# PRECAUTIONARY SAVINGS AND

# THE DEMAND FOR ANNUITIES

Michel Strawczynski

Discussion Paper No. 96.05

June 1996

# Precautionary Savings and the Demand for Annuities

Michel Strawczynski*

June 1996

## Abstract

Two well-known results in the field of annuities are: (i) egoistic agents should annuitize all their wealth and (ii) altruistic agents should segment their savings between riskless bonds (for bequests) and annuities (for consumption). Given these two results, it is puzzling to note that private annuity markets are thin. In this paper we show that in the presence of precautionary savings, altruistic agents reduce the demand for annuities. Thus, the lack of private annuity markets becomes less than a puzzle, and maybe explained by the existence of a low demand, which is satisfied by existing pension arrangements.

## 1. Introduction

A well-known puzzle is that private annuity markets are thin. The accepted explanation for the lack of private annuities was formally presented by Abel (1986), based on adverse selection. If individual survival probabilities are private information, competitive firms will fail to provide the actuarially fair rate of return that would be guaranteed by a government-managed compulsory system. Leaving aside the issue of the significance of the adverse selection argument (which is seriously challenged by the existence of various insurance markets more suitable to the adverse selection story), note that the explanation assumes a positive demand for annuities, both for egoistic and for altruistic individuals. If individuals are egoistic, all their wealth should be annuitized and, clearly, social security plans would not suffice to satisfy demand.[1] If individuals are altruistic, they should segment their savings, with riskless bonds serving as bequests and annuities serving for future consumption.[2] But if all consumers have a positive demand for annuities, it is hard to believe that an appropriate market would fail to emerge, even after taking into consideration the existence of compulsory (annuity-type) social security systems.

The aim of this paper is to characterize the demand for annuities in the presence of precautionary savings. After presenting a brief look at the literature (Section 2), we perform this characterization in two stages. Section 3 presents a theoretical analysis that includes different types of income uncertainty. The purpose of this analysis is to understand whether we still obtain the results stated above under different types of income uncertainty. In section 4 we discuss the implications of the model for empirical purposes, and perform a stylized simulation. The purpose of the simulation is to assess the allocation of savings between riskless bonds and actuarially fair annuities in the presence

---

[1] Auerbach, Kotlikoff and Weil (1994) stress that in the U.S. increases in social security and pensions raised the fraction of the income of the elderly represented by these annuitized resources from 40 percent in 1967 to 55 percent in 1988.

[2] This result is characterized in Section 3 for the case of a utility function that includes a child's utility as an argument in its father's utility (Barro, 1974). Sheshinski and Weiss (1981) and Abel (1986) have shown this result for the case of bequests as an argument in the utility function.

of precautionary savings against future generation's income uncertainty. Conclusions (and technical appendixes) appear at the end of the paper.

## 2. A Brief Survey of the Literature

Brugiavini (1994) summarizes data for both U.K. and U.S. households, which show that annuities are a rarity in both economies. Estimates for the U.S. give a figure of 2 percent, while new annuity holders in the U.K. in 1987 accounted for 4 percent of total single insurance premia (which include life insurance and personal pensions as well). Kotlikoff and Spivak (1981) attribute this phenomenon to the (imperfect) mechanism of the family: parents guarantee transfers to sons in exchange for support in the case of a long retirement period. Although, as the authors claim, this mechanism is an imperfect substitute for annuities, it could account — at least in part — for the lack of demand for annuities.

Eckstein, Eichenbaum and Peled (1985) show that the existence of adverse selection considerations imparts a Pareto-improving role to mandatory non-discriminatory social security programs that coexist with residual discriminatory private annuities. According to this study, a limited social security system will improve the allocation of resources, given that a private annuity market will not emerge owing to the adverse selection problem. Abel (1986) presented a model with an explicit solution for an annuity firm that copes with the adverse selection problem: given positive demand for annuities by all individuals, an actuarially fair rate of return will yield negative profits.

Friedman and Warshawsky (1990) calculated the rate of return to annuities, finding that even after accounting for adverse selection, the yields of annuities are lower than yields on plausible alternative investments. An interesting finding of their paper is that annuity purchasers are characterized by a higher probability of survival than the average population, which is in line with the adverse selection argument.

The present paper proposes a different explanation, one that is in line with the described empirical stylized facts. It is based on altruistic agents and precautionary behavior, which, of course, may be seen as complementary to the ideas surveyed in this section.

## 3. The Model

### a. Specification of the model

We start by introducing an overlapping generations model that allows for both egoistic and altruistic agents, and analyze different cases under income certainty and income uncertainty. Two types of income uncertainty are considered in this section: (a) Second-Period Income Uncertainty (SPIU) and (b) Future Generation's Income Uncertainty (FGIU).[3] A discussion of the relevance of each type of uncertainty for empirical purposes appears in Section 4.

Assuming a two-period world, an individual's maximization problem is:

$$\text{Max } U(c_1) + \delta(1 - p)EU(c_2) + \alpha EU(c_K) , \tag{1}$$

where $U$ is the utility function [$U' > 0$, $U'' < 0$, $U'(0) = \infty$, $U'(\infty) = 0$, $U''' > 0$[4]], $\delta$ — the subjective discount rate, $p$ — period, $c_i$ ($i = 1, 2$) — the consumption in period i, $E$ is the expectation operator, $\alpha$ is the (discounted) altruism coefficient, and $c_K$ — the consumption of a single offspring ('kid').

To complete the formulation of the problem we must consider budget constraints. For the sake of expositional convenience, we distinguish at this stage between different cases.

### b. Egoistic agents ($\alpha = 0$)

Note, first, that for egoistic agents FGIU is irrelevant, since individuals do not care about the future generation.

Allowing for SPIU through an additive shock, the budget constraints are:

$$\begin{align}
\text{(i)} \quad & Y = c_1 + s + a , \\
\text{(ii)} \quad & Rs + Aa + \epsilon^s = c_{2H} \tag{2} \\
& Rs + Aa - \epsilon^s = c_{2L} ,
\end{align}$$

where $Y$ is a certain income, $s$ ($\geq 0$) is the demand for riskless bonds, $a$ ($\geq 0$) is

---

[3] The relevance of future generations' income uncertainty is stressed by Strawczynski (1994, 1995).

[4] The assumption of a positive third derivative is essential for the main result of the paper. It denotes precautionary behavior, as first explained by Leland (1968).

the demand for annuities, R is the return to a riskless bond, A is the return to annuities, $\epsilon^s$ is the additive shock on second-period income, and sub-indexes H and L represent high and low states of nature, respectively. In all the cases considered in the paper we will assume that annuities are actuarially fair; i.e., $A=R/(1-p)$.

### Income certainty ($\epsilon^s = 0$)

Income certainty implies $c_{2H} = c_{2L} = c_2$. A consumer decides between allocating savings between riskless bonds and actuarially fair annuities. Since the only relevant case from the consumer's point of view is the state of nature in which he is alive, riskless bonds clearly constitute a dominated asset, since its return equals R, compared to $A=R/(1-p)$ for actuarially fair annuities. Consequently, all saving resources are allocated to annuities, and the first order condition is:

$$U'(c_1) = R\delta U'(c_2) \ . \tag{3}$$

From equation (3) we observe that the first-order condition (f.o.c.) does not include the term $(1-p)$, i.e., actuarially fair annuities provide full insurance against the existence of undesired savings at the end of the first period.

### SPIU ($\epsilon^s > 0$)

The introduction of income uncertainty does not alter the fact that riskless bonds are a dominated asset, since they provide a better return in the only relevant state of nature (being alive) in the second period. The f.o.c. is:

$$U'(c_1) = \delta R[qU'(c_{2H}) + (1-q)U'(c_{2L})] \ , \tag{4}$$

where q and $(1-q)$ represent the probabilities of a positive and negative income shock, respectively. The f.o.c. remains the same, with the difference that now, at the optimum, an individual will save more — owing to income uncertainty (precautionary savings). As in the certainty case, since annuities provide full insurance against accidental

savings[5] at the end of the first period, it is optimal to allocate all one's savings to annuities.

Hence, both under income certainty and under income uncertainty, optimizing egoistic individuals should annuitize all their wealth.

### c. Altruistic agents ($\alpha > 0$)

In order to allow for a tractable presentation, we introduce a model that assumes only FGIU. The case of SPIU is deferred to the next subsection.

The budget constraints of this problem for the parents are:

$$Y = c_1 + a + s , \tag{4.i}$$

$$Rs = B^D , \tag{4.ii}$$

$$Rs + Aa = B^S + c_2 , \tag{4.iii}$$

where B represent bequests, and superscripts D and S represent the cases of demise and survival, respectively. The budget constraints for the offspring are:

$$B^D + Y^K + \epsilon^k = c_{KH}^D , \tag{4.iv}$$

$$B^D + Y^K - \epsilon^k = c_{KL}^D , \tag{4.v}$$

$$B^S + Y^K + \epsilon^k = c_{KH}^S , \tag{4.vi}$$

$$B^S + Y^K - \epsilon^k = c_{KL}^S , \tag{4.vii}$$

where $Y^k$ and $\epsilon^k$ are, respectively, the child's income and the additive shock on his income.

Income certainty ($\epsilon^k = 0$)

In this case, $c_{KH}^D = c_{KL}^D = c_K^D$ and $c_{KH}^S = c_{KL}^S = c_K^S$.

---

[5] For an example of accidental savings with both life- and income-uncertainty see Strawczynski (1993).

To solve the problem we substitute $B^S$ and $B^D$ from equations (4.ii) and (4.iii) in equations (4.iv)–(4.vii), and then plug the result into the utility function:

$$\text{Max } U(c_1) + \delta(1 - p)U(c_2) + \alpha p U[R(Y - c_1 - a) + Y^K] +$$
$$\alpha(1 - p)U[R(Y - c_1 - a) + Aa - c_2 + Y^K] \, . \tag{1'}$$

In order to obtain the f.o.c., we differentiate with respect to a, $c_1$ and $c_2$:

(i) $\quad RpU'(c_K^D) = (1 - p)(A - R)[U'(c_K^S)]$

(ii) $\quad U'(c_1) = R\alpha[pU'(c_K^D) + (1 - p)U'(c_K^S)]$ $\qquad$ (5)

(iii) $\quad \delta U'(c_2) = \alpha[U'(c_K^S)] \, .$

If annuities are actuarially fair, we know that $(A - R)/R = p/(1 - p)$. Then, from equation (5.i), we get that $c_K^S = c_K^D$. Substituting this expression in the budget constraint gives the well-known result that *altruistic agents should segment their savings* (Sheshinski and Weiss, 1981); thus, bequests are provided through riskless bonds, while second-period consumption is financed by annuities. The intuition of this result is straightforward: at the optimum, the life-contingent asset (annuity) serves the life-contingent good (second-period consumption).

We now introduce income uncertainty.

## FGIU ($\epsilon^k > 0$)

In order to obtain the solution we substitute $B^S$ and $B^D$ from equations (4.ii) and (4.iii) in equations (4.iv)–(4.vii), and insert the result into the utility function:

$$U(c_1) + \delta(1 - p)U(c_2)$$
$$+ \alpha p\{qU[R(Y - c_1 - a) + Y^K + \epsilon^k] + (1 - q)U[R(Y - c_1 - a) + Y^K - \epsilon^k]\} \tag{1''}$$
$$+ \alpha(1 - p)\{qU[R(Y - c_1 - a) + Aa - c_2 + Y^K + \epsilon^k]$$
$$+ (1 - q)U[R(Y - c_1 - a) + Aa - c_2 + Y^K - \epsilon^k\}$$

To obtain the f.o.c. we differentiate according to a, $c_1$ and $c_2$.

$$Rp[qU'(c_{KH}^D) + (1 - q)U'(c_{KL}^D)] =$$
$$= (1 - p)(A - R)[qU'(c_{KH}^S) + (1 - q)U'(c_{KL}^S)] \quad (6.i)$$

$$\delta U'(c_2) = \alpha[qU'(c_{KH}^S) + (1 - q)U'(c_{KL}^S)] \quad (6.ii)$$

$$U'(c_1) = R\alpha p[qU'(c_{KH}^D) + (1 - q)U'(c_{KL}^D)]$$
$$+ R\alpha(1 - p)[qU'(c_{KH}^S) + (1 - q)U'(c_{KL}^S)] . \quad (6.iii)$$

We analyze the result using the following propositions.

**PROPOSITION 1:** IF THE OPTIMAL SOLUTION IS INTERIOR, THEN THE RESULT OBTAINED BY SHESHINSKI AND WEISS (1981) HOLDS UNDER INCOME UNCERTAINTY; I.E., RISKLESS BONDS SERVE BEQUESTS AND ANNUITIES SERVE SECOND-PERIOD CONSUMPTION.

**Proof:** An interior solution implies that (6.i), (6.ii) and (6.iii) are satisfied as equalities. Given actuarially fair annuities, $(A-R)/R = p/(1-p)$, and consequently, from (6.i), we get:

$$[qU'(c_{KH}^D) + (1 - q)U'(c_{KL}^D)] =$$
$$[qU'(c_{KH}^S) + (1 - q)U'(c_{KL}^S)] . \quad (7)$$

Substituting this result in (6.iii) yields:

$$U'(c_1) = \alpha [qU'(c_{KH}^D) + (1 - q)U'(c_{KL}^D)]$$
$$= \alpha [qU'(c_{KH}^S) + (1 - q)U'(c_{KL}^S)] . \quad (8)$$

This equality holds if $c_{KH}^S = c_{KH}^D$ and $c_{KL}^S = c_{KL}^D$. Substituting these equalities in the budget constraint 4ii to 4vii allows us to complete the proof, since we get that $c_2 = Aa$ and $B^D = B^S = Rs$.

**PROPOSITION 2:** AN INCREASE IN THE FUTURE GENERATION'S INCOME UNCERTAINTY DECREASES THE DEMAND FOR ACTUARIALLY FAIR ANNUITIES RELATIVE TO THE DEMAND FOR RISKLESS BONDS.

**Proof:** Using proposition 1, we know that savings are used in order to provide bequests and annuities to finance second-period consumption. For our purposes, therefore, it suffices to show that an increase in the future generation's income uncertainty results in an increase of bequests. We show this by introducing the budget constraint in (6.ii):

$$\delta U'(c_2) = \alpha[qU'(B^S + Y^K + \epsilon^k) + (1 - q)U'(B^S + Y^K - \epsilon^k)] . \qquad (9)$$

Clearly, under precautionary behavior (i.e., a convex marginal utility function) an increase in $\epsilon^k$ reduces the right-hand side, and in order to maintain maximization we must reduce $c_2$. Reducing $c_2$ implies a reallocation of resources towards $B^S$, i.e., we have obtained an increase in bequests.

A particular case that characterizes the implications of the existence of FGIU for the demand for annuities is obtained when the result is a corner solution with minimal allocation to savings. This case is analyzed in Appendix A.

*d. Altruistic agents and SPIU*

The budget constraints for the parents are:

i)  $Y = c_1 + a + s$

ii)  $Rs = B^D$

iii)  $Rs + Aa + \epsilon^s = B^{SH} + c_{2H}$

iv)  $Rs + Aa - \epsilon^s = B^{SL} + c_{2L}$ ,

where $\epsilon^s$ is the shock to second-period income. The budget constraints of the offspring are:

v)  $B^D + Y^K = c_K^D$

vi)  $B^{SH} + Y^K = c_{KH}^S$

vii)  $B^{SL} + Y^K = c_{KL}^S$ .

Using these budget constraints, the problem is to maximize the following utility function:

$$\text{MAX } U(c_1) + \alpha p U[R(Y-c_1-a)+Y^K]+(1-p)\delta[qU(c_{2H})+(1-q)U(c_{2L})] \quad (10)$$

$$+ (1-p)\alpha\{qU[R(Y-c_1-a)+Aa-c_{2H}+Y^K+\epsilon^s]+(1-q)U[R(Y-c_1-a)+Aa-c_{2L}+Y^K-\epsilon^s]\}$$

The f.o.c.'s are obtained by calculating derivatives with respect to a, $c_1$, $c_{2H}$ and $c_{2L}$:

(i):  $RpU'(c_K^D) = (1-p)(A-R)[qU'(c_{KH}^S) + (1-q)U'(c_{KL}^S S)]$

(ii):  $U'(c_1) = R\alpha p [U'(c_K^D)] + R\alpha(1-p)[qU'(c_{KH}^S) + (1-q)U'(c_{KL}^S)]$  \quad (11)

(iii):  $\delta U'(c_{2H}) = \alpha U'(c_{KH}^S)$

(iv):  $\delta U'(c_{2L}) = \alpha U'(c_{KL}^S)$ .

In order to characterize the solution, assume that $\alpha = \delta$. In this case (assuming that the solution for the low state of nature is interior):[6]

$$c_{KH}^S = c_{2H} = (Rs + Aa + Y^K + \epsilon)/2, \quad B^{SH} = (Rs + Aa - Y^K + \epsilon)/2$$

$$c_{KL}^S = c_{2L} = (Rs + Aa + Y^K - \epsilon)/2, \quad B^{SL} = (Rs + Aa - Y^K - \epsilon)/2$$

$$B^D = Rs .$$

Hence, *the result obtained by Sheshinski and Weiss (1981) does not hold for an interior solution under SPIU*. The intuition is as follows: Under FGIU, it does not matter whether the individual dies or survives — in either case, there are shocks to the offspring's consumption; the introduction of income uncertainty therefore does not change the fact that it is optimal to use annuities for second-period consumption and riskless bonds for bequests. Not so under SPIU: if the individual dies, there are no shocks at all; but if he survives, the shock influences both second-period consumption and the offspring's consumption (through bequests). Since annuities provide better protection against shocks, it is optimal *in the case of survival* to use annuities both for second-period

---

[6] By an 'interior solution' we mean that the equation 11iv holds as an equality. This is true if for $b = 0$, $\delta U'(c_{2L}) < \alpha U'(c_{KL}^S)$, i.e., that there are positive bequests at the optimum.

consumption and for bequests.[7]

An interesting result is obtained when, in the low state of nature, the solution is a corner solution [i.e., for $b = 0$, $\delta U'(c_{2L}) > \alpha U'(c^s_{kL})$]. If the individual survives to the second-period and has low income, it is optimal for him to use all his saved resources (including annuities and riskless bonds) for own consumption, in contrast to the segmentation result above.

## 4. Precautionary Savings and the Demand for Annuities

An empirical application of the model requires explicit discussion of its implications. The first issue relates to the relevance of the altruism model. Since the literature does not seem to have reached a consensus on whether the relevant model should be the egoistic set-up or the altruistic one, we will not discuss this topic in detail.[8] In any case, since it is not clear whether altruistic behavior is strong, we will consider only a range of values for the altruism coefficient that is lower than 1.

The second issue is related to the timing of annuity purchases. According to our model, annuities are purchased under complete lack of private information. In reality, it is not unlikely that individuals acquire private information about their survival probabilities in the course of their lifetime. This problem is less restrictive if, at the optimum, individuals do not change their decisions, as shown by Brugiavini (1994). According to her model, if we take into account the possibility of a reallocation of savings in advanced periods of the life cycle, it can be shown that it is optimal not to change the amount purchased early on in life.

The third issue is the difference between kinds of uncertainty. Assume that the two periods of the model are equal in length: the first period is from age 25 to 50 and the

---

[7] In terms of Sheshinski and Weiss (1981), the covariance between the kid's income and consumption is not zero (in contrast to FGIU, where the covariance *is* zero). This is so because income uncertainty is contingent on being alive.

[8] Some evidence against altruism is provided by Altonji, Hayashi and Kotlikoff (1992); for evidence supporting altruism see Bernheim (1991).

second — from 51 to 75[9]. Then, SPIU includes both an active period of life (50–65) and a passive one (66–75). It is well known that in the passive period of life income uncertainty is very low (see, e.g., Deaton and Paxson, 1994). Thus, it seems that the relevant uncertainty for an empirical application is not *income* uncertainty, but other kinds of uncertainty, such as health-costs uncertainty (Palumbo, 1994). Since the consideration of health-costs uncertainty is beyond the scope of the present paper,[10] we assume (for the purpose of the simulation) that the only relevant uncertainty is FGIU.

With respect to FGIU, Appendix D includes a summary tabulation of the findings of different works on the income-formation process of fathers and sons. The results show that the son's income uncertainty is at least as high as the father's second-period income uncertainty (according to some estimates it is even higher). This result is consistent with the fact that after retirement parents' income depends on their occupation during their active period, whereas the son's income represents an unconditional expected value.[11]

In order to understand the magnitude of income uncertainty, note (as a representative example) the results obtained by Barsky, Mankiw and Zeldes (1986): the coefficient of variation is 0.63 for future generation's income uncertainty, compared with 0.5–0.55 in the case of parents' income uncertainty. These results stress that once we accept the altruistic model as the relevant framework, the future generation's income uncertainty should play an important role in determining the allocation of savings and consumption.

Table 1 shows the results of a simulation of the allocation of savings for different cases of risk aversion and altruism coefficient. The implications of these results are extremely significant in terms of the (non-) emergence of private annuity markets. For a coefficient of altruism of 0.6 and a coefficient of relative risk aversion of 4, sixty percent of savings are allocated to riskless bonds. A look at the results cited by Auerbach, Kotlikoff and Weil (1994, p. 2) according to which in 1988 social security and

---

[9] The choice of the periods corresponds to the life expectancy in developed countries (see appendix C).

[10] As stated by Auerbach, Kotlikoff and Weil (1994), a broader definition of annuities should take health expenses into account.

[11] The last sentence will not be true if the income process is characterized by intergenerational links. In that case, the offspring's expected incomey is conditional on his parents' income.

private pension arrangements represented 55 percent of 'elderly income', supports the hypothesis that *net* (i.e., after deduction of compulsory social security and private pension arrangements) demand for private annuities is quite low. This result suggests that the absence of a private annuity market maybe explained by the low demand for private annuities.[12]

Table 1. Precautionary Behavior and the Demand for Annuities[a]

| | Average propensity to consume | Precautionary premium | Percentage in total savings | |
|---|---|---|---|---|
| | (% of income) | (% of income) | Bonds | Annuities |
| $\theta = 2$ | | | | |
| $\alpha = 0.6$ | 74 | 7.2 | 43 | 57 |
| $\alpha = 0.8$ | 72 | 7.1 | 52 | 48 |
| $\theta = 4$ | | | | |
| $\alpha = 0.6$ | 78 | 10.3 | 60 | 40 |
| $\alpha = 0.8$ | 77 | 10.0 | 64 | 36 |
| $\theta = 6$ | | | | |
| $\alpha = 0.6$ | 78 | 11.5 | 67 | 23 |
| $\alpha = 0.8$ | 78 | 11.0 | 70 | 30 |

[a] The benchmark simulation is based on the following parameters: $\delta = 1$, $q = 0.5$, p=0.8 (using actuarial data as appears in Hubbard, Skinner and Zeldes, 1994) and a coefficient of variation of 0.6 (Barsky, Mankiw and Zeldes, 1986).

## 5. Conclusions and Further Directions of Research

This paper has shown that in the presence of precautionary savings altruistic agents substantially reduce their demand for annuities. According to our simulation, for a coefficient of altruism of 0.6 and coefficients of risk aversion equal to or higher than 4,

---

[12] Auerbach, Kotlikoff and Weil (1994, Table 2, p. 28) show that for men and women aged 50–54 the annuitized share of resources in 1983 was 80 percent.

the *net* demand for annuities (i.e., after correcting for existing compulsory arrangements) is low, in fact — virtually nonexistent.

Further research should concentrate on the empirical validity of the argument. More specifically, it would be useful to assess the impact of future generations' income uncertainty on consumer behavior and to study the distribution of different saving channels as a function of idiosyncratic income uncertainty in the economy.

# APPENDIX A

## A Corner Solution with Minimal Allocation to Savings

Assume a corner solution with a minimal allocation to savings (say, one unit). We ask: will the unit of savings be allocated to annuities or to riskless bonds?

**PROPOSITION A.1:** IF THERE IS NO PRECAUTIONARY BEHAVIOR ($U''' = 0$), INDIVIDUALS ARE INDIFFERENT TO THE ALLOCATION OF THE FIRST UNIT OF SAVINGS BETWEEN RISKLESS BONDS AND ANNUITIES.

**Proof:** See Appendix B.

**PROPOSITION A.2:** GIVEN PRECAUTIONARY BEHAVIOR ($U''' > 0$), INDIVIDUALS STRICTLY PREFER RISKLESS BONDS TO ANNUITIES, I.E., THERE IS NO SEGMENTATION OF SAVINGS, AND *ALTRUISTIC INDIVIDUALS ALLOCATE THEIR (MINIMAL) WEALTH TO RISKLESS BONDS.*

**Proof:** We must prove that if we have a corner solution with a one unit of savings to allocate, the allocation will be to riskless bonds and not to annuities.

The first step is to characterize the corner solution by noting that in a corner solution f.o.c. (6.i) and (6.ii) are satisfied as inequalities. One unit of resources allocated to first-period consumption yields $U'(c_1)$. One unit allocated to savings yields $R[\delta p EU'(\theta_1) + \alpha EU'(\theta_2)]$, where $\theta_i$ (i = 1, 2) are the optimal shares of second-period consumption and kid's consumption [according to equation (6.iii)]. Since it is usually accepted that the altruism coefficient and the rate of time preference are such that agents prefer first-period consumption in the allocation of the first unit of resources, we assume that the values of R, $\delta$ and $\alpha$ are such that the corner solution is obtained when $U'(c_1)$ is higher than the marginal utility of savings, i.e., when $U'(c_1) > R[\delta p EU'(\theta_1) + \alpha EU'(\theta_2)]$.

Without loss of generality, assume that income Y (equal to 2 units) is low enough to allow — at the optimum — a one-unit allocation to one of the two available saving

assets.[13] This means that in the case of a corner solution one unit is allocated to first-period consumption and one unit is allocated to savings. We must prove that it is optimal to allocate the latter unit to riskless bonds rather than to annuities (i.e., $s = 1$ and $a = 0$).

We simply need to look at the allocation of the first unit of savings. Assuming $\alpha = \delta$ (for simplicity), by applying equation (6.iii) if we allocate the unit of savings to riskless bonds, the right-hand side is:

$$p[qU'(R + Y^K + \epsilon^k) + (1 - q)U'(R + Y^K - \epsilon^k)] +$$

$$(1 - p)\left[qU'\frac{R + Y^K + \epsilon^k}{2} + (1 - q)U'\frac{R + Y^K - \epsilon^k}{2}\right] .$$

If we allocate the unit to annuities, the right-hand side is:

$$(1 - p)\left[qU'\frac{Y^K + A + \epsilon^k}{2} + (1 - q)U'\frac{Y^K + A - \epsilon^k}{2}\right] .$$

The comparison is between a higher return in the case of survival against a lower return in both states of nature, survival and demise. Under the assumption of precautionary behavior, the expected marginal utility obtained from the higher return to annuities (in the case of survival) is lower than the expected marginal utility obtained through riskless bonds. The point is illustrated in Figure 1: if an individual allocates the unit to riskless bonds, his marginal utility is substantially reduced regardless of whether he survives or not. But if the unit is allocated to annuities, the additional return $(A - R)$ lowers marginal utility by less than the reduction that occurs in the case of riskless bonds. Clearly, the result is obtained as a consequence of the convexity of marginal utility, which is the

---

[13] A rationale for this assumption could be the existence of a fixed operational cost for each transaction on different assets.

condition for precautionary behavior.[14]

## FIGURE 1



Survival          Demise

Riskless bond

Actuarially fair annuity

---
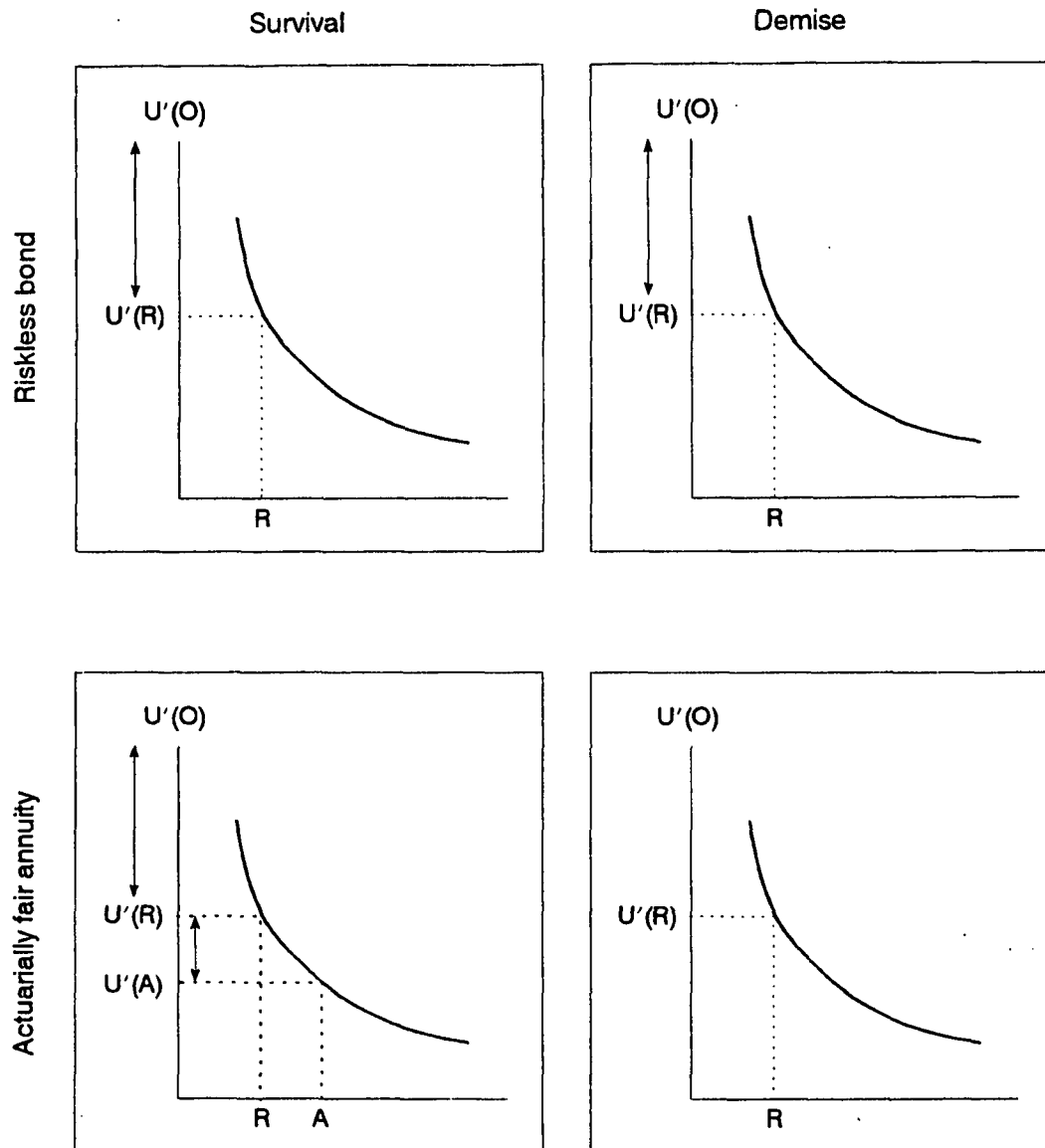
[14] Note that if the $U'$ were linear, the reduction in expected marginal utility would be the same, since $p(A-R) = (1-p)R$; see Appendix B.

## APPENDIX B

## The Case of Linear Marginal Utility

Assume a linear marginal utility function:

$$U' = K + k \times (\text{resources}) . \tag{B.1}$$

If the consumer allocates one dollar to riskless bonds (making the same assumptions as in proposition 4), marginal utility is:

$$p\{q[K + k(R + Y^K + \epsilon^k)] + (1 - q)[K + k(R + Y^K - \epsilon^k)]\} +$$

$$(1-p)\left\{q\left[K + k\frac{R + Y^K + \epsilon^k}{2}\right] + (1 - q)\left[K + k\frac{R + Y^K - \epsilon^k}{2}\right]\right\} \tag{B.2}$$

$$= K + k(R + Y^K) .$$

If the consumer allocates one dollar to annuities, marginal utility is:

$$(1 - p)\left[q\left[K + k\frac{Y^K + A + \epsilon^k}{2}\right] + (1 - q)\left[K + k\frac{Y^K + A - \epsilon^k}{2}\right]\right]$$

$$+ p(K + kY^K) \tag{B.3}$$

$$= K + kY^K + k(1 - p)A .$$

Recall that actuarially fair annuities imply that $A = R/(1 - p)$, so that the last expression becomes:

$$K + k(R + Y^K) , \tag{B.3'}$$

which is equal to the marginal utility of riskless bonds as shown in (B.2).

APPENDIX C

**Life Expectancy**

Table C.1 summarizes the statistics of life expectancy in 1992, according to the *World Bank Atlas*.

Table C.1

|  | Number of economies | GNP per capita (US$) | Population |
|---|---|---|---|
| Less than 55 | 39 | 280 | 611 |
| 55-64 | 24 | 510 | 1,468 |
| 65-69 | 39 | 1,930 | 846 |
| 70-72 | 41 | 1,110 | 1,614 |
| 73 or more | 51 | 20,590 | 900 |
| No data | 13 | 1,510 | 4 |

# APPENDIX D

## Evidence on Income Uncertainty

Table D.1 present results on income uncertainty according to different studies.

Table D.1. Evidence on Income Uncertainty of Fathers and Sons

| | Coefficient of variation | | Standard deviation of log income | | Sample, Survey |
|---|---|---|---|---|---|
| | Father | Son | Father | Son | |
| Jencks (1972) | | | | | |
| Unconditional[a] | | 0.65 | | | 37 mill., census |
| Conditional[a] | | 0.63 | | | |
| Behrman and Taubman (1985)[b] | 0.81 | 1.12 | | | 3,768, Twin sample |
| Barsky, Mankiw and Zeldes (1986)[c] | 0.5-0.55 | 0.63 | | | PSID and Census |
| Solon, Corcoran, Gordon and Laren (1991) | | | | 0.591 | 854, PSID |
| Solon (1992) | 0.68 | 0.67 | | | 322, PSID |
| Zimmerman (1992)[d] | | | | | |
| Earnings | | | 0.418 | 0.502 | 192, NLS |
| Wages | | | 0.412 | 0.406 | 188, NLS |

Notes:
(i) Some of the figures were calculated using the estimates reported in the cited works.
(ii) PSID — Panel Study of Income Dynamics; NLS — National Longitudinal Survey.

[a] Based on 1968 full-time year-round annual earnings of male workers. The unconditional coefficient of variation for all workers is 0.72.

[b] Based on 1980 yearly earnings (families with reported offspring).

[c] Father's estimate was calculated using Hall and Mishkin (1982) data. Son's estimates are according to Jencks (1972).

[d] Based on four-year average of father's earnings (Table 6, p. 421).

**Bibliography**

Abel, A. B. (1986). "Capital Accumulation and Uncertain Lifetimes with Adverse Selection," *Econometrica*, 54 (No. 5, September): 1079-97.

Auerbach, A., L. Kotlikoff and D. Weil (1994). "The Increasing Annuitization of the Elderly — Estimates and Implications for Intergenerational Transfers, Inequality and National Saving," mimeo.

Altonji, J. H., F. Hayashi and L. Kotlikoff (1992). "Is the Extended Family Altruistically Linked? Direct Tests Using Micro Data," *American Economic Review*, 82 (5): 1177-98.

Barro, R. J. (1974). "Are Government Bonds Net Wealth?" *Journal of Political Economy*, 82 (6): 1095-1117.

Barsky, R., G. Mankiw and S. Zeldes (1986). "Ricardian Consumers with Keynesian Propensities," *American Economic Review*, 76: 676-691.

Behrman, Jere R. and P. Taubman (1985). "Intergenerational Earnings Mobility in the United States: Some Estimates and a Test of Becker's Intergenerational Endowments Model," *Review of Economics and Statistics*, 67 (No. 1, February): 144-151.

Bernheim, D. (1991). "How Strong are Bequests Motives? Evidence Based on Estimates of the Demand for Life Insurance and Annuities," *Journal of Political Economy*, 99: 899-927.

Brugiavini, A. (1994). "Uncertainty Resolution and the Timing of Annuity Purchases," *Journal of Public Economics*, 50 (1): 31-62.

Deaton, A. and C. Paxson (1994). "Intertemporal Choice and Inequality," *Journal of Political Economy*, 102 (3): 437-467.

Eckstein, Z., M. Eichenbaum and D. Peled (1985). "Uncertain Lifetimes and the Welfare Enhancing Properties of Annuity Markets and Social Security," *Journal of Public Economics*, 26: 303-326.

Friedman, B. M. and M. J. Warshawsky (1990). "The Cost of Annuities: Implications for Saving Behavior and Bequests," *Quarterly Journal of Economics*, 105 (No. 1, February): 135-154.

Hall, R. E. and F. S. Mishkin (1982). "The Sensitivity of Consumption to Transitory Income: Estimates from Panel Data on Households," *Econometrica*, 50 (No. 2, March): 461–481.

Hubbard, R. G., J. Skinner and S. P. Zeldes (1994). "The Importance of Precautionary Motives in Explaining Individual and Aggregate Saving," *Carnegie-Rochester Conference Series on Public Policy*, 40 (June): 59–125.

Jencks, C. (1972). *Inequality — A Reassessment of the Effect of Family and Schooling in America*. Basic Books.

Kotlikoff, L. J. and A. Spivak (1981). "The Family as an Incomplete Annuities Market," *Journal of Political Economy*, 89 (No. 2, April): 372–391.

Leland, H. (1968). "Saving and Uncertainty: The Precautionary Demand for Saving," *Quarterly Journal of Economics*, 82: 465–473.

Palumbo, M. (1994). "Uncertain Medical Expenses and Precautionary Saving Near the End of the Life Cycle." University of Houston, Manuscript (November).

Sheshinski, E. and Y. Weiss (1981). "Uncertainty and Optimal Social Security Systems," *Quarterly Journal of Economics*, 96 (No. 2, May): 189–206.

Solon, G. (1992). "Intergenerational Income Mobility in the U.S.," *American Economic Review*, 82 (No. 3, June): 393–408.

———, M. Corcoran, R. Gordon and D. Laren (1991). "A Longitudinal Analysis of Sibling Correlations in Economic Status," *Journal of Human Resources*, 26(3): 509–534.

Strawczynski, M. (1993). "Income Uncertainty, Bequests and Annuities," *Economics Letters*, 42: 155–158.

——— (1994). "Government Intervention as a Bequest Substitute," *Journal of Public Economics*, 53: 477–495.

——— (1995). "Income Uncertainty and Ricardian Equivalence," *American Economic Review*, 85 (No. 4, September): 964–967.

Zimmerman, D. J. (1992). "Regression Toward Mediocrity in Economic Stature," *American Economic Review*, 82 (No. 3, June): 409–429.

- 22 -

## רשימת המאמרים בסדרה

R. Melnick and Y. Golan - Measurement of Business Fluctuations in Israel.   91.01

91.02   י. ארטשטיין, צ. זוסמן – דינאמיקה של עליות שכר בישראל: כוחות שוק
והשוראות בינענפיות.

M. Sokoler - Seigniorage and Real Rates of Return in a Banking Economy.   91.03

E.K. Offenbacher - Tax Smoothing and Tests of Ricardian Equivalence:   91.04
Israel 1961-1988.

91.05   ג. עופר, ק. פלוג, נ. (קלינר) קסיר, – קליטה בתעסוקה של עולי בריה"מ בשנת
1990 והלאה: היבטים של שמירה והחלפת משלחי יד.

91.06   צ. זוסמן, ד. זכאי, – פערים בין בכירים וזוטרים ומשברים במערכת ציבורית:
שכר הרופאים בשנים 1974 עד 1990.

M. Beenstock, Y. Lavi and S. Ribon - The Supply and Demand for Exports   91.07
in Israel.

R. Ablin - The Current Recession and Steps Required for Sustained   91.08
Sustained Recovery and Growth.

91.09   צ. הרקוביץ, ל. (רובין) מרידור – ההשלכות המקרו-כלכליות של עלייה המונית
לישראל.

M. Beenstock - Business Sector Production in the Short and Long Run in   91.10
Israel: A Cointegrated Analysis.

91.11   א. ארנון, ר. עמיחי, – ההפרטה וגבולותיה.

91.12   ק. פלוג, נ. קסיר (קלינר) – עלות העבודה בתעשייה הישראלית.

A. Marom - The Black-Market Dollar Premium: The Case of Israel.   91.13

A. Bar-Ilan and A. Levy - Endogenous and Exogenous Restrictions on   91.14
Search for Employment.

M. Beentstock and S. Ribon- The Market for Labor in Israel.      91.15

91.16    ד. אלקיים, – השפעת המדיניות המוניטרית על פער הריביות במגזר השקלי הלא
צמוד 1986 עד 1990.

92.01    מ. דהן , – בחינת מדד הדחף הפיסקאלי של ה-IMF עבור המשק הישראלי לשנים
1964 עד 1990.

O. Bar Efrat - Interest Rate Determination and Liberalization of      92.02
     International Capital Movement: Israel 1973 - 1990.

Z. Sussman and D. Zakai - Wage Gaps between Senior and Junior      92.03
     Physicians and Crises in Public Health in Israel, 1974-1990.

92.04    צ. ויס, ע. לויתן – התפתחות תשלומי העברה בישראל, 1965 עד 1989.

O. Liviatan - The Impact of Real Shocks on Fiscal Redistribution and      92.05
     Their Long-Term Aftermath.

A. Bregman, M. Fuss and H. Regev - The Production and Cost Structure      92.06
     of the Israeli Industry: Evidence from Individual FiRm Data.

M. Beenstock, Y. Lavi and A. Offenbacher - A Macroeconometric Model      92.07
     for Israel 1962-1990: A Market Equilibrium Approach to
     Aggregate Demand and Supply.

92.08    ס. ריבון , – מודל חודשי לשוק הכסף.

R. Melnick - Financial Services, Cointegration and the Demand for      92.09
     Money in Israel.

92.10    מ. ברון , – העליות לארץ והשפעתן על הפסיפס הדמוגרפי של האוכלוסייה והן
ההון האנושי.

92.11    ד. זינגר, – גורמים הקובעים את ההסתברות של פירמות להיסגר.

R. Melnick - Forecasting Short-Run Business Fluctuations in Israel.      92.12

K. Flug, N. Kasir and G. Ofer - The Absorption of Soviet Immigrants    92.13
     into the Labor Market from 1990 Onwards: Aspects of
     Occupational Substitution and Retention.

א. ארנון, ח. פרשטמן, - הפרטת מונופולים טבעיים: הריצה אחר הבלתי מוכח.    92.14

B. Eden - How to Subsidize Education and Achieve Voluntary Integration:    93.01
     An Analysis of Voucher Systems.

א. ברגמן, א. מרום, - גורמי צמיחה בסקטור העסקי בישראל (1958 עד 1988).    93.02

מ. דהן, - צמיחה כלכלית תחת איום ביטחוני.    93.03

ק. פלוג, נ. (קלינר)קסיר - קליטה בתעסוקה של עולי חבר המדינות - הטווח הקצר.    93.04

מ. דהן, - האם קיימת יריבות בין שיויון בחלוקת ההכנסות להתפתחות כלכלית?    93.05
     המקרה של ישראל.

צ. הרקוביץ, ל. מרידור - ההשלכות המקרו-כלכליות של עלייה המונית לישראל:    93.06
     עדכון ובחינה מחודשת.

A. Arnon, D. Gottlieb - An Economic Analysis of the Palestinian Economy:    93.07
     The West Bank and Gaza, 1968-1991.

צ. הרקוביץ, ל. מרידור, נ. קנטור - הגירה וצמיחה בתנאים של ניידות הון    93.08
     בלתי משוכללת: גל העלייה לישראל בראשית שנות התשעים.

K. Flug, N. Kasir - The Absorption in the Labor Market of Immigrants    93.09
     from the CIS - the Short Run.

R. Ablin - Exchange Rate Systems, Incomes Policy and Stabilization    94.01
     Some Short and Long-Run Considerations.

B .Eden - The Adjustment of Prices to Monetary Shocks When Trade is    94.02
     Uncertain and Sequential.