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THE HOUSING MARKET IN ISRAEL 2008-2010: ARE HOUSE PRICES A "BUBBLE"?*

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Abstract

Housing prices have risen rapidly over the past three years. Between December 2007 and August 2010, prices rose by 35 percent in real terms – an average annual rise of 12 percent, much faster than their long-term rate of increase. Given this surge in prices, concerns have been raised that the price rise is unrelated to market fundamentals and has been driven by expectations of capital gains, which suggests the development of a bubble in house prices. The purpose of this article is to assess whether a bubble has developed in house prices in this period. To that end, we first review the course of prices retrospectively over the last few decades relative to rents and income; and then use three different approaches to derive the "fundamental price" from a standard asset-pricing equation. Our analysis suggests that while the prices at the end of the examined period are somewhat high relative to market fundamentals, from a long-term perspective their level is not exceptional in comparison with past episodes of rising house prices. According to various measurements, the actual price is between 3 percent below and 10 percent above the fundamental price. We therefore do not find evidence of a bubble in house prices (as of August 2010). If a bubble exists, it is at an early stage, and cannot yet be detected from the data.

1. INTRODUCTION

Housing prices have risen rapidly over the past three years. Between December 2007 (the low point before the recent surge in prices) and August 2010, prices rose by 35 percent in real terms – an average annual rise of 12 percent, much faster than the 1.5 percent multi-year average annual rate of increase (starting in 1973).[1] Given this surge, assertions have been made in public discourse that a bubble is forming in house prices. The purpose of this

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[1] The August 2010 figure was the most recent observation when this study was written.

study is to assess whether a bubble in house prices in Israel is developing, according to the accepted economic meaning of the term in the literature. An analysis of the data yields ambivalent findings; some of the measurements show that the level of prices is indeed high in comparison with the market fundamentals, but the level is not exceptional in comparison with past episodes in the Israeli economy, and other measurements indicate that the prices correspond closely to the market fundamentals. In general, we do not find evidence of a bubble in house prices; if a bubble exists, it is at an early stage, and cannot yet be detected from the data.

If a bubble is developing, this means that because the price surge is not supported by market fundamental, prices will fall precipitously at some stage. This would bring about a crisis in the domestic real estate market, and a drop in the value of the securities held by the banks. Such a development, beyond the fall in the value of the property owned by households and a slowdown in real estate activity, would have at least some degree of negative impact on stability in the banking system.

As in standard asset pricing theory, a bubble in housing prices is defined as a gap between the actual price and the "fundamental price," where the fundamental price reflects the discounted value of the flow of expected housing services which depends on supply and demand factors in the housing market, and on the interest rates that influence the discount rates. In a bubble, the rise in property prices is driven by expectations feeding on themselves, without any connection to the market fundamentals.[2] In such a situation, the expectation of an increase in property prices causes a rise in the demand for them, fueled by the investors' desire for capital gains. Burgeoning demand pushes up property prices, thereby fulfilling the expectations, even if the market fundamentals do not support a price rise. Our analysis is based on this definition of a price bubble; we examine whether a bubble in housing prices exists in Israel, according to the accepted approaches in the literature.

As noted, houses are an asset whose price should represent the discounted value of a flow of expected housing services, which are represented by the rent. Rent is paid for the right to use a home for a given period, i.e., for the housing services that it provides. Rent does not confer any ownership of housing, and it therefore cannot generate capital gains for the tenant. It therefore follows that according to the above definition of a bubble, development of a bubble in rent is impossible. This insight provides the basis for the empirical methodology presented below.

Similar to the approach taken by Campbell and Shiller (1989), we do not attempt to analyze the factors that shift demand and supply of housing such as demographics, construction costs, supply of land etc., nor do we try to evaluate the contribution of such factors to house prices or rents. Instead, the asset pricing methodology suggests an equilibrium relationship between interest rates, rents and house prices. Absent a bubble this relationship should hold, regardless of demand and supply factors that affect rents and prices, and is therefore suitable and most straightforward for providing an answer to the question this paper poses.

[2] For example, see Flood and Garber (1980) and Blanchard (1979).

The article first examines developments in the Israeli housing market retrospectively, and presents various indicators of the connection between house prices, rent, and income in the economy. We then use three alternative methods of assessing the question of the existence of a bubble by separating the fundamental element and the residual defined as a bubble: direct measurement, the Kalman filter, and econometric estimation. The three methods are based on standard asset pricing theory,[3] as described above, meaning that the asset price should be equal to the present value of the expected income it generates.

The direct measurement calculates the fundamental price by discounting a future flow of rent using the yield curve observed at that point.[4] The bubble element in this method is calculated as the difference between the actual price and the fundamental price. This measurement is based on an assumption of market efficiency and rational expectations. According to these assumptions, the actual price of property is the most efficient forecast for individuals of the expected cash flow from it (Shiller, 1981). Since rational expectations do not assume systematic forecasting errors, a sustained deviation of prices from the calculated value would indicate the existence of a bubble.

The Kalman filter method calculates the fundamental price by deriving the expected future flow of rent at each point in time on the basis of information available to players in the economy. This method also assumes that the bubble element develops according to an autoregressive process derived from the asset pricing equation.

The third approach estimates an econometric equation for the ratio of rent to the price of houses. This ratio, which is derived from the asset pricing equation, reflects the real return on home ownership. This is determined by expectations of individuals for the market interest rate and for the growth rate in profits resulting from property ownership (Campbell and Shiller, 1989). At equilibrium, this ratio equals the imputed cost of renting for the homeowners (Himmelberg, Mayer, and Sinai, 2005). In this method, we estimate the ratio of rent to the price of houses as a function of the cost resulting from home ownership, where the residual obtained from the estimate gives an indication of the existence of a bubble in house prices.

The structure of the article is as follows: the next section (Section 2) presents the key facts about the housing market in Israel. Section 3 briefly presents a theoretical framework for examining the existence of a bubble in house prices. Sections 4-6 examine the question of whether actual price development is consistent with the economic fundamentals for the three approaches described above: direct measurement, the Kalman filter, and econometric estimation. Section 7 concludes.

[3] For example, see Gordon (1962), Blanchard (1979), Shiller (1981), Flood and Hodrick (1986), and Campbell and Shiller (1989).

[4] Obviously there are no data for future rent after the sample ends, it is therefore estimated using an ARMA process.

2. THE HOUSING MARKET IN ISRAEL – THE FACTS

This section surveys indicators of the state of the housing market in the Israeli economy for the purpose of considering the question of whether house prices have become unrelated to the basic conditions prevailing in the economy, represented by rent and wages. We portray the development of house prices in recent decades and indicators of the degree to which they correspond to the fundamental factors. The indicators are house prices in comparison with rent and the average wage.

Figure 1 displays the development of the real price of housing in 1973-2010.[5] House prices rise over time at a faster rate than general prices. Real housing prices rise by an average of 1.5 percent annually.[6] The existence of a long-term rising trend should be no surprise, at least to the extent that demand increases over time, due to an increase in the population, while the supply of available land for construction is inelastic (although high-rise housing can be built), while the technological improvements in the sector are relatively slow.[7] The rising trend in the real price has not been uniform in recent decades; long cycles of boom and bust are distinguishable. In particular, a prominent significant and sustained rise in house prices began in 1989 with the beginning of the wave of immigration from the countries of the former Soviet Union, and continued until 1996. The entry of one million immigrants increased the population of the country by 20%, thereby boosting the demand for housing, which was visibly reflected in the real price of housing. Prices surged by almost 80 percent, 7.5 percent annually, during this period. From the mid-1990s until 2007, the real price declined consistently, with an aggregate fall in prices of over 20 percent (2.3 percent annually). Over the past three years, house prices have risen steeply – between December 2007 (the low point before the recent surge) and August 2010, prices jumped by 35 percent beyond the rise in the Consumer Price Index, exceeding the 1996 peak by 2 percent.

The recent rapid increase in house prices gives rise to the question of whether this price increase is supported by supply and demand factors in the housing market, and constitutes a correction of the consistent decline in prices during the preceding decade, or whether the price increases are feeding themselves, and are supported by expectations of further rises, with a bubble developing in house prices. In order to obtain an initial indication with respect to this question, we examine the development of house prices in comparison with rent and the average wage.

[5] The house price index is based on a Central Bureau of Statistics survey of prices of owner occupied houses. Figures for the house prices survey exist from 1994. However, since house prices were used until 1998 to measure the cost of owner occupied housing services in the CPI, this series (which is very similar to a survey of house prices in 1994-1998, when there are data for both series) can be used to obtain a historical series of house prices. The prices in this index are adjusted for the quality and size of the home.

[6] The average house price over the past 12 months, August 2009-August 2010, compared with the 1973 average.

[7] It is also possible that the improvement in the quality of houses over the years is not fully deducted from the measured price, and the price rise therefore also reflects a change in quality.

Figure 1
The Real Price of Houses (in comparison with the Consumer Price Index)
(January 1973-August 2010, monthly figures, January 2000 = 1)

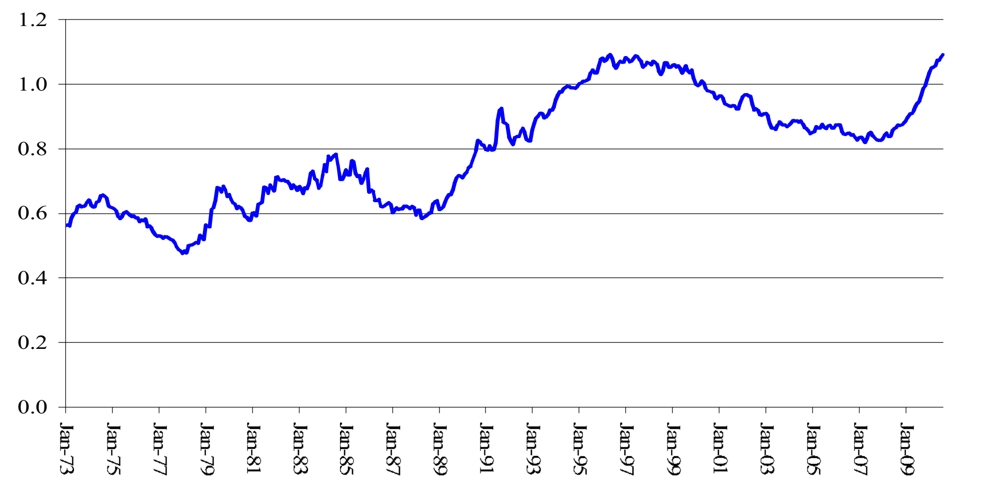


Figure 2 displays the development of the ratio of the price of houses to rent, where rent is measured according to new and renewed leases starting in 1999.[8,9] The price-rent ratio is a common measure for evaluating stability in price increases in the real estate market.[10] In the long run, this ratio should be fairly stable, since it (or, to be more exact, its inverse) reflects the real return on home ownership. Figure 2 clearly shows that since the beginning of 2009, house prices have risen more steeply than rent. At the same time, comparing the development of this ratio in Israel to its development in selected countries where it was known in retrospect that a bubble had developed indicates that the situation in Israel differed greatly from the situation in those countries (Figure 3). In Israel, the ratio is relatively constant; it has been rising in recent years, but does not deviate from its long-term level. In the selected countries, on the other hand, the graph clearly illustrates that house prices were unrelated to rent during the bubble years, and afterwards fell precipitously.

[8] Rent is measure according to both new and renewed leases and according to existing leases. Where methodology is concerned, rent in new leases is a better indicator for a comparison with house prices, because it reflects the housing market situation in real time, and is not affected by leases signed during the year preceding the measured month. Unfortunately, measurement of rent according to new leases began only in 1999. In the years before 1999, we therefore use rent according to existing leases. Note that until 2007, there was no significant difference between the two ways of measuring. Starting in the second half of 2007, however, a gap between them opened as a result of the accelerated increase in rent.

[9] The sample begins in 1994, with the change in the method of measuring the owner occupied housing price index. Measurement could have begun earlier, but the demand for housing during these periods was also affected by motives of hedging against inflation, particularly in the 1980s, and the ratio of house prices to rent was therefore much higher at that time than its present value (Rubinstein, 1999).

[10] For example, see McCarthy and Peach (2004) and Shiller (2007).

Figure 2
The Ratio of House Prices to Rent
(January 1994-August 2010, January 2000 = 1)

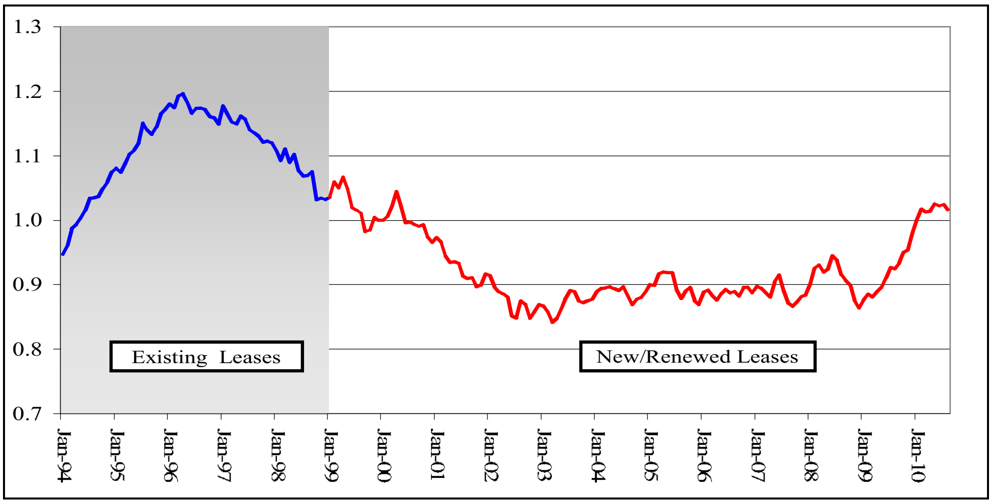
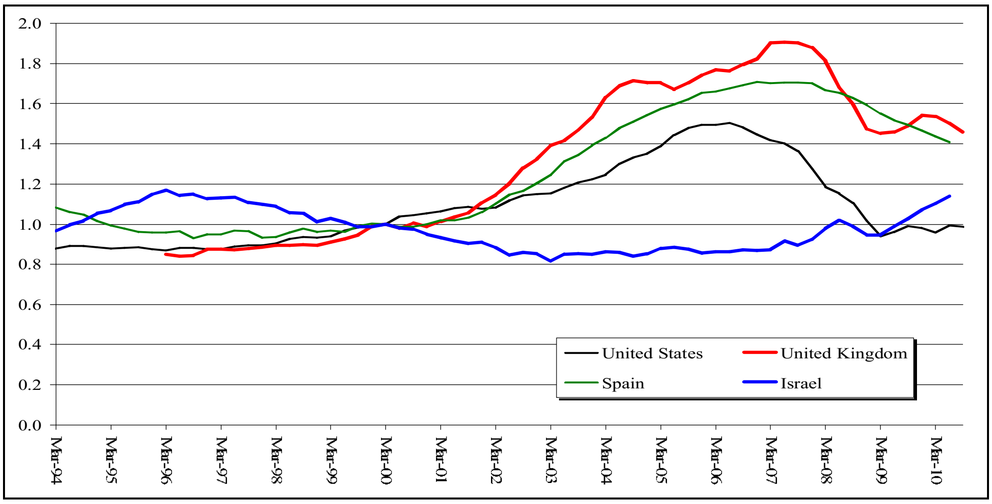


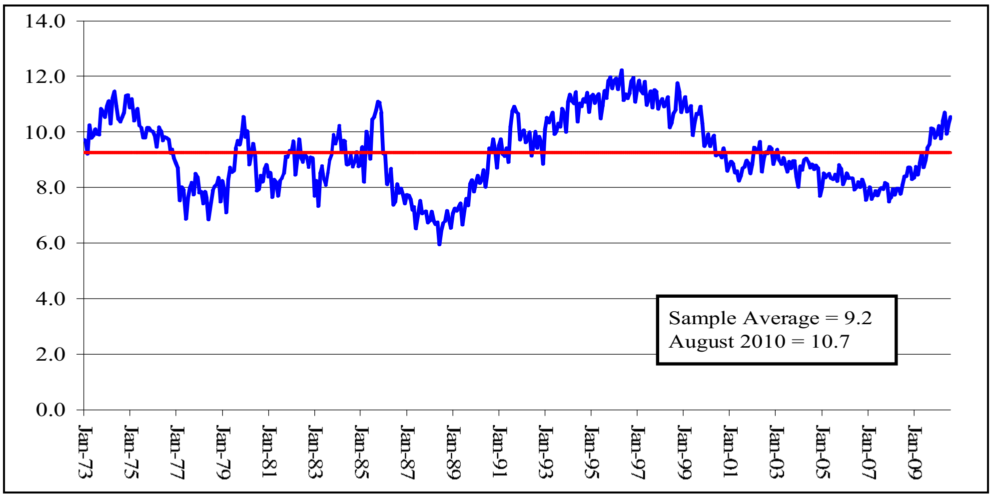
Figure 3
The Ratio of House Price to Rent – An International Comparison*
(1994 Q1-2010 Q2, quarterly figures, January 2000 = 1)



* For the American data: the Case-Shiller Index of home prices (source: S&P), divided by the rent item in the American Consumer Price Index (source; BLS). UK: the owner occupied house prices index (source: Bloomberg), divided by the rent item in the UK Consumer Price Index (source: UK Office for National Statistics). Spain: OECD figures. Israel: the owner occupied housing prices index, divided by the rent item in the Consumer Price Index (source: Central Bureau of Statistics).

Figure 4 displays the development of the ratio of the price of houses to the average wage of employed persons. This ratio represents the number of years that a person in an average job must work in order to purchase a home, and its fluctuation reflects changes in an employed person's ability to purchase a home. It is usually assumed that there is a positive connection between house prices and the average wage. Case and Shiller (2004) found that in various states in the US, per capita income was the most important variable in explaining changes in house prices. An exceptional deviation from the long-term average in the ratio would therefore signal a deviation from the basic factors. Housing has also clearly become more expensive according to this ratio: in January 2008-August 2010, house prices rose by a cumulative 43 percent beyond the rise in the average wage. As a result, the gap between its level and the sample average widened; according to this calculation, 10.7 years of work at the average (gross) wage are now needed to purchase a home at the average price (Figure 4).[11] Furthermore, an international comparison shows that the number of wage years needed to purchase a home in Israel was significantly higher than in other developed countries.[12] At the same time, this comparison does not indicate

Figure 4
The Ratio of the Average Home Price to the Average Wage Per Employee Post (January 1973-August 2010, monthly figures, wage years)

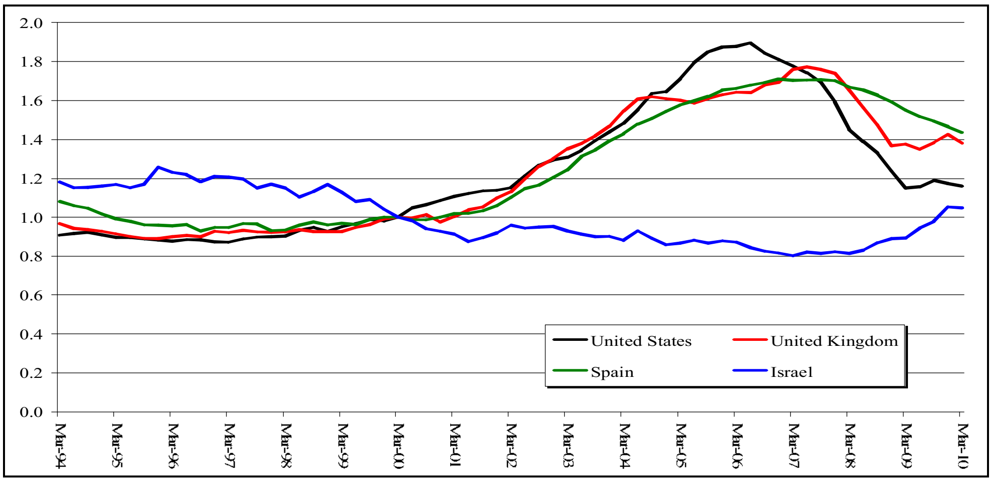


[11] The conversion of the index of house prices in the survey to shekels was based on data of the first quarter of 2008, when the average price of a home was NIS 755,000. The average monthly wage at that time was NIS 7,900 (NIS 95,000 per year).

[12] The "Annual Demographia International Housing Affordability Survey" (2010) presents the number of income years that a median-income household needed to purchase a home in the following countries: US, Australia, Canada, Ireland, New Zealand, and UK. The survey shows that the number of work years for purchasing a home in Israel was much higher than in all the countries in the sample. For example, the median number of income years per household needed to purchase a home in Israel was 7.7, compared with 2.9 years in the US, 5.1 years in the UK, and 6.8 years in Australia (the highest number of years in the sample was for Australia).

that prices were unrelated to the fundamental factors, since housing is a commodity that is not tradable between countries, and market forces therefore do not act to equalize prices or purchasing power between countries. In addition, the overall rise in the ratio of the price of houses to wages in Israel is still lower than in the developed countries in which a bubble was observed (Figure 5).

Figure 5
The Ratio of House Prices to Average Wage – An International Comparison*
(1994 Q1-2010 Q2, quarterly figures, 2000 Q1 = 1)



* For the American figures: the Case-Shiller home prices index (source: S&P), divided by the average weekly wage (source: BLS). UK: house prices index, divided by the average wage per salaried position (source: Bloomberg). Spain: OECD figures. Israel: the owner occupied house prices index, divided by the average wage per salaried position (source: Central Bureau of Statistics)

From the figures presented in this section, we conclude that while house prices have indeed risen steeply since the beginning of 2008, this development can still be regarded as a correction of the continuous drop in prices in real terms that began in the middle of the preceding decade, and it does indicate that prices are unrelated to the fundamental economic factors.

3. A RATIONAL BUBBLE IN THE ASSET PRICING EQUATION

This section presents the methodological basis for the analysis in the article. We use the basic asset pricing equation, which appraises the economic value of an asset (a home) as the present value of the dividends (rent) that the asset is expected to yield.[13] This value is called the fundamental price of the asset. We show, according to what is presented in the literature,

[13] This equation is discussed extensively in the literature. For example, see Shiller (1980), Campbell and Shiller (1989), Flood and Hodrick (1986, 1990), and many others.

that a prolonged deviation of the actual price from the fundamental price does not necessarily contradict this asset pricing equation.[14] Such a deviation will be called the bubble component of the asset price.

We use P to denote the real price of homes, RR to denote the real rent, and r to denote the real risk-free interest rate. A risk-neutral investor will be willing the pay the present value of ownership for a home; this value is composed of the rent and the discounted value of the expected price of the home in the ensuing period:

$$(1) \quad P_t = RR_t + \frac{E_t(P_{t+1})}{1+r_t}.$$

Note that this equation assumes that an investor has all the capital needed to buy a home, or alternatively that the mortgage interest rate available to him is equal to the interest rate on savings. In Section 4.3 of the article, we dispense with this assumption.

Moving Equation (1) forward in time, we derive the real fundamental price of homes, P_t^f, which equals the present value of the flow of rent payments that the asset is expected to yield:

$$(2) \quad P_t^f = E_t \sum_{s=0}^{\infty} \gamma_{t,t+s} RR_{t+s} ,$$

where $\gamma_{t,t+s} = \Pi_{j=0}^{s-1}(1+r_{t+j})^{-1}$ for $s \geq 1$, $\gamma_{t,t} = 1$, and we assumed that $\lim_{s\to\infty} \gamma_{t,t+s} P_{t+s} = 0$. P_t^f , as displayed in Equation (2), constitutes a solution for the price of homes in Equation (1), but this is not the only solution. Any price in the form of:

$$(3) \quad P_t = P_t^f + B_t$$

$$(4) \quad B_t = \frac{E_t(B_{t+1})}{1+r_t}$$

is a solution for (1), where B_t is the bubble component of the price. Equation (4) places restrictions on the stochastic process that produces the bubble, so that it is consistent with Equation (1), the asset pricing equation, and is therefore also consistent with rational expectations.

These equations indicate that the bubble component in the price of homes can be estimated as the difference between the actual price and the fundamental price. To be exact, assuming that measurement errors add white noise to Equation (3), the bubble component is measured as the autoregressive part of the difference between P and P^f. The autoregressive process is derived from Equation (4).

This analytical framework is attractive for examining for the existence of a bubble because in our view a bubble cannot exist in rent. In general, the bubble component results

[14] For example, see Blanchard (1979), Flood and Garber (1980), and Flood and Hodrick (1990).

from inertia of expectation of a rise in **asset** prices, independently of market fundamentals, and a rational investor will be willing to pay a higher price for an asset in the expectation of future capital gains, even if the dividends (the rent) from it are low. On the other hand, a tenant cannot generate profits from a rent increase; if the rent in the economy rises, this reflects a shortage of housing services compared with the existing supply. The willingness of tenants to pay higher rent reflects the benefit that they derive from **the housing services** that the home that they rent gives them.

Based on the above methodology, we use three different methods to measure the deviation of the actual price from the fundamental price. The first method measures the fundamental price directly from actual rent figures and the yield curve. The second method uses the Kalman filter to derive the bubble component according to Equation (4). The third method estimates an econometric equation for the ratio of the price of a home to the rent, where the residual obtained from the estimation gives an indication of the existence of a bubble in home prices.

a. Limitations of the analysis

Equations (1) through (4) provide a simple theoretical basis for analyzing house prices. This framework may be too simple, and it is therefore worthwhile discussing, at least briefly, the limitations of the analysis. The main assumptions in the above framework are risk neutrality, ignoring maintenance and depreciation costs, and purchasing with no leverage.

Risk: Risk has two contrary effects on house prices, and the direction of the bias resulting from the assumption of risk neutrality, if any, is therefore unclear. For a risk-averse *investor*, risk increases the cost accompanying asset ownership, and the price that he is willing to pay will therefore be less than that offered by a risk-neutral individual. The effect of this factor is to make the bubble component measured using Equation (1) smaller than its actual size. On the other hand, homes are also used for housing, not merely as an asset. For a person purchasing a home for housing purposes, home ownership hedges the risk resulting from fluctuations in rent and the costs of an unexpected move in living quarters.[15] For those purchasing homes for use as living quarters, a category that includes most of the buyers in the market, home ownership also has a risk-moderating effect, and they will therefore be willing to pay a premium for home ownership. This factor causes the bubble component measured using Equation (1) to be larger than its actual size.

Note that in the econometric estimation, presented in Section 6 below, variables reflecting the risk, relative to other assets, are added to the regression equation. The direct measurement and Kalman filter, however, rely on the risk-neutrality assumption.

Depreciation and maintenance costs: Depreciation reduces the return on home ownership, and therefore lowers the price that an investor will be willing to pay for a given home. This factor therefore makes the bubble component derived from Equation (1) smaller than its actual size. With the Kalman filter and the econometric estimation, however, the

[15] See Sinai and Souleles (2005).

equations include an intercept (see the description below), which is used to capture the bias at the price level.[16]

Leverage: Equation (1) assumes that the equity of the home buyers pays completely for the purchases. In practice, of course, many home buyers are obliged to resort to mortgages to finance their purchase. Since mortgage interest is usually higher than the alternative yield for a home buyer (the discount rate), taking a mortgage incurs a cost equal to the interest rate spread. A higher mortgage interest rate means a higher transaction cost for the buyer, and the price that he will be willing to pay for it will therefore be lower. It therefore follows that this factor causes the bubble component estimated using Equation (1) to be lower than its actual size. We will later expand the basic asset pricing equation, and use the direct measurement to evaluate the sensitivity of the results to leverage.

4. DIRECT MEASUREMENT OF THE FUNDAMENTAL PRICE

In this approach, we measure the fundamental price using actual rents and yield curves according to Equation (2) above. We denote the discounted value of actual rent as $\widetilde{P}^f$, i.e.:[17]

(5) $$\widetilde{P}_t^f = \sum_{s=0}^{\infty} \gamma_{t,t+s} RR_{t+s} .$$

From Equations (2) and (5), we obtain $P_t^f = E_t(\widetilde{P}_t^f)$, and under rational expectation, the gap between the actual price, $\widetilde{P}_t^f$, and the expected price, $E_t(\widetilde{P}_t^f)$, is therefore white noise, i.e.:

(6) $$\widetilde{P}_t^f = P_t^f + u_t \qquad u_t \sim WN .$$

From Equations (1) and (6), we derive that the asset price is composed of its fundamental ex post price, $\widetilde{P}_t^f$, the forecasting error, and the bubble component:

(7) $$P_t = \widetilde{P}_t^f - u_t + B_t .$$

Since u_t is white noise, a persistent deviation of P_t from $\widetilde{P}_t^f$ gives an indication of the existence of a speculative bubble, as derived from Equation (4).

a. Measuring the fundamental price of houses in Israel

Equations (1) through (7) provide the theoretical framework for measuring the fundamental price of houses and its deviations from the actual price. In particular, for measuring $\widetilde{P}_t^f$, we use the actual rent starting from time *t*, with interest rates from the smoothed forward

[16] The estimates do not take into account fluctuations in depreciation over time, but their level is adjusted through the constant in the equation.

[17] Because this expression involves an infinite summation, it cannot be calculated directly from the data. For dealing with the endpoint for measurement purposes, see Section 4.1.

yield curve of government bonds.[18] The sample period is from January 1996 until August 2010; the selection of the sample period was dictated by data availability.

Equation (5) discounts rent payments in an infinite horizon; in order to make it operative, we therefore make assumptions about the development of rent beyond the end of the sample period, and about the interest rates in the distant future. We estimate the future rent beyond the most recent available observation using an ARMA estimation process (see Appendix 1 for details).[19] Similarly, we use yield curve data for a horizon of up to 15 years. Beyond that, we assume that the curve is flat. These assumptions make the asset pricing equation operative, and the fundamental price is calculated using:

$$(5') \qquad \hat{P}_t^f = \sum_{s=0}^{T} \gamma_{t,t+s} RR_{t+s} + \sum_{s=1}^{K} \gamma_{t,T+s} RR_{T+s}^{est.} + \gamma_{t,T+K+1} \frac{RR_{LR}^{est.}}{r_{LR}},$$

where T is the time index of the most recent available observation, $RR_{T+s}^{est.}$ is the expected rent on date $T+s$ derived from an ARMA process, $RR_{LR}^{est.}$ is the expected long-term value, r_{LR} is the long-term real interest rate, and K is the forecast horizon from the ARMA process. We arbitrarily set K at 50 years. The last element in Equation (5') discounts a console obtained starting on date $T+K+1$.

Estimating Equation (5') gives a criterion for comparing changes in actual house prices with the changes in the fundamental price derived from the expected real interest rate and rent. A price rise that is inconsistent with the expected development of rent and real interest indicates the development of a bubble.

Figure 6A depicts the series of actual house prices against the fundamental price as measured from Equation (5'), and Figure 6B displays the bubble component calculated as a percentage of the deviation of the actual price from the fundamental price. It is clear from the graph that in the first part of the sample, following the years of mass immigration, the actual price was higher than the fundamental price for a prolonged period, but the gap between them continually shrank as a result of both a drop in the actual price and a rise in the fundamental price, starting in mid-2002 at least. The gap was eliminated in mid-2006, and actually changed direction in the following two years, as reflected in a negative deviation of almost 30 percent. During the final sample years, it was clear that the actual price did not significantly deviate from the fundamental price. At the endpoint in August 2010, the actual price was slightly lower than the level derived from the market fundamentals, at least according to this measurement.

[18] Shiller (1981) and Flood and Hodrick (1986) used a similar approach for estimating the fundamental price in the American stock market. They used actual dividends as an estimate of the dividend expectations in the preceding periods.

[19] In principle, a forecast for rent can be created through more complex econometric estimation that takes into account additional factors affecting rent beyond its own lags. The difference in the forecasts derived from such estimation will be primarily in the period close to the end of the sample period, and less in the long-term estimates. It therefore appears that use of an autoregressive process is not reflected in significant differences in results in comparison with a more complex estimation.

Figure 6A
The Real Price of Houses and their Fundamental Price (direct measurement)

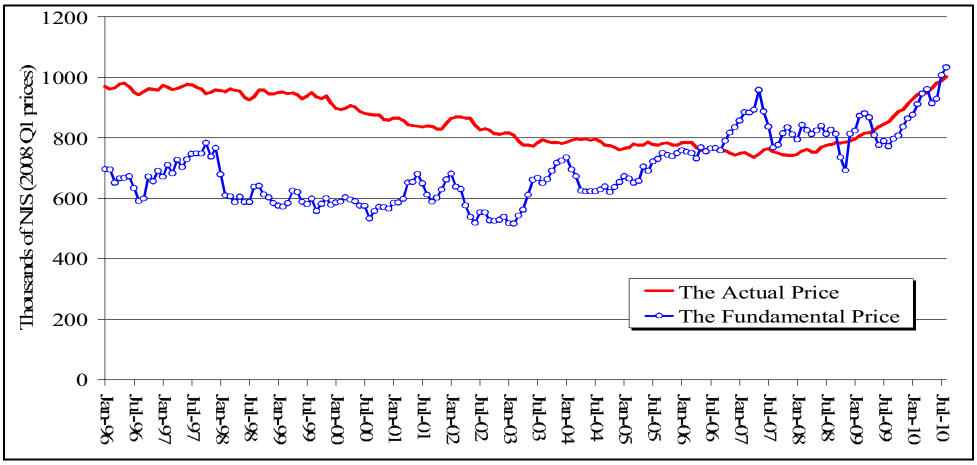
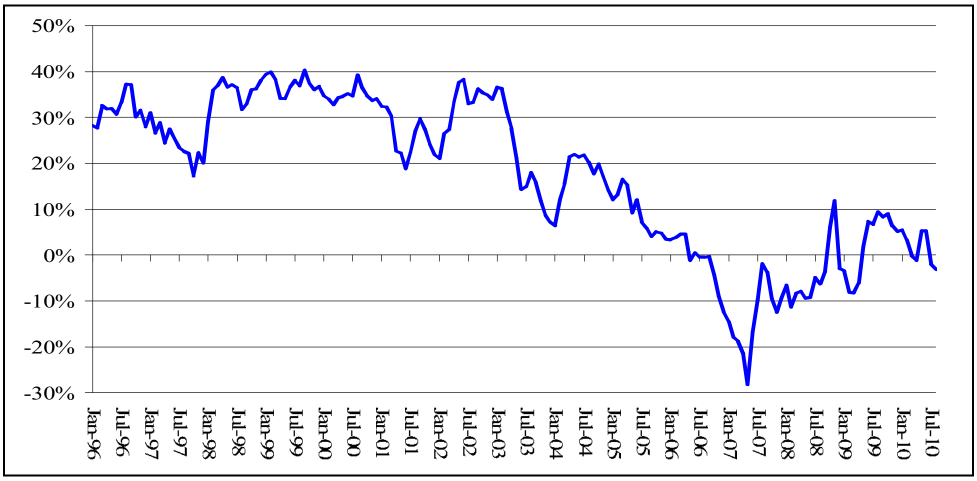


Figure 6B
The Bubble Component in House Prices (direct measurement)



We emphasize that measurement of the fundamental price in this section of the article does not use the actual price figures; the estimate is based solely on rent and interest rates. In the results for recent years, the proximity of the actual price to the fundamental price is therefore not due to an attempt to fit the model to the data, as usually occurs in econometric estimation.

It is clear from the graphs that when the prices were higher than the fundamental price, their downward adjustment process took over a decade, while when the fundamental factors supported a price increase in 2009-10, prices rose rapidly. It is possible that this finding indicates downward stickiness in house prices.

b. The interest rate and rent

Figure 6A clearly shows that the series of the fundamental component of house prices is more volatile than the series of actual prices over the entire sample period. The volatility of the fundamental price is obviously due to the volatility of its components: the yields for various horizons and rent. An analysis of the development of the components shows that most of the volatility in the fundamental price during the sample period resulted from fluctuations in the real long-term interest rate. Short-term interest rates and rent had less influence, particularly rent, whose effect on the volatility of the price was almost negligible. As a rule, rent primarily determined the price level, at least insofar as the fluctuations in it are temporary and do not reflect a permanent change in the level (or trend, if one exists). Figure 7A displays the fundamental price in comparison with the real 15-year forward interest rate. The graph shows a clear negative correlation between the two series: the correlation coefficient is –0.95 during the sample period. In general, the absolute value of the correlation coefficient of shorter-term interest rates is smaller, although the relationship is not monotonic. The correlation with the 1-year interest rate (the shortest-term interest rate we used) is of particular interest, since it is most directly influenced by the Bank of Israel interest rate. The correlation coefficient between it and the fundamental price was lower in absolute value than the correlation with the long-term interest rate, although its value, -0.79, is not negligible. Figure 7B displays the fundamental price in comparison with rent; rent has a positive impact on the fundamental price as derived from Equation (5’), but for the sample period as a whole, its correlation coefficient is –0.34. This result indicates that most of the fluctuation in the fundamental price is due to interest rate fluctuations, particular those of long-term interest rates.

Figure 7A
The Fundamental House Price (direct measurement) and the Real 15-Year Forward Interest Rate

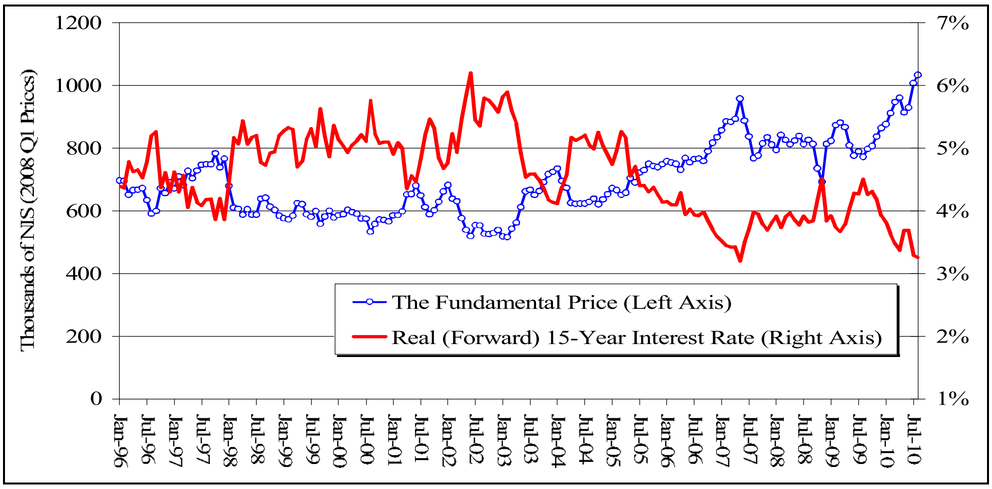


Figure 7B
The Fundamental House Price (direct measurement) and Rent

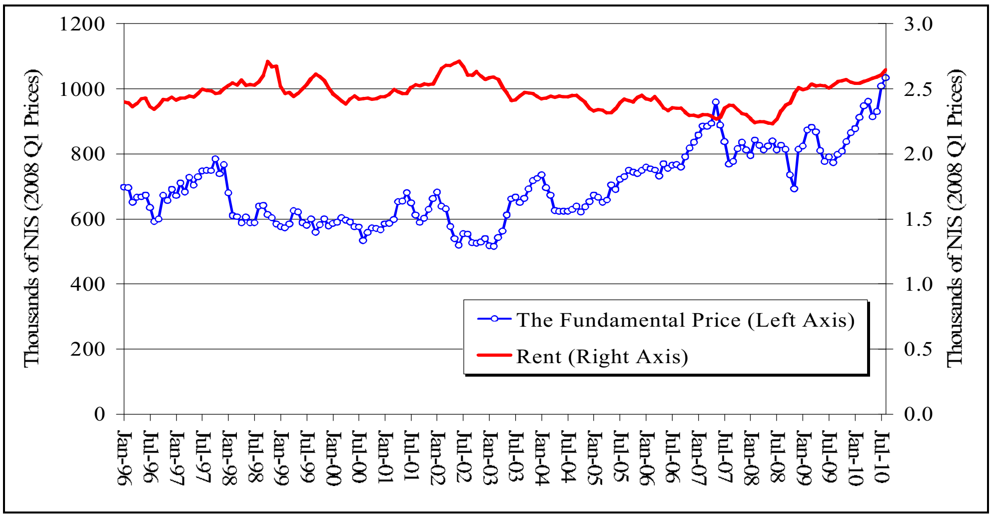
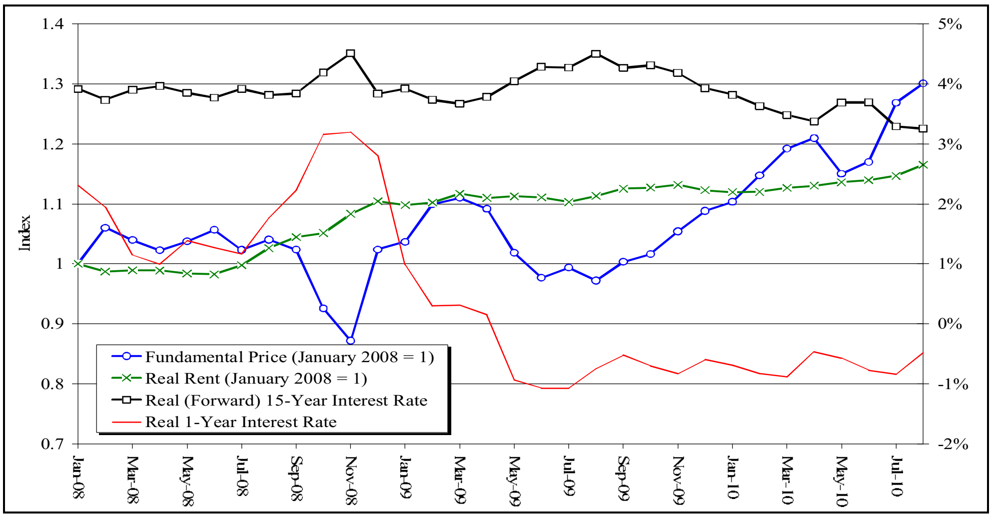


Figure 8
The Fundamental Price, Rent, and Interest Rates Starting in January 2008



It is interesting to examine the correlation between the variables starting January 2008, both because from this point onwards, housing prices began to rise, and because during this period, the fundamental price does a good job of explaining the actual prices. The yield curve remained a dominant factor during this period, but less so than for the entire sample period. In particular, the short-term part of the curve has less effect. The rise in real rent, especially at the beginning of the period, also supports the rise in the fundamental price, and starting in January 2008, the correlation coefficient between the variables is positive 0.45 (in contrast to the negative correlation coefficient during the sample period as a whole).

The 1-year and 15-year interest rates, rent, and the fundamental price starting in January 2008 are displayed in Figure 8. The graph clearly shows the negative correlation between the long-term interest rate and the fundamental price, and the rising trend in rent that supports the price increase. On the other hand, the prices became unrelated to the short-term interest rate, at least in the second half of the period, starting in mid-2009, when the steepest price rise occurred. A significant drop in the long-term interest rate began in mid-2009, which supported a rise in prices, while the short-term interest rate remained almost unchanged during this period.

To conclude this section, we evaluate the effect of monetary policy, i.e., the cutting of the short-term interest rate in the wake of the global crisis, on the fundamental price. The Bank of Israel interest rate began to fall in October 2008 in response to the global crisis; we ask how the fundamental price would have behaved had the interest rate not fallen. The Bank of Israel interest rate affects mostly the short-term part of the curve; it has less effect on the long-term part, and it is reasonable to assume that the long-term real interest rate would have fallen in any case, given the decrease in the corresponding returns around the world, even if the Bank of Israel interest rate had remained unchanged. In order to estimate the effect of monetary policy, we calculate the fundamental price while setting the short-term side of the yield curve from October 2008 until the end of sample period at its average level during the year preceding the interest rate cut. Since it is difficult to estimate the exact range in which the Bank of Israel interest rate has a significant effect on the real return, we fixed the yields up to a relatively long 10-year range, and in our opinion, the resulting estimate is an upper bound for the effect of the policy.

Figure 9
The Fundamental Price – What If the Interest Rate Had Not Fallen?

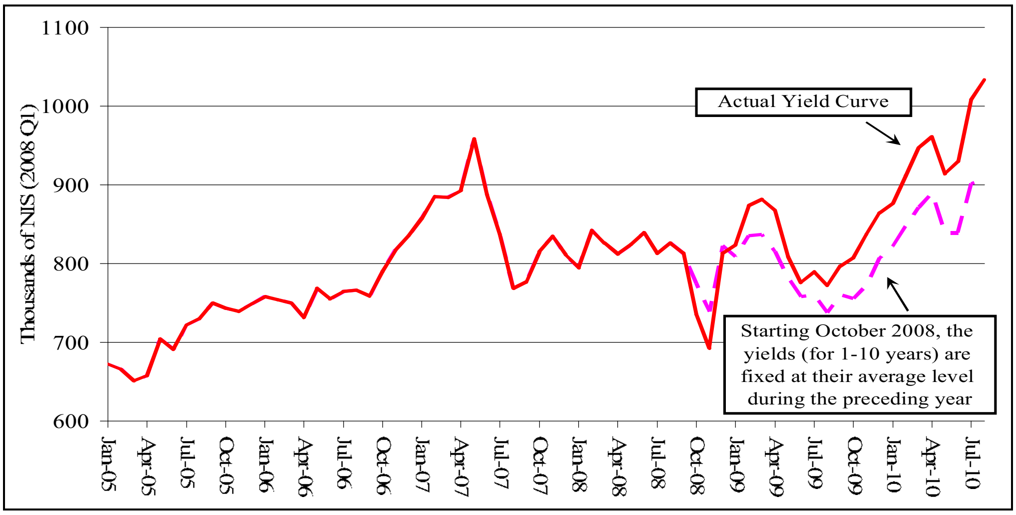
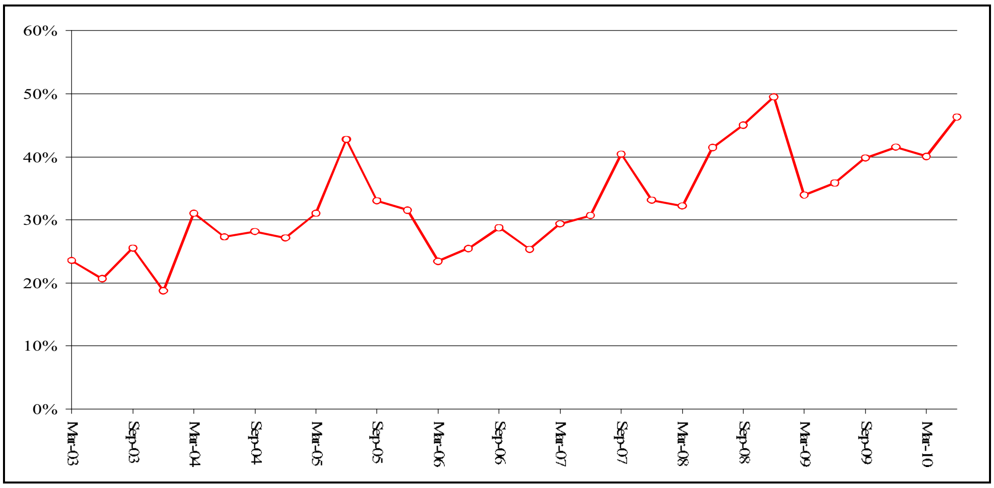


Figure 9 displays the result of the exercise. It shows that it took about one quarter for the short-term interest rate to begin affecting the fundamental price; starting in January 2009, the contribution of the short-term interest rate to the price continually increased. The estimate derived from this measurement is that by the endpoint of the sample in August 2010, the cumulative contribution of monetary policy to the real price was up to 14 percent.[20]

c. Leverage

The rate of leverage among home buyers has risen in recent years (Figure 10). When the mortgage interest rate is higher than the alternative interest rate for the investor, the cost of buying a home rises as a result of financing expenses, and the price that an investor will be willing to pay for a given home is lower. A rise in the leverage rate increases the financing expenses and therefore also the fundamental price of housing. Up until now, the analysis has ignored leverage; the asset pricing equation, Equation (1), on which the analysis is based, assumes that the investor has all the capital needed to buy a home, or alternatively that the mortgage interest rate that he must pay is equal to the interest rate on savings. It is of interest to test the sensitivity of the results to this assumption.

Figure 10
The Average Rate of Leverage of Home Purchasers (2003 Q1-2010 Q2)



* The leverage rate is calculated as the ratio of total mortgages granted to the value of the homes purchased. The value of the homes is calculated as the product of the average transaction value by the number of transactions during the period.

[20] Fixing the real interest rates for periods of 1-5 years (instead of up to 10 years) gives almost the same results, except for the last four months of the sample, in which the medium-term interest rate (6-10 years) contributed almost 4 percent to the fundamental price. According to this measurement, the cumulative contribution of monetary policy to the price was about 10 percent.

(i) Methodology

We use λ to denote the leverage rate, and $-r^m$ to denote the real interest rate on mortgages. The amount that a risk-neutral investor will be willing to pay out of his pocket for a home, $(1–λ)P$, equals the net present value resulting from owning it. This value is composed of rent and the discounted value of the expected price of the home in the coming period, minus the discounted value of the mortgage payments during the coming period:

$$(8)\quad (1-\lambda)P_t = RR_t + \frac{E_t(P_{t+1})}{1+r_t} - \frac{(1+r_t^m)\lambda P_t}{1+r_t}.$$

Reordering the equation gives:

$$(9)\quad \begin{aligned} P_t &= \frac{1+r_t}{1+\tilde{r}_t} RR_t + \frac{E_t(P_{t+1})}{1+\tilde{r}_t} \\ 1+\tilde{r}_t &\equiv (1-\lambda)(1+r_t) + \lambda(1+r_t^m) \end{aligned}.$$

From this expression, it is easy to see that if $\tilde{r}_t = r_t$, then Equation (9) is identical to Equation (1). Equality of interest rates is obtained in two cases: one is obviously when there is no interest rate spread, i.e., when $r^m=r$, and the other is when there is no leverage, i.e., when $\lambda=0$. These two cases do not exist in reality; in particular, the mortgage interest rate is usually higher than the available interest rate on savings, and therefore typically $\tilde{r}_t > r_t$. As a result, the fundamental price derived from Equation (9) is typically **lower** than that obtained from Equation (1). The fundamental price is now given by:

$$(10)\quad P_t^f = E_t\left\{ \sum_{s=0}^{\infty} \tilde{\gamma}_{t,t+s} \frac{1+r_{t+s}}{1+\tilde{r}_{t+s}} RR_{t+s} \right\},$$

where $\tilde{\gamma}_{t,t+s} \equiv \Pi_{j=0}^{s-1}(1+\tilde{r}_{t+j})^{-1}$ for $s \geq 1$, $\tilde{\gamma}_{t,t} = 1$.

As in the above analysis, here too the fundamental price is measured using actual rent and the entire yield curve. In addition, however, we use the mortgage interest rate for various horizons and the leverage rate.[21]

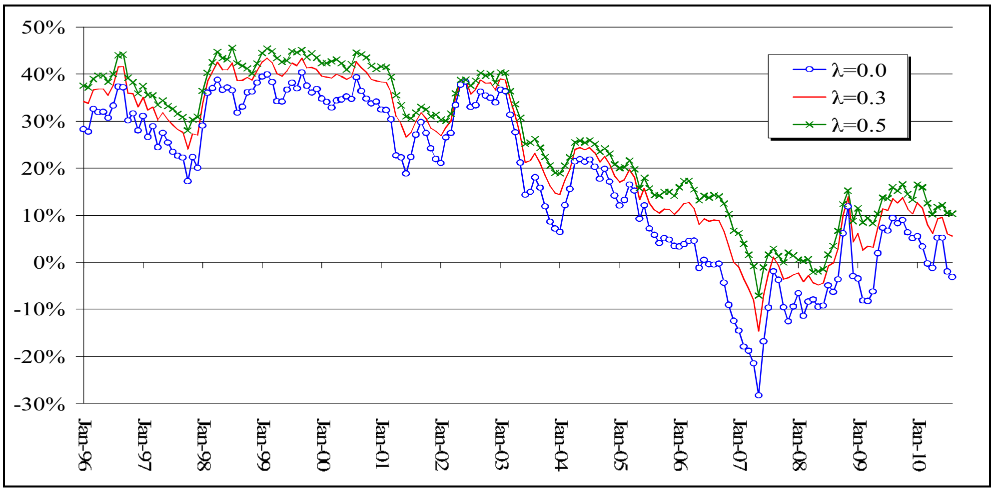
(ii) Results

We estimated the bubble component for various leverage rates, where the relevant range for the Israeli economy is 20-50 percent, as can be seen in Figure 10. Figure 11 displays the

[21] Equation (10) uses the forward interest rate (as derived from the discounting coefficient $\tilde{\gamma}_{t,t+s}$); in order to make the equation operative, the mortgage interest rate should therefore be turned into the forward interest rate. In practice, data exist for the real mortgage rate for periods of up to five years, 5-12 years, 12-15 years, 15-17 years, 17-20 years, and over 20 years. We derive the forward interest rate using the Nelson-Siegel-Svensson Model. See Nelson and Siegel (1987) and an extension of their model in Svensson (1994).

results for leverage rates of 30 percent and 50 percent and also for the case of no leverage, for the sake of comparison to the results presented until now. As expected, the bubble component is higher for higher leverage rates, because the financing costs (which are higher than the yield on savings) lower the fundamental price. At the same time, the differences are not large, and are not enough to alter the conclusions of the analysis with no leverage.

Figure 11
The Bubble Component in House Prices for Various Leverage Rates
(direct measurement, January 1996-August 2010)



5. ESTIMATION USING A KALMAN FILTER

a. The econometric methodology

As noted above, Equations (1) through (4) provide a theoretical framework for estimating the bubble component in house prices. Based on these equations, in this section of the article, we estimate B_t using a Kalman filter (KF) as an unobservable variable, while using the data for house prices, rent, and the interest rate.[22]

Using KF requires writing a system estimated in State Space Form (SSF), i.e., writing a system of equations as a backward looking linear system, with a description of the stochastic process of the unobservable variables. It is therefore necessary to make the system of equations linear, solve for the expectations (for rent and future interest rates), and

[22] Burmeister and Wall (1982) used KF to identify a bubble in asset prices in their examination of hyperinflation in Germany in the 1920s, and Wu (1995, 1997) used KF in an examination of foreign currency markets and the American stock market.

express the equations as a system in reduced form. The following sections briefly present the approach we adopted in order to obtain a system of equations in SSF.

Linearity

In order to obtain a linear system, we use a log-linear approximation for Equation (1). This approximation is carried out around a fixed ratio of the price of a home to rent, representing the long-term ratio of these prices. From this equation, we derive the log-linear version of Equations (2) through (4) presented above.

A log-linear approximation of Equation (1) yields:

(1’) $$\log(Q_t^H) \cong \tilde{\kappa} + \frac{\kappa}{1+\kappa} E_t[\log(Q_{t+1}^H)] + \frac{1}{1+\kappa}\log(RR_t) - \frac{\kappa}{1+\kappa}\log(1+r_t)$$

$$\tilde{\kappa} \equiv \log(1+\kappa) - \frac{\kappa}{1+\kappa}\log(\kappa) \qquad \kappa \equiv \frac{\Pi^{RR}}{1+r}\cdot\frac{Q^H}{RR},$$

where κ is a parameter expressed as a function of the long-term values of the ratio of the price of a home to the rent, Q^H/RR, the real interest rate is r, and the gross rate of change of the real rent is Π^{RR}.[23]

A log-linear expression for the fundamental price can be derived from Equation (1’):

(2’) $$\log(Q_t^{H,f}) = \tilde{\kappa}(1+\kappa) + \frac{1}{1+\kappa}\sum_{s-0}^{\infty}\left(\frac{\kappa}{1+\kappa}\right)^s E_t\left[\log(RR_{t+s}) - \kappa\log(1+r_{t+s})\right],$$

where Equation (2’) assumes that $\lim_{s\to\infty}\left(\frac{\kappa}{1+\kappa}\right)^s E_t\left[\log(Q_{t+s}^H)\right] = 0$.

Here too, the dynamic equation, Equation (1’), has multiple solutions, and the fundamental price as presented in (2’) is only one of them; every solution of the form:

(3’) $$\log(Q_t^H) = \log(Q_t^{H,f}) + \log(B_t)$$

(4’) $$\log(B_t) = \frac{\kappa}{1+\kappa}E_t\left[\log(B_{t+1})\right],$$

solves Equation (1’). Note that $\log(B_t)$ measures approximately the percentage of deviation of the price of houses from the fundamental price.

A solution for the expectations

The fundamental price in Equation (2’) is a function of the expectations for rent and interest rates in the future. In order to express the expectations in terms of observable variables, we assume for the sake of simplicity that these variables are determined in an *ARMA*(*p,q*)

[23] In the event that the relative price of rent is constant in the long term, then Π^{RR}=1. We take no position with respect to the size of Π^{RR}, since we estimate κ, which contains additional long-term values.

process. From such a process, the expectations of the variables can be easily expressed for any future horizon, given the present data.

In addition to expectations for the fundamental variables, we want to express the bubble as an autoregressive process. Moving Equation (4') one period backwards and reordering it yields:

(4'') $$\log(B_t) = \frac{1+\kappa}{\kappa}\log(B_{t-1}) + \eta_t$$

$$\eta_t \equiv \log(B_t) - E_{t-1}\left[\log(B_t)\right],$$

where η_t is the forecast error of the bubble component. Given rational expectations, η_t is white noise that is not correlated with variables from the preceding periods.

The SSF

The system of equations in SSF is composed of two types of equations: measurement and transition. The measurement equations describe the development of the observable variables: the price of housing described in the asset pricing equation – Equation (1'), and the rent and real interest rate described as ARMA processes. The transition equations describe the development of the unobservable variables: the bubble component in the house price described in Equation (4'') and the shocks in the MA processes in the rent and real interest rate equations.[24]

b. The data and results

For the purpose of estimation, we use three data series: real house prices measured on the basis of an **owner occupied house prices index**, net of the Consumer Price Index, real rent measured through the owner occupied **housing services** item in the Consumer Price Index, net of the general index,[25] and the real interest rate on government bonds. The obvious question is which interest rate to use, the short-term or long-term rate.

From Equation (2), we see that the relevant interest rates for the analysis are those obtained from the entire yield curve, meaning interest rates for all horizons. On the other hand, the model formulation in Equations (1') through (4') requires only one interest rate, from which the entire curve is constructed. It is also clear that the estimated fundamental price will tend to be lower if we use a higher level of interest, and the bubble component obtained will therefore be greater. Since long-term interest rates are typically higher than short-term interest rates, the choice of which series to use will affect the results. For these reasons, we chose to run the model twice: once using a short-term interest rate – the 1-year

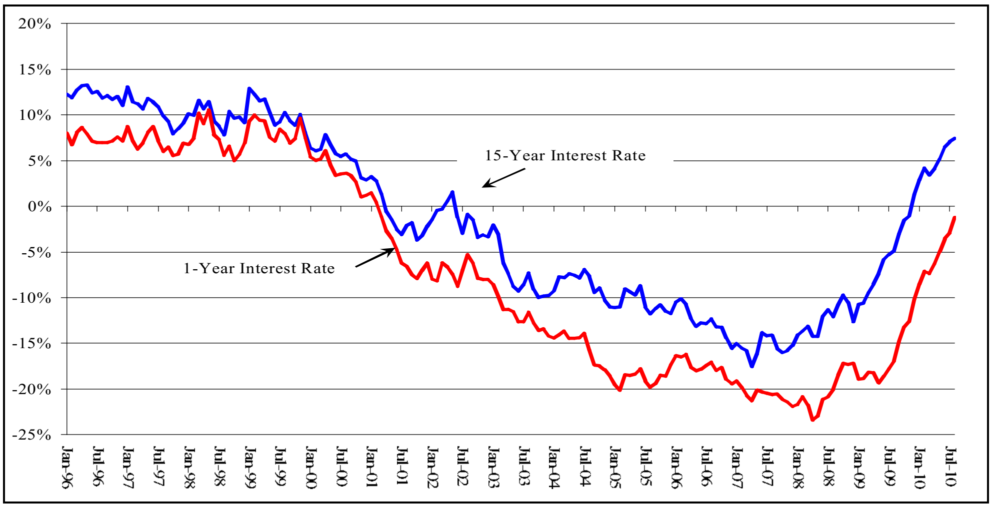
[24] The full formulation of the system of equations can be obtained from the authors upon request.

[25] The owner occupied housing services item measures the rent in new and renewed leases. These data are available only from 1999 onwards. For the period before 1999, we use the rent item that measures the average rent in existing leases, i.e. the item for month *t*, including leases mostly signed during the year preceding time *t*. The ARMA estimation is based solely on new leases, but the coefficients were used for the entire sample period.

rate, and a second time using a long-term interest rate – the 15-year rate. In this way, we obtained a range of estimates for the bubble component.

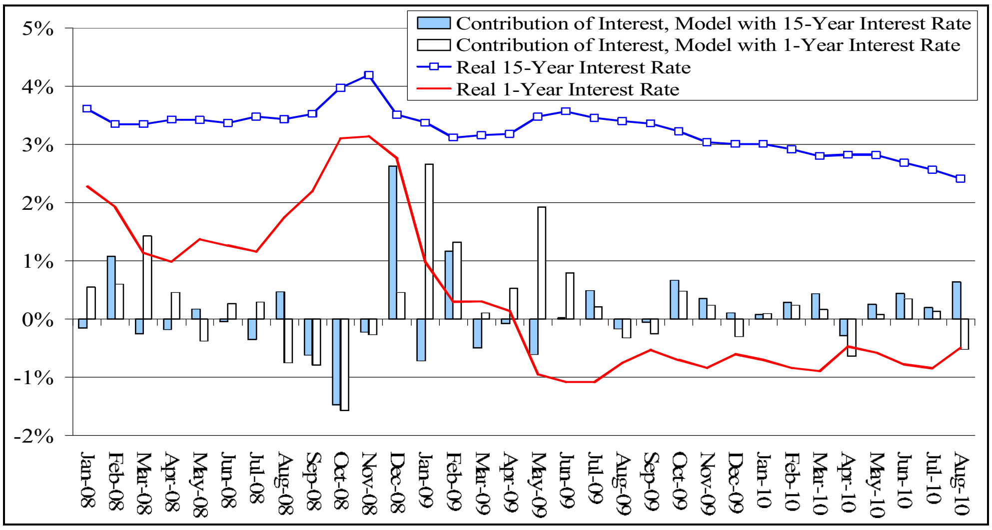
As noted, the rent and interest rate series are presented in the model as ARMA processes. The estimation results are displayed in Appendix 1.[26] Figure 12 displays the Kalman filter estimates for the bubble component from the model with a short-term interest rate in comparison with a long-term interest rate. The graph indicates that at the beginning of the sample period, house prices were higher than the prices derived from the model, but the gap between them narrowed, and became negative already during 2001, five years before the estimate obtained from direct measurement. The gap reached its peak between the second half of 2007 and the first half of 2008, at the beginning of the recent surge in prices. At their lowest point, the prices were 20 percent lower than the price derived from the model. According to the Kalman filter estimates from early 2008, prices rise at a faster pace than that derived from the market fundamentals, and the bubble component shrinks accordingly. By the end of the sample period, actual prices range from 1 percent below to 7 percent above the fundamental price. In view of this pattern, the recent price surge appears to be nothing more than a correction of the price downturn in real terms during the past decade, and a closing of the gap with the fundamental price.

Figure 12
The Bubble Component in House Prices in Kalman Filter Estimation



[26] We used the same estimate for rent in the direct measurement.

Figure 13
Contribution of the Interest Rate to the Change in the Fundamental Price



The effect of interest

As a result of the linearity of the model, the estimation framework in this section makes it easy to calculate the effect of each variable on the fundamental price. Figure 13 displays the contribution of the interest rate to the change in the fundamental price during the past three years for the two estimations we ran. It is of particular interest to examine the effect of the interest rate starting in October 2008, when monetary policy became expansionary.

It is evident from the graph that the short-term interest rate contributed to the price rise in the first months after it was cut, until June 2009. During this period, the interest rate contributed a cumulative 6.0 percent to the real price. Afterwards, when it was stabilized, its contribution to the price was negligible (actually, at the end of the sample period, its contribution was slightly negative). When the model was estimated using the long-term interest rate, the overall estimate was 3.6 percent lower. As a result of the continual lowering of the long-term interest rate since mid-2009, however, a large part of the contribution to the price accumulated at the end of the sample period. This result is consistent with the findings from the direct measurement, which showed that most of the contribution of interest to prices was about a year after the beginning of monetary expansion, although the estimate for the contribution from the Kalman filter estimation was much lower than that obtained through the direct measurement.

6. EMPIRICAL TESTS FOR THE YIELD ON HOUSING – THE RATIO OF RENT TO HOUSE PRICES

In this section, we present additional empirical tests that evaluate the behavior of the ratio of rent to housing prices. This ratio constitutes an indicator of the yield on housing, in accordance with the asset-pricing equation presented in Section 3 of this article. We first present the behavior of this ratio in comparison to the trend that characterizes it, and then examine the factors affecting it and its deviation from the trend. This analysis rests on a methodology that expands the asset-pricing equation upon which we have hitherto relied. An examination of the divergence in the ratio of rent to the price from the expected ratio provides further indications regarding the existence of a bubble in the housing market. The examinations show that a deviation from the trend in the ratio of rent to the price of housing is distinguishable; however, at least according to some of the estimations, this deviation is mostly explained by short-term factors, particularly the interest rate.

a. A comparison of the ratio of rent to house prices to its trend

A number of articles that examine the development of house prices, and others that analyze the development of prices in the capital markets detect the existence of booms and busts using statistical methods that test the development of the target variable over time with respect to its long-term trend.

Bordo and Jeanne (2002) present a very simple method that examines whether the average of the three most recent periods deviates significantly from the long-term average. The degree of deviation that will be considered large depends on the standard deviation of the series, and is determined by calibration. Another method concerns the stationarity of the ratio of rent to the price of a home; Taipalus (2006) examines whether a unitary root exists in this ratio. Assuming that rent and the interest rate are stationary variables, the existence of a unitary root in the ratio is consistent with the presence of a bubble. The problem with this method stems from the short samples examined each time, while the trait of stationarity characterizes a variable in the long term, and is difficult to test using short samples. For this reason, that article also did not use the common tests for stationarity, and the approach utilized was much simpler.

The method we chose to use here examines the development of the above-mentioned variable with respect to the trend calculated using an HP filter. Note that this approach does not identify the existence or non-existence of a bubble, because it does not examine the causes of the deviation of the price from the trend. The deviation from the trend can be explained by fundamental economic reasons; deviation from the trend that is not explained by such factors can be suspected as a bubble, in the event that it is "big enough". In the second part of the section, we examine the factors affecting the deviation from the trend. This method is vulnerable to criticism for arbitrariness in the selection of parameters, but it can provide support for the results obtained from other methods.

Adalid and Detkin (2007) and other articles present a very similar analysis method. For each point in time examined, a very smooth trend is generated (the selection of a smoothing parameter of 100,000 is common, in contrast with the 1,600 accepted in quarterly data) calculated on the basis of the data until that point. The next step is testing whether the actual price of the variable being tested deviates from the trend with the addition of a confidence interval based on the standard deviation. The results of an analysis of this type depend on a large number of parameters, the selection of which is not anchored in theory: for example, the exact formulation of the variable being tested, the degree to which the trend is smoothed, the length of the period according to which the trend is constructed or whether it is calculated on the basis of the entire sample (i.e., a fixed trend line for the entire sample), and the size of the confidence interval, relative to the standard deviation. We conducted a number of sensitivity analyses for these parameters, and in general it can be stated that the results are not sensitive to the exact formulation of the dependent variable (the logarithm of the ratio, the inverse of the ratio), but they are more sensitive to fixing the length of the period for which the moving trend is calculated and the smoothing parameter. Nevertheless, the evaluation for the most recent period remains intact for different definitions.

b. A sensitivity test for the selection of the smoothing parameter

In calculating an HP trend, a single parameter determines the degree of smoothing of the estimated trend line. We use λ to denote the smoothing parameter. The larger λ the smoother the trend and the closer it will be to a linear trend. A smaller λ means that the smoothed series will be closer to the original series, and the deviations of the original series from the derived trend will be smaller. Selection of the value of λ depends on the frequency of the data and the length of the cycle characterizing the variable being tested. For quarterly data, it is common to use λ=1,600; however, if the data feature a longer cycle, and when we want to describe longer-term trends, we will use a larger parameter.

An examination of the ratio of rent to house prices over almost four decades shows that this ratio has relatively long cycles of 10-15 years (Figure 1). Maravell and Del Rio (2001) examine the selection of λ for data with different frequencies and cycle lengths. The table that they attach to their article shows that for variables with long cycles, it is best to select a λ larger than the accepted 1,600 for quarterly data. In particular, for cycles of around 15 years, λ is in the neighborhood of 10,000. Since there is no definite rule for selecting the smoothing parameter, we tested the deviation of the ratio of rent to house prices from its trend by using three alternative parameters for the HP procedure: 1,600 – as usually accepted for quarterly data; 10,000 – which is appropriate for long cycles; and 100,000, which was selected in a number of articles dealing with identification of boom and bust periods in asset prices.

Figure 14
The Deviation of the Ratio of Rent to the House Price from HP Trend
Moving Period of 60 Quarters, Confidence Interval of 1 Standard Deviation

λ = 100,000 λ = 10,000 λ = 1,600

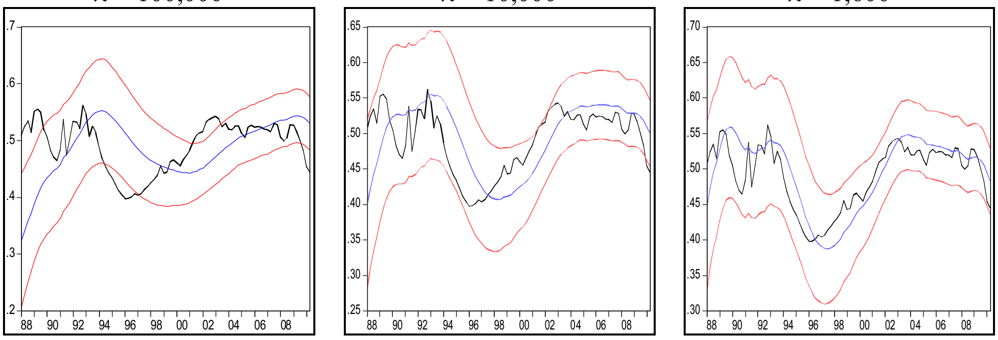
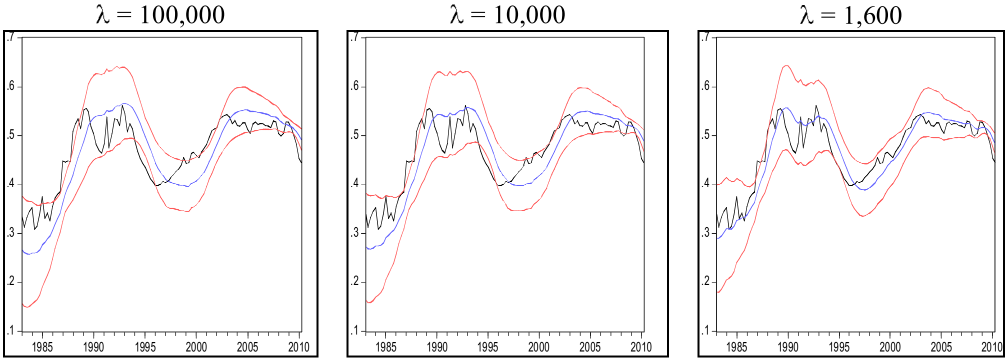


Figure 14 displays the HP trend calculated for the ratio of rent to the price of houses for 60 periods (15 years) up until the examined point in time, with three possible smoothing parameters, and with one standard deviation around the trend. The graphs show that a larger smoothing parameter means a smoother trend, with fluctuations in the measured ratio tending to deviate outside the defined confidence interval. For λ=100,000, an upward deviation can be seen, i.e., rent is high, relative to the price of houses (or the price of houses is low, relative to rent) at the end of the 1980s and the beginning of the millennium. On the other hand, high house prices, relative to rent, are obtained in the mid-1990s and in the past year. The picture obtained for λ=10,000 is similar, although the deviation remains outside the confidence interval only during the past year. When λ=1,600, the trend is very similar to the development of the ratio actually measured, and there is therefore no deviation outside the confidence interval, although a relatively large deviation from the trend line in early 2010 is visible, even for a low value of the smoothing parameter.

Figure 15
The Deviation of the Ration of Rent to House Prices from HP Trend
Moving Period of 32 Quarters, Confidence Interval of 1 Standard Deviation

λ = 100,000 λ = 10,000 λ = 1,600

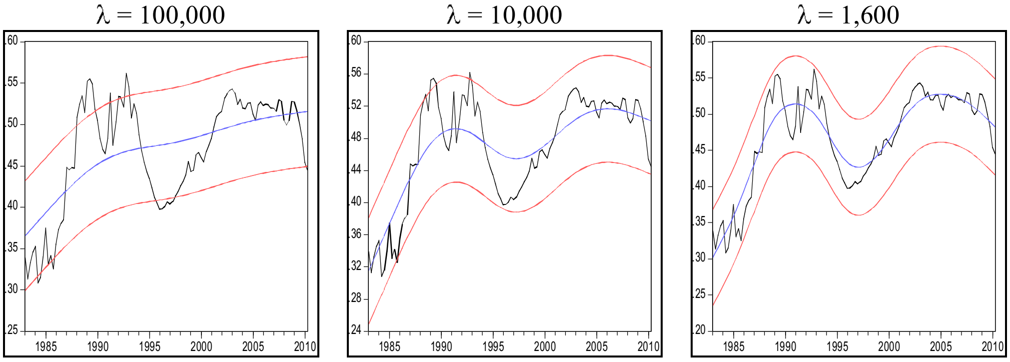


We repeated the exercise for a shorter moving period of 32 quarters (instead of 60). Such a period, 8 years, does not include a complete cycle, but it is long enough to describe a trend in this ratio. The standard deviation calculated for a shorter period is expected to be smaller, and the probability that the actual ratio will deviate from its trend is therefore greater. The picture that emerges from Figure 15 is similar to that in the previous test: a relatively large deviation from the trend in the periods that were also identified for a longer moving sample, in particular a larger deviation than 1 standard deviation in 2010 for all three of the tested parameters. Beyond this, even with a more lenient definition of a confidence interval of 1.65 standard deviations on either side (a 90 percent confidence interval), the first half of 2010 remains lower and outside the confidence interval (not displayed).

Another test we ran was calculating the HP trend on the basis of the entire sample. Such an examination retrospectively evaluates the entire period, given also the development of the series after the period under discussion, and does not require detection of a deviation solely on the basis of the developments known at the time. Here, too, we examined the deviations for three different values of λ. When the HP trend is calculated on the basis of the entire sample from 1983 until 2010, the influence of the choice of smoothing parameter on the HP trend is palpable. Due to the large-scale fluctuations during these years, the standard deviation is very large, and all the fluctuations in the actual ratio of rent to house prices are within the confidence interval, even though at the beginning of 2010, they are on the edge of the confidence interval (Figure 16). When we shorten the sample period, the variance is smaller, and this ratio is above the trend's confidence interval in the recent period for the larger values of λ. It should be mentioned again that this constitutes evidence of a deviation from the trend, not a bubble; this deviation may be explained by economic fundamentals, without any deviation from the ratio explained by those fundamentals. In order to answer the question of whether a bubble exists, we must examine whether economic factors can explain the deviation.

Figure 16
The Deviation in the Ratio of Rent to House Prices from HP Trend According to the 1983-2010 Period, Confidence Interval of 1 Standard Deviation

λ = 100,000 λ = 10,000 λ = 1,600



c. The factors affecting the ratio of rent to the price of houses

(i) Methodology

In order to learn whether the ratio of rent to house prices, or its deviation from the trend of this ratio, reflects the existence of a bubble, we must examine whether it is explained by fundamental factors, or whether the ratio deviates significantly from what can be explained by these factors.

Himmelberg, Mayer, and Sinai (2005), like others in this field, calculated the imputed rent as representing the cost of owning a home.[27] This approach expands the asset pricing equation that we employed in the preceding sections of this study, while stressing the factors that affect the connection between actual rent and short-term house prices. In this approach, the derived rent must be equal to the cost of ownership at equilibrium:[28]

$$(11) \quad \textit{Imputed Rent} = \textit{Cost of Ownership} = P_t \times (r_t + t_t + \delta_t - g_{t+1} + \gamma_t),$$

where P is the present price of a home, r is the risk-free interest rate, t is the tax rate on home ownership, δ is the depreciation rate (or the cost of maintenance), g is the rate of capital gains resulting from the expected change in the price of a home during the period, and γ is the risk premium. All the factors in the parentheses, some of which are unobservable, affect the ratio of the derived cost of home ownership to the price of a home.

Ahuja and Porter (2010) and Ahuja et al. (2010) examine the housing markets in China and Hong Kong by substituting values for all the components of Equation (11), and evaluate whether the actual prices match the implied values from this equation. The difficulty with this method lies in estimating the expected capital gains and the risk premium. Incorrect estimation of these amounts is liable to result in a mistaken conclusion about deviation or a lack of deviation from the fundamental factors. In this paper, we adopt a more general approach by estimating these relationships without forcing identity, due to the difficulty in assessing some of its components.

The relevant <u>interest rate</u> should be that which reflects the cost for the coming year, i.e., the short-term interest rate. However, since the expected price also appears in the equation, and is affected by the expected interest rate in the following period, the annual cost can be presented using the long-term interest rate. If home ownership also involves mortgage payments at some interest rate, and the mortgage interest rate differs from the long-term

[27] McCarthy and Peach (2004), who tested for the existence of a bubble in the US housing market, also adopt a similar approach that includes additional variables, such as the interest rate and household income, in calculating the affordability of purchasing a home, and in comparing its price to the rent. Weeken (2004), who examines the housing market in the UK, also emphasizes the need to consider additional factors, particularly the risk-free interest rate and the risk premium, in explaining the development of the ratio of rent to house prices.

[28] If we write Equation (1) so that the rent is received in period t+1 instead of t (at a monthly or quarterly frequency, the timing of rent payments is not very important empirically), we obtain: $E_t(RR_{t+1}) = P_t(r_t - g_{t+1})$, where $g_{t+1} \equiv E_t(P_{t+1}/P_t) - 1$ is the expected rate of capital gain. From this, it is easy to see that Equation (1) is a special case of Equation (11).

risk-free interest rate, both the mortgage interest rate and the leverage rate should be included. In the estimation, we examine a number of specifications with different interest rates, but we did not take the leverage rate into account.

In Israel, there is no property tax, only tax on sale of a home under certain circumstances, and on a home purchase. The effect of (one-time) taxation, related to a sale or purchase transaction, on the expected yield becomes smaller as the length of time during which the home is owned becomes longer. It would have been possible to also take into account taxation relative to alternative assets, such as financial assets. Because there is a lack of good data on the effective tax rates on various assets over time, we did not take this component into account in the estimation.

Maintenance cost represents the actual cost (renovations and repairs), or the depreciation (in the absence of the necessary renovations). It can be assumed that the depreciation rate of a home is constant over time, and it can therefore be ignored. It is possible that a change in the composition of the homes with respect to their quality and size could affect the average depreciation rate in the economy, but we do not take this into account in the analysis.

The expected capital gains are unobservable. In order to capture their movement over time, it is possible to take into account the variables influencing the **expected** price of houses in the coming period, especially the supply and demand variables. Here, it is possible to take into account the supply of houses (investing in housing construction, inventory, building starts or completions in comparison with population) and its cost – inputs prices, and factors affecting demand – the business cycle (unemployment). In selecting these variables, it should be noted that it will be possible to interpret them as affecting the *future* price, and not (mainly) the current price, because then it will be more difficult to predict what the direction of their effect should be. Use of a change in the lagged price as an indicator of the expected change in the future, as was done in some of the articles, is liable to lead to erroneous conclusions if there is a change in the market conditions or the expectations of one.

The risk premium is also unobservable, but it can be presented through the relative variance of the housing price. Larger fluctuations in the historic yield on other assets, securities for example, mean that a smaller yield on the price of a home will be required. On the other hand, larger fluctuations in house prices, for example as a result of sharp fluctuations in the exchange rate, especially in the past, when the connection between house prices and the dollar was stronger, act to increase the required yield, i.e., to increase the ratio of rent to house prices. At the same time, since a large proportion of homes are also used as residences (not merely as an investment asset), home ownership is also used as a hedge against fluctuations in the price of housing services over time.

The two sides of Equation (11) can be divided by P, giving:

$$(12) \qquad \frac{Rent_t}{P_t} = r_t + t_t + \delta_t - g_{t+1} + \gamma_t \cdot$$

The interest rate is an observable variable, and its effect can therefore be estimated directly. The other components of the equation are unobservable. The expected capital gains should be presented using indicators of the expected changes in supply and demand that will affect the future price of housing, and the risk premium using fluctuations in alternative assets. We therefore write:

$$(13)\qquad \frac{Rent_t}{P_t} = f(\overset{(+)}{r^{long}_t}, \overset{(+)}{x^{expected}_{\sup ply}}, \overset{(-)}{x^{expected}_{demand}}, \overset{(+)}{\sigma^P_t}, \overset{(-)}{\sigma^j_t})\ .$$

(ii) Estimation

Equation (13) can be estimated. We estimate the ratio of actual rent to actual house prices. If there is a large deviation (downward) of this ratio from the estimated value, then the house prices are higher than what is reflected by the value of their housing services. This means that the expected price of houses in the coming period deviates from what is supported by equilibrium, and from what is explained by the economic factors.

We estimated equations in the form of Equation (13) from mid-1996 (because of data limitations) through mid-2010 in various specifications.[29] The equation was estimated with three alternative interest rates: the 10-year yield on bonds, the average mortgage interest rate, and the real 1-year interest rate (Specifications (1), (3), and (4) in Table 1). The results for the three specifications are very similar.

As expected, the interest rate appears with a positive sign in all the formulas. A higher interest rate means more discounting of expected income, and the present value of the homes is therefore lower, as reflected in a higher present return. Alternatively, the interest rate can be regarded as representing a higher alternative yield, which acts to increase the required yield on homes, and therefore has a positive impact on the estimated ratio. The relative risk of investment in a home is reflected by the standard deviation of the yield on shares, and standard deviation of inflation expectations. The two variables appear in the various specifications with a negative sign. A higher variance in shares and inflation reduces the relative risk of investing in a home, and consequently the required yield on investment in this asset, and therefore this variable also has a negative sign. Economic activity, as reflected by the change in GDP, is expected to have a positive impact on future demand, and therefore on the future price of houses.[30] Higher expected capital gains act to depress the required current yield (from rent), meaning that the effect of GDP on this ratio is expected to be negative.

[29] Estimation was carried out by ordinary least squares. All the components included in Equation (13) are expected to be stationary, but ADF tests do not reject the hypothesis that some of the variables contain a unit root, i.e. are not stationary. At the same time, the power of unit root tests is low in short samples, meaning that they tend to accept the null hypothesis – an absence of stationarity – even when this is incorrect. An indication of the validity of the estimation results in this context is obtained from a stationarity test for the regression residuals. With all the specifications, stationarity was obtained at a high level of significance (less than 1 percent) – a result that supports the validity of the estimation.

[30] Here it is necessary to assume that these variables no longer affect the present price of rent, and will affect only the future prices.

Table 1
Estimation of the Factors Affecting the Ratio of Rent to Price of a Home

	(1)	(2)	(3)	(4)	(5)
Sample Period	1996q3-2010q2	1996q3-2010q2	1996q3-2010q2	1996q3-2010q2	1996q3-2010q2
Dependent Variable	Rent/PH	Rent/PH	Rent/PH	Rent/PH	Diff. from HP λ = 10,000
Constant	-0.81 (0.00)	-2.55 (0.00)	-0.63 (0.02)	-0.60 (0.01)	-0.09 (0.17)
Real long-term interest rate (2-quarter moving average)	0.072 (0.00)	0.17 (0.00)			0.033 (0.00)
Mortgage interest rate (2-quarter moving average)			0.043 (0.00)		
Real 1-year interest rate (2-quarter moving average)				0.028 (0.00)	
Standard deviation (8 quarters) of the lagged change in share prices	-0.70 (0.00)	-1.22 (0.01)	-0.68 (0.01)	-0.43 (0.11)	-0.38 (0.08)
Standard deviation (6 quarters) of lagged inflation expectations	-0.06 (0.01)	-0.17 (0.00)	-0.04 (0.09)	-0.05 (0.01)	
Rate of change in GDP (3-quarter moving average)	-2.52 (0.01)	-7.35 (0.00)	-2.24 (0.02)	-2.66 (0.00)	
Inventory of homes relative to population at a lag (4-quarter moving average)	1.5E-0.5 (0.00)	4.9E-0.5 (0.00)	1.2E-0.5 (0.00)	1.2E-0.5 (0.00)	
Change in exchange rate multiplied by the proportion of leases denoted in dollars	0.012 (0.01)	0.009 (0.04)	0.012 (0.00)	0.014 (0.00)	0.013 (0.00)
Dummy variable for the first quarter	-0.06 (0.00)	-0.05 (0.07)	-0.07 (0.00)	-0.07 (0.00)	-0.05 (0.00)
Dummy variable for the second quarter	-0.03 (0.08)	-0.03 (0.22)	-0.03 (0.05)	-0.02 (0.09)	
Lagged dependent variable	0.69 (0.00)		0.76 (0.00)	0.79 (0.00)	0.82 (0.00)
	R^2 = 0.98 D.W. = 1.61	R^2 = 0.94 D.W. = 0.70	R^2 = 0.98 D.W. = 1.70	R^2 = 0.98 D.W. = 1.93	R^2 = 0.95 D.W. = 1.55

The inventory of homes relative to population is expected to affect the future price of houses.[31] A higher ratio of homes is expected to moderate house prices, and the required yield from rent will therefore increase. Accordingly, this variable has a positive sign.

The equation also included the rate of change in the exchange rate, adjusted by the proportion of leases denoted in dollars. Its sign and significant effect indicate that a rise in the exchange rate is reflected to a greater extent (especially in the period in which leases

[31] An attempt to include the rate of change in investment in residential construction from the national accounts produced no significant results.

were dollar-linked) in the rent than in house prices. A separate examination that we conducted shows that the immediate transmission to rent was three times as large as the immediate transmission to home prices – the result obtained is therefore reasonable.

The ratio of rent to the price of a home features a high serial correlation. In order to correct for this correlation, the ratio at a lag was also included in the equation. The correlation obtained – between 0.6 and 0.8 – was significantly lower than 1, but indicates the existence of a high serial correlation in the series.[32] In order to correct for the serial correlation, we also included the dependent variable at a lag in the estimation. For the sake of comparison, Column (2) displays a specification similar to (1), without the variable at a lag. It can be seen that there is no significant change in the effect of the other explanatory variables in the equation, and that the fit of the regression is very high, even without including the variable at a lag. At the same time, the standard deviation of the equation is higher without the lag, as is the serial correlation mentioned above. The inclusion of the dependent variable at a lag can be justified by pointing out that individuals use it building their expectations for the future behavior of the price.

Estimation using deviations from the HP trend

We also evaluated the factors affecting the cost of home ownership using deviations from the HP trend presented in Section 6.1. Because the trend of this ratio represents the medium- and long-term effects, such as business cycles and the effect of housing inventory per population, we do not expect to find an effect of variables of this type on the gap between the actual ratio and its trend.[33] We therefore wish to examine which short-term variables – interest rate, risk, exchange rate – affect the gap between the actual ratio of rent to the price of a home and its trend. Column (5) in the following table displays the estimation results, where the dependent variable is the difference between the actual ratio of rent to housing prices and its HP trend for λ=10,000 presented in the preceding section. The table shows that the (long-term) interest rate, the standard deviation of share prices, and the development of the exchange rate explain a significant part of the deviation from the trend,[34] and affect it in the expected direction, as indicated by estimation of the ratio itself.

Indications of the existence of a bubble

In order to evaluate whether the prices correspond to the market fundamentals, we examined whether the ratio of rent to house prices deviated from what was expected according to these explanatory factors. Figure 17 displays the deviations of the actual ratio (and of the difference from the trend) from the estimated value as obtained from the

[32] Estimation using AR(1) produces similar results, although the significance of some of the variables was affected.

[33] The possible effect of these variables (the change in GDP and housing inventory relative to population) was tested and found not to be significant.

[34] The equation also includes the dependent variable at a lag in order to correct for the serial correlation. Even without the dependent variable at a lag, the explanation rate of the equation is relatively high, at least 80%, with a rise in the standard deviation of the equation.

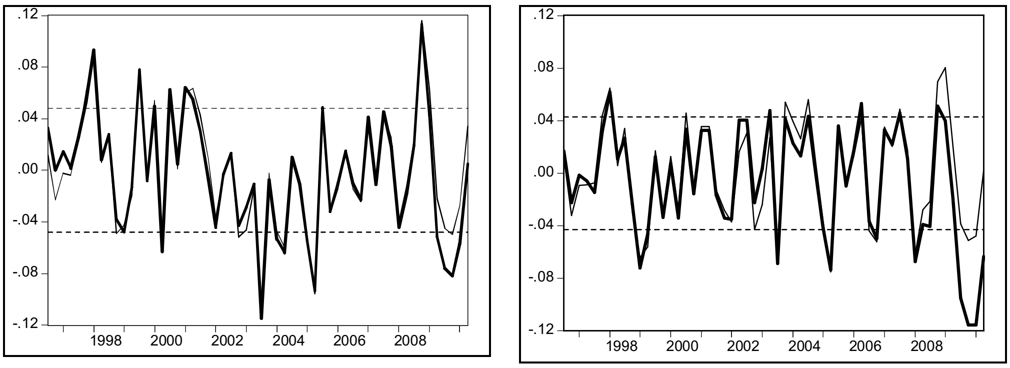
regression residuals obtained for specifications (1) and (5) above in the full sample (the thin line). The deviations obtained in specifications (3) and (4), which include alternative interest rates, are very similar to those displayed, while the deviations in the estimation that does not include the dependent variable at a lag (Column (2)) are larger during the entire sample period, but have a similar development.

It can be seen that in both specifications, there are deviations larger than one standard deviation during certain periods, but there are no major deviations in the estimated ratio. In specification (1), which concerns the actual ratio of rent to house prices, there is a relatively large deviation at the end of 2009 that narrows in mid-2010, and therefore does not exceed the size of past deviations. In specification (5), which estimates the factors explaining the deviation from the trend, the factors appearing in the equation explain all of the deviation from the trend, and there is no significant estimation error during the recent period.

At the same time, note that the coefficients obtained in these equations are based on the entire sample, which also includes the recent period, when the ratio of rent to house prices fell precipitously. In order to evaluate the stability of the results, specifications (1) and (5) were estimated for the sub-period ending in mid-2009, and based on the coefficients obtained, the deviations of the actual ratio from what was expected according to this estimation were examined. The thick line in Figure 17 represents these deviations.

Figure 17
The Residuals from the Estimation until 2009.2 (thick line) and until 2010.2 (thin line)

Specification (5) | Specification (1)



The graph clearly shows that when the sample does not include the recent period, the deviations of the actual ratio from what was expected differ from those during the rest of the period. The difference between the estimation for the full sample and that for the sub-period obtained for the past year (which is outside the sample) is substantial, while until then, the deviations behaved similarly in the two estimations. It therefore follows that the relationships between the explanatory variables and the estimated variable changed during the recent period, indicating a deviation in the ratio of rent to the price of houses from the level explained by the relationships prevailing over the past decade.

The average value of the ratio of rent to the price of a home (the average yield from rent) in the sample was 3.52 percent. The largest deviation obtained from the partial estimation in specification (1), which is displayed in Figure 17, is 0.1 percent – only a small deviation from the average yield. In other words, while at the end of the sample displayed here, the deviations exceed those characterizing it as a whole, the quantitative deviation of the actual ratio of rent to the price of an apartment from the ratio explained by the economic factors is not very large. When we examine the deviation from the HP trend, the difference between the two estimations is smaller, and the deviation during the recent period does not differ from that in other periods, even in estimation for the sub-period.

7. SUMMARY

In this article, we examined the question of whether a bubble exists in house prices in Israel using three different methods: direct measurement, a Kalman filter, and econometric estimation. The three methods are based on standard asset pricing theory, which we used to estimate the fundamental component of house prices, where the difference between the actual and the fundamental price was defined as a bubble. The findings using the first two methods presented, direct measurement and the Kalman filter, indicate that the level of house prices does not deviate significantly from the fundamental level, even though the price increase beginning in January 2008 would have been different, both historically and in comparison with the market fundamentals. On the other hand, the conclusions from the findings in the econometric estimation were less clear-cut than those obtained in the first two approaches, even though the size of the price deviation did not appear to be large.

In the direct measurement, the real price increase in 2008–10 was supported by a rise in the fundamental value of homes, and the bubble component in house prices during this period was accordingly low. An examination of the bubble component, taking into account the leverage of households, shows that as the level of leverage rises, the bubble component becomes larger, since financing costs reduce the fundamental price, and therefore inflate the bubble component. At the same time, the gaps are not large, and are not enough to alter the conclusions of the analysis without leverage.

The results of the estimation with the Kalman filter method were consistent with those obtained in the first method; they indicate that the recent price rise is no more than a correction of the slide in prices over the preceding decade, and appear to be a closing of the gap with the fundamental price.

An examination of the contribution of the expansionary monetary policy to the real price increase from 2008 until August 2010 using the two methods shows a broad span of estimates ranging from 3.6 percent to 14 percent, and we therefore have little confidence in quantification of the effect.

Estimation of the equations under the econometric approach indicates that as of the second half of 2010, the downward deviation in the ratio of rent to the price of a home is not large in comparison with the other deviations during the sample period. However, estimating the deviations based on the coefficients obtained by estimating the equations for the sub-period ending in mid-2009 indicates that the deviations in the actual ratio differ from the deviations prevailing during the rest of the period. This difference between the two estimations shows that the relationships between the explanatory variables and the estimated variable may have changed recently, indicating a deviation in the ratio of rent to the price of homes from the level explained by the relations prevailing over the preceding decade. At the same time, an examination of the equation that estimates the deviation in the ratio of rent to the price of a home from the HP trend as an explained variable substantially moderates this last conclusion. In this equation, the difference between the two estimations is smaller, and the deviation in the recent period does not exceed that prevailing in other periods, even in estimation for the sub-period.

In conclusion, the findings of the examinations presented in this study lead us to the conclusion that the level of housing prices in Israel does not differ significantly from a level reflecting the market fundamentals, and we therefore find no evidence of a bubble in the price. If such a bubble exists, it is in the initial stages, and cannot yet be detected in the price.

Appendix 1: Estimation Results of ARMA Processes

For the purpose of estimating the bubble component using a Kalman filter, we adapted ARMA processes to the real 1-year interest rate, the real 15-year interest rate, and the logarithm of the rent. The estimated rent was also used in the direct measurement. This appendix presents the results of the estimation.

The number of autoregressive lags (the AR component) and the lags as a moving average (the MA component) for each variable were selected while taking into account three information criteria that assign weights to the model's goodness of fit on the one hand (as expressed in the logarithm of the likelihood function) and the degrees of freedom on the other hand. The indices that we used were the Akaike Information Criteria, the Schwarz Criterion, and the Hannan-Quinn Information Criterion. In order to select the number of lags, we ran ARMA processes from ARMA(0,0) to ARMA(18,18). The three information criteria were calculated for each regression, and after weighting the results and exercising judgment, we selected the appropriate number of lags for each variable.

Rent

The series we used was owner occupied housing services, net of the Consumer Price Index. This series expresses rent in new and renewed leases at each point in time. The period estimated is 1999-2010, depending on the availability of the data. Before 1999, the rent data express the average price in existing leases at each point in time (including those signed during the year preceding the relevant period). We assumed that the stochastic process that we found for the new leases was also valid for the existing ones. Obviously, this assumption introduces some distortion into the bubble indices at the beginning of the period (1996-1999), but an examination of the rent series (in new leases, compared with existing ones) clearly shows that they were very similar until at least 2003; only starting in 2005 did the connection between them loosen. From a weighting of the information criteria, we found ARMA(5,3) to best represent the process of the logarithm of rent. The estimation results are displayed in Table A.1.

Table A.1
Estimation Results of ARMA(5,3) Process for the Logarithm of Real Rent

Dependent variable: the logarithm of real rent in new/renewed leases (owner occupied housing services)
Sample Period: June 1999-October 2010

Variable	Coefficient	Standard Deviation	Prob.
Constant	0.0260	0.0141	0.0644
AR(1)	2.3660	0.1499	0.0000
AR(2)	-2.3441	0.4083	0.0000
AR(3)	1.0677	0.5215	0.0406
AR(4)	-0.1420	0.3656	0.6977
AR(5)	0.0234	0.1153	0.8389
MA(1)	-0.8354	0.1964	0.0000
MA(2)	0.1798	0.2164	0.4062
MA(3)	0.5634	0.2111	0.0076
SE of Regression	0.0087		

Real interest rates

For the Kalman filter estimation, we used the real interest rate on 1-year and 15-year bonds derived from a smoothed yield curve. From a weighting of the information criteria, we chose to estimate the 1-year interest rate as an ARMA(2,5) process, and the 15-year interest rate as an ARMA(8,4) process. The estimation results are displayed in Table A.2.

Table A.2
Estimation Results of ARMA Processes for Real Interest Rates

Real 1-Year Interest Rate: ARMA(2,5)				**Real 15-Year Interest Rate: ARMA(8,4)**			
Sample Period: January 1996-October 2010				Sample Period: January 1996-October 2010			
Variable	Coefficient	STD	Prob.	Variable	Coefficient	STD	Prob.
Constant	0.0018	0.0019	0.3665	Constant	0.0008	0.0027	0.7637
AR(1)	0.4347	0.3181	0.1717	AR(1)	0.3877	0.2477	0.1176
AR(2)	0.5235	0.3047	0.0858	AR(2)	0.1392	0.1887	0.4606
MA(1)	0.4007	0.3245	0.2169	AR(3)	0.0187	0.1914	0.9220
MA(2)	-0.1239	0.0964	0.1988	AR(4)	-0.0558	0.1967	0.7767
MA(3)	-0.0895	0.0926	0.3339	AR(5)	0.5148	0.1221	0.0000
MA(4)	-0.0498	0.0870	0.5672	AR(6)	-0.2281	0.0953	0.0167
MA(5)	-0.1359	0.0859	0.1133	AR(7)	0.1675	0.0763	0.0281
SE of Reg.	0.0025			AR(8)	0.0272	0.0739	0.7125
				MA(1)	1.0338	0.2352	0.0000
				MA(2)	0.7176	0.3723	0.0539
				MA(3)	0.5065	0.3317	0.1267
				MA(4)	0.5875	0.1426	0.0000
				SE of Reg.	0.0015		

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