# MIND THE GAP: STRUCTURAL AND NONSTRUCTURAL APPROACHES TO ESTIMATING ISRAEL'S OUTPUT GAP

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This paper reviews various methods of estimating the output gap, and applies two of them to Israel's economy—the production-function method, and estimating structural vector autoregression (SVAR), both structural methods. The productionfunction method focuses on dividing factors of production into a trend and a cyclical component, and also breaks down the Solow residual into productivity and capital utilization. The SVAR method also breaks output down into trend and cyclical components, via long-term constraints. As there are various definitions of potential output (that which does not produce inflationary pressures, and long-term output), the paper shows how it can be measured in accordance with the chosen definition. An estimate of the Phillips curve equation yields the result that estimates of the output gap by both methods have a positive effect, consistent with economic theory, on price volatility. The results of the estimate give rise to several conclusions: (1) the annual rate of growth of potential output in the second half of the 1990s declined by about one percentage point from the rate in the first half. (2) Estimates of the output gap including start-ups do not differ significantly from estimates excluding them. This result must be treated with caution, however, due to the small number of observations with data on start-ups. (3) It is clear that the business cycle at the beginning of the 1990s derived mainly from supply shocks (in particular the influx of immigrants), while the recession that started in 1996 was due to demand shocks.

## 1. INTRODUCTION

The output gap is an important index used by policymakers in general and central banks in particular. It is therefore important to find reliable estimates of the output gap that will help, for example, in the determination of monetary policy, taking the output gap as an indication of inflationary pressures. The output gap is defined as the difference between actual businesssector product and (unobserved) potential output. Thus, potential output must be defined at the outset. Okun (1962) defines it as output that can be produced under full employment (long-term output). Another widespread approach in economic literature, and on which De Masi (1997) concentrates, views potential output as the maximum that can be produced without causing inflationary pressures. These differing definitions lead to the situation in which the

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output gap derived from the latter definition constitutes an indication of inflationary pressures, whereas the former definition expresses business cycles in terms of long-term business-sector product.

In addition to the different definitions of potential output and hence the different output gaps derived from them, there are several approaches to the analysis of the output gap, and these can sometimes lead to different conclusions regarding policy. One example of this is the difference between the approach that considers that potential output reflects the long-term trend of actual business-sector product completely independent of policy measures, and that the only effect that policy measures have is to reduce the deviations of that product from the trend, and the approach based on the assumption that policy can increase potential output. In this context it is important to consider the time scale in measuring the output gap, as if, for example, a positive shock causes a surge in economic activity and inflationary pressures in the short run, in the longer run investments may increase, thereby raising potential output and not only actual business-sector product.

One of the main approaches currently to the explanation of the output gap is that which assumes that permanent long-term shocks affect potential output, while temporary shocks affect only the output gap. Demand-side shocks may be viewed as affecting actual businesssector product, and thereby the output gap, and supply-side shocks as affecting potential output. The distinction between the different shocks and their effects on the economic variables, however, is in many cases not clear cut.

This paper will show quarterly estimates of the output gap in Israel (based on the two definitions given above) from the second half of the 1980s until 2001, obtained by using three different methods of estimating, including structural approaches reflecting economic relationships: (1) measuring the output by means of the production function (PF); (2) estimating the SVAR by Blanchard and Quah's (1989) model that imposes long-term constraints on the estimate, and (3) a purely statistical approach using the Hodrick-Prescott (HP) filter.

The estimates of the output gap refer to business-sector product excluding start-ups. In any case, a parallel empirical examination of output-gap estimates including the contribution of start-ups yielded similar results. Specifically, estimates of output gap are not sensitive to the removal of start-ups (see Appendix 3).

The paper is organized as follows: Part 2 presents a brief review of a range of methods of estimating the output gap that can be found in the relevant economic literature, with reference to the three most widespread approaches to estimation in that literature. Part 3 measures the output gap in Israel using the production-function approach, along the lines of Menashe and Mealem (2000), but unlike their study, the current one includes a measurement of capital utilization. Part 4 deals with the estimate of the output gap using the SVAR model, based on Blanchard and Quah's (1989) model that imposes long-term constraints on the estimate. In Parts 3 and 4 the estimate is based on the definition of the output gap as an indicator of inflationary pressures. Part 5 compares the estimates of the output gap using the two methods above and the estimate of the trend using the HP filter. In the comparison of the various estimates, several empirical criteria are examined to establish which estimates give the best results, for instance the correlation between the various estimates of the output gap and their use in estimating the Phillips curve. Part 6 describes the difference between potential output that does not create inflationary pressures and long-term business-sector product for the two structural estimation methods. Part 7 summarizes and concludes.

## MIND THE GAP: ESTIMATING ISRAEL'S OUTPUT GAP 81

# 2. METHODS OF MEASURING THE OUTPUT GAP: THE CURRENT SITUATION

The different interpretations of and approaches to the output gap led to a wide variety of methods for measuring it. These can by classified into three main groups: direct measurement via surveys of companies' activity, statistical (nonstructural) methods, and structural methods based on economic theory. Each method has advantages and disadvantages, described below.

• *Direct measurement of the output gap*: in the short term production technology is a given, and there is a supply-side constraint deriving from the factors of production —capital stock or the labor force. In this situation the output gap can be measured directly by using data obtained from surveys of companies' capital utilization and comparing the data received with a certain critical value, generally the average utilization during the sample period. The drawbacks of this method are: the data on utilization generally relate to only some of the companies in the economy (mainly manufacturing companies); the definition of utilization in the survey are naturally subjective and problematic; and there is no exact critical threshold for utilization.

• *Nonstructural methods of measuring the output gap*: this group includes all the methods of estimation based on particular statistical procedures and not on specific economic theory, such as those that measure potential output as a simple linear trend or as a flexible smoothed trend (the HP filter) of data on actual business-sector product. Another method, developed by Baxter and King (1995) and called the Band Pass Filter, views the business cycle as a situation in which business-sector product is either below or above the trend for a minimum number of consecutive quarters, and the output gap is not calculated until those periods have passed. The advantage of these methods is that they do not require much information, and they can be applied even when only one data series is known. The major disadvantage of these methods is their inability to distinguish between supply shocks and demand shocks, and in the absence of an economic framework, there is no reference to the different effects of different shocks on potential output. The distinction between structural and nonstructural methods is not unequivocal, however, as in many instances structural methods, for example measuring business-sector product via the production function, employ trends and statistical methods to measure some of the components of the output gap. Nevertheless, the case can be argued on principle that a distinction should be drawn between the two systems of estimation.

• *Structural methods for measuring the output gap*: this group assumes that a particular theory exists that describes economic conduct properly. Two central approaches may be singled out: one based on statistical estimation of several series, on the assumption of long- and shortterm relations derived from economic theory (SVAR), and the other based on measuring the output gap by means of a cumulative production function. The estimation of the output gap via SVAR is based on the work of Blanchard and Quah (1989). The underlying assumption is that demand-side shocks are neutral in the long term. In that sense it is supply-side shocks and a deterministic component of the model that determine potential output. The productionfunction approach for calculating the output gap and potential output require relatively much information: an assumption about production technology, labor-market data on labor input, unemployment rates and the participation rate, as well as capital stock and total productivity. Another problem in using this approach is the exogenous assumption of the unobserved natural unemployment rate. There are two other methods of estimation in the group of structural models in addition to the above: (1) estimation by multivariate unobserved component models, based on a number of equations that create a certain economic structure and that are estimated

together with unobserved variables. The first study in which this method was used to estimate potential output was that of Kuttner (1994), which was extended by Apel and Jansson (1999), who presented a model that incorporated, in addition to the stochastic process of the unobserved variables (the NAIRU (non-accelerating-inflation rate of unemployment) and potential output), also the equations of the Phillips curve that relates inflation to the output gap and the unemployment gap (the gap between the actual rate of unemployment and the NAIRU) and another equation that relates these two gaps in accordance with Okun's Law. (2) The multivariate HP filter: this approach estimates potential output as that which would yield a minimum of the loss function that includes two main elements—the data of the smoothed function of actual business-sector product as given by the estimate of the standard HP filter, and a second element that gives a minimum for the squares of the sums of the errors of several economic equations such as the Phillips curve and the relation between the output gap and the unemployment gap, in which the output gap is the explanatory variable.

## 3. MEASURING THE OUTPUT GAP VIA THE PRODUCTION FUNCTION

One of the major structural approaches to the measurement of the output gap is that of the production function (PF). This method is simple to apply, and it has been the subject of previous studies of Israel's economy: Hercowitz and Bar Gil (1997), Djivre and Ribon (2000), and Mealem and Menashe (2000). In this section the authors have adopted the measurement of the business-sector output gap using the cumulative PF as described in detail in the study by Mealem and Menashe (2000), incorporating an update and the addition of data measuring capital utilization in the sample period, 1986:I to 2001:I (seasonally adjusted quarterly data). Adding the index of capital utilization enables the two components of the deviation of the Solow residual to be separated: the deviation of total factor productivity (TFP) and the deviation of capital utilization.

The production function is of the Cobb-Douglas type with constant returns to scale and business-sector product of  $(Y_t)t$  at time *t* dependent of total factor productivity of  $(A_t)$  and production inputs of capital  $(K<sub>i</sub>)$  and of labor  $(L<sub>i</sub>)$ .

$$
Y_t = A_t (Z_t K_t)^{\alpha} L_t^{1-\alpha}
$$

where  $(Z<sub>i</sub>)$  represents capital utilization.

Furthermore, it is assumed that the markets are competitive, so that capital elasticity a and labor elasticity 1–a are measured by the share of capital and labor in business-sector product. Set  $a = 0.32$ , which is the common share of capital in business-sector product, and the production function can be shown in log terms as:

$$
\log Y_t = \log A_t + 0.32 \log Z_t + 0.32 \log K_t + 0.68 \log L_t.
$$

 $(3.1)$   $y_t = a_t + 0.32z_t + 0.32k_t + 0.68l_t$ .

Potential output is defined as:

$$
\log Y_t^P = \log A_t^P + 0.32 \log Z_t^P + 0.32 \log K_t^P + 0.68 \log L_t^P.
$$

$$
(3.2) \t y_t^P = a_t^P + 0.32z_t^P + 0.32k_t^P + 0.68l_t^P.
$$

The deviation from potential output (in log terms) is given by the equation:

$$
(3.3) \t y_t^C = y_t - y_t^P = a_t - a_t^P + 0.32(z_t - z_t^P) + 0.32(k_t - k_t^P) + 0.68(l_t - l_t^P).
$$

It is assumed that total capital stock is potentially available for companies' use (so that  $k_{t} = k_{t}^{P}$ ), and thus every deviation from potential output based on capital derives from its non-utilization,<sup>1</sup> so that:

$$
(3.4) \t y_t^C = a_t^C + 0.32z_t^C + 0.68l_t^C.
$$

Similarly, it is assumed that capital utilization in the potential output is normal, i.e.,

$$
z_t^C = z_t - z_t^P = z_t - \log(Z_t^P = 1) = z_t.
$$

Total productivity reflects the supply side, so that after deducting capital utilization, the cyclical share of productivity [6] does not need to be included in the output gap under its definition as an indicator of inflationary pressures. Moreover, the cyclical share of productivity in the sample shows that it behaves as a white noise (with no cycle). Thus:

$$
(3.5) \quad gap_t = 0.32z_t^C + 0.68l_t^C.
$$

In the study by Mealem and Menashe (2000) there is a detailed discussion of the cyclical share of labor input  $l_t^C$  that describes the gap between actual labor input and potential input due to the differences between unemployment rates, participation rates, and utilization of labor input; at the same time elasticities to changes that took place in the labor market in the 1990s are analyzed (the influx of immigrants, the rise in the share of foreign workers in the labor force, and the possible rise in the natural unemployment rate). On the other hand, the components of the Solow residual (total factor productivity and capital utilization) are not separated. This separation is essential for the measurement of the output gap as an indication of inflationary pressures, because as mentioned above the productivity component expresses a supply shock, and it must be removed from the cyclical element of business-sector product as derived from equation (3.4). The resolution into the different elements is carried out by estimating the utilization, with productivity obtained from the difference between utilization and the Solow residual.

Utilization is estimated by means of a quarterly index (monthly average) of electricity consumption in business-sector manufacturing (CU) divided by business-sector equipmentcapital stock. Mealem and Strawczynski (1998) showed that this index is correlated with the business cycle, and Marom and Bergman (1998) showed that it is correlated also with manufacturing data.

<sup>1</sup> Alternatively it was assumed that the potential gross capital stock is the actual long-term capital stock measured by the HP filter, but the deviation of the capital stock was minimal and hardly changed the estimates of the output gap.

Capital utilization is measured by the log of the index of electricity consumption in the business sector. As the business-sector output gap (without correcting for capital utilization) in 1995 was equal to zero on average (Djivre and Ribon, 2000, and Mealem and Menashe, 2000 came up with a similar finding), it may be stated that the index of electricity consumption averaged 100 in 1995. It must be emphasized that for purposes of calculating the quarterly capital stock it was assumed that the increase was uniform over the year. GDP and gross capital stock are expressed at constant prices, seasonally adjusted (NIS million, at 1995 prices).

Figure 1 shows the business-sector output gap without start-ups, and the contribution to it made by the deviation of labor input and by capital utilization (the data are in terms of percent of GDP in the period from 1986:I to 2001:I).

# 4. ESTIMATING THE OUTPUT GAP USING SVAR

The estimation applies an econometric method that uses SVAR as a general framework for resolving variables into their permanent and transitory components. It should be stressed that the approach is an econometric one in which no specific economic model plays a central part. More than one economic model could fit the bill, and Appendix 2 presents one such model. The method was first shown by Blanchard and Quah (1989). The current estimation follows in their footsteps, using data on Israel's economy.

The concept underlying their method is that various economic variables are affected by common permanent and transitory







shocks, i.e., all shocks in the economy affect all the variables. The influx of immigrants, for example, or the collapse of the NASDAQ, the outbreak of the *intifada*, changes of government, and similar shocks experienced by the economy affect the whole economic system and not just one specific economic variable. Nevertheless, a given shock will have a different type and intensity of impact on different economic variables. This leads to the observation that the random disturbances in every regression are in effect a function of the same shocks to the economy. The problem therefore is to break the random disturbance in the regression equation down into its permanent and transitory components. If this can be done for the equation of business-sector product, it will enable the business cycle to be identified by means of the transitory element.

The resolution is performed via the long-term constraints on the transitory disturbances. The statistical model imposes the condition that their effects on business-sector product disappear in the long term. The following section describes in detail the method of resolving the disturbance into its distinct elements.

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## **The econometric methodology**

Assume that the following autoregressive relation (AR) exists:

$$
(4.1) \tX_t = \Phi_p(L)X_t + \varepsilon_t,
$$

where  $X<sub>i</sub>$  is the vector  $mxl$  of stationary covariance variables with expected value zero. Assume that  $m = 2$  so that the first variable is business-sector product and the second variable includes the information on its breakdown into two elements.

Φ*p (L)* is a polynomial of degree *p* in the lag operator.

$$
\Phi_p(L) = \sum_{j=1}^p \phi_j L^j.
$$

The  $\phi_j$  are coefficient matrices of order 2 x 2.

 $\varepsilon$ <sub>t</sub> is a 2 x 1 vector of random disturbances that satisfies

$$
E(\varepsilon_{t}) = 0 \t E(\varepsilon_{t}\varepsilon_{t}) = \Sigma \t \forall t
$$
  

$$
E(\varepsilon_{t}\varepsilon_{s}) = 0 \t \forall t \neq s
$$
  

$$
E(\varepsilon_{t}X_{t-j}) = 0 \t \forall j > 0.
$$

Equation (4.1) defines the VAR model. As the same explanatory variables appear in all the equations, the OLS estimate will yield consistent and efficient results. After the estimation, estimates of the matrices and of the random disturbances will be available.

Under the assumptions of equation  $(4.1)$ ,  $X<sub>t</sub>$  may be shown as the moving average (MA) of the random disturbances (called the Wold Representation):

$$
(4.2) \qquad X_t = C(L)\varepsilon_t = \sum_{j=0}^{\infty} c_j \varepsilon_{t-j} ,
$$

where the  $c_j$  are matrices of order 2 x 2. Estimates of these matrices may be found via the  $\phi_j$ of equation  $(4.1)^2$ 

 $\varepsilon$ <sub>i</sub> is the disturbance from the AR, every term of which contains permanent and transitory components, and the aim is to resolve the  $\varepsilon$ <sub>*t*</sub> into these components. Let  $v$ <sub>*t*</sub> represent the resolved disturbance, i.e., it is the 2 x 1 vector of disturbances each term of which represents a permanent or transitory disturbance. To perform the resolution use will be made of the property that the presentation of the MA is not unique.

$$
(4.3) \qquad X_{t} = C(L)\varepsilon_{t} = \underbrace{C(L)\mathcal{Q}\mathcal{Q}^{-1}\varepsilon_{t}}_{S(L)} = S(L)v_{t} \quad \Leftrightarrow \quad \sum_{j=0}^{\infty} c_{j}\varepsilon_{t-j} = \sum_{j=0}^{\infty} s_{j}v_{t-j},
$$

where  $Q$  is the inverse matrix of coefficients of order  $2 \times 2$ , so that

 $E(v_t) = 0, E(v_t v_t) = I, E(v_t v_s) = 0 \quad \forall t \neq s.$ 

The assumption  $E(v_i v_i') = I$  derives from the fact that the variance of  $v_i$  must be a diagonal matrix, or, in other words, the permanent shocks are not correlated with the transitory ones. The comparison to the unitary matrix is only a normalization that will be reflected in the terms of the matrix *Q.*

Equation 94.30 shows that a matrix  $Q$  must be found that will express the  $\varepsilon_i$  in terms of  $v_i$ , as  $\varepsilon_i = Qv_i$ . This equality and equation (4.3) also show that  $c_j Q = s_j$  (derived by substituting  $\varepsilon_{t-j} = Qv_{t-j}$  in (4.3).

The presentation of MA in equation (4.2) shows, however, that  $c_0 = I$  (Appendix 1 shows the calculation that yields this result), so that  $Q = s_0$ . This leads to the following equations:

$$
(4.4) \qquad \varepsilon_{t} = s_{0}v_{t}.
$$

$$
(4.5) \qquad s_j = c_j s_0.
$$

Equation (4.40 shows that knowing  $s_0$  enables the  $\varepsilon$ <sub>*t*</sub> to be broken down into its permanent and transitory elements, and equations (4.3) and (4.5) show that knowing  $s_0$  enables  $X<sub>t</sub>$  to be broken down, since for that resolution the correct weights (the  $s_j$ ) must be assigned to the  $v_i$ disturbances. Hence, at this point attention must be focused on identifying the  $s_0$  matrix.

The matrix  $s_0$ :  $s_0$  is a 2 x 2 matrix, so that four parameters must be identified. Equation (4.4) yields the following:

$$
Var(\varepsilon_t) = Var(s_0v_t) = s_0Var(v_t)s_0 = s_0s_0,
$$

so that

(4.6)  $\Sigma = s_0 s_0'$ .

<sup>&</sup>lt;sup>2</sup> The method of calculation is shown in Appendix 1.

But *Var*( $\varepsilon$ <sub>*t*</sub>) =  $\Sigma$  was estimated in equation (4.1).  $\Sigma$  is a symmetrical matrix, so that equation (4.6) gives 3 equations with 4 unknowns. In other words, another constraint is required so that the  $s_0$  can be identified.<sup>3</sup>

Till now  $v_t$  has been considered a vector of permanent and transitory disturbances, but the model has not yet imposed any condition that makes them into such. To make this distinction apply, long-term constraints must be set on the  $s_0$  so that the resolution into permanent and transitory disturbances will be reflected in the *S*(1).

Before imposing the constraints, one must clarify the significance of a transitory shock in the given context. Clearly the effect of a transitory shock is expected to disappear in the long run, i.e.,

$$
\lim_{j\to\infty} s_j=0\,,
$$

as  $s_j$  is the effect of a shock applied today to  $X_{t+j}$ . However, as  $X_t$  is a vector of stationary variables, it is clear that this applies, and it need not be imposed on the system, so that a stronger constraint is required. Define

$$
S(1)=\sum_{j=0}^{\infty}S_j.
$$

*S*(1) is the total effect of  $v_t$ , the series  $\left\{X_{t+j}\right\}_{j=0}^{\infty}$ . Imposing the constraint that certain terms in  $S(1)$  are equal to zero means that the total effect of the appropriate terms in  $v<sub>t</sub>$  on part of the terms of *X* offset each other over time. If the appropriate terms in  $X_t$  are the first order differential of a particular variable, the constraint requires the variable to return to its original level in the long run. So that this constraint turns the appropriate terms in  $v<sub>t</sub>$  into transitory shocks with respect to that variable.

*S*(1) is a 2 x 2 matrix, so that one of its terms may be made equal to zero (clearly the location of the zeros in *S*(1) must be determined with discretion according to the variables constituting  $X_i$ . These constraints may now be used to obtain the missing equations required for the identification.

Bearing in mind that according to equation (4.5)  $s_j = c_j s_0$ , so that summing over *j* gives:

$$
(4.7) \t s(1) = c(1)s_0.
$$

The matrix  $C(1)$  is obtained from the estimate of the  $c_j$ , so that setting one of the terms of  $c(1)s_0$  equal to zero (in accordance with the location of the zeros in  $S(1)$  will provide the missing identification equations for  $s_0$ .

To summarize: identifying  $s_0$  enables  $v_t$  to be identified by means of (4.4). Estimating the  $c_j$ , together with  $s_0$  enables the  $s_j$  to be identified via (4.5). Now  $X<sub>t</sub>$  can be resolved into its permanent and transitory components by using equation (4.3).

<sup>&</sup>lt;sup>3</sup> Note that (4.6) in fact gives a set of second order equations, so that it would seem that several solutions are obtained. However, the different solutions are in fact identical in absolute values. The only difference between them is the interpretation regarding the sign of random disturbance. Thus for example in the system of business-sector product and unemployment, a positive shock may be defined as one that increases businesssector product or one that raises unemployment. The user of the system will choose the solution that suits him best.

## **Estimating the output gap**

The estimation of the output gap is based as stated on an econometric model that assumes a background of economic motivation which does not actually feature in the estimation.<sup>4</sup> For purposes of the estimation data of business-sector product, excluding start-ups, at constant prices (*y*) and an unemployment rate of (*u*) were used. The quarterly data cover the period from 1980:II to 2001:I. Both series were seasonally adjusted by the 'multiplicative moving average' method. Since the application of the Wold representation requires the use of stationary series with zero expected value, the variables underwent the following transformations: the log of business-sector product was taken and the first-order difference was calculated, and the average was deducted from the series of the differences. A linear trend was deducted from the unemployment rate, despite the fact that it is reasonable to assume that unemployment is stationary. The authors are of the opinion that the existence of the trend is a product of the sample period rather than a statistical property of unemployment.

For this application of the estimation method, the approach of Blanchard and Quah (1989) will be adopted. They considered the transitory shocks to be demand shocks, and the permanent ones to be supply shocks. The long-term constraint requires that the sum of the coefficients of the demand (transitory) shocks in the MA is zero. If the result that is being sought is that in the long term these shocks will have no effect on the level of business-sector product, this means that not only does their effect on ∆*y* disappear, but that it is actually offset over time (i.e., a positive effect today requires a future negative effect). It is not enough that the effect of the demand shock on ∆*y* disappears in the long term, as that means that *y* reverts to its original rate of growth only, but its level might have changed due to past growth rates that differed from the long-term rates. Hence, the effect of a demand shock on the level of *y* means that the effects on ∆*y* total zero. Thus ∆*y* has to be included in the estimation so that the long-term constraint can be imposed to help the identification of the  $s<sub>o</sub>$  matrix. Equation (4.1), the VAR model, was therefore run on

$$
X'_t = [\Delta y_t u_t].
$$

As stated,  $v_t$  is the vector of disturbances resolved into permanent (supply) components and transitory (demand) ones. In the current case,  $v<sub>t</sub>$  is a 2 x 1 vector, i.e.,  $m = 2$ . If the demand shock is made the first term in the vector (row 1, column 1), then since ∆*y* is located as the first term in  $X_t$ , the long-term constraint on  $S(1)$  has the form

$$
S(1) = \begin{bmatrix} 0 & \sum_{j=0}^{\infty} \binom{j}{j=0} \\ \sum_{j=0}^{\infty} \binom{j}{j=0} & \sum_{j=0}^{\infty} \binom{j}{j=0} \end{bmatrix},
$$

in other words, the effects of the demand shock on ∆*y* total zero.

As stated, additional identification equations are obtained from the variance matrix. In the current case the following equations are obtained:

4 One such a model (taken from Blanchard and Quah, 1989) is described in Appendix 2.

 $(4.8)a$   $(s_{11}^0)^2 + (s_{12}^0)^2 = \Sigma_{11}$  $(s_{11}^0)^2 + (s_{12}^0)^2 = \Sigma_{11}$ 

$$
(4.9) \qquad \left(s_{21}^0\right)^2 + \left(s_{22}^0\right)^2 = \Sigma_{22}.
$$

- $(4.10)$   $s_{11}^0 s_{21}^0 + s_{12}^0 s_{22}^0 = \Sigma_{12}.$
- $(4.11)$   $C_{11}(1)s_{11}^{0} + C_{12}(1)s_{21}^{0} = 0,$

where the  $C_{ij}$  are terms of the matrix  $C(1)$  obtained by summing over the  $C_{j}^{s}$  and  $\Sigma_{ij}$  are the terms of the matrix  $\Sigma$  obtained from the initial estimation of the VAR. The unknowns  $s_{ij}^0$  are the terms of the matrix  $s_0$ . Equations (4.8) to (4.10) derive from equation (4.6), while (4.11) derives from (4.7). Solving the set of equations in effect identifies  $s_0$ . The equations are of the second order, and it yields four solutions. Nevertheless, the absolute values of the solutions are practically identical to each other. As was pointed out in the econometric methodology review, the choice of a solution is of no essential significance, so that the one most convenient for the purposes of the study may be selected. In the case in question, the signs of the terms of the matrix  $s_0$  were chosen such that their interpretation should be in line with economic intuition:

$$
s_0 = \begin{bmatrix} Demand\ effect\ on\ \Delta y : + & \text{Supply effect on}\ \Delta y : + \\ Demand\ effect\ on\ u : - & \text{Supply effect on}\ u : + \end{bmatrix}.
$$

A positive demand shock is expected to boost growth and reduce unemployment, so that  $s_{11}^0$  is positive and  $s_{21}^0$  negative. A positive supply shock will increase growth, so the  $s_{12}^0$  is positive, but its effect on unemployment is not clear. Specifically, during the sample period the main supply shock was the huge influx of immigrants into Israel in the early 1990s, and it appears that its long-term effect was to actually to increase unemployment, so that  $s_{22}^0$  is positive.<sup>6</sup>

Once  $s_0$  has been identified,  $v_t$  and the matrices  $s_j$  can be calculated from equations (4.4) and (4.5), and also ∆*y* can be broken down into demand (transitory) and supply (permanent) components:

$$
\Delta y_t = \Delta g a p_t + \Delta y_t^P = \sum_{j=0}^{\infty} s_{11}^j v_{t-j}^D + \sum_{j=0}^{\infty} s_{12}^j v_{t-j}^S,
$$

where

is the term in row *r* and column *c* in the matrix  $s_j$ ;  $s^j_{rc}$ 

<sup>&</sup>lt;sup>5</sup> Note that  $C(1)$  is the infinite sum of the matrices, so that to calculate it many  $c_j$  were summed, till the summation met the convergence criterion.

<sup>&</sup>lt;sup>6</sup> The choice of a different solution would not change the results, but would make it more difficult to interpret them. For example, a positive demand shock could be defined as increasing unemployment and lowering growth. In that case the signs of the terms in the first column would be reversed.

is the demand shock;  $v_t^D$ 

is the supply shock;  $v_t^S$ 

*gap*, is the output gap; and

is potential output.  $y_t^P$ 

Equation (4.12) cannot be calculated exactly because it contains an infinite summation. In practice, summation was carried out only over sixteen quarters, as the estimate of the effect of the tail of the series did not exceed one-tenth of a percent of business-sector product.

All that remains to be done to calculate the output gap is to sum  $\Delta gap$ . Clearly, this calculation is sensitive to the starting point, i.e., it is based on the assumption that in the period prior to the sample period the output gap was zero; the level of the output gap was therefore adjusted so that an average output gap of zero would be obtained for 1995. The choice of this point in time is consistent with the findings of several previous studies, such as Menashe and Mealem (2000), Djivre and Ribon (2000), and the IMF estimate of 2001. Figure 2 shows the output gap obtained from this estimate.



# 5. A COMPARISON OF THE METHODS

In addition to a comparison of the two methods discussed in this paper, the results will also be examined in relation to the estimated output gap obtained via the HP filter. The latter is intended to analyze whether the methods make any contribution beyond the simple estimate of removing

## MIND THE GAP: ESTIMATING ISRAEL'S OUTPUT GAP 91

the trend from the GDP. Table 1 shows the coefficients of correlation between the estimates of output gap obtained by the different methods. The estimates also relate to the correlation between the moving average four-quarter output gaps, due to the high volatility of the original variable (as shown in Figure 3).

**Table 1 Coefficient of Correlation between the Different Methods of Estimating the Output Gap, 1986.4–2001.1**

	Original series			Moving 4-quarter average			
	HР	$\overline{PF}$	<b>SVAR</b>		HР	РF	<b>SVAR</b>
HP	1.00	0.36	0.75	HP	1.00	0.35	0.79
PF	0.36	1.00	0.42	PF	0.35	1.00	0.46
<b>SVAR</b>	0.75	0.42	1.00	<b>SVAR</b>	0.79	0.46	1.00

Table 1 shows that that there is no specially high correlation between the output gaps measured by the different methods, which leaves open the possibility of comparing and contrasting the methods. It is notable that the SVAR estimate and the HP estimate are correlated, while the correlation with the PF estimate is weaker. Figure 3 shows the four-quarter moving average output gap obtained by the various methods. Most clearly visible is the high level of the estimate from the HP filter relative to the estimates obtained from the other two methods. This derives from its sensitivity to he choice of sample period, since for any given sample the HP estimate will be around zero. A reminder is in order at this point that the level of the SVAR estimate is adjusted such that the average output gap in 1995 was zero. Thus from the aspect of determining the level of the gap, the PF model has an advantage over the other two methods. Another prominent feature is the correlation between the HP and SVAR estimates, while the correlation with the PF estimate is weaker.

The estimates of the output gap must exhibit some cyclical features, since the correlation coefficients of the estimates with their lags must show that the correlation coefficient with short lags is positive, while that after a sufficiently long lag (after half of the cycle) is negative, reverting thereafter to being positive, and so on. As a rule, some consistency of estimates over several years is desirable, as it is not reasonable to consider business cycles as too short. Figure 4 shows the correlation between the different estimates of the output gap. It can be seen clearly that the HP filter estimate is the least cyclical. The correlation coefficient becomes negative after only five quarters (half the cycle), compared to eight quarters for the PF estimate and ten for the SVAR. In other words, the estimates of the output gap from the structural methods of measurement are more consistent, and apparently indicate a more reasonable length of cycle.

Economic theory correlates the output gap in the short term with inflation, which is the reason for central banks' concern with the output gap. It is of interest to discover therefore which of the methods gives a better indication of the pressure that the output gap exerts on prices. The sample period was a period characterized by many changes that affected Israel's economy—the change in the inflation environment, the heavy influx of immigrants, the change









in the exchange-rate regime, various reforms intended to increase competition, etc. It is reasonable to assume that all these changed various economic parameters, in particular the way inflation reacted to economic variables. Also, in the early periods in the sample period the coefficient of correlation of (annual average) inflation with the (four-quarter moving average) output gap was in effect zero, but the later the period examined, the higher its value. This was apparently due to the fact in the second half of the 1980s inflation was affected mainly by the Stabilization Program, and the output gap did not play a major role in determining inflation. As a result of these considerations, it was decided to start the sample in the last quarter of 1992, while ensuring a certain minimum number of observation. That point in time was when inflation went down to a level of about 10 percent. The estimates of the output gap and the change in price is shown are shown in Figure 5.

The connection between the estimates of the output gap and prices was examined in two ways: simple coefficients of correlation between inflation and the output gap were calculated, and the Phillips curve was estimated.

Correlation coefficients: Table 2 gives the coefficients of correlation between the various estimates of the output gap and the change in prices. It can be seen that the estimates obtained by the PF method are correlated with prices throughout the whole sample period, whereas the SVAR estimate is not correlated with inflation at the beginning of the sample period, but the later the period examined, the higher the correlation.<sup>7</sup> The HP estimate emerged as having the lowest correlation with prices. A similar result was obtained with regard to the correlation between the estimates of the output gap and unexpected inflation.

<sup>7</sup> This applies to all the estimates.

# Figure 5 **Output Gap vs Inflation**





Output gap (4-quarter moving average)

 $\overline{3}$ 

 $\overline{A}$ 

# **Table 2**

**Correlation Over Time between Output Gap (4-quarter moving average) and Inflation (annual average)**



The Phillips curve: as a rule, a positive relation would be expected between unexpected inflation and the output gap. This approach assumes that the output gap expresses demand-side shocks only, and is consistent with Blanchard and Quah's (1989) approach, which interpreted permanent shocks as supply shocks, and transitory ones as demand shocks. Moreover, the effect of supply factors on unexpected inflation must be examined by introducing

variables representing supply shocks into the regression, as such a shock is likely to affect prices in the opposite direction to that of the output gap itself.

The specification of the estimate is shown in equation (5.1). The rate of change in the price of imported inputs with a one-quarter lag was chosen as representing the supply side. It should be noted that the use of supply shocks derived from the estimation methods or of other variables (wages and total productivity) with the appropriate lags were found to be not statistically significant.

Apart from the level of the gap, the speed limit was also introduced into the regression (the hypothesis that the sum of the coefficients of the first and second lag is zero cannot be rejected), the specification used assumes that the effect of the change in the output gap is faster than its level; this derives from the assumption that the public does not quickly revise its view about the level of the output gap, but the change in it gives a first signal regarding its trend. Similarly, the specification incorporates the output gaps of the last three periods, as the variable [66] includes the gap with a one-period lag and that with a two-period lag.

It should also be noted that if inflation expectations are taken over to the right side of the equation, the hypothesis that the coefficient of this variable is different from 1 cannot be rejected, and this equality was therefore imposed on the estimate (as suggested by economic theory), so that the dependent variable is unexpected inflation. Equation 5.1 is estimated by the least squares method for the three methods of measuring the output gap. The results are given in Table 3.

$$
(5.1) \quad PI_t - EXP4Q_t/4 = \lambda_1 DGAP_{t-1} + \lambda_2 GAP_{t-3} + \delta D98Q4_t + \sigma PIMIN_{t-1} + \varepsilon_t,
$$

where

PI is the rate of change in prices (quarterly average)

EXP4Q is inflation expectations four quarters forward (quarterly average)

DGAP is the change in the output gap

GAP is the output gap



PIMIN is the rate of change in prices of imported inputs.





a Numbers in parentheses are the *t* statistic.

In general no significant differences were observed between the different equations regarding the goodness of fit and regarding their effect on the supply variable and on the dummy variable. The coefficient of the change in the prices of imported inputs was negative, as expected. Nevertheless, in all equations it does not have a high level of significance, and does not significantly improve the goodness of fit, and it is concluded therefore that the estimates of the output gap available do not represent excess demand, and supply factors do not constitute a significant component of the estimates.

It can also be seen that the HP and SVAR estimates succeed in explaining price changes by means of the speed limit component, while in the PF estimate the explanatory power derives from the level of the output gap, as reflected from he values of the *t*-statistic.

In additional analyses performed, in which output gaps obtained from one of the methods were introduced into one regression equation, no method was found to take power to explain price changes from other estimation methods, nor was any evidence found that the structural methods succeed in explaining inflation better than does the HP estimate.

## 6. LONG-TERM POTENTIAL OUTPUT

In the previous sections of this paper potential output has been taken as the level that does not create inflationary pressures. In both the PF and the SVAR methods of estimation this was

reflected by the fact that in measuring the output gap only demand factors were taken into account (at least methodologically). However, as stated at the outset, there are other definitions of potential output, and thus also of the output gap. Clearly a change in the definition requires a change in the measurement method.

One generally accepted definition of potential output is the long-term level of output given the current economic situation. In other words, the level of output after the effects of transitory shocks on both the demand side and the supply side have been exhausted. This section shows the changes that need to be made to the PF and SVAR methods of measurement so that the estimate of the output gap obtained from them reflects the deviation of actual output from the long-term level.

For the sake of convenience and terminological consistency, the difference between actual and long-term output will be called the business cycle (the term output gap will be reserved for the gap that reflects inflationary pressures).

## **The production function (PF) method**

In this method, as stated, potential output is calculated indirectly, i.e., first the output gap is calculated, and potential output is measured as the difference between it and the log of actual output. If business cycles are defined as the difference between actual and long-term output, the definition will include not only the transitory shocks on the demand side (as reflected in the output gap), but also the transitory supply-side shocks. These supply-side shocks are measured in the cyclical part of total productivity  $a_t^c$ . The results obtained for potential output and long-term output according to each of these two definitions will be examined:

*Results*. a) The output level that does create inflationary pressures. b) The long-term level of output. A comparison of the quarterly growth rate of potential output (moving four-quarter average) according to the first definition with its level in the equivalent period a year earlier (Figure 6) shows that the growth rate of potential business-sector output excluding start-ups averaged 5.6 percent from 1996 compared to a rate of 6.4 percent in the first half of the 1990s. A similar picture emerges also if start-ups are included. An alternative, as stated, is to calculate the growth rate of long-term potential output (output *minus* of transitory shocks, including supply-side shocks, and to obtain a smoother series as shown in the figure.

### **The SVAR estimate**

As mentioned above, in the SVAR estimate Blanchard and Quah's (1989) approach was adopted. In this approach, demand shocks are considered to be transitory, and thus their effect disappears in the long term, whereas supply shocks are taken to be permanent. It should be stressed at this point that this estimate has no constraint that prevents the effect of supply shocks from disappearing in the long run or from changing over time. Within the framework of the model, supply shocks are permanent in the sense that in the long run they still have some effect on output. The impulse response shown in Figure 7 describes the effect of the two types of shock on output and on unemployment. A one-standard-deviation effect of the supply shock on output in the long term is assessed at 1.9 percent; however, its effect at any point in time is different from this. This result suggests that supply shocks comprise permanent and transitory components, and to obtain the long-term output the permanent effect must be isolated.

Figure 6 Growth Rate of Potential Output by Two Measurement Methods, 1990.1-2001.1 (annual average)







Equation (4.12) divided the change in output into demand and supply components. To obtain an estimate of the business cycle, the transitory part of the supply shock must be added to the demand component. Generally, the permanent effect of the supply shock is measured by [67]. To enable the separation of the components to be performed, this factor will be removed from equation (4.12), yielding:

(6.1) 
$$
\Delta y_t = \underbrace{\sum_{j=0}^{\infty} s_{11}^j v_{t-j}^D + \sum_{j=0}^{\infty} s_{12}^j v_{t-j}^S - S_{12} (1) v_t^S}_{\Delta cycle_t} + \underbrace{S_{12} (1) v_t^S}_{\Delta y_t^{IR}},
$$

where  $y_t^{LR}$  denotes long-term output, and *cycle*<sub>t</sub> denotes the deviation from it. The expression

$$
\sum_{j=0}^{\infty} s_{12}^j v_{t-j}^S - S_{12}(1) v_t^S
$$

denotes the transitory part of the supply shock on [71]. Now the change in the business cycle can be calculated:

$$
(6.2) \qquad \Delta cycle_t = \Delta y_t - S_{12}(1)v_t^s,
$$

where  $\Delta y_t$  is obtained from the data, and  $S_{12}(1)$  and  $v_t^s$  are calculated in the estimate of the output gap (see Section 4 above). It should be noted that the shortcoming of this method of separating the components lies in the high variance of the estimate of  $S_{12}(1)$ , as it depends on an infinite scheme of coefficients, each of which has a standard deviation. This means that any specific estimate obtained by this method should be treated with caution, and the general trend of the business cycle should be examined. Figure 8 shows the business cycles obtained from this estimate and compares them with the output gap shown above in Figure 2.8

*Results.* Figure 8 shows clearly that at the beginning and end of the sample period the level of the business cycle was similar to the level of the output gap. The correlation between the two series can also be seen clearly, especially the fact that the timing of the turning point is the same in both. However, the low level of the business cycle in the middle of the sample period (from 1989 to 1993) also stands out. This difference highlights the conceptual difference between the two series. The output gap is not affected by changes in long-term output, and permanent supply shocks therefore will not affect it, whereas the business cycle is defined as the difference between actual and long-term output. It is therefore of interest to examine how supply shocks create the difference between the two series in the middle of the sample period. Again, it must be borne in mind that due to their high variance it would be a mistake to assign too much significance to estimates at particular points in time, such as the recession towards the end of 1992, and that the general trend obtained from them must be examined.

In 1989 mass immigration into Israel from the former Soviet Union started, bringing some 200,000 immigrants into the country in 1990, and 175,000 in 1991, increasing the population by about 4 percent in those years.

The rate declined in 1992 relative to the previous two years, but remained at the high level of 75,000 a year until 1995, increasing the population by 1.5 percent a year. The significance of such high population growth rates is the immediate rise in the economy's long-term potential production. Actual production, however, does not react as rapidly as does long-term output because of the time it takes for new immigrants to be absorbed into the labor market and to find employment appropriate to their skills. The difference between the reactions of longterm and actual output explains the picture of the recession at the beginning of the 1990s, which was deeper in the measurement of the business cycle than in the output gap, because the increase in potential output, that does not create inflationary pressures, is slower than that

<sup>8</sup> In the case of the business cycle too the level was adjusted so that the average of 1995 was zero.



of long-term output. Immigrants contribute their potential production to long-term output immediately on arrival in the country. The rise in demand that they bring, on the other hand, is smaller in the period of their absorption, at least until they merge into the labor market.

The closing of the gap between the business cycle and potential output in 1993 was the result of a slowing in the rate of increase (and even a decrease) in potential output concurrent with relatively rapid growth of actual output (Figure 9). The moderation of potential output derived from the combination of several factors at one point in time: (1) the slowdown in the number of mew immigrants arriving in Israel. 1992 The number of immigrant arrivals fell from over 150,000 a year to about 75,000 a year each in 1992 and 1993. (2) A 23 percent reduction in residential construction due to the ending of government-initiated construction intended to find a housing solution for the immigrants, after several years in which such public building had risen considerably (and with no change in 1992). (3) In 1993 closures were imposed on the Palestine Autonomy areas, reducing in one year the number of Palestinians working in Israel in the business sector from about 110,000 to about 80,000. During that time other foreign workers were being absorbed relatively slowly (the number in the business sector rose from 14,000 to 27,000), and did not make up for the shortage of Palestinian workers. As stated, the gap between estimated output gap and the business cycle resulted from supplyside shocks. Hence, the estimate enables the business cycle to be broken down into supply and demand components. The similarity between the estimates at the beginning of the sample period and at its end indicates that the supply shocks in those periods did not constitute major elements in the creation of the cycle. This finding shows that the recession that started in the second half of the 1990s derived from demand factors, as the difference between the measures at this time were not significant (Figure 8).



Figure 9 also shows that similar to the result obtained from the production function method, the average rate of growth of potential output declined by about one percentage point from the first half of the 1990s to the second half, from about 5.3 percent to 4.5 percent.

## 7. SUMMARY AND CONCLUSIONS

In this paper two of the structural approaches to the estimation of potential output and the output gap (or the business cycle) derived from it were employed—the production-function (PF) approach, and the SVAR estimate. Potential output has two generally accepted definitions (the output that does not crate inflationary pressure, and long-term output), so that the estimation method must be appropriate to the desired definition. The paper shows how this can be done for the two estimation methods. The PF approach is based on the decomposition of each factor of production into a permanent and a cyclical component; the contribution of this paper is in the decomposition of the Solow residual into productivity and capital-utilization components by using data on business-sector electricity consumption. This breakdown enables potential output, with its different definitions, to be measured as it distinguishes between demand and supply factors. Both demand-side shocks and supply-side shocks enter into the measurement of the business cycle, whereas only the former feature in the measurement of the output gap. The SVAR approach is an econometric one, with an economic model in the background which does not however directly affect the estimate. This approach concentrates on breaking down the shocks to output into demand components perceived as transitory and

supply components perceived as permanent. This approach also enables the potential output, using its different definitions, to be estimated. As this method treats the supply shock as permanent, in the sense that some of its effect is also felt in the long term, with the extent of the effect varying over time, the effect of the shock can be broken down into transitory and permanent components. This separation enables potential output, in its different definitions, to be estimated. This study records the various stages of the econometric estimation so that interested readers can apply this method in the future.

The estimates of the output gap obtained via the two approaches were compared to the HP estimate, which served as a benchmark and represented the non-structural approaches, based on the tests carried out, it seems that the structural methods (i.e., the SVAR and PF methods) yielded better results than did the HP method. An examination of the degree of consistency of the output gap by means of the correlation coefficients showed that the cycle derived from the HP estimate was the shortest, so much so that its length seemed unreasonable; the structural methods provided more consistent estimates. The HP estimate was also inferior to the others with regard to the coefficient of correlation with inflation. Despite these findings, no evidence was found that the structural methods had greater explanatory power with regard to inflation, as obtained from the estimate of the Phillips curve. It was also found that in this estimation the PF estimate explains inflation via the level of the output gap, while the other two methods explain it via the change in it (i.e., via the speed limit).

The last section of the paper showed the estimates of the business cycle obtained from the long-term estimate, and how various economic developments in the sample period explain the difference between the changes in the business cycle and those of the output gap.

# APPENDIX 1

#### **The switch from AR to MA**

The AR presentation:

$$
(1) \qquad X_t = \sum_{j=1}^p \phi_j X_{t-j} + \varepsilon_t
$$

The MA presentation:

$$
(2) \qquad X_t = \sum_{i=0}^{\infty} c_i \varepsilon_{t-i}
$$

from which the following derives:

$$
X_{t-j} = \sum_{i=0}^{\infty} c_i \varepsilon_{t-j-i}
$$

Introducing  $X_{t-i}$  into (1) yields

$$
(3) \qquad X_t = \sum_{j=1}^p \phi_j \sum_{i=0}^\infty c_i \varepsilon_{t-j-i} + \varepsilon_t
$$

From  $(2)$  and  $(3)$  the following is obtained:

(4) 
$$
\sum_{i=0}^{\infty} c_i \varepsilon_{t-i} = \sum_{j=1}^{p} \phi_j \sum_{i=0}^{\infty} c_i \varepsilon_{t-j-i} + \varepsilon_i
$$

Rearranging (4) gives:

(5) 
$$
(c_0 - I)\varepsilon_t + (c_1 - \phi_1 c_1)\varepsilon_{t-1} + (c_2 - \phi_2 c_0 - \phi_1 c_1)\varepsilon_{t-2} + \dots
$$

$$
\dots + (c_p - \phi_p c_0 - \phi_{p-1} c_1 - \dots - \phi_1 c_{p-1})\varepsilon_{t-p} + \sum_{j=p+1}^{\infty} (c_j - \sum_{k=1}^p \phi_k c_{j-k})\varepsilon_{t-j} = 0
$$

As the equality must hold for all  $\varepsilon$ <sub>*i*</sub>, every coefficient in (5) must equal zero. As estimates of  $\phi_j$  are available, estimates for  $c_j$  can be derived:

$$
c_0 = I
$$
  
\n
$$
c_1 = \phi_1 c_0
$$
  
\n
$$
c_2 = \phi_2 c_0 + \phi_1 c_1
$$

etc.

In general, for  $k \geq 1$ , the following obtains:

$$
c_k = \sum_{i=1}^{\min\{k,p\}} \phi_i c_{k-i}
$$

# APPENDIX 2

## **An economic model for SVAR**

The specification for the econometric model in the SVAR approach can be based on various economic models. This appendix shows a simple Keynesian model that provides an economic application for the econometric approach. The model shown below is from Blanchard and Quah (1989). It serves as an example only, and other, richer, models can be proposed that would give similar econometric specifications. The equations in the model are as follows:

Cumulative demand:

$$
(1) \t Y_t = M_t - P_t + a\theta_t
$$

The production function

$$
(2) \t Yt = Nt + \thetat Yt = Nt + \thetat
$$

The price equation:

$$
(3) \t P_t = W_t - \theta_t
$$

The equation of wage determination:

$$
(4) \qquad W_t = E_{t-1}\big(W_t\big|N_t = \overline{N}\big)
$$

The variables Y, N and q denote the logarithms of output, employment, and productivity respectively. The log of full employment is denoted by  $\overline{N}$ . P, W, and M denote the log of the price level, the nominal wage, and the money supply respectively. Note that aggregate demand is directly affected by productivity (for instance via investment demand(, and the wage is determined on period in advance to give an expectation of full employment

Similarly, the money supply and productivity derive from stochastic processes determined by:

$$
(5) \qquad M_t = M_{t-1} + v_t^T
$$

$$
(6) \qquad \theta_{t} = \theta_{t-1} + v_{t}^{P}
$$

It is assumed that the shock to the money supply is transitory and the shock to productivity is permanent. Define unemployment as:

$$
(7) \qquad U_t = \overline{N} - N_t
$$

The solution of the model for the growth rate and the unemployment rate is:

$$
(8) \qquad U_t = -v_t^T - av_t^P
$$

(9)  $\Delta y_t = v_t^T - v_{t-1}^T + a(v_t^P - v_{t-1}^P) + v_t^P$ 

From equations (8) and (9) it can be seen that the effect of the two shocks on unemployment and on the rate of growth disappears in the long run (in this model as soon as after two periods the shocks no longer affect these variables). Note that the sum of the coefficients of the transitory shock in equation (9) is zero (unlike the sum of the coefficients of the permanent shock); in other words, after the transitory shock output reverts to its original level (and after the permanent shock the rate of growth reverts to its original rate, but the level of output changes).

## APPENDIX 3

## **Differences between estimates of the output gap including and excluding the contribution of start-ups**

In the last few years start-ups made a significant contribution towards explaining output. In 2000, for example, their contribution doubled the rise in productivity in the business sector. The major contribution of knowledge-producing start-ups lies in the significant increase in productivity reflected by the rise in supply. On the other hand, the method of making entries in the national accounts does not ensure that the added value of this know-how in fact reflects technological advance and increased productivity, as entries are made according to investment in venture capital funds without regard to whether the investment will be profitable.

In addition to this problem in the use made of data on the contribution of start-ups' output with regard to the supply side, the demand side must be taken into account too, particularly the question of what is the output gap relevant to the analysis of inflation—does it or does it not include the output of startups? In this context it is important to bear in mind that the demand for staff in start-ups exerts upward pressure on the nominal wage, part of which at least is not reflected in increased productivity but in increased prices, so that it is must be measured in the inflation equation.

Figure 10 shows the differences in output gaps with and without the contribution of startups according to estimates obtained by two of the estimation methods described in the paper, SVAR and HP. The results indicate that as expected there are no differences between estimates of the output gap with or without the contribution of start-ups until 1995, and thereafter the differences relate mainly to the level of the output gap and not to its trend, with the output gap excluding start-ups exceeding (in absolute terms) the output gap including start-ups. In the production-function method the inclusion or exclusion of start-ups did not affect the output gap (and therefore does not appear in the figure). This is because the differences between the two output series will be reflected in the total productivity component (the supply side), which reflects potential output, and not the output gap (the demand side). The use of estimates of the output gap including and excluding start-ups in each of the estimation methods in explaining inflation does not yield significant differences either, not in the an examination of the coefficients of correlation between inflation and the output gap and not in the estimate of the Phillips curve.

Finally, the question of which estimate of





output is relevant to the output gap remains an open one from the methodological aspect, unanswerable empirically at this stage due to the relatively small number of observations in which there is a difference between the output gaps.

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