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PIECEWISE LINEAR INCOME TAX**

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ABSTRACT

This paper calculates optimal linear income taxes when differences in income are caused by random factors ('luck') rather than by unobserved individual abilities, as assumed in the classical theory of income taxation. As first shown by Varian (1980), in the former case income taxation acts as social insurance. By introducing life uncertainty and precautionary behavior, we find higher optimal marginal tax rates than those found by Varian. We also find that — in the context of a piecewise two-bracket linear tax schedule — the second marginal tax is always higher than the first, and equals 100 percent. This last finding contrasts with results recently obtained in the framework of classical income taxation theory, which show a lower second marginal tax. For the parameters used in the simulation, we find that a Rawlsian social planner chooses a higher first marginal tax rate than a utilitarian planner would.

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1. Introduction

Following Mirrlees (1971), the main stream of the literature on optimum income taxation ascribes differences in income to unobserved differences in individual abilities. In his seminal paper, Mirrlees showed that using plausible parameters for income distribution in the economy and assuming a Benthamite social welfare function,¹ the optimum income tax is approximately linear, and the marginal tax rate is low (approximately 20 percent).² These findings led a number of authors to check the sensitivity of the results to different assumptions underlying in Mirrlees' analysis. Atkinson (1973) found higher optimal marginal tax rates using alternative specifications of the social welfare utility function. Stern (1976) showed that the optimum marginal tax rate is substantially increased when making realistic assumptions about the elasticity of substitution between leisure and consumption. Sheshinski (1989) found slightly higher marginal tax rates for a linear system, while in the framework of a two-bracket system marginal rates were similar to those of Mirrlees.

More recently, Slemrod *et al.* (1994) found a wide range of optimal marginal tax rates (for both linear and two-bracket systems) as a function of the elasticity of substitution, the degree of inequality aversion of the social planner, and the revenue requirements of the system. A remarkable result of their paper is that assuming the same

¹ The Benthamite ('utilitarian') social welfare function is the sum of individual utilities. Mirrlees (1971) shows results for the transformation $W = e^{-U}$ as well, where U represents individual utility.

² Tax rates are sensitive to the assumption on the required revenue of the government. In his simulations Mirrlees assumed a low revenue requirement of up to 9 percent (i.e., his figures are not far from a purely redistributive system).

income distribution as Mirrlees, their simulations show that in a two-bracket system the optimal second marginal tax rate is lower than the first.³ Consequently, the usual progressivity feature of rising marginal tax rates is replaced by a softer one, which considers rising average tax rates.

An alternative way of checking the sensitivity of these results to the underlying assumptions of the analysis is to compute optimal marginal rates according to alternative theories of income formation.⁴ One of the main candidates for this kind of examination is Varian's (1980) approach to the income formation process, which states that differences in income are mainly due to unobservable random factors, which may be called 'luck'. According to this view, the desirability of the income tax rests on its social insurance characteristics; these are obtained, for example, by providing an equal demogrant to all individuals, as in the case of the linear income tax. If we perform this examination by looking at Varian's results on optimal linear marginal tax rates (1980, p.

³ A similar result appears in simulations by Kanbur and Tuomala (1994) for a non-linear income tax. Note that these results are not general. Sadka (1976) shows that it is optimal to set a zero marginal tax rate for the individual with the highest ability. Since a reduction of the marginal tax from a positive rate to nearly zero increases the economic activity of this individual, his utility is clearly enhanced (otherwise he would have not chosen to work more) and we obtain a Pareto improvement; moreover, an infinitesimal marginal tax rate would increase tax revenue, allowing for further resources to be distributed among low-income agents. However, it is not clear whether marginal taxes should start to fall before reaching the top of the income distribution. Diamond (1996) shows that in the absence of income effects, optimal marginal taxes rise until the top of the distribution of abilities, and only at the top itself should it be reduced to zero.

⁴ Tuomala (1984, 1990) provides an interesting extension by adding luck to the classical model of abilities (his results for the linear case are shown in Appendix B). However, he leaves open the question of the sensitivity of optimal tax rates when income is formed by the interaction of savings (investment) and luck.

57, Table 1) we find that his simulations support *low* tax rates that are quantitatively close to those found by Mirrlees. If we consider a coefficient of variation of 0.6 as empirically plausible, the optimum tax rate computed using Varian's model is 25 percent,⁵ i.e., not far from the 20 percent tax rate found by Mirrlees.⁶ This finding raises the question of the sensitivity of Varian's results to the underlying assumptions of his own analysis, most notably two assumptions that seem to affect the results of his simulations: (i) complete certainty about the length of life, and (ii) the use of a quadratic utility function, which implies increasing absolute risk-aversion. The latter assumption has been seriously challenged by the modern theory of insurance, since it implies that an individual's aversion to a fixed bet increases with income. Moreover, since marginal utility is linear under a quadratic utility function, the consumer does not exhibit precautionary behavior,⁷ an element that would clearly affect the desirability of income taxation.

The paper is organized as follows: Section 2 introduces the setup for the optimum linear income tax schedule. Sections 3 and 4 characterize the effects of life uncertainty and precautionary behavior on computed optimal linear tax rates. Section 5 characterizes

⁵ Varian's Table 1 shows the sensitivity of the results to different levels of the random component of income. Calculating the coefficients of variation for each case (as a percentage of the expected second-period income, x) we obtain that the empirically plausible range is between $n = 0.2$ and $n = 0.3$ (where the coefficients of variation are 0.44 and 0.73, respectively). Assuming that a coefficient of variation of 0.6 represents a benchmark for empirical purposes (see, e.g., Barsky, Mankiw and Zeldes, 1986), we calculated the optimum tax using Varian's methodology for $n = 0.25$, which corresponds to a coefficient of variation of 0.58.

⁶ Varian shows even lower marginal rates — of 4 percent — in the framework of the continuous case (see Varian, 1980, Table 2, p. 65).

⁷ Precautionary behavior has become a standard element of consumption analysis both on the micro and macro level; see Hubbard, Skinner and Zeldes (1994).

the optimum piecewise system using a two-bracket schedule. Section 6 summarizes and concludes the paper.

2. An Optimum Linear Income Tax With Life Uncertainty

Each agent solves the following problem:

$$\text{MAX}_{x_i} u(w_i - x_i) + p_i E u[c(x_i + \epsilon_i)] , \quad (1)$$

where u is a utility function ($u' > 0$, $u'' < 0$), w_i is person i 's income in the active period of life, p_i ($0 < p \leq 1$) is the probability of his survival to the second and last period of life, E is the expectation operator, defined on uncertain second-period income $x_i + \epsilon_i$ (where x_i are the savings at the end of the first period and ϵ_i is a non-observable random shock), and c represents consumption.

Since one of the main aims of this paper is to compare its results with those obtained by Varian (1980), we will assume that individuals are identical ex-ante, i.e., that $w_i = w$, $\epsilon_i = \epsilon$ and $p_i = p$ for all i . A discussion of the general case (where w_i is different for each i) and the implications for the results and for the existence of a market failure is presented in Appendix A.

If we could actually observe luck (ϵ) occurring, the optimal policy of the government ex-ante would be a 100 percent tax rate on luck. But we cannot observe luck — we can only observe actual second-period income, $x + \epsilon$. Note also that the definition of the problem implies that $E c(x + \epsilon) = x$, and since individuals are egoistic they consume all the available income in the second period. Moreover, the consumption function c is linear in the proposed model, since we consider only the case of a linear income tax. The F.O.C. for an individual is:

$$u'(w - x) = pEu'[c(x + \epsilon)]c'(x + \epsilon) , \quad (2)$$

It is assumed that the government collects a linear income tax from all individuals, and its revenues are distributed only to individuals who are alive in the second period:⁸

$$\begin{aligned} T = Ety = tx ; \quad S = pD \\ S = T \end{aligned} \quad (3)$$

where T represents taxes collected, S is the benefit distributed to consumers, t is the tax rate, y is the income subject to taxation (in our case, savings, x) and D is the demogrant given to consumers. As shown in equation (3), the government budget must be balanced, and therefore the demogrant equals tx/p . By substituting the government budget equation in the individual maximization problem, the government's problem is how to choose the optimum tax rate t such that:

$$\text{MAX}_t u[w - x^*(t)] + pEu \left[c[x^*(t) + \epsilon] + \frac{tx^*(t)}{p} \right] , \quad (4)$$

where $x^*(t)$ is the optimum amount of savings chosen by the individual [by using equation (2)]. The F.O.C. for the government's problem is:⁹

⁸ In a sense, there is an annuity component in government intervention, since the government returns all the collected taxes back to the consumers, including taxes collected from individuals who died in the first period. But the annuity component does not have the effect of annuities in the usual sense, since it is used to increase the demogrant (i.e., it does not imply an increase in the available rate of return of savings as in the usual case considered for annuities, which are a function of the amount of savings, x). For the effect of annuities in the usual sense see Abel (1986).

⁹ Note that under a linear one-bracket income tax, $c[x(t) + \epsilon] = (1 - t)(x + \epsilon) + D$, where $D = tx(t)/p$.

$$v'(t) = -u'(w - x)x'(t) + pEu' \left[\frac{x(t) + p(1-t)\epsilon - (1-t)x(t)(1-p)}{p} \right] \left[(1-t)x'(t) + x + \epsilon + \frac{tx'(t) - x}{p} \right] = 0. \quad (5)$$

After some manipulations we can re-write equation (5) as follows:

$$\frac{Eu'[(1-t)(x(t) + \epsilon) + tx(t)/p](1-t)x'(t)}{p\epsilon + x'(t) - (1-t)x'(t)(1-p) - x(t)(1-p)} = Eu' \left[\frac{x(t)[1 - (1-t)(1-p)]}{p} + (1-t)\epsilon \right]. \quad (6)$$

Before interpreting the first-order condition (6), note that equation (11) in Varian (1980, p. 55) is a particular case of this equation, for $p = 1$. The left hand side represents the 'efficiency' effect (i.e., the effect on savings), while the right-hand side represents the 'insurance' effect (changes in the demogrant). Using the same procedure as the one used by Varian, we interpret this F.O.C. by assuming a small decrease in the tax rate, Δt . The numerator on the left hand side increases by $\Delta tx'(t)Eu'\{(1-t)[x(t) + \epsilon] + tx(t)/p\}$, an increase caused by the effect of the tax-cut on savings, multiplied by the marginal utility. But note that this increase in the numerator is reinforced by a decrease in the denominator, of $\Delta t(1-p)x'(t)$.¹⁰ At the limit, when p tends to 0 (i.e., the consumer is almost certain that he is not going to live in the second period), the decrease in the denominator is maximal (i.e., the tax cut induces a high reaction of savings owing to 'efficiency' factors).

On the right hand side, the insurance effect is caused by the fact that instead of facing a risk of $(1-t)\epsilon$ the consumer now faces an increased risk of $[1 - (t + \Delta t)]\epsilon$.

¹⁰ This effect is not present in Varian's analysis, given the assumption of $p = 1$.

The insurance effect is therefore:

$$Eu' \{x(t)[1 - (1 - t)(1 - p)]/p + (1 - t)\epsilon\} \epsilon \Delta t .$$

It is easy to verify that the derivative of the argument of marginal utility with respect to p is negative:¹¹ since marginal utility is a decreasing function of its argument, the lower the probability of survival, the less important is this effect since the consumer does not enjoy the insurance effect of the income tax. In the limit, when p tends to 0, the argument of expected marginal utility tends to infinity, and consequently marginal utility tends to zero.¹²

Note that the strength of the insurance effect is a function of the third derivative of the utility function. This is so, because the impact of the change Δt on the insurance effect depends on the magnitude of the *change* in marginal utility (i.e, it depends on the convexity of the marginal utility). As well known, convexity of the marginal utility implies a positive third derivative of the utility function (as in the logarithmic utility function, and unlike the quadratic utility function, where marginal utility is linear).¹³

In order to isolate the importance of life uncertainty and precautionary behavior we proceed in the following two sections by comparing the results of calibrated simulations with Varian's benchmark: (i) we calculate optimal taxes for a quadratic utility function (as in Varian) under life uncertainty, and (ii) we calculate optimal taxes with life certainty (as in Varian) but with a logarithmic utility function.

¹¹ This derivative is equal to $-t/p^2$.

¹² This is true for $u'(\infty) = 0$ (logarithmic utility function).

¹³ For a graphical explanation of this point see Strawczynski (1994, p. 490).

3. The Effect of Life Uncertainty on the Optimum Linear Marginal Tax

Assume a quadratic utility function:

$$u = -\frac{c^2}{2} + bc, \quad (7)$$

where c is consumption and b is a parameter to be calibrated in the simulation.

The F.O.C. is:

$$-[b - (w - x)] + \frac{p(1-t)}{2} \left[b - \left[(1-t)(x+n) + \frac{tx}{p} \right] + b - \left[(1-t)(x-n) + \frac{tx}{p} \right] \right] = 0, \quad (8)$$

where n is the income shock which takes a positive value ($+n$) with probability 0.5 and a negative value ($-n$) with probability 0.5. After some manipulation the following formulae for $x(t)$ and $x'(t)$ are obtained:

$$x(t) = \frac{b[p(1-t)-1]+w}{1 + p(1-t) - t(1-p)/p}, \quad (9)$$

$$x'(t) = \frac{(2b-w)p + b[2(1-p)t-(1-p)^2/p] + w(1-p)/p}{[1 + p(1-t) - t(1-p)/p]^2}$$

Using these results and the first-order condition (5) (see Appendix C), the formula for the optimal tax t is a function of the following parameters:

$$t = t(p, n, w, b). \quad (10)$$

As in Varian, increases in n (risk) and w (income)¹⁴ and a decrease in b (which

¹⁴ This result is specific to the quadratic utility function, which implies increasing absolute risk aversion. Thus, the higher the wage, the more averse is the consumer to a given bet and con-

corresponds to the case of an increase in risk aversion) result in an increase of the optimum tax rate. The new element is related to p ; the higher p (i.e., the higher the probability of survival), two opposite effects are at work: on the one hand, the greater is the adverse effect of the tax, since marginal utility of savings is weighted by a higher probability of survival; on the other hand, if he survives, the tax-transfer system provides insurance. Since it is not clear which of these effects dominates, it is important to choose parameters that are in line with existing evidence in order to assess the importance of these effects by means of a calibrated simulation.

Table 1 shows the results of a simulation for different values of n , the income risk. The range of n values was chosen so as to correspond to empirically accepted coefficients of variation. The choice of p is related to the limits of economic decisions concerning active and passive periods of life. Assuming that retirement age is 65 and that life expectancy is 75,¹⁵ and using an actuarial table to calculate the probability of survival,¹⁶ we estimate that $p = 0.8$. For the purpose of comparison, we also present the optimum tax computed using Varian's methodology, which corresponds to the case $p = 1$. As in Varian, we assume that $w = b = 1$, implying that when $t = 1$, the optimum amount of savings x , equals zero.

sequently he prefers a higher tax (demogrant).

¹⁵ This figure corresponds to the category with a high life expectancy, which is also the relevant category for developed countries (The World Bank Atlas, 1992).

¹⁶ For this purpose we used a table for 1980, which appeared in Hubbard, Skinner and Zeldes (1994).

Table 1. Quadratic Utility Function and Life Uncertainty

n	x		Coefficient of variation		p = 0.8		p = 1		t*-t**
	p=0.8	p=1	p=0.8	p=1	t*	D*/x	t**	D**/x	
0.1	0.39	0.48	0.26	0.21	0.26	0.32	0.07	0.07	0.19
0.2	0.38	0.45	0.53	0.44	0.30	0.37	0.19	0.19	0.11
0.22	0.37	0.44	0.59	0.50	0.31	0.39	0.22	0.22	0.09
0.25	0.37	0.43	0.67	0.58	0.32	0.40	0.25	0.25	0.07
0.275	0.37	0.42	0.75	0.66	0.34	0.42	0.28	0.28	0.06
0.3	0.36	0.41	0.83	0.73	0.35	0.44	0.31	0.31	0.04

According to Table 1, the optimum tax under life uncertainty is higher than the tax under life certainty by a percentage that declines as the income risk increases. The reason for this result is that under increasing absolute risk aversion the relative desirability of income taxation rises as a function of the size of the risk. When $n = 0.25$ (which implies coefficients of variations of 0.58 and 0.67 for $p = 0.8$ and $p = 1$, respectively) is taken as the benchmark, we find that life uncertainty raises the optimum tax rate and the share of the demogrant in savings by 7 and 15 percentage points, respectively.

4. The Effect of Precautionary Behavior on the Optimum Linear Marginal Tax

Assume now that the utility function is $u = \ln c$. The first-order condition in this case is:¹⁷

$$\frac{1}{w-x} = \frac{p(1-t)}{2} \left[\frac{1}{x + (1-t)n} + \frac{1}{x - (1-t)n} \right]. \quad (11)$$

Equation (12) shows the formula for x , which is obtained after solving a second-order equation:

$$x = \frac{w(1-t)p + \sqrt{p^2w^2(1-t)^2 + 4[1 + (1-t)p](1-t)^2n^2}}{2[1 + (1-t)p]}. \quad (12)$$

Note that when $t = 1$ (a 100 percent tax), the individual chooses not to save. Note, too, that x rises as p rises,¹⁸ showing that the individual is willing to save for second-period consumption.

From (11) we can calculate $x'(t)$:

$$x'(t) = \frac{\left[wp + \frac{2p^2w^2(1-t)^2 - 12p(1-t)^2n^2 - 8(1-t)n^2}{\sqrt{(1-t)^2[p^2w^2 - 4n^2 - 4p(1-t)n^2]}} \right]}{4[1+(1-t)]^2} \times$$

$$\left[1 + (1-t)p - 2p[w(1-t)p + \sqrt{p^2w^2(1-t)^2 + 4[1+(1-t)p](1-t)^2n^2}] \right].$$

Finally we use private first-order condition (6) to compute an equation for the

¹⁷ For simplicity we explicitly neglect in this section the annuity effect of the demogrant (in the simulations we will consider only the case $p = 1$).

¹⁸ The derivative of x with respect to p is equal to $\{2w(1-t) + [1+(1-t)p] [2pw^2n^2(1-t)^2] / [p^2w^2n^2(1-t)^2 + 4(1 + (1-t)p)(1-t)^2n^2]^{0.5}\} / \{4[1 + (1-t)c]^2\}$.

optimum tax rate:

$$\frac{x(t)[1 - p(1 - t)]x'(t)}{1 - t} = n^2. \quad (13)$$

In order to isolate the effect of assuming a function with decreasing absolute risk aversion (logarithmic) instead of increasing absolute risk aversion (quadratic), we consider only the case of $p = 1$. Comparing the results using these two functions allows us to estimate the effect of precautionary behavior. The results of the simulation for different levels of income risk are shown in Table 2. Again, all simulations are performed for $w = 1$.

Table 2. Logarithmic Utility Function and Life Certainty^a

n	x		Coefficient of variation		t* (log)	t** (quadr.)	t*-t**
	log	quadr.	log	quadr.			
0.1	0.50	0.48	0.20	0.21	0.04	0.07	-0.03
0.2	0.47	0.45	0.43	0.44	0.22	0.19	0.03
0.22	0.45	0.44	0.49	0.50	0.29	0.22	0.07
0.25	0.39	0.43	0.65	0.58	0.45	0.25	0.20
0.252	0.38	0.43	0.67	0.59	0.47	0.26	0.21
0.255	0.36	0.43	0.70	0.60	0.50	0.26	0.24

^a Note that with life certainty the optimal D/x is equal to the optimal tax.

The results show that the effect of insurance rapidly increases as the size of income risk rises. The optimal tax rate rises from 30 to 50 percent for a range of coefficients of variation between .5 and .7. Note also that the difference between the optimal taxes in the two cases increases as income risk increases, a result that may be attributed to the

fact that with precautionary behavior, the importance of insurance increases as a function of the income risk. Taking the case of $n = 0.25$ as a benchmark (as in Section 3), we find that the optimum tax rate for the logarithmic utility function is 20 percent higher than the one obtained by Varian. It is important to note that a logarithmic utility function implies a coefficient of relative risk aversion of 1. Higher coefficients of relative risk aversion¹⁹ clearly imply even higher optimal marginal tax rates.

5. An Optimum Piecewise Linear Schedule

In this section we design the simplest stylized setup for a two-bracket linear schedule. The main purpose of the analysis is to compare the results to those obtained in the framework of classical income taxation theory, and especially to see whether optimal taxes follow the usual pattern of rising marginal tax rates or a declining pattern as shown by Slemrod *et al.*

Assume that the income formation process emerges in the following four possible states of nature (each with probability 0.25):

- I) $y_1 = x - \epsilon_2$
- II) $y_2 = x - \epsilon_1$
- III) $y_3 = x + \epsilon_1$
- IV) $y_4 = x + \epsilon_2$

where y represents second-period resources, x are first-period savings and ϵ_1 and ϵ_2 are fixed random shocks ($\epsilon_2 > \epsilon_1$). Further assume that first-period income w is equal for all individuals, and consequently the only difference in second-period resources

¹⁹ Unfortunately we were unable to handle the more general case of $u = c^{1-\theta}/(1-\theta)$, $\theta > 1$, which implies equations with an order higher than 2.

is caused by 'luck'.²⁰ For simplicity, we assume that although the social planner cannot observe low occurrences of luck (ϵ_1), high occurrences of luck are observable ($\epsilon_2 - \epsilon_1$).²¹ Since by assumption there are two types of shocks, the income tax schedule in this setup is clearly defined as follows:

- On the first d dollars, $0 < d < x + \epsilon_1$, the marginal tax rate is t_1 .
- On the next $(\epsilon_2 - \epsilon_1)$ dollars, the marginal tax rate is t_2 .

Recalling the law of large numbers, we sort the individuals in the economy into four groups, according to the different idiosyncratic shocks. We further impose a restriction on the social planner, according to which he must set a common demogrant for all individuals, i.e., $ET = S = 4D$ (where T are taxes collected, S are transfers, and D is the equal demogrant). The following is the expression for the demogrant:

$$D = t_1 x + \frac{t_2(\epsilon_2 - \epsilon_1)}{4} . \quad (14)$$

Using this expression we can write after-tax second-period resources y_i^* for each state of nature ($i = 1, 2, 3, 4$):

²⁰ For simplicity we do not take into account the annuity effect on the demogrant (as in Section 3).

²¹ The rationale of this assumption can be seen in a world where individuals have different initial endowments ex-ante (i.e., not equal as assumed here). In that case, if the difference between the initial endowments (x) is similar to the effect of luck (ϵ), it will be difficult for the social planner to differentiate between the luck component and the endowment component. This is not the case when we allow for *extreme* realizations of luck (ϵ_2): if the only way of becoming a millionaire is to be lucky, then $\epsilon_2 - \epsilon_1$ should be high enough compared to differences in initial endowments.

$$\text{I) } y_1^* = x - \epsilon_2 + \epsilon_2 t_1 + \frac{t_2(\epsilon_2 - \epsilon_1)}{4}$$

$$\text{II) } y_2^* = x - \epsilon_1 + \epsilon_1 t_1 + \frac{t_2(\epsilon_2 - \epsilon_1)}{4}$$

$$\text{III) } y_3^* = x + \epsilon_1 - t_1 \epsilon_1 + \frac{t_2(\epsilon_2 - \epsilon_1)}{4}$$

$$\text{IV) } y_4^* = x + \epsilon_2 - t_1 \epsilon_1 - t_2 \frac{3(\epsilon_2 - \epsilon_1)}{4}$$

The problem of a utilitarian government is to set optimal tax rates t_1^* and t_2^* such that the representative individual ex-ante utility is maximized:

$$\text{MAX}_{t_1, t_2} u(w - x) + p \frac{\sum_{i=1}^4 u(y_i^*)}{4} \quad (15)$$

Note that x is equal for all individuals, since we assume that ex-ante all individuals have the same first-period income, w .

PROPOSITION 1: In this setup, the second optimum marginal tax rate, t_2 , for a utilitarian social planner is higher than t_1 and equal to 100 percent.

PROOF: The individual's first-order condition is:

$$u'(w - x) = p \sum_{i=1}^4 \frac{u'(y_i)}{4}, \quad (16)$$

where u' is the derivative with respect to x , and tax rates in equation (16) are adjusted

to their optimum level. By contradiction, assume that the optimum t_2 equals k ($0 < k < 1$). An increase in k implies that y_4 goes down while y_1 , y_2 and y_3 increase. Note that $y_4 \geq y_3$ for all t_1 and t_2 :

$$y_4 = x + \epsilon_2 - \epsilon_1 t_1 - t_2 \frac{3(\epsilon_2 - \epsilon_1)}{4} \geq y_3 = x + \epsilon_1 - t_1 \epsilon_1 + t_2 \frac{(\epsilon_2 - \epsilon_1)}{4} \quad (17)$$

$$\epsilon_2 \geq \epsilon_1 + t_2(\epsilon_2 - \epsilon_1) .$$

Clearly, an increase in k is welfare-improving, and consequently $k \neq 1$ is not an optimum, contradicting the initial hypothesis.

We now show that $t_1 < t_2 (=1)$. By contradiction, assume that for the optimum $x > 0$, t_1 equals 1. Since in the present case x is a function of t_1 , by using equation (6) we show that the efficiency effect on savings is a function of t_1 . It follows immediately that when $t_1 = 1$ optimal savings equal zero, which contradicts the initial hypothesis of $x > 0$.

This result contrasts with the findings of Slemrod *et al.*, since optimal marginal tax rates rise with income. The intuition of the result runs as follows: if the only way to become rich, as in the present framework (i.e., achieving y_4), is to have good luck, government intervention through a well-defined two-bracket system succeeds in transferring resources from good states of nature to bad ones through the second marginal tax. Efficiency effects are confined to the first marginal tax rate, which is the only relevant marginal tax for saving decisions in the present context.

Table 3 shows the results of a simulation of optimal two-bracket tax rates for $n = 0.25$ and $n = 0.3$, assuming a quadratic utility function.²² Simulations are

²² Clearly we would like to run a simulation with a function that exhibits precautionary behavior. Unfortunately the solution for the logarithmic case implies a fourth-order equation for

presented for both life certainty ($p = 1$) and life uncertainty ($p = 0.8$).

Table 3. Optimum Two-Bracket Income Tax

n^a	x	Coefficient of variation	t_1^*	t_2^*	D^*/x	Average marginal tax
$p = 1$						
0.25	0.43	0.58	0.25	1	0.35	0.44
0.3	0.41	0.73	0.31	1	0.41	0.48
$p = 0.8$						
0.25	0.41	0.61	0.12	1	0.25	0.34
0.3	0.39	0.77	0.20	1	0.33	0.40

^a For $n=0.25$ it was assumed that $n_1=0.15$ and $n_2=0.35$; for $n=0.3$: $n_1=0.2$ and $n_2=0.4$.

Note that the figures for optimum savings and the optimal tax t_1 are the same as those shown in Section 3. The reason is related to optimum behavior with a quadratic utility function: although the second marginal rate reduces the variance of second-period after-tax income, provided no precautionary behavior exists, the optimum amount of savings x remains unchanged.²³ The share of the demogrant in savings, is higher than in Section 3, because the existence of a second marginal tax allows for an improved insurance mechanism.

Note that when the case of $p = 0.8$ and $n = 0.25$ is taken as a benchmark — as in previous sections — the optimum average marginal tax rate is 34 percent, which is

$x(t)$, which is analytically untractable.

²³ See, e.g., Abel (1988, p. 49).

quite high in view of the fact that precautionary behavior is absent. These findings strengthen the ones obtained under the linear system, i.e., optimal marginal tax rates are higher than 25 percent, as computed by Varian.

PROPOSITION 2: For the parameters used in the simulation, a Rawlsian social planner will adopt a higher first marginal tax than will a utilitarian social planner. As in the utilitarian case, the second optimum tax rate will be 100 percent.

PROOF: A Rawlsian social planner will maximize the income of an individual with the worst realization ($-\epsilon_2$):

$$\text{MAX}_{t_1, t_2} y_1^* = x(t_1) - \epsilon_2 + \epsilon_2 t_1 + \frac{t_2(\epsilon_2 - \epsilon_1)}{4}.$$

This equation immediately shows that the optimum policy is to set $t_2 = 1$. With respect to t_1 , let us denote by x^* and t_1^* the optimal values in the utilitarian case. First note that if $x^* < (1 - t_1^*)\epsilon_2$, a Rawlsian planner will prefer the policy $t_1 = 1$ rather than the utilitarian solution ($t_1 = t_1^*$), since in the former case y_1 is higher (note that for $t_1 = 1$, in the optimal solution $x = 0$, and the tax-transfer policy leads to equalization of after-tax income).

After noting that for the parameters used in our simulation $x^* > (1 - t_1^*)\epsilon_2$, we ask whether t_1^* is an optimum for a Rawlsian planner. The answer to this question depends on the reaction of x to changes in t_1 , compared to the change in the term $-(1 - t_1)\epsilon_2$ ('bad luck'). Raising t_1 lowers the effect of 'bad luck' because the most unlucky individual is less exposed to the negative shock ϵ_2 . At the same time, raising t_1 discourages savings (the higher t_1 , the higher this effect). As shown in Table 4, which is based on the same parameters as Table 3 (for $n = 0.25$), the choice of the optimum

t_1 for a Rawlsian planner depends on the parameters of the simulation:

Table 4. The Max-Min Solution

t	x	Δx	$-(1-t_1)\epsilon_2$	$\Delta-(1-t_1)\epsilon_2$	Δy_1
0.1	0.474	-0.026	-0.36	0.04	0.014
0.2	0.444	-0.030	-0.32	0.04	0.010
0.3	0.412	-0.032	-0.28	0.04	0.008
0.4	0.375	-0.037	-0.24	0.04	0.003
0.5	0.333	-0.042	-0.20	0.04	-0.002

The last column of Table 4 shows the impact of raising the tax rate on the income of the individual with the lowest income. The optimum tax is somewhere between 0.4 and 0.5, compared with 0.25 in the utilitarian case.²⁴

6. Summary and Conclusions

This paper considers the sensitivity of the optimal marginal linear income tax when differences in income are caused by random factors rather than by unobserved abilities, as in the classical theory of optimum income tax. We found that the optimum tax is higher than 25 percent, which is the figure obtained by Varian in the linear tax framework. For the benchmark parameters used in the simulation, we found that both life uncertainty and precautionary behavior substantially raise the optimum linear income tax. The effect of precautionary behavior would probably have been greater had we used more realistic assumptions about relative risk aversion.

²⁴ Helpman and Sadka (1978) have shown that in the context of the classical model of income taxation, a Rawlsian planner chooses a higher linear optimal tax rate than a Benthamite planner.

A remarkable finding, which contrasts with the one recently obtained by Slemrod *et al.* in the context of the classical model of income taxation, is that for a utilitarian social planner and a piecewise two-bracket linear income schedule, the second marginal rate is higher than the first, and equal to 100 percent. This pattern is maintained when the social planner is Rawlsian, whereas for the parameters used in the simulation we found a higher first marginal tax rate than the one obtained in the utilitarian case.

APPENDIX A

A Model With Heterogeneous Agents

Assume that w_i is different for each i , so that individuals are different ex-ante. The agent solves the following problem:

$$\text{MAX}_{x_i} u(w_i - x_i) + p_i \text{Eu}[c(x_i + \epsilon_i)] , \quad (1)$$

where w_i is the initial endowment (ability) and ϵ_i is the independently distributed idiosyncratic random shock. Assuming a Benthamite social utility function, the government's problem is to maximize:

$$\text{MAX}_t \sum_{i=1}^n \left[u[w_i - x_i^*(t)] + \text{Ec}[x_i^*(t) + \epsilon_i] + \sum_{i=1}^n \frac{tx_i^*(t)}{n} \right] . \quad (4')$$

Clearly, the solution depends on the distributions of both abilities and luck. The existence of a demogrant implies that redistribution is both from the lucky to the unlucky and from high-ability to low-ability individuals.

Concerning government intervention, note that this general model provides an example of market failure. Assuming that abilities are not observable and that there is asymmetric information (i.e., individuals know which type they belong to, but the government or the insurance companies do not), a private insurance company will fail to provide insurance based only on the 'luck' component, because redistribution occurs as a consequence of both luck and ability. Therefore, as a result of adverse selection, we may encounter a situation in which high-ability individuals will prefer not to buy such insurance in order to avoid redistribution to low-ability individuals. This problem could be solved by making universal participation compulsory, which is the case that calls for government intervention.

APPENDIX B
Optimal Linear Tax Rates

<i>Source</i>	<i>t (%)</i>	<i>Model</i>
Mirrlees(1971), p.202-203	20	Classical (abilities)
Stern(1976), p.161	54	Classical, elasticity of substitution
Atkinson(1973)	Up to 50	Classical, social planner
Sheshinski(1989), p.201	20-35	Classical
Slemrod <i>et al</i> (1994),Table 1	12.5-76	Classical
Tuomala (1990), p. 145	26	Classical, income uncertainty
Varian (1980)	25	Luck as a source of income
Present paper	Over 32	Luck, life uncertainty and precautionary behavior

APPENDIX C

Optimum Tax Formula Under Life Uncertainty

Equation (10) is obtained by using first-order condition (5) and the formulae obtained for $x(t)$ and $x'(t)$, as shown in (9):

$$[b - (w - x)]x'(t) =$$

$$p0.5 \left[b - \frac{x(t) + p(1-t)\epsilon - (1-t)x(t)(1-p)}{p} \right] \left[(1-t)x'(t) + x(t) + \epsilon + \frac{tx'(t) - x(t)}{p} \right] +$$

$$p0.5 \left[b - \frac{x(t) - p(1-t)\epsilon - (1-t)x(t)(1-p)}{p} \right] \left[(1-t)x'(t) + x(t) - \epsilon + \frac{tx'(t) - x(t)}{p} \right]$$

This equation serves as the basis for the simulation in Table 1.

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