of d based on the standard normal distribution
i	=	the domestic interest rate
$\theta \in [0,1]$	=	the coefficient
μ_1, σ_1	=	the expectation and the standard deviation in the normal distribution (1)
μ_2, σ_2	=	the expectation and the standard deviation in the normal distribution (2)
X	=	the option's strike price
S	=	the spot price.

The following equation weights by θ two of G&K's equations, each one of which contains one expectation and standard deviation that determine the shape of the distribution. According to this model, the two equations combined give one double-log-normal distribution described by five parameters: two means (μ_1,μ_2), two standard deviations (σ_1,σ_2), and the weight (θ).

These variables enable the calculation of the average expectation of the whole distribution of changes in the exchange rate, μ_e, by weighting the two estimated means by θ:

(9) $\mu_e = \theta\mu_1 + (1-\theta)\mu_2$.

The standard deviation of changes in the exchange rate, σ_e can theoretically be calculated in a similar way, by weighting the two estimated standard deviations and their covariance:

(10) $\sigma^2{}_e = \theta^2\sigma^2{}_1 + (1-\theta)^2\sigma^2{}_2 + 2\theta(1-\theta)\sigma^2{}_{1,2}$

The significance of the two expectations and the two standard deviations can constitute an explanation of different world states. Assume, for example, that within a month only one change is expected in the market—*ceteris paribus*—namely a change in the central bank interest rate, which affects the exchange rate. Assume, too, that it is not known whether the change will be made or not. A change in the interest rate is described by mean and standard deviation 1 (‘world state 1’), whereas the absence of a change in the interest rate is described by mean and standard deviation 2 (‘world state 2’). According to the model, each of these world states will be given a weight describing the chances of being in one of the two possibilities.

The example can be extended to a more complex case if we choose to describe a situation in which many expected changes are concentrated in two separate possibilities. Thus, for example, to the example of expectations of a change in the interest rate we can add expectations of a reform in the taxation of the domestic capital market. This example is wider than the previous one, and yields four world states[4]. However, the four world states may be described by means of the two groups, which are distinguished by their effect on the development of the exchange rate, each group having one expectation and one standard deviation appropriate for all its natural states. Naturally, reality is more complex and contains an infinite number of expected world

[4] An increase in interest with no change in taxation; an increase in interest with a change in taxation; a reduction of interest with no change in taxation; and a reduction of interest with a change in taxation.

states. Hence, the significance of the two expectations and standard deviations should be wider, representing two groups which incorporate many world states.

The variables in the model and their significance are presented in Table 1.

Table 1

The Variables in the Model and their Significance

Variable	Possible range*	Calculation	Significance
μ_1	$\Re$	Estimate	Average of expectations of change in exchange rate, given world state 1
μ_2	$\Re$	Estimate	Average of expectations of change in exchange rate, given world state 2
σ_1	$0 > \sigma_1$	Estimate	Standard deviation of change in exchange rate, given world state 1
σ_2	$0 > \sigma_2$	Estimate	Standard deviation of change in exchange rate, given world state 2
θ	$0 \leq \theta \leq 1$	Estimate	Weight ascribed to world states
μ_e	$\mu_2 \leq \mu_e \leq \mu_1$	$\theta\mu_1 + (1-\theta)\mu_2$	Average of expectations of change in exchange rate
$\sigma^2{}_e$	$\sigma_2 \leq \sigma_e \leq \sigma_1$	$\sigma^2{}_e = \theta^2\sigma^2{}_1 + (1-\theta)^2\sigma^2{}_2 + 2\theta(1-\theta)\sigma^2{}_{1,2}$	Estimate of exchange-rate volatility implicit in market
Skew			Extent of asymmetry of distribution
Kurt			Thickness of tails

*In building the estimation there is no a priori constraint requiring the five parameters to be within the possible range.

2. The estimation method

The method of estimating the double-log-normal distribution is based on the loss function whose aim is to minimize the squared deviations between the prices of options as assessed and priced by investors and the estimates obtained from the pricing equation. The idea behind the method is to give a general description of the options market by means of a limited number of variables. In practical terms, we are trying to find a set of variables[5] which will minimize the following objective function:

$$(11)\qquad \underset{\{\mu_1,\mu_2,\sigma_1,\sigma_2,\theta\}}{Min} \sum_{i=1}^{N} \left[\frac{\hat{c}_i - c_i}{c_i}\right]^2 + \left[\frac{\hat{p}_i - p_i}{p_i}\right]^2$$

Where:

c_i – the price of a call option.

p_i – the price of a pt option.

$\hat{c}_i$ - the estimator of the price of a call option.

$\hat{p}_i$ - the estimator of the price of a put option.

The estimators estimated by means equations 7 and 8.

The objective function is minimized using the Gauss-Newton method, which is based on changes in the gradient of the objective function.

In contrast with other methods, which restrict the sphere of variance of the parameters, we enable the parameters to change in all possible spheres. Nevertheless, the values of the parameters obtained via this estimation method are within reasonable spheres.

3. The added value of the estimation method

In addition to the components in equation (11), the loss function of Bhara (1997) includes another component which takes into account the deviations of the theoretical forward rate from the forward rate derived from the interest rate and the spot rate. The change we introduce into the loss function includes relinquishing this component in order to use it later as a control for the estimation results. The change we make in the loss function in this model is consistent with the purpose of the estimation, which is to minimize only the gap between the actual prices of options and those obtained from the pricing equation.

As stated, equation (9) weights the two means of the distribution functions, and calculates one mean, which represents expectations of a change in the exchange rate, as derived from the model. Note that these expectations are an estimation derived from the model, without any constraint at the base of the estimation method. Expectations of devaluation can also be estimated from another source, described by means of the UIP (Uncovered Interest-rate Parity) equation, and according to which the interest-rate spread is equal to estimated expectations of changes in the exchange rate. Other studies, on the other hand (Fama, 1984; Taylor, 1995; Stein, 2002), which estimate the interest-rate spread according to expectations of changes in the exchange rate, find that there is a consistent skew in the spread which is explained by the risk premium. Accordingly, they claim, the risk premium has to be deducted from the interest-rate spread in order to create an unbiased estimate of expectations of changes.

The reliability of the model can be examined, and the expectations of changes in the exchange rate derived from it can be compared with those derived from other sources. This is done in Section IV.

[5] The vector of the five parameters which determine the shape of the double-log-normal distribution.

III. THE DATA AND THE SAMPLE

An NIS/$ option traded on the stock market is characterized as an option series, in accordance with its redemption dates[6]. In each series call and put options are traded at several fixed strike rates. The strike dates of the option series are monthly, so that at any point in time there are four series of options for the next three months and to the end of the next quarter. In 2002 another option series was added, redeemable at the end of the current year. The fixed strike rate of the options is the last exchange rate published by the Bank of Israel before the strike date. The options traded on the stock market are called 'off-the-shelf' and are homogeneous in character, in contrast with the options of the commercial banks and those issued by the Bank of Israel, which are not characterized as option series. For the purposes of the model, which estimates the price of the NIS/$ options at identical redemption intervals but at different strike rates, we use the data of the options traded on the TASE.

In order to estimate the prices of options, in addition to those known at the time of the sample, we also sample the NIS/$ exchange rate and the NIS interest rate for the lifetime of the options—the yield on Treasury bills redeemed at or near the date the options expire (Appendix B). Note that the dollar interest rate was not sampled in the present framework, because this variable is endogenous to the model (see Section II.3).

Prices of options are sensitive to the price of the underlying asset—the NIS/$ exchange rate, market interest rates, and players' expectations regarding the future development of the underlying asset. Hence, at any point during the trading day a

[6] The rules for recording the NIS/$ options series on the TASE are as follows:

- Beginning of period: the range of strike prices is fixed at 70 agorot (70/100 of NIS)—eight options at different strike prices. One option at a strike price equivalent to $ 1, four options at a strike price that is higher than the dollar exchange rate, and three options at a strike price that is lower than the dollar exchange rate (at intervals of 10 agorot).
- Continuation of period: additional options are issued on every trading day (except in the last week of the strike date), in accordance with the development of the dollar exchange rate, where the strike price of the options exceeds the dollar exchange rate by 30 agorot and is 20 agorot less than the dollar exchange rate (a minimum range of 50 agorot).
- When the term of the options is 4 months or less: four options are issued with strike prices close to the dollar exchange rate, at intervals of 5 agorot (this rule applies only when the annual implied volatility does not exceed 15 percent).

change in one of these factors will affect the prices of options, provided that at least one transaction in them has been executed. Consequently, it is very important to have uniformity as regards the time the data are sampled for the purposes of the model. In sampling the prices of options on the basis of transactions executed, such as closing prices on the TASE, it is not obvious that all the options are traded near the closing time, and some of the options with different strike prices could be traded at different times of the day on the basis of information that is not relevant when sampling the prices. Another alternative to sampling option prices which solves the problem of time differences is to calculate the average of the supply and demand of option prices, as recorded in the TASE register at a specific point in time, and also to sample the dollar exchange rate and NIS interest rates at the same point. All the options of the various strike prices have sale and purchase offer prices which can be implemented at that point in time. Thus, the average of the offer prices reflects the nearest price to that at which the transaction could be implemented for all the options for which offer prices exist[7].

In accordance with these characteristics, we decided to sample the best supply and demand for each option, as recorded in the TASE register, on each trading day and at a specific time. At the same time we gathered the other data—the NIS/$ exchange rate known at that time and the yield on T-bills. The number of call/put options at the various strike prices for which prices are quoted is not constant throughout the trading day. However, this refers to an average of 20 options at different strike prices—all the liquid options in the market (see Appendix 2).

The extent of trading in options traded on the TASE is concentrated in the strike prices closest to the representative exchange rate and short strike terms. Hence, a sufficient amount of information can be obtained from relatively short-term options—up to 60 days. In these redemption terms the extent of trading is greater and is published for a greater number of strike prices, while for the longer term only three options on average are traded at different strike prices, generally around the known NIS/$ exchange rate.

[7] The prices are determined by the player at the margin at every strike price.

These data serve as the basis for estimating the expected distribution of the NIS/$ exchange rate at a given point in time, by means of the double-log-normal distribution function. Several dates were chosen close to various events, including the two key interest-rate hikes in June 2002.

We make a distinction between four different approaches which relate to the distribution of the expected exchange rate and interact with one another.

1. ***The true distribution of the future exchange rate*** is the only distribution which describes the behavior of the future exchange rate. This distribution is not identifiable.

2. ***The subjective distribution received by each of the various market players*** is the distribution by which the players price derivatives transactions. This distribution is not necessarily the true distribution of the future exchange rate, and there may be other subjective elements that are connected with decision-making under risk, in accordance with the Prospect Theory, as noted by Kahneman and Tversky (1979) and Levy and Levy (2002). Furthermore, different market players see different ‘true’ distributions, so that the distribution perceived by the various market players is a conglomerate of all these distributions. Nevertheless, the distribution perceived by the various players affects their behavior, and so this distribution might be of more interest than the true distribution.

3. ***The estimated distribution of the exchange rate: the average of the subjective distributions*** is the result of the empirical examination of the market data using various techniques. The result relates to a specific period, a pre-determined frequency, specific currencies, etc.

4. ***The distribution based on the theoretical assumption regarding the behavior of the exchange rate*** is the function (or set of functions) that describes the distribution of the exchange rate. The distributions can be derived from the theoretical stochastic process of the exchange rate and/or the theoretical process by which market equilibrium emerges. This distribution, which does not need to describe the situation precisely, is more convenient for analyzing the behavior of the exchange rate. Thus, for example, Krugman (1988) assumes a theoretical distribution of the exchange rate, on the basis of which he presentes an exchange-rate regime with a band.

There is interaction between the various approaches. For example, the true distribution of the exchange rate (1) affects the distribution perceived by the players (2). However, changes in players' expectations could affect the true distribution (1). The means used by the players to estimate their expected distribution (2) are usually the estimated distribution of the exchange rate (3), and the distribution might be based on a theoretical assumption (4).

An examination of the interaction between the various approaches enables the estimated distribution to be improved. A well-known phenomenon is the Smile Volatility: the estimated distribution shows that the implicit volatility of options increases as the strike rate recedes from the ATMF. The Smile phenomenon is examined by means of the Black-Scholes equation, in which the distribution of the exchange rate is based on theoretical assumption (4), which is a log-normal distribution. This phenomenon contradicts the model's theoretical assumption (4). The conclusion from this is that the distribution of the exchange rate expected by the players (2) is not log-normal, and hence possibilities of improving the theoretical distribution are proposed.

The regime involving an exchange rate within a band ("Target zone"), as is the case in Israel, is likely to affect the distribution of the exchange rate expected by the market players. This effect is expected to grow stronger as the exchange rate gets closer to the limits of the band. Campa, Chang, and Refalo (1998) examined the reliability of the exchange-rate band in Brazil in 1994–97 by means of options. Their findings indicate that before 1996 the reliability of the band was smaller than it was subsequently. The effect of the band on the distribution of the exchange rate has not been examined in the current study. However, the exchange-rate band did not have a notable effect on the distribution of the exchange rate in Israel when the NIS/currency-basket exchange rate was relatively far away from the limits of the band. Nevertheless, at the end of 1996 and in the first half of 1997 the NIS/currency-basket exchange rate was near the lower limit of the exchange-rate band, remaining there in the second half of 1997 and the first half of 1998. Even though market data for the period of the sample used in this study show that there is very little likelihood that the exchange rate will return to the lower limit of the band, that limit could become effective again at some future point. In that case, the double-log normal distribution will reflect the distribution of the exchange rate more faithfully, providing a good index of the reliability of Israel's exchange-rate regime as perceived by investors. When the exchange rate remains within the band but close to either of its limits its double-log-normal distribution will indicate both the probability that it will deviate appreciably from the limits of the band and the extent of the deviation—constituting an index of the reliability of exchange-rate policy.

IV. RESULTS

In this section we examine how the information implicit in the expected distribution of exchange-rate *changes*[8] can serve as a means of analyzing the future development of the exchange rate. We begin by calculating statistical data that reflect the distribution function, and find that they reflect the structure of investors' expectations regarding future changes in the NIS/$ exchange rate. We examine these statistical data several times on dates close to important economic events. Using the model presented here, we estimate the extent of their effect on expectations.

The information implicit in the distribution of the future exchange rate is obtained from five parameters which are estimated in the framework of the model: two pairs of means and a standard deviation and their weights in the total distribution. These parameters determine the shape of the distribution of the future NIS/$ exchange rate expected by investors.

From the results it is possible to calculate the goodness of fit between the theoretical prices of the options obtained from the distribution and actual prices. We compare the goodness of fit between two different models: one with a double-log-normal distribution of the exchange rate, and one with is a log-normal distribution (Table 2). As expected, since the double-log-normal distribution has more degrees of freedom, a comparison of the two sets of results shows that the double-log-normal one yields a better fit with actual prices.

[8] The distribution of exchange-rate changes is a combination of two normal distributions assuming the exchange rate itself has a double log-normal distribution.

Given the form of the double-log-normal distribution, we calculate several statistical parameters (Table 3) which provide a more detailed description of the structure of investors' expectations: the *median* of the distribution, reflecting the point at which the accumulative probability is 50 percent. The *most frequent change*, reflecting the most probable change. The *inter-quartile range*, i.e., the difference between the change of the exchange rate obtained at a probability of up to 75 percent and that obtained at a probability of up to 25 percent. The inter-quartile range is measured in percentage points and the greater the range, the greater the uncertainty. In addition, the statistical data include the four first moments of the distribution, namely, *mean, standard deviation, skewness,* and *kurtosis*. We calculate the critical Jarque-Bera value (henceforth, JB) on the basis of the four moments, in order to examine the similarity between the distribution and normal distributions.

The mean of the exchange rate, which is the first moment of the distribution, might also be obtained from another source, making it possible to examine the reliability of the model by comparing them. The mean of the exchange-rate changes, according to the UIP assumption, equals the forward premium obtained from the NIS/$ interest-rate spread *less* the risk premium (as explained in Section III), which constitutes an estimate of market expectations regarding exchange-rate changes. If the mean of the exchange rate obtained according to the working model is close to that obtained from another source of information, we may conclude that both its reliability and its ability to describe investors' expectations are good (Table 4). Note that the objective function in other studies includes another parameter, which also minimizes the deviations of the mean of the exchange rate obtained from other sources than the one obtained from the model. Hence, in those studies the comparison between two different sources of information is not relevant. Table 4 shows that the gaps are small and localized, and that the difference between the means is not significant.

It is customary to calculate the expected standard deviation of exchange-rate changes with the aid of the Black-Scholes equation, which assumes—in contrast with the assumption made here—that the distribution function of these changes is normal. Hence, comparing the standard deviation implicit in NIS/$ options obtained by means of the Black-Scholes equation with that obtained here will indicate the error due to the assumption that the distribution is normal (Table 4). As the table shows, the comparison of the two standard deviations for the same options data yields positive differences. The comparison of the two standard deviations for the options issued by the Bank of Israel yields even larger positive differences. These findings support the assumption that the distribution of exchange-rate changes is normal biases the estimate of the implicit standard deviation downwards.

We describe the structure of investors' expectations in greater detail by means of the remaining statistical data, making it possible to compare the probability that deviations will develop in Israel's foreign-currency market.

In this section we calculate the distribution on several dates close to economic events that affected the NIS/$ exchange rate, thereby examining the extent to which they influenced investors' expectations. We can also calculate the probability of a specific depreciation and equivalent appreciation, parameters which reflect the level of uncertainty and asymmetry in market expectations of the future NIS/$ exchange rate. Below we present an example of information derived in this way, using options data for six dates around the series of key interest-rate hikes made by the Bank of Israel in June 2002.

1. On 9 June 2002 the NIS/$ exchange rate was NIS 5 for $ 1, after sharp depreciation since the beginning of 2002 which amounted to 19 percent (see Chapter 2, *Annual Report 2002*, Monetary Department, Bank of Israel). In the course of the trading day the Bank of Israel took the unusual step of raising its key interest rate by 1.5 percentage points. Before the announcement the distribution of exchange-rate changes was symmetrical and normal (Figure 1), with a standard deviation of 11.5 percent and a mean of 5.8 percent. These findings indicate that the level of uncertainty regarding future developments in the foreign-currency market was high, with average depreciation expectations of 5.8 percent in annual terms—which is also the most frequent change. Following the announcement of the 1.5 percentage-point interest-rate hike, the exchange rate depreciated slightly, and the distribution was not normal: the exchange-rate mean dipped slightly, to 4.7 percent, the median declined more significantly, to 2.3 percent, and the most frequent change was negative –6.4 percent. Alongside the decline in expectations of a change in the exchange rate, the level of

uncertainty rose in the wake of the announcement: the standard deviation rose, to stand at 13 percent (compared with 11.5 percent beforehand), and the probability of depreciation of 20 percent or more also rose, and reached 14.3 percent (compared with 10.5 percent beforehand). The inter-quartile range of the distribution, which constituted estimate of the level of uncertainty, expanded after the announcement and reached 20 percentage points, compared with 15.3 percentage points beforehand. Note that according to the JB test, the distribution (after the announcement) was similar to a normal distribution at a significance level of 5 percent (the critical value was 1.2 percent). Despite this result, the distribution was not normal, as the statistical parameters show.

2. On 10 June, the trading day after the announcement about the interest-rate hike, the exchange rate did not decline, and even rose slightly to NIS 4.962 for $ 1. The distribution was symmetrical and normal (Figure 1), with a standard deviation of 10.8 percent and mean of 8.4 percent. This indicates that after internalizing the new information, which surprised investors, their expectations did not moderate, and even rose slightly. The probability of depreciation of 20 percent or more was 14.1 percent, compared with 10.5 percent before the interest-rate announcement.

3. On 17 June, after the publication of the Consumer Price Index (CPI) for May which was up by 1 percent, the NIS/$ exchange rate remained above NIS 4.9 for $ 1. As Figure 2 shows, the distribution can be characterized as a mixture of two normal distributions: one with a very high probability that the exchange rate would depreciate by 8.8 percent with a standard deviation of 11 percent, and the other with a relatively low probability of exchange-rate appreciation—6.4 percent—and a standard deviation of only 4.7 percent. This situation created almost identical distributions for 5 percent appreciation and 12 percent depreciation, indicating that there was considerable uncertainty regarding the future development of the foreign-currency market, and that the probability of sharp exchange-rate changes, especially towards depreciation, was also high. Thus, the inter-quartile range of the distribution was estimated at a relatively high rate—17.5 percentage points. Note that the mean of the distribution indicated depreciation of 6 percent, and that the most frequent change was even higher—8.5 percent.

4. On 25 June, after a 2 percentage-point interest-rate hike by the Bank of Israel, the NIS/$ exchange rate remained high, and the distribution of the changes continued to constitute a mixture of two normal distributions, with a moderation of the level of expectations (Figure 2). Although the mean of the exchange rate rose and reached 6.7 percent, the median of the distribution declined to 3.8 percent, and the most frequent change was only 2.3 percent, compared with 8.5 percent on 17 June. Alongside a slight dip in the level of expectations, the level of uncertainty remained unchanged. On the one hand, the inter-quartile range of the distribution declined to 12.7 percentage points, while on the other, the standard deviation rose to 13.9 percent, with the probability of depreciation of 20 percent or more remaining 12.7 percent.

5. On 10 July, after some stability in the exchange rate and actual appreciation to NIS 4.724 for $ 1, the distribution of the changes continued to constitute a mixture of two normal distributions (Figure 2). The most frequent probability was that the exchange rate would depreciate by only 1 percent with a specially fat right-hand tail, indicating that sharp depreciation was still a reasonable possibility. Thus, the inter-quartile range remained high, at 13.4 percentage points, and the probability that the exchange rate would depreciate by 20 percent or more was relatively high—15.3 percent. This finding indicates that investors assessed that the high local-currency interest rate, which increased the alternative cost of holding foreign currency, would exert pressure for moderate exchange-rate appreciation. At the same time, investors feared relatively sharp depreciation due to the continuation of the negative economic developments evident since the beginning of 2002.

6. On 12 August, after the exchange rate had stabilized at 4.67, the distribution continued to remain a combination of two normal distributions. The distribution remained similar to that of 10 July, with a decline in the level of uncertainty, expressed in both the inter-quartile range and the probability of depreciation of 20 percent or more. However, the left-hand tail of the distribution remained unchanged at the high level of 10 July.

Thus, the model provides a considerable amount of important information about the expected development of the NIS/$ exchange rate which cannot be obtained from other sources. This kind of examination of the foreign-currency market enables both economic agents and policymakers to assess future exchange-rate developments in greater detail, in the same way as investors do.

Table 2
Goodness of Fit between the Theoretical Option Prices and Actual Prices

		Goodness of fit*	
	No. of options	Double-log-normal	Log-normal
9/6	24	0.121	0.121
9/6**	20	0.057	0.059
10/6	24	0.060	0.060
17/6	26	0.011	0.022
25/6	23	0.004	0.070
10/7	19	0.008	0.038
12/8	16	0.005	0.022

*The squared sum of deviations between the theoretical and actual prices of options *divided by* the actual prices. There is an inverse relation between the value denoting the goodness of fit and the goodness of fit itself: the lower the value of the goodness of fit, the higher the goodness of fit itself.

**After the Bank of Israel's exceptional announcement regarding the rise in its key interest rate.

Table 3
Statistical Indices of the Shape of the Distribution of the Exchange Rate Changes
Selected Dates, June-August 2002

	NIS/$	Mean	Frequent	Median	(1) Inter-quartile range	Implied Volatility	SKEW	KURT	(2) J.B.	Prob. to revaluation by 20%	Prob. to devaluation by 20%
9/6	5.007	5.8	5.8	5.8	15.3	11.4	0.0	3.0	1E-10	1.2%	10.5%
9/6*	4.939	4.7	-6.4	2.3	20.3	12.9	0.6	2.7	4.19	0.7%	14.3%
10/6	4.962	8.4	8.4	8.4	14.6	10.8	-0.0	3.0	7E-11	0.4%	14.1%
17/6	4.944	6.0	8.5	5.7	17.5	11.6	0.2	2.6	1.002	0.4%	12.4%
25/6	4.924	6.7	2.3	3.8	12.7	13.9	1.6	6.1	57.3	0.2%	12.7%
10/7	4.724	6.9	1.0	3.6	13.4	11.4	1.1	3.8	15.8	0.0%	15.3%
12/8	4.668	6.5	0.7	2.9	12.8	10.9	1.1	4.1	17.1	0.2%	13.7%

NOTES:

(1) The inter-quartile range: the difference between the rate of change of the exchange rate was obtained with a probability of 75 percent and the rate of change with a probability of 25 percent. The range is measured in percentage points.

(2) JB, the Jarque-Bera value: a statistical test of whether the estimated distribution has the characteristics of a normal distribution. The critical value at a significance level of 5 percent is 5.99, and at a significance level of 1 percent it is 9.21 percent.

Table 4
Comparison between Statistical Indices of the Model and Indices Received from Other Sources of Information, Changes of NIS/$

	(A) Average of the Means (%)	(B) Interest rate Different (%)	(C) Gap (%points)	(D) Implied Volatility according to the Model (%)	(E) Implied Volatility from the options in TASE (%)	(F) GAP (%points)	(G) Implied Volatility from the options in B.o.I (%)	(H) Gap (%points)
9/6/02	5.8	4.8	1.0	11.4	11.4	0.0	10.2	1.2
9/6/02*	4.7	4.8	-0.1	12.9	10.9	2.0	10.2	2.7
10/6/02	8.4	5.8	2.6	10.8	10.8	0.0	10.6	0.2
17/6/02	6.0	6.7	-0.7	11.6	10.3	1.4	10.4	1.2
25/6/02	6.7	7.9	-1.3	13.9	10.7	3.2	8.1	5.8
10/7/02	6.9	7.1	-0.2	11.4	8.6	2.8	7.7	3.7
12/8/02	6.5	7.3	-0.8	10.9	9.3	1.6	8.4	2.5
Average	6.4	6.3	0.1	11.8	10.3	1.6	9.4	2.5
S.E.	1.1	1.2	1.3	1.1	1.0	1.2	1.2	1.9

NOTES:

(a) The average mean according to the model: assuming that the distribution of changes in the exchange rate is mixture of two normals.

(b) The interest-rate spread: the difference between the yields on T-bills and the Libid dollar interest rate for three months, in annual terms.

(c) The difference between them: the difference between (a) and (b).

(d) The standard deviation according to the model: assuming that the distribution of changes in the exchange rate is mixture of two normals.

(e) The standard deviation implicit in stock-market options: the standard deviation implicit in options traded on the TASE (with the same options data as in section 4), assuming that the distribution of exchange-rate changes is normal.

(f) The difference between them: the difference between (d) and (e).

(g) The standard deviation implicit in the Bank of Israel's options: assuming that the distribution of exchange-rate changes is normal.

(h) The difference between them: the difference between (d) and (g).

*After the Bank of Israel's unusual announcement about the key interest-rate hike.

Figure 1
Distribution of NIS/$ Exchange-Rate Changes, Assuming the Distribution is a combination of two Normal Distributions, Before and After the Bank of Israel Raised the Interest Rate by 1.5 Percentage Points in Mid-June 2002 (percent)

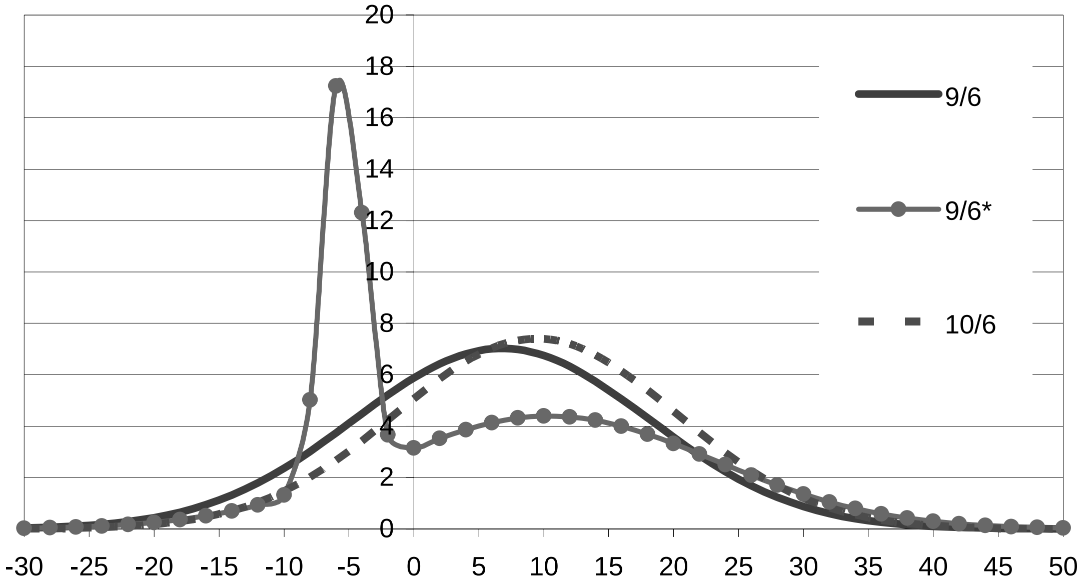
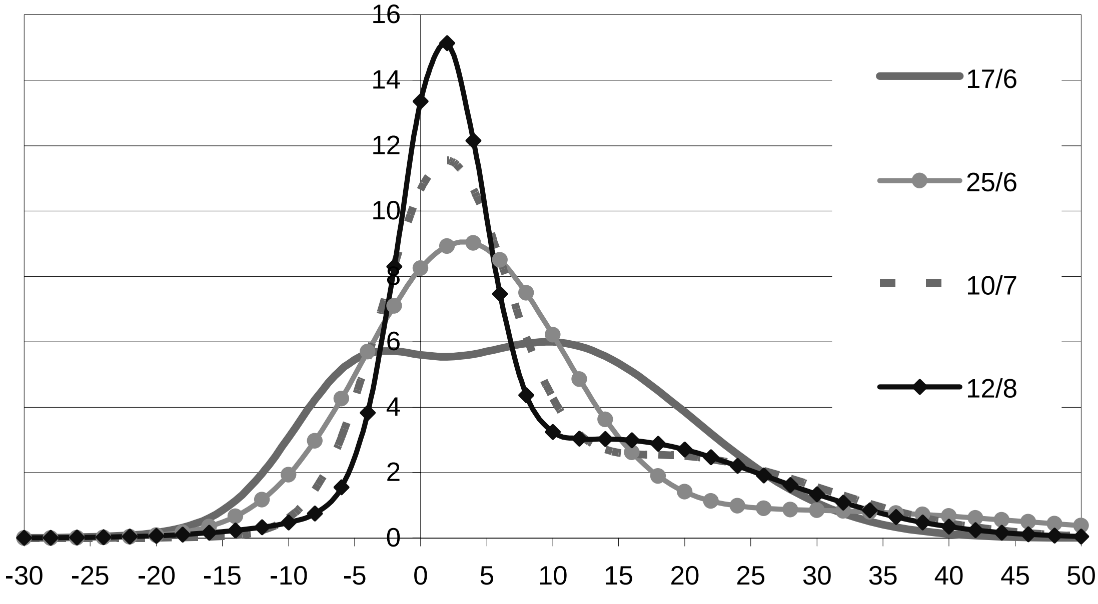


Figure 2
Distribution of NIS/$ Exchange-Rate Changes, Assuming the Distribution is a combination of two Normal Distributions, Before and After the Bank of Israel Raised the Interest Rate by 1.5 Percentage Points at the end of June 2002 (percent)



5. CONCLUSION

We estimated the implied distribution of future NIS/$ exchange-rate changes based on the NIS/$ options traded on the TASE, using options with the same maturity and different strike prices. The estimated distribution of the exchange rate is a double-log-normal Distributions, making it possible to identify the expectations of players in the market regarding the future course of the exchange rate, as well as the possibility that it will change in a non-continuous manner. The data derived from the exchange-rate options are integrative, and include several parameters that are concentrated within one distribution anticipated by the various players.

The application to the NIS/$ exchange rate showed that changes in the Bank of Israel’s key interest rate affected the level of expectations of an exchange-rate shift risk, and changes in the expected distribution of the exchange rate. In Israel there is a statistical correlation between the exchange rate and inflation. This correlation is expected to obtain also between expectations of exchange-rate shifts and inflation expectations. Thus, an application of this work is important for formulating monetary policy, which requires price stability.

A possible further direction for study is to examine and quantify the effect of economic events - such as changes in the Bank of Israel’s key interest rate - on the distribution of the exchange rate. An additional possibility is to examine the reliability of the model by examining the differences between the actual prices of the various options and those indicated by the model. An examination of this kind will make it possible to create a confidence interval in the distribution and undertake a statistical review to test whether the changes in the shape of the distribution at different dates are significant.

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Appendix A.
Non-Technical summary

In recent years the information implicit in the prices of sophisticated financial assets, especially options, has attracted the attention of the business sector and the academic community, as well as central banks, as an input in determining monetary policy. This information reflects market expectations regarding the future price of underlying assets when options are struck, as the income obtained at that time depends on the price of the underlying asset then. The prices of options depend, therefore, on market expectations regarding the mean of the underlying asset, the standard deviation of that mean, the possibility of a sharp change (jump) in the price of the underlying asset, and other parameters which characterize the distribution. Thus, for example, options on the NIS/$ exchange rate at different strike rates provide an indication not only of the expected average rate and its volatility, but also of the probability of various changes in it.

Various methods of estimating the distribution of the exchange rate expected by market players on the basis of options at different strike rates and other market data are described in the literature. The methods can be divided into two main groups: parametrical and non-parametrical. In order to undertake a parametrical estimation, assumptions must be made about the stochastic process of the price of the underlying asset and/or the shape of the distribution. The assumptions dictate the parameters that need to be estimated in order to determine the shape of the distribution. Thus, for example, Black and Scholes assume that the distribution of the price of the underlying asset is log-normal, and that two parameters should be estimated from it—the mean and the standard deviation of the underlying asset. In the present study we have chosen to assume that the distribution of the NIS/$ exchange rate consists of a mixture of two log-normal distributions, in common with the study undertaken by Bahra (1997). The method weights two distributions so as to obtain one, which is double-log-normal. In the framework of the double-log-normal distribution, an individual log-normal distribution is obtained as a private case, as assumed by Black and Scholes in their options-pricing equation. The main reason for estimating the more complex distribution is the increase in events worldwide that attest to inconsistency between the log-normal distribution and that estimated from the markets.

The double-log-normal distribution answers two different assumptions. The first, which relates to the underlying asset, is that changes in this asset are random and accompanied by jumps.[9] The second, concerning the true distribution of the changes in the underlying asset, is that it consists of a large number of normal distributions.

According to the latter assumption, the options-pricing equation in this paper can be interpreted as the weighted average of two Black and Scholes equations, using five estimating parameters: two pairs of means and standard deviations and their weights in the total distribution. The estimation method is based on the loss function whose aim is to minimize the squared deviation of options prices as priced by investors on the basis of the options pricing equation, using the estimated distribution.

The options data sampled for the purposes of this study are those of NIS/$ options traded on the TASE with identical maturity dates and different strike rates. The prices of options are sensitive to the price of the underlying asset, i.e., the NIS/$ exchange rate, to interest rates in the market, and to the expectations of options players as regards the future behavior of the underlying asset. At any point in time during the trading day a change in one of these factors will affect the prices of the options (provided at least one transaction has been performed in them). Hence, the synchronization of the data is very important, especially the assessment of options prices at a specific point in time. The sampling of options prices at different strike prices, which solves the problem of unity of time, is based on the average of the best supply and demand prices for each traded option, as recorded in the TASE register at a specific point in time. Concurrently, the other data required by the model—the dollar exchange rate and the appropriate nominal interest rate—are sampled at the same point in time. The average of quotations for all put and call options at different strike prices reflects the nearest price at which the transactions could be performed in all the options simultaneously.

The data in the model make it possible to calculate, *inter alia,* the mean of the distribution obtained from weighting the two means, representing the expectations of changes in the exchange rate, as derived from this model. Note that these expectations are an estimate derived from the model, without any constraint underlying the estimation method. The expectations of depreciation may also be calculated from another source of data, described by means of the UIP equation, according to which the NIS/$ interest-rate spread is equal to the estimate of expectations of changes in the

[9] Mixed diffusion process with jumps.

exchange rate.[10] Comparing these two sources of information as regards the mean of the exchange rate shows that the differences are relatively small and random. It can therefore be concluded that the reliability of the model is good, and its ability to describe investors' expectations is adequate. The expected standard deviation of exchange-rate changes is usually calculated by means of the Black-Scholes equation, which assumes that the distribution function of changes in the exchange rate is normal, unlike the assumption made in this study. Hence, a comparison between the standard deviation implicit in NIS/$ options obtained by means of the Black-Scholes equation and that obtained here will point to the error caused by the assumption that the distribution of the changes is normal. The main finding of the comparison between these two sources of information regarding the implicit standard deviation is that the assumption that the distribution is single-log-normal is biased downward. The size of the bias depends on the extent to which this assumption is wrong.

By means of the remaining statistical data we describe the structure of investors' expectations in greater detail, making it possible to examine the probability of exceptional developments in Israel's foreign-currency market. In addition, it is possible to calculate the distribution on several dates close to economic events that affect the NIS/$ exchange rate, and thus to examine the extent to which they influence investors' expectations as reflected in the results of the model. It is also possible to calculate the probability of a specific depreciation and equivalent appreciation, as these parameters reflect the level of uncertainty and asymmetry of market expectations regarding the future NIS/$ exchange rate. It can thus be seen that this model provides a great deal of important information about the expected development of the NIS/$ exchange rate which could not be obtained from other sources. This kind of examination of the foreign-exchange market enables both economic agents and policymakers to make a better assessment of future NIS/$ exchange-rate developments, as expected by investors.

[10] Assuming that the risk premium is zero.

Appendix B.
The Data Used in the Study

The table below shows the data sampled for the purpose of the model. The best quoted prices for demand and supply are for put and call options at various strike prices, days to maturity, the NIS/$ exchange rates known at the time of the sample, and the yield on Treasury bills known at the time of the sample and for a period equivalent to the lifetime of the options. Averages of quoted prices, for both put and call options, at the various strike prices are those nearest to the price at which the transactions could be closed at the time of the sample, and hence are the actual prices used for estimating the model (see Section III).

Date	9/6/02				9/6/02				10/6/02			
Time	14.00				16.55				14.12			
NIS/$	4.9972				4.9972				4.962			
Interest Rate	6.3				6.3				7.4			
Days to Maturity	51				51				50			
	Put		Call		Put		Call		Put		Call	
Strike Price	Ask	*Bid*	Ask	*Bid*	Ask	*Bid*	Ask	*Bid*	Ask	*Bid*	Ask	*Bid*
4.5			0.626	0.5			0.59	0.408	0.0045		0.548	0.4
4.6			0.546	0.384			0.434	0.322	0.0085	0.0001	0.4	0.284
4.65			0.446	0.332			0.378	0.282	0.008	0.0001	0.376	0.3
4.7	0.025	0.0002	0.344	0.212			0.286	0.222	0.0085	0.0002	0.326	0.254
4.75	0.035	0.0005	0.296	0.234			0.454	0.185	0.0095	0.0002	0.26	0.244
4.8	0.045	0.006	0.248	0.2	0.024	0.007	0.334	0.152	0.0155	0.005	0.214	0.196
4.85	0.015	0.0095	0.204	0.19	0.03	0.0165	0.172	0.127	0.021	0.018	0.172	0.164
4.9	0.025	0.021	0.163	0.125	0.046	0.036	0.148	0.095	0.033	0.03	0.134	0.127
4.95	0.046	0.034	0.128	0.107	0.062	0.054	0.095	0.08	0.053	0.047	0.1	0.096
5	0.074	0.052	0.097	0.092	0.189	0.073	0.093	0.055	0.075	0.068	0.075	0.07
5.05	0.188	0.081	0.074	0.061	0.158	0.086	0.053	0.043	0.109	0.096	0.055	0.05
5.1	0.228	0.096	0.056	0.043	0.206	0.101	0.04	0.025	0.167	0.112	0.041	0.035
5.2	0.298	0.152	0.037	0.023			0.034	0.016			0.027	0.0105
5.3			0.025	0.0055								

Date	17/6/02				25/6/02				10/7/02			
Time	13.01				14.17				13.28			
NIS/$	4.9442				4.9242				4.7238			
Interest Rate	8.65				10.03				9.07			
Days to Maturity	43				35				48			
	Put		Call		Put		Call		Put		Call	
Strike Price	Ask	*Bid*	Ask	*Bid*	Ask	*Bid*	Ask	*Bid*	Ask	*Bid*	Ask	*Bid*
4.5	0.0014		0.578	0.372			0.458	0.45	0.0025	0.0001	0.374	0.202
4.6	0.0025		0.392	0.326			0.362	0.3	0.0075	0.0035	0.176	0.153
4.65	0.003	0.0001	0.382	0.244			0.312	0.252				
4.7	0.004	0.003	0.292	0.232	0.005	0.0001	0.264	0.244	0.029	0.022	0.093	0.089
4.75	0.0065	0.005	0.262	0.204	0.0055	0.0025	0.216	0.204	0.045	0.035	0.085	0.051
4.8	0.013	0.0115	0.218	0.16	0.0105	0.01	0.171	0.16	0.097	0.068	0.043	0.033
4.85	0.02	0.0185	0.178	0.148	0.02	0.0185	0.125	0.116	0.129	0.099	0.032	0.023
4.9	0.035	0.032	0.142	0.109	0.033	0.032	0.09	0.087	0.164	0.14	0.03	0.016
4.95	0.055	0.051	0.09	0.076	0.053	0.051	0.063	0.058	0.23	0.176	0.013	0.0115
5	0.079	0.077	0.059	0.057	0.086	0.077	0.042	0.038	0.254	0.181	0.0125	0.008
5.05	0.128	0.104	0.041	0.039	0.131	0.112	0.031	0.028	0.356	0.232		
5.1	0.152	0.142	0.028	0.027	0.178	0.151	0.022	0.0185			0.006	
5.2	0.332	0.185	0.0145	0.0095			0.0095	0.008				
5.3	0.458	0.228	0.0065	0.001			0.0095	0.0011				

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